University of Washington, Bothell

CSS 342: Data Structures, Algorithms, and Discrete Mathematics

Induction Problem Examples

Some practice problems to help with learning induction.

1) Find a formula for $1/2 + 1/4 + 1/8 + \ldots + 1/2^n$ by examining the values of this expression for small values of n. Use mathematical induction to prove your result.

2) Find a formula for $1/(1*2) + 1/(2*3) + \ldots + 1/n(n + 1)$ by examining the value of this expression for small values of n. Use mathematical induction to prove your result.

3) Prove that $7^n – 1$ is divisible by 6, for = 1, 2, …

4) Show that postage of six cents or more can be achieved by using only 2-cent and 7-cent stamps.

5) What is wrong with this “proof” by strong induction?

“Theorem” For every nonnegative integer n, 5n = 0. Basis Step: 5 * 0 = 0. Inductive Step: Suppose that 5j = 0 for all nonnegative integers j with 0 <= j <= k. Write k + 1 = i + j, where i and j are natural numbers less than k + 1. By the induction hypothesis, 5(k + 1) = 5(i +j) = 5i + 5j = 0 + 0 = 0.

SOLUTION

1) Find a formula for $1/2 + 1/4 + 1/8 + \ldots + 1/2^n$ by examining the values of this expression for small values of n. Use mathematical induction to prove your result.

$P(1) = ½$
$P(2) = ½ + ¼ = ¾$
$P(3) = ½ + ¼ + 1/8 = 7/8$
….
$P(n) = ½ + ¼ + 1/8 + \ldots 1/2^n = (2^n - 1)/ 2^n$

1) Proof using induction:
   $P(1): ½ = (2^1 - 1)/ 2^1$

2) Assume P(k) is true and prove P(k+1).
   P(k) -> P(k+1).

   Given: $P(K) = ½ + ¼ + 1/8 + \ldots 1/2^k = (2^k - 1)/ 2^k$
   Prove: $P(K+1) = ½ + ¼ + 1/8 + \ldots 1/2^k + 1/2^{k+1} = (2^{k+1} - 1)/ 2^{k+1}$

Substitution:

$$(2^k - 1)/ 2^k + 1/2^{k+1} = (2^{k+1} - 1)/ 2^{k+1}$$

Multiply both sides by 2^{k+1}

$$2(2^k - 1) + 1 = (2^{k+1} - 1)$$

Simplify

$$2^{k+1} - 2 + 1 = 2^{k+1} – 1$$
$$2^{k+1} - 1 = 2^{k+1} – 1$$

qed.