## Geometry of Aerial Photographs

## Aerial Cameras

- Aerial cameras must be (details in lectures):
- Geometrically stable
- Have fast and efficient shutters
- Have high geometric and optical quality lenses
- They can be classified according to type of frame:
- Single lens Frame
- Multiple frame
- Strip Frame
- Panoramic
- They can also be either film or digital cameras.


THIS COURSE WILL DISCUSS ONLY SINGLE FRAME CAMERAS


Aerial camera with viewfinder and electronic control


## Aerial Camera in action



## Single lens Frame Cameras

- Standard is $9^{\prime \prime}(23 \mathrm{~cm})$ frame size and $6^{\prime \prime}$ $(15.24 \mathrm{~cm})$ focal length( f ).
- They can be classified, as mentioned before, according to the field of view to:
- Normal angle ( up to $75^{\circ}$ )
- Wide angle (up to $75^{\circ}$ )
- Super-wide angle (greater than to $75^{\circ}$ )


## Components of a single frame film camera.

Three main parts:

1. Magazine
2. Body
3. Lens cone assembly

Components and notes to be discussed din lecture







## Components of a single frame film camera.



Fiducial Marks

## A vertical Photograph

## Components of a single frame film camera.



Example of a corner Fiducial Mark under Magnification


Figure 2.31 The LH Systems ADS 40 digital camera.

## 800 km / 500 miles



## Spaceborne Sensors

12 km / 40,000 ft

## Geometry of aerial cameras



## Geometry of aerial cameras

- Identify the following:
- L: perspective center.
- Fiducial Center F.C.
- Principal Point (P.P) or O: the point where the perpendicular from the perspective center intersects the photograph. Usually deviates from the F.C by a very small distance.
- Principal axis: the line perpendicular from the principal center on the plane of the photograph (negative).
- f the focal length, equals the Principal Distance.


## Types of Photographs (by tilt)

## Aerial Photos are:

 1- Vertical or tilted- Vertical photos are taken with the optical axis (Principal Line) vertical or tilted by no more than $2^{\circ}$
- Tilted: if the optical axis is tilted by no more than $3^{\circ}$



## Types of Photographs (by tilt)

## 2- Oblique Photos

 If the optical axis is intentionally strongly tilted to increase coverage, they are:- Low oblique: if the tilt is not enough to show the horizon, usually 3 to $30^{\circ}$



## Types of Photographs (by tilt)

## 2- Oblique Photos

- Low oblique
- High oblique: if the horizon is shown on the photograph



High oblique


Low oblique


Example of vertical, high, and low oblique photos


## Types of Photographs (by tilt)

## 3- Convergent

 PhotographsLow oblique photos in which camera axis converge toward one another

(a)


## Comparison between vertical and oblique photos

- Coverage
- Geometry
- Low cloud?
- View: issues with tall features and views of sides of features.
- Others


## Photo Coordinates (film)

- We use positives for ease of geometry and familiarity of feature shapes, negatives may be used in certain applications
- Lines connecting middle fiducials ON THE POSITIVE define a photo coordinate system, in which $x$ is in the direction of flight, $A$ RIGHT-HAND coordinate system accurate as 1 micron = $1 / 1000 \mathrm{~mm}$



## Photo Coordinates (digital)

- Pixels in a digital image represent coordinates of rows and columns.

|  | Column |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 1 | 2 |  |  |  | 6 |  |
| 0 | $+$ | + | +3 | + | + | $+$ | + | + |
| 1 | + | + | + | + | + | + | +4 | + |
| 2. | + | + | + | + | + | + | + | + |
| 3 | + | $+$ | + | + | + | + | + | + |
| 4 | + | + | $+$ | 9 | + | + | $+$ | + |
| 5 | + | $+$ | $+$ | + | + | + | $+$ | $+$ |
| 6 | + | $+1$ | + | + | + | $+$ | +2 | + |
| 7 | $+$ | $+$ | $+$ | + | + | + | + | + |
|  |  |  |  |  |  | ima |  | enced |


(b)


FIGURE 9-3
(a) Nonreferenced satellite image. (b) Georeferenced image. (Courtesy University of Florida.)


## Column



## Geometry of a vertical photographs

- The line LoO, the optical axis is assumed truly vertical



Figure 3.8 Geometry of vertical photographs.

## Geometry of a digital frame camera

- Similar geometry is assumed in case of a digital camera
- Uses a two dimensional array of CCD elements mounted at the focal plane of the camera.
- The image is a grid of picture elements (pixels)
- Similar geometry is assumed in case of a digital camera
- Uses a two dimensional array of CCD elements mounted at the focal plane of the camera.
- The image is a grid of picture elements (pixels)
- The size of pixels in one image represent the resolution of the image, the smaller the pixel, the higher (better) the resolution.
- A mega (million) pixel image includes one million pixel
- Number of pixels can be as low as $500 \times 500=250,000$ pixels, or megas of pixels for commercial cameras, or gegapixels in classified cameras.
- What is the size of image of a camera that includes a CCD frame sensor, that is 1024 X1024 elements, if the size of a pixels is $5 \mu$ ? How many megapixels are there?
Answer: image size $=\quad=\quad \mathrm{mm}$,

$$
\text { No of pixels = ........ }=\text { mega pixel }
$$

## Digital image resolution


1.6 m

0.10 m

0.80 m

0.05 m

0.40 m

0.03 m

0.20 m

0.01 m

image taken from a firstgeneration Landsat satellite over Milwaukee, Wisconsin.

## 1-m

## resolution

## image

obtained

## from the

 IKONOS
## satellite

showing
San

## Francisco.

## Concept of image pyramids




Figure 2.32 Raw ADS 40 image.


Figure 2.33 Rectified ADS 40 image.



Flying spot scanner: The geometry is such that the after a row is finished being scanned, the vehicle advanced to the beginning of the next row.

- Which digital system is suitable for what purpose??
- Why??


## Image coordinate corrections

Our goal is to measure photo coordinates and relate them to ground coordinates by equations to obtain ground coordinates. Once you have ground coordinates, you can draw a map, establish cross section, etc.
Measured photo coordinates need to be corrected prior to substation in equations, for the following:

- Film shrinkage
- Principal point location
- Lens distortions
- Atmospheric refraction
- Earth curvature
- Which of the corrections inapplicable for digital images??



Jigure 3.27 Atmospheric refraction.

## Vertical Photographs

## Scale of a Vertical Photograph

- Scale of a photograph is the ratio of a distance on a photo to the same distance on the ground.
- Photographs are not maps, why?
- Scale of a map and scale of a photograph.
- Orthphotos (orthophoto maps), what are they?
- Scale (s) at any point:

$$
S=\frac{f}{H-h}
$$

-Average scale of a photograph:

$$
\mathrm{S}_{\mathrm{avg}}=\frac{f}{\mathrm{H}-\mathrm{h}_{\mathrm{avg}}}
$$

If the $f, H$, and $h$ are not available, but a map is available then:

$$
\text { Photo Scale }=\frac{\text { photo distance }}{\text { map distance }} \mathrm{X} \text { map scale }
$$



The scale of a vertical photograph approximately equals to the ratio of the flying height above the ground $(H)$ and the focal length of the camera lens (f)

$$
\text { Scale }=\frac{\text { imageDist }}{\text { surfaceDist }}=\frac{f}{H}
$$

## Scale of a vertical photograph.



## Ground Coordinates from a Single Vertical Photograph

- With image coordinate system defined, we may define an arbitrary ground coordinate system parallel to ( $x, y$ ) origin at nadir.
- That ground system could be used to compute distances and azimuths. Coordinates can also be transformed to any system
- In that ground system:

$$
\begin{aligned}
& X_{a}=x_{a} * \text { (photograph scale at a) } \\
& Y_{a}=y_{a}{ }^{*} \text { (photograph scale at a) } \\
& \hline
\end{aligned}
$$



Figure 27-8 Ground coordinates from a vertical photograph.

## Relief Displacement on a Vertical Photograph

- The shift of an image from its theoretical datum location caused by the object's relief. Two points on a vertical line will appear as one line on a map, but two points, usually, on a photograph.
- The displacement is from the photgraphic nadir point. In a vertical photo, the displacement is from the principal point, which is the nadir in this case.
- Photographic Nadir point is where the vertical from the Exposure Station intersects the photograph.


## Relief

## displacement

Towers A and B are equally high, but placed at different distances from the nadir point, thus have different relief displacements. A tower, depicted beneath nadir point has no relief displacement



Relief displacement from Nadir (enlarged)


$r_{a} / R=f / H$

Figure 27-9 Relief displacement on a vertical photograph.

- Relief displacement (d) of a point wrt a point on the datum :

$$
d=\frac{r h}{H}
$$

where:
$r$ : is the radial distance on the photo to the high point
$h$ : elevation of the high point, and H is flying height above datum

- Assuming that the datum is at the bottom of vertical object, H is the
flying height above ground, the value $h$ will compute the object height.


## Or, in general:

Assume that point $C$ is vertically above $B$, they are shown on the photograph as (c) and (b).
Measured radial distances from the center to points $c$ and $b\left(r_{c}\right.$ and $r_{b}$ ), then

$$
\begin{aligned}
& d_{c}=r_{c}-r_{b} \quad \text { and; } \\
& d_{c}=\left(\begin{array}{lll}
r_{c} & * & \left.h t_{c}\right) /\left(\text { flying height above ground }=H-h_{b}\right)
\end{array}\right.
\end{aligned}
$$

Example 6-7. A vertical photograph taken from an elevation of 535 m above mean sea level (MSL) contains the image of a tall vertical radio tower. The elevation at the base of the tower is 259 m above MSL. The relief displacement $d$ of the tower was measured as 54.1 mm , and the radial distance to the top of the tower from the photo center was 121.7 mm . What is the height of the tower?

## Answer:

$\mathrm{d}=\mathrm{rh} / \mathrm{H}^{\prime}$, then

$\mathrm{H}=$


## Tilted Photographs



## Basic elements of a tilted photographs

- The optical axis is tilted from the vertical
- Identify the following:
- $t=$ angle of tilt between the plumb line and the optical axis LO
 principal line in the isocenter.
- no $=$ the principal line joining the nadir point $(\mathrm{n})$ and the principal point (0).

- Lno = the principal plane: it is the vertical plane containing $\mathrm{o}, \mathrm{L}$ and n (shaped plane).
- im = axis of tilt: it is the line perpendicular to the principal line from the isocenter $i$ in the plane of the photograph.
- $\mathrm{S}=$ the swing angle: it is the angle measured from the positive photographic $y$-axis clockwise to the principal line (on).
- x'y' axes are the auxiliary coordinate system of the tilted photograph where:
$\mathrm{y}^{\prime} \quad$ is the principal line (no).
$x^{\prime} \quad$ is the perpendicular to $y^{\prime}$ from point $n$.
$\theta=$ the rotation angle between $y$ and $y^{\prime}$ axes in $a$ counterclockwise direction.



Figure (3-6) Basic elements of tilted photograph

## What and why an auxiliary coordinate

 system?- A step to relate photo coordinates to ground, because the photograph is tilted.
- Thus, photo and ground coordinates are not parallel any more.
- You need a system in between as a step to transfer photo coordinates to ground, specially that tilt is variable.




## Relationship between Photo and Auxiliary coordinate system

$$
\begin{aligned}
& x_{a}^{\prime}=x_{a} \cos \theta-y_{a} \sin \theta \\
& y_{a}^{\prime}=x_{a} \sin \theta+y_{a} \cos \theta+f \tan t
\end{aligned}
$$

## Scale of a tilted Photograph

The tilt of a photograph occurs around the axis of tilt in the direction of the principal line.


## Scale of a tilted Photograph

- Scale = horizontal distance on the photo / horizontal distance on the ground =



Also,

## LK $=\mathrm{H}-\mathrm{h}_{\mathrm{A}}$

= flying height above ground

Then:

Scale of a tilted photograph $\mathrm{S}_{\mathrm{A}}=f(\sec t)-y^{\prime} \sin t /\left(\mathrm{H}-\mathrm{h}_{\mathrm{A}}\right)$

## Example

## Example 3-1:

A tilted Photo is taken with a 6 inch focal length camera from a flying meight of 8200 feet Tilt and swing angles are $3^{\circ} 30^{\circ}$ and $218^{\circ}$ respectively. Point (A) has an elevation of 1435 feet and its image coordinates are $\mathbf{x a}=-285$ inch. ya 3.43 inch . What is the scale at point (a)?

Solution
$\theta=$

## Ground Coordinates from a tilted photograph

- Coordinates of point A in a ground coordinate system $X^{\prime}, Y^{\prime}$ where:
- $X^{\prime}, Y^{\prime}$ are parallel to $x^{\prime}$ and $y^{\prime}$ (auxiliary system)
- Ground Nadir N is the origin of the ground system
- Note that in the auxiliary coordinate system, lines parallel to $x^{\prime}$ are horizontal, thus $x^{\prime}$ on the photo is horizontal and directly related to ground X by the scale, or

$$
X_{A}^{\prime}=x^{\prime} / S_{A}
$$



- But in the auxiliary system, $y^{\prime}$ is in the direction of maximum tilt and not horizontal, the scale is ratio between horizontal projections.
- Ka: Horizontal projection of $y^{\prime}=y^{\prime} \cos t$
- Then,
- $Y^{\prime}=y^{\prime} \cos t / S$


## Example

## Example 3-1:

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If the image coordinates of another point (b) are $\mathrm{xb}=3.09$ inch, yb 1.78 inch. and the elevation of (B) is 1587 feet calculate ground coordinates of (A) and (B).

