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Analysis of uniformly and linearly distributed mass dampers under harmonic and earthquake excitation

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Abstract

The effectiveness of multiple mass dampers has been investigated by Igusa and Xu [Dynamic characteristics of multiple tuned mass substructures with closely spaced frequencies. Earthq. Engng Struct. Dynam. 21 (1992) 1050–70], Yamaguchi and Harnpornchai [Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillations. Earthq. Engng Struct. Dynam. 22 (1993) 51–62], Abe and Fujino [Dynamic characterization of multiple tuned mass dampers and some design formulas. Earthq. Engng Struct. Dynam. 23 (1994) 813–35] and Kareem and Kline [Performance of multiple mass dampers under random loading. J. Struct. Engng 121 (1995) 348–61]. In this paper, we extend the results of these previous investigations to examine the performance of uniformly and linearly distributed multiple mass dampers, respectively. These systems were selected to ascertain whether the distribution of masses located close to the central mass damper would influence the performance of the entire system in reducing vibration. We evaluate performance numerically through assessing the effectiveness and robustness of each system, as well as considering the effects of redundancy, under harmonic excitation. In this regard, we evaluate the performance of the system when certain individual dampers do not function. We show that the uniformly distributed mass system is more effective in reducing the peak dynamic magnification factor. The linearly distributed system is more robust under mistuning. It is more robust to damping variation for low damping values but the effectiveness of the two systems converges as damping increases. The uniformly distributed system is slightly more reliable when an individual damper fails. The eleven mass system is optimum for both configurations for harmonic excitation. The 21-mass system is more effective in structural vibration decay in both cases for the El Centro earthquake simulation.

Keywords: Structural control; Structural dynamics

1. Introduction

Multiple mass dampers have been investigated by several researchers such as Igusa and Xu [1], Fujino and Abe [5], Yamaguchi and Harnpornchai [2], Abe and Fujino [3] and Kareem and Kline [4]. Its superior performance relative to the single mass damper (SMD) is well-known. In this paper, we numerically evaluate the performance of multiple dampers with uniformly and linearly distributed masses, respectively, under harmonic excitation. These systems were selected to ascertain whether the distribution of masses located close to or away from the central mass damper would influence the effectiveness in reducing vibration. After this thorough analysis, we briefly consider the effectiveness of the damping systems under earthquake excitation. In the harmonic analysis, we develop an algorithm to identify the optimum tuning of the individual dampers, and we assess the performance as characterized by effectiveness, robustness and redundancy. Effectiveness is defined as the reduction of the peak structural response under a given loading. For harmonic excitation, this is the peak dynamic magnification factor (DMF). Robustness is defined as the ability of the system to behave properly under slight mistunings or parameter variations. Redundancy is defined as the ability of the system to be effective when one or more of the dampers does not function.

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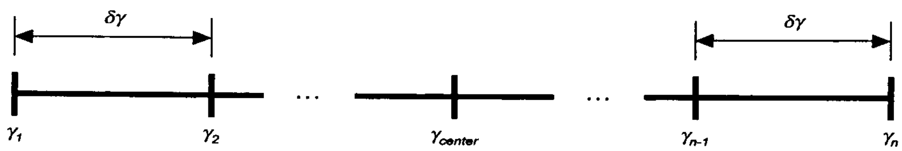

Fig. 1. Frequency ratio distribution.

2. Basic assumptions

For this analysis, we assume that the structure is a single degree of freedom system; the structure and the dampers are linearly elastic; the inherent damping ratio of the structural system is 0.01; and the total mass ratio of the MMD system is 0.01. The equation of motion of the structure–MMD interaction contains a number of parameters that govern the behavior of the structural vibration reduction. One of the goals of this investigation is to find a combination of the parameters that generates the minimum peak response of the structure. The following parameters are used:

n	total number of mass dampers
μ_{total}	ratio of the mass dampers' total mass to the structural mass
μ_{k}	ratio of each mass damper mass to the structural mass
ζ_{s}	damping ratio of the structure
ζ_{d}	damping ratio of each mass damper
ω_{s}	natural frequency of the structure
ω_{k}	natural frequency of each mass damper
ω_{e}	frequency of the forcing function
γ_{k}	frequency ratio of ω_{k} to ω_{s}
γ_{center}	tuning ratio of the central mass damper natural frequency to the structure's natural frequency, $(\omega_{(n+1)/2}/\omega_{\text{s}})$
λ	frequency ratio of ω_{e} to ω_{s}
FR	frequency range of the mass damper, $(\omega_n-\omega_1)$, where ω_n=natural frequency of the nth mass damper and ω_1=natural frequency of the 1st mass damper

The natural frequencies of the mass dampers are equally spaced over a range, FR, as shown in Fig. 1.

The equations of motion of the structure–MMD system shown in Fig. 2 can be expressed in a matrix form as

$$\mathbf{M}\cdot\ddot{\mathbf{u}}+\mathbf{C}\cdot\dot{\mathbf{u}}+\mathbf{K}\cdot\mathbf{u}=\mathbf{F} \tag{1}$$

where the vector $\mathbf{u}$ is the absolute displacement vector of the structure and the mass dampers:

$$\boldsymbol{u}=[u_{\text{s}},u_1,u_2,\ldots u_{n-1},u_n]^T \tag{2}$$

The mass matrix is diagonal:

$$\mathbf{M}=\text{diag}[m_{\text{s}},m_1,m_2,\ldots m_{n-1},m_n] \tag{3}$$

The stiffness matrix $\mathbf{K}$ is formed as

$$\mathbf{K}=\begin{bmatrix} k_{\text{s}}+\sum^{n}_{\text{k}=1}k_{\text{k}} & -k_1 & -k_2 & \cdots & -k_{n-1} & -k_n \\ -k_1 & k_1 & 0 & 0 & \cdots & 0 \\ -k_2 & 0 & k_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k_{n-1} & 0 & \cdots & 0 & k_{n-1} & 0 \\ -k_n & 0 & 0 & \cdots & 0 & k_n \end{bmatrix} \tag{4}$$

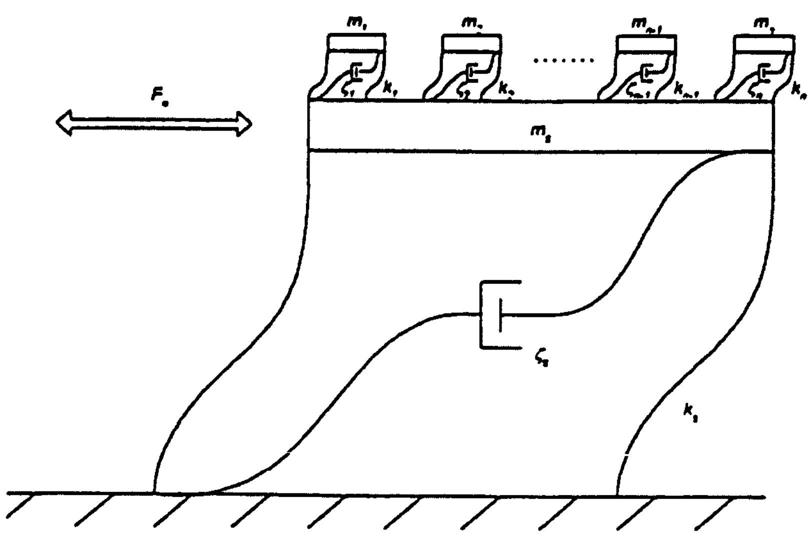

Fig. 2. Multiple mass dampers attached to the main structure.

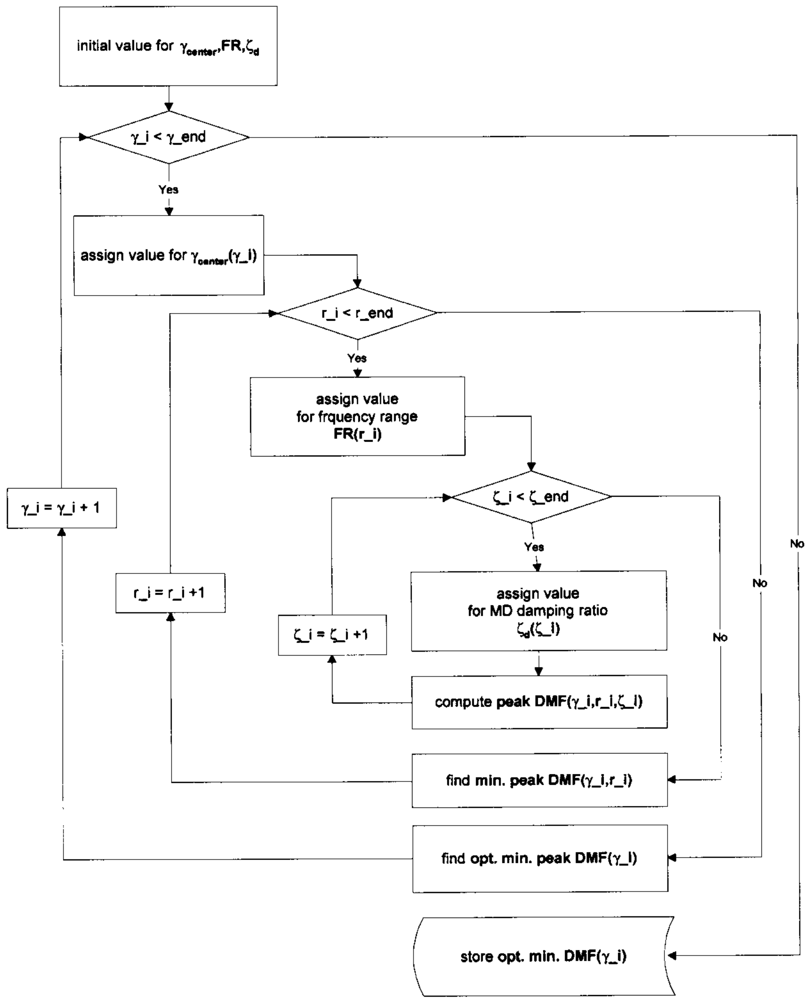

Fig. 3. Flowchart for the numerical search.

Table 1
Optimum parameters for the uniformly (UDMRS) and linearly (LDMRS) distributed mass damper systems with various n

Number of mass dampers, n	Central mass damper frequency tuning ratio		Frequency range		Mass damper damping ratio		Minimum peak DMF	
	UDMRS	LDMRS	UDMRS	LDMRS	UDMRS	LDMRS	UDMRS	LDMRS
1	0.9886	0.9886	N/A	N/A	0.065	0.065	11.3864	11.3864
3	0.9907	0.9871	0.085	0.100	0.035	0.035	10.0596	10.3912
5	0.9912	0.9895	0.115	0.145	0.025	0.030	9.7633	10.1483
7	0.9924	0.9882	0.120	0.170	0.025	0.025	9.6880	10.0824
9	0.9941	0.9875	0.130	0.200	0.020	0.020	9.6406	9.9932
11	0.9932	0.9889	0.135	0.200	0.020	0.015	9.5879	9.9115
21	0.9937	0.9882	0.140	0.230	0.020	0.015	9.5696	9.8424
31	0.9955	0.9884	0.150	0.240	0.015	0.010	9.5332	9.7817
41	0.9927	0.9887	0.145	0.240	0.020	0.010	9.5794	9.8074
51	0.9935	0.9882	0.155	0.250	0.015	0.010	9.5898	9.7713
61	0.9938	0.9883	0.155	0.250	0.015	0.010	9.5792	9.7775
71	0.9940	0.9883	0.155	0.250	0.015	0.010	9.5717	9.7778
81	0.9942	0.9884	0.155	0.250	0.015	0.010	9.5673	9.7835
91	0.9943	0.9884	0.155	0.250	0.015	0.010	9.5624	9.7842

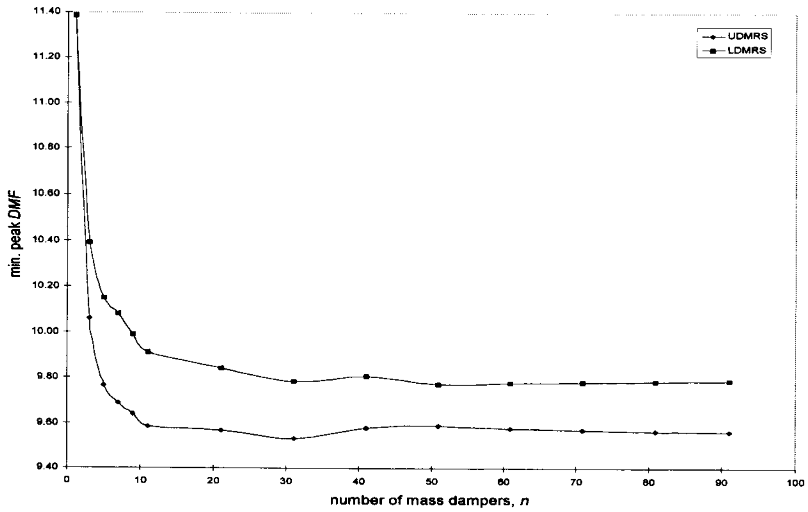

Fig. 4. Minimum peak DMF vs number of mass dampers.

The damping matrix C can be formed in a similar manner. If the forcing function is harmonically oscillating and acts only on the structure, it is given by

$$\mathbf{F}=[F_e \cdot e^{i\cdot\omega_e\cdot t}\ 0\ 0\ \ldots\ 0]^T \quad (5)$$

Assuming a harmonic solution with a frequency identical to the forcing function frequency, ω_e, allows the following steady state solution for u_s:

$$u_s=\frac{F_e/(m_s\cdot\omega_s^2)}{\mathrm{Re}(Z)+\mathrm{Im}(Z)\cdot i} \quad (6)$$

where

$$\mathrm{Re}(Z)=1-\lambda^2-\sum_{k=1}^{n}\frac{\mu_k\cdot\lambda^2\cdot[\gamma_k^2\cdot\{\gamma_k^2-\lambda^2\}+\{2\cdot\zeta_d\cdot\gamma_k\cdot\lambda\}^2]}{\{\gamma_k^2-\lambda^2\}+\{2\cdot\zeta_d\cdot\gamma_k\cdot\lambda\}^2}$$

$$\mathrm{Im}(Z)=2\cdot\zeta_s\cdot\lambda+\sum_{k=1}^{n}\frac{2\cdot\mu_k\cdot\zeta_d\cdot\gamma_k\lambda^5}{\{\gamma_k^2-\lambda^2\}+\{2\cdot\zeta_d\cdot\gamma_k\cdot\lambda\}^2}$$

Then, by definition, the Dynamic Magnification Factor can be expressed as

$$\mathrm{DMF}=\frac{1}{\sqrt{\mathrm{Re}(Z)^2+\mathrm{Im}(Z)^2}} \quad (7)$$

Two types of mass ratio distributions are considered: *Uniformly* (UDMRS) and *Linearly Distributed* (LDMRS) systems. The frequency ratio distribution is identical for the two cases. For a given total number of mass dampers, n, the mass ratio of each damper for the UDMRS is

$$\mu_k=\frac{\mu_{\mathrm{total}}}{n} \quad (8)$$

The frequency ratio, γ_k, is equally spaced over a frequency range, FR. The central mass damper frequency is determined first. The central mass damper fundamental

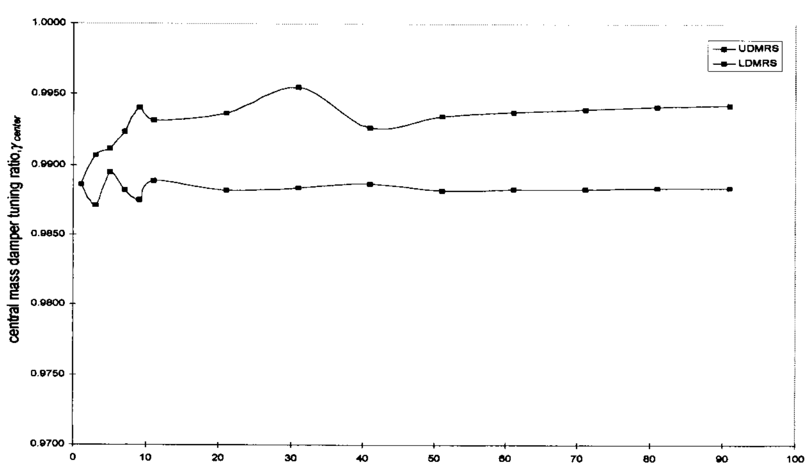

Fig. 5. Optimum central mass damper tuning ratio vs number of mass dampers.

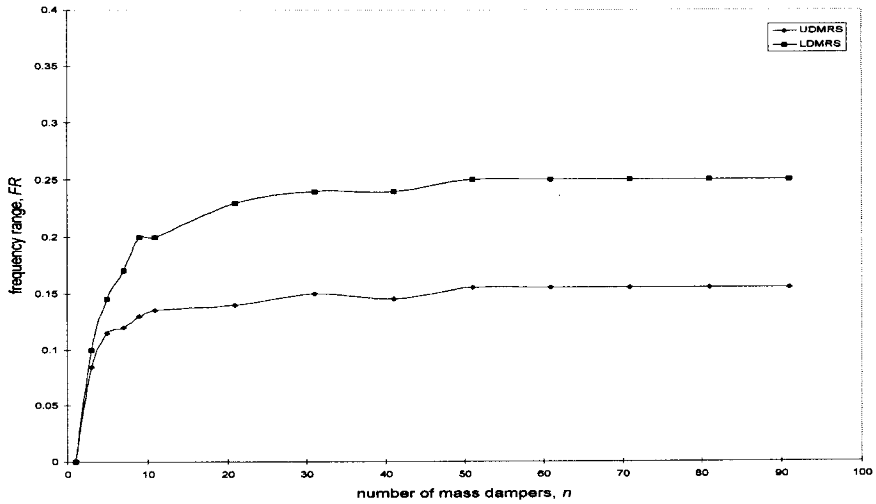

Fig. 6. Optimum frequency range vs number of mass dampers.

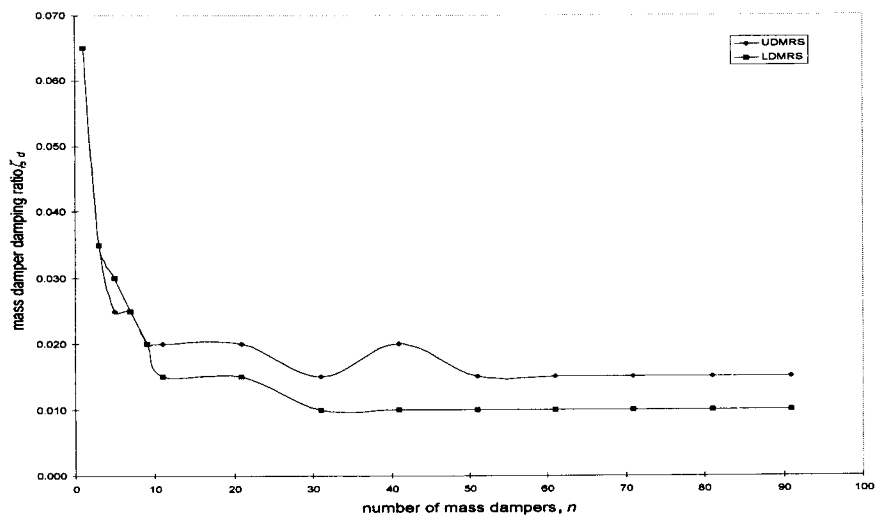

Fig. 7. Optimum mass damper damping ratio vs number of mass dampers.

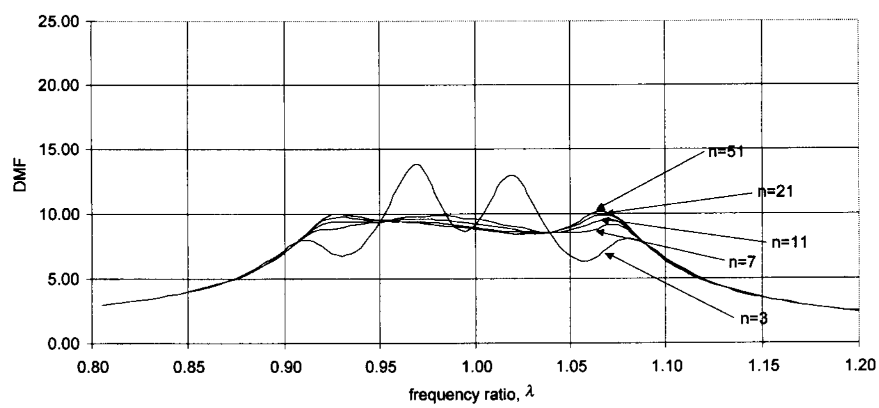

Fig. 8. Effect of number of dampers n: γ_{center}=0.9932, FR=0.135, ζ_d=0.02.

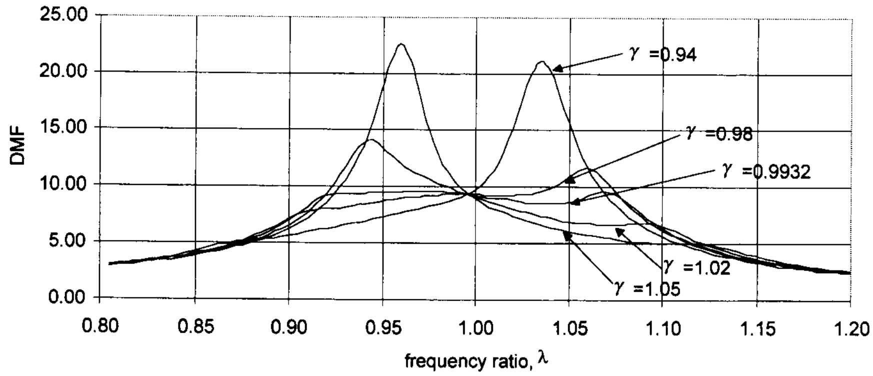

Fig. 9. Effect of γ: n=11, FR=0.135, ζ_d=0.02.

frequency is tuned to the structure's fundamental frequency according to a certain frequency ratio, γ_{center}. If $d\gamma$ is the constant frequency interval, then the frequency of the remaining mass dampers can be obtained as:

$$\gamma_k=\gamma_{(n+1)/2}-d\gamma\cdot\left(\frac{n+1}{2}-k\right) \quad (9)$$

$k=1,2,\ldots,n$

where $d\gamma=\frac{FR}{(n-1)}$.

For the *Linearly Distributed Mass Ratio System* (LDMRS), the central mass damper has the highest mass ratio which linearly decreases towards both ends. This distribution system allows a higher concentration of mass in the dampers whose frequencies are close to the frequency of the structure. Let $d\mu$ be the constant mass ratio increment; therefore, it follows that

$$\mu_k=\mu_{(n+1)/2}-|d\mu\cdot\left(\frac{n+1}{2}-k\right)|,\ k=1,2,\ldots,n \quad (10)$$

where $d\mu=\frac{4\cdot\mu_{total}}{(n+1)^2}$ and

$$\mu_{(n+1)/2}=d\mu\cdot\frac{(n+1)}{2}\ \sum_{k+1}^{n}\mu_k=\mu_{total}$$

The frequency ratio is distributed in the same manner as in the UDMRS. The stiffness ratio of each mass damper, κ_k, is defined as:

$$\kappa_k=\gamma_k^2\cdot\mu_k \quad (11)$$

where $\kappa_k=k_k/k_s$.

The search for the set of the three design parameters, ζ_d, FR, and γ_{center}, that yields the lowest peak DMF for a given system is one of the major objectives of this numerical investigation. An algorithm shown in Fig. 3 was developed to evaluate the optimum set of values of the design parameters for a system of n mass dampers. This algorithm consists of loops to search for each of the three design parameters. It is noted that this standard optimization may have been performed using mathematics packages; it is shown here for completeness. The root loop is built to determine γ_{center}. The first sub-loop is executed with a range of FR values. Under this sub-loop, the second sub-loop is performed with varying ζ_d. This process produces a number of combinations of the design parameters depending on the number of input values of each design parameter. Each run of this algorithm computes the peak DMF value. The minimum peak DMF can be obtained when all the peak DMF values are compared. In this manner, the set of the design parameters that yields the minimum peak DMF is obtained for the system of n mass dampers. The results of the

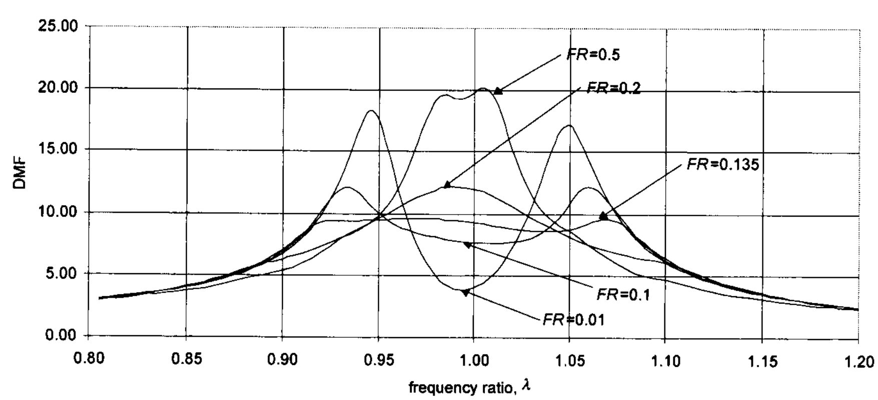

Fig. 10. Effect of FR: n=11, γ_{center}=0.9932, ζ_d=0.02.

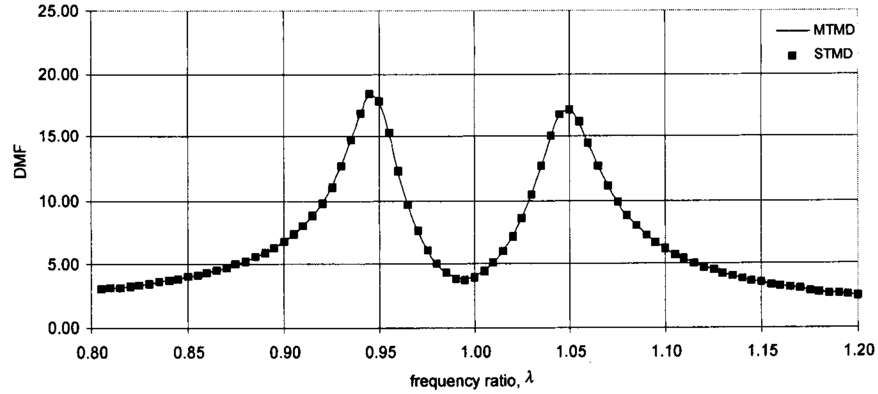

Fig. 11. MMd and SMD comparison (MMD: FR=0.01, n=11, γ_{center}=0.9932, ζd=0.02, μ_{MMD}=0.01, SMD: γ=0.9932, ζ_{d}=0.02, μ_{SMD}=0.01).

search will be presented in the next section. After determining this optimum set, we then assess the sensitivity to each parameter.

3. Effectiveness analysis for harmonic excitation

For a given number n of mass dampers, the three parameters, γ_{center}, FR, and ζ_{d}, have a combined effect on the response of the main structure. Therefore, the search for optimum conditions compares all the possible combinations of the parameters simultaneously for each set of n dampers. Previous investigations did not clearly define the manner in which the tuning ratio was determined. Yamaguchi and Harnpornchai [2] set the central mass damper frequency to be identical to the structural frequency. Abe and Fujino [3] applied the tuning equation developed by Den Hartog [6] to tune the central mass damper. Abe and Fujino [3] distributed the frequencies of the mass dampers over a certain range centered at the natural frequency of the structure. For this study, the numerical investigation was conducted for the initial condition that the central mass damper frequency had a value close to the natural frequency of the structure. Then, the central mass damper frequency was varied over a given range to allow for a search for the optimum frequency tuning ratio. The optimum parameters of the two mass ratio distributions, UDMRS and LDMRS, for various n values are shown in Table 1.

Fig. 4 shows that the minimum peak DMF converges as the number of mass dampers increases. There is no significant additional reduction in the peak DMF for MMDs with $n \geq 1$. For any n, the UDMRS reduces the peak DMF more than the LDMRS. The central mass damper tuning ratios also converge to their optimum values as shown in Fig. 5. It is interesting to note that the optimum γ_{center} for LDMRS shows some variation when $n<11$ but converges to the single mass damper (SMD) tuning ratio when $n>11$ ($\gamma^{p1}_{\mathrm{center}}$=0.9884,$\gamma_{\mathrm{SMD}}$=0.9886). Because the central mass damper has the highest concentration of mass for LDMRS, more mass is closely tuned to the central damper tuning ratio. Hence, the LDMRS with a large number of mass dampers acts similar to the SMD.

Fig. 6 illustrates that the optimum frequency ranges are greater for LDMRS. Fig. 7 shows that the optimum mass damper damping ratio decreases as the number of mass dampers increases. The optimum value of ζ_{d} for LDMRS is slightly less than for UDMRS.

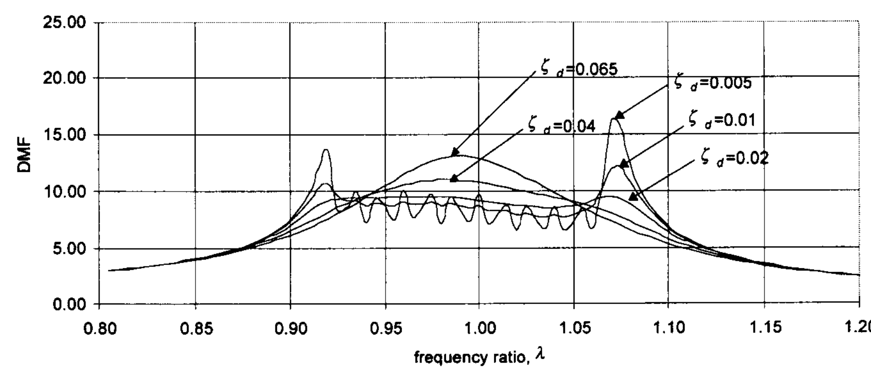

Fig. 12. Effect of ζ_{d}: n=11, γ_{center}=0.9932, FR=0.135.

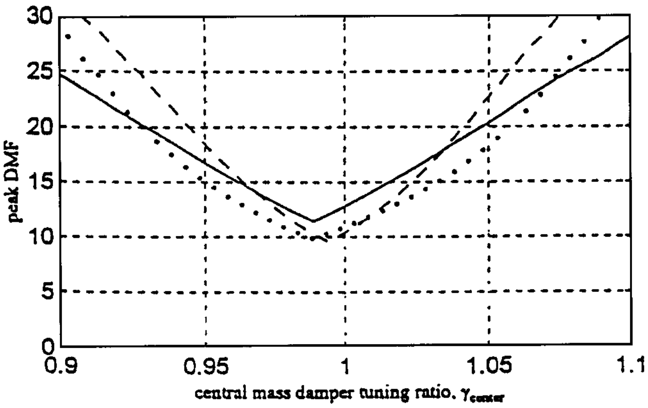

Fig. 13. Comparison of robustness for the tuning ratio: SMD (———): ζ_d=0.065; UDMRS (– – –) FR=0.140, ζ_d=0.02; LDMRS (· · ·) FR=0.230, ζ_d=0.015.

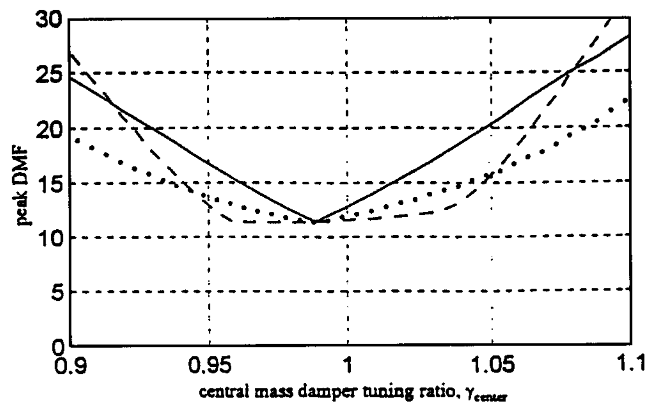

Fig. 14. Comparison of robustness for the tuning ratio.: SMD (———), ζ_d=0.065; UDMRS (– – –) FR=0.190, ζ_d=0.02; LDMRS (· · ·) FR=0.335, ζ_d=0.015.

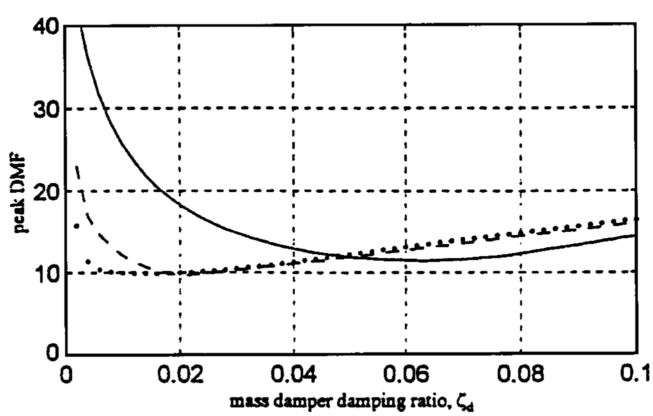

Fig. 15. Comparison of robustness for the mass damper damping ratio: SMD (———) γ_{SMD}=0.9886; UDMRS (- - -) FR=0.135, γ_{UDMRS}=0.9937; LDMRS (· · ·) FR=0.23, γ_{LDMRS}=0.9882.

3.1. Effect of the number of mass dampers

The peak responses of the MMD system with the number of mass dampers n as a variable are shown in Fig. 8. The values of n considered are 3, 7, 11, 21, and 51. The other variables are kept unchanged for the five curves (γ_{center}=0.9932, FR=0.135, ζ_d=0.02). These values for the parameters are the optimal ones for UDMRS 11 mass damper case. It can be seen in Fig. 8 that the influence of the n value is not significant as long as n>3. The minimum peak response is found for n=11.

3.2. Effect of the central mass damper tuning ratio

Fig. 9 shows the influence of the central mass damper tuning ratio, γ_{center}, on the structural response. The values for γ_{center} are 0.94, 0.98, 0.9932, 1.02, and 1.05. The other variables remain constant (n=11, FR=0.135, ζ_d=0.02). These values for the parameters are the optimum ones for UDMRS 11 mass damper case. The change of γ_{center} yields a shift of the frequencies of all mass dampers when the FR value is fixed. As γ_{center} increases, each mass damper frequency increases by the same amount of γ_{center} increment.

The peak response of the curve with γ_{center}=0.94 occurs at λ greater than unity. When γ_{center} is increased to 0.98, the peak of the response curve is reduced and a flat region of the response curve is developed around λ=1. As γ_{center} is increased to 0.9932, the peak responses do not occur at a particular point but spread out over a wide range of λ. The minimum peak response can be found in the flat region. The opposite behavior is observed as γ_{center} increases further. Fig. 9 illustrates that effectiveness can be optimized through minimizing the peak DMF as a function of γ_{center}.

3.3. Influence of frequency range

Five different values are assigned to the frequency range FR and the corresponding response curves are plotted in Fig. 10. The values for FR are 0.01, 0.1, 0.135, 0.2, and 0.5. The other variables are held constant for the five curves (n=11, γ_{center}=0.9932, ζ_d=0.02). The values for the variables for the UDMRS 11 mass damper case are given in Table 1. When FR is the lowest (0.01), the curve shows two high local peaks. The two local peaks are reduced when FR is increased to 0.1. The minimum peak response is obtained with FR to be 0.135 and the peak responses form a wide flat region around λ=1 as seen previously.

The response curve becomes a single peak curve and the peak value increases with higher FR. The two extreme cases (FR=0.01 and 0.5) illustrate the relationship between the MMD and the SMD. The behavior of the MMD with FR=0.01 is almost identical to the

Table 2
Mass and frequency ratios for the 11 mass systems

No.	UDMRS Mass ratio	Frequency ratio	LDMRS Mass ratio	Frequency ratio
1	0.00091	0.9257	0.0003	0.8889
2	0.00091	0.9392	0.0006	0.9089
3	0.00091	0.9527	0.0008	0.9289
4	0.00091	0.9662	0.0011	0.9489
5	0.00091	0.9797	0.0014	0.9689
6	0.00091	0.9932	0.0017	0.9889
7	0.00091	1.0067	0.0014	1.0089
8	0.00091	1.0202	0.0011	1.0289
9	0.00091	1.0337	0.0008	1.0489
10	0.00091	1.0472	0.0006	1.0689
11	0.00091	1.0607	0.0003	1.0889

behavior of the SMD using the same tuning ratio of the MMD as shown in Fig. 11.

This behavior implies that if the frequencies of mass dampers are very closely spaced, the MMD behaves like a SMD regardless of the number of mass dampers. This phenomenon has been observed by others; e.g. Fujino and Abe [5] and Igusa and Xu [1]. On the other hand, when the frequencies are widely spread out (FR=0.5), the peak response of the MMD (μ_{MMD}=0.01) is close to that of the SMD with a smaller mass ratio (μ_{SMD}=0.003). All frequency domain response plots show that the peak response occurs at λ somewhere between 0.9 and 1.1. The DMF curves rapidly decrease outside the range. The size of this range (0.2), however, is not a universal value

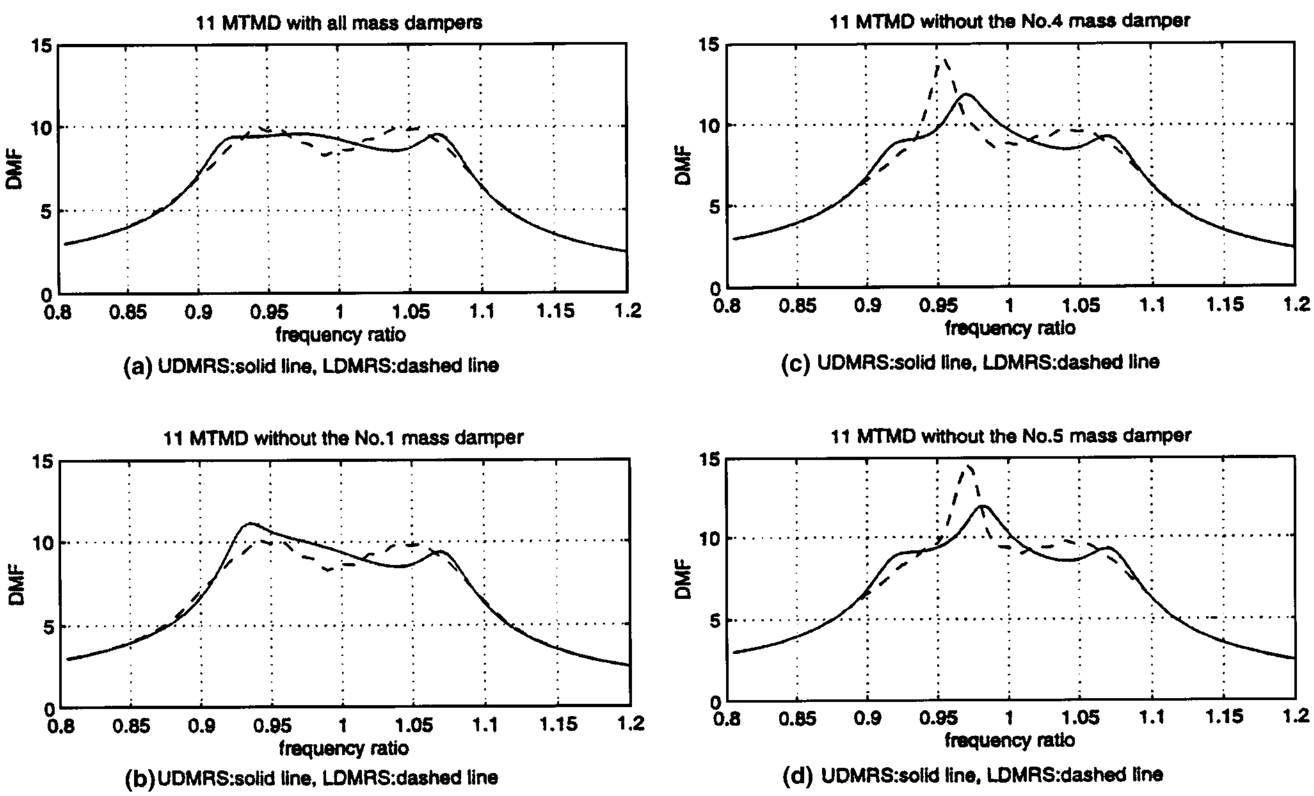

Fig. 16. (a) Comparison of the effectiveness of the 11 mass damper systems with all dampers. (b) Comparison of the effectiveness of the 11 mass damper system without the No. 1 mass damper. (c) Comparison of the effectiveness of the 11 mass damper systems without the No. 4 mass damper. (d) Comparison of the effectiveness of the 11 mass damper systems without the No. 5 mass damper.

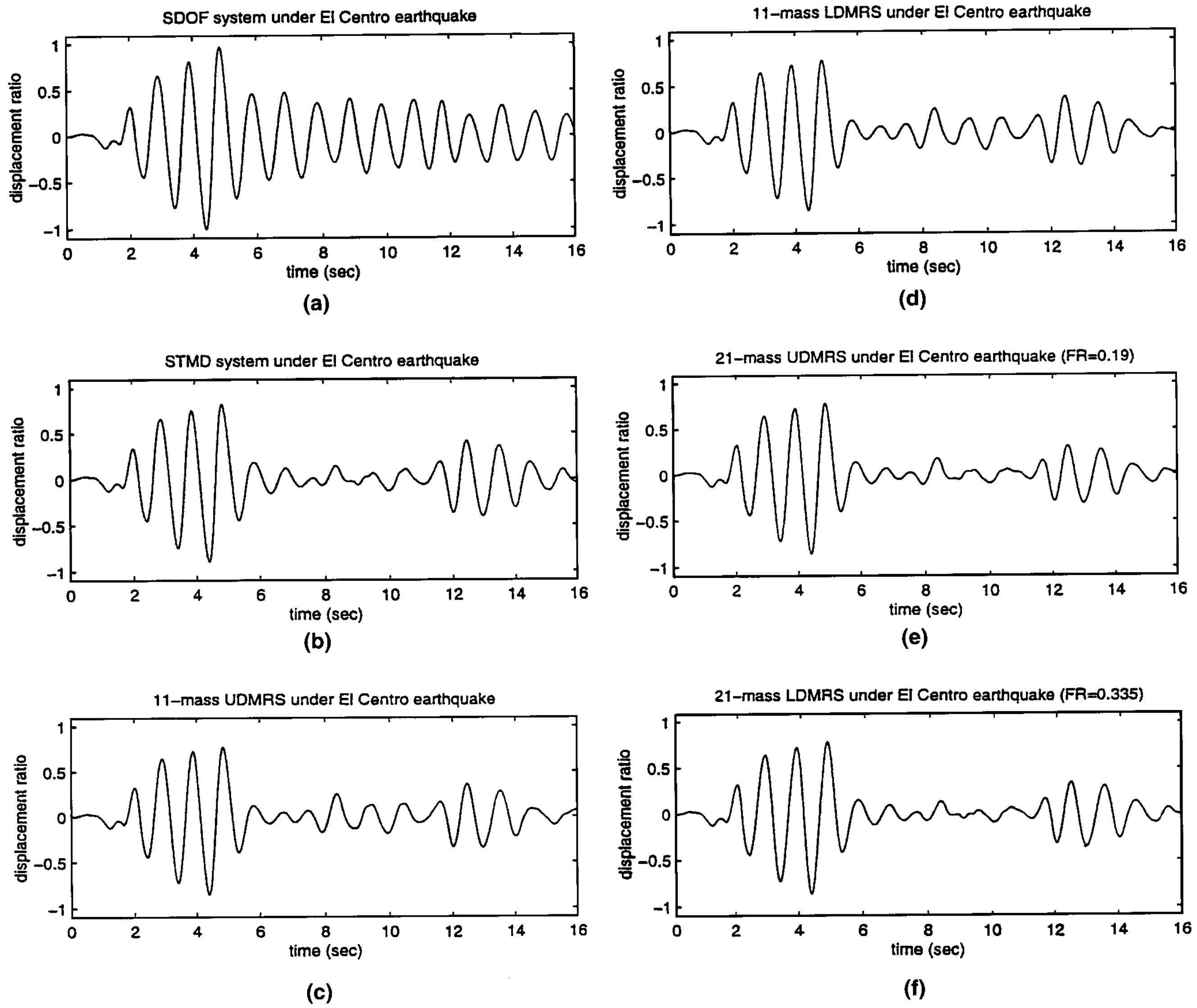

Fig. 17. (a) The response of the single degree of freedom (SDOF) structure without dampers attached to the El Centro ground excitation. (b) Response of the SDOF system with a single tuned mass damper. (c) Response of the SDOF system with an 11 uniformly distributed mass system. (d) Response of the SDOF system with an 11 linearly distributed mass system. (e) Response of the SDOF system with an 21 uniformly distributed mass system. (f) Response of the SDOF system with an 21 linearly distributed mass system.

for all cases examined. Most of the figures show the peak response or the top flat region in a λ range less than 0.2. It can be concluded that the mass dampers for which the frequencies are far from the central tuned frequency do not play significant roles in reducing the peak response.

3.4. *Influence of damper damping*

The influence of the damper damping ratio, ζ_d, is examined by varying the value of ζ_d (0.005, 0.01, 0.02, 0.04, and 0.065) while holding the other variables constant (n=11, γ_{center}=0.9932, and FR=0.135). The values are optimum for the 11-mass UDMRS damper case. Fig. 12 shows the responses of MMD with varying ζ_d. A fluctuation of peak responses is observed with small ζ_d (0.005). When ζ_d is 0.005, it appears that mass dampers are not effective in dissipating energy.

The fluctuation of the peak responses of the main structure diminishes as the mass dampers begin to dissipate energy through their damping elements. However, with excessive damping, higher than an optimum value, the vibration amplitude of the mass dampers is reduced. The minimum peak response is obtained with ζ_d=0.02 as shown in Fig. 12.

4. Robustness of the MMD under harmonic excitation

The sensitivity of a system to a certain parameter is determined by comparing the ideal case with those

Table 3
Parameters used in the vibration control for the single degree of freedom system under El Centro and Imperial Valley loading conditions

Figure	System	γ or γ_{center}	FR	ζ_d
17(b); 18(b)	Single mass damper	0.9886	–	0.065
17(c); 18(c)	11-mass uniformly distributed	0.9932	0.135	0.02
17(d); 18(d)	11-mass linearly distributed	0.9889	0.20	0.015
17(e); 18(e)	21-mass uniformly distributed	0.9937	0.19	0.02
17(f); 18(f)	21-mass linearly distributed	0.9882	0.335	0.015

obtained using variations of the parameters of interest. In this manner, the robustness of the MMD system is examined for γ_{center} and ζ_d. The case of n=21 is considered for this robustness investigation, i.e. the rest of the parameters except the one examined are the optimum values for the 21 MMD case.

4.1. Robustness for γ_{center}

The central mass damper tuning ratio of the 21 MMD is 0.9932 for UDMRS and 0.9889 for LDMRS. The rest of the parameters are taken from Table 1 for each case of the UDMRS and LDMRS. In Fig. 13, the three models shown are the SMD, the UDMRS and the LDMRS.

Although the UDMRS provides the lowest peak DMF values, it is not significantly different from the LDMRS. The region in which the UDMRS shows higher effectiveness is very small (approximately 0.02). Furthermore, the two cases of the MMDs do not show significantly greater robustness than the SMD when the optimum parameters are used. The peak DMF values of the MMDs become even higher than that of the SMD when the offset of the central mass damper tuning ratio goes beyond certain ranges. It is noticed that this range is greater for the LDMRS than the UDMRS. This behavior can be explained by the mass ratio distribution: as the central mass damper tuning ratio begins to shift, more mass dampers have frequencies that are further away from the optimum tuning frequency in the UDMRS than in the LDMRS.

Therefore, the UDMRS loses its effectiveness more rapidly than the LDMRS, i.e. the UDMRS is less robust than the LDMRS in terms of the frequency tuning ratio variation. One way to improve the robustness of the MMD is to increase the frequency range. Fig. 14 shows the improved robustness of the MMDs. The peak DMF curve of the UDMRS forms an almost a flat region in a wide section ([$0.95<\lambda<1.04$]). By increasing the frequency range from 0.135 to 0.190, the system becomes less sensitive to the shift of the frequency tuning ratio. However, the minimum peak DMF value is increased since the optimum value for the frequency range is not used. The new frequency range (FR_{UDMRS}=0.190) allows the UDMRS to obtain high robustness and keep the minimum peak DMF of the UDMRS less than that of the SMD. For the same reason, the new frequency range (FR_{LDMRS}=0.335) is applied to the LDMRS.

4.2. Robustness for ζ_d

Fig. 15 shows the comparison of the peak DMF curves of the SMD, UDMRS, and LDMRS. It illustrates that the SMD is less sensitive to the variation of the mass damper damping ratio in a broad region centered at its optimum damping ratio (ζ_{SMD}=0.065). Though the two MMDs seem less robust under the variation of ζ_d, the range in which the peak DMF values of the MMDs are lower than the minimum peak DMF of the SMD is quite broad. The two MMDs also show the flat region in their curves around their optimum values (ζ_{UDMRS}=0.02, ζ_{LDMRS}=0.015). In particular, the LDMRS shows very robust behavior when the damping is low.

5. Redundancy analysis under harmonic excitation

The major advantage of the MMD over the SMD is its ability to function when one or more dampers fail. For example, when the damper whose frequency ratio is the minimum among the dampers is inactive, the maximum DMF using the 3 mass UDMRS is increased by 63% while that of the 11 mass damper UDRMS is only increased by 16%. In this section, the 11 mass system is considered. The dampers are numbered such that the smallest frequency is assigned to No.1 and the greatest is assigned to No. 11. The details appear in Table 2.

Fig. 16(a–d) show the DMF curves of the two cases (UDMRS and LDMRS) of the 11-mass systems. Fig. 16(a) shows the DMF curves of the two cases with their optimum parameters. In Fig. 16(b), the systems are evaluated with the loss of damper No. 1, and this process is continued for all of the dampers. Fig. 16(c,d) show the effect of 'losing' other dampers for comparison. In summary, the UDMRS shows less fluctuation of the

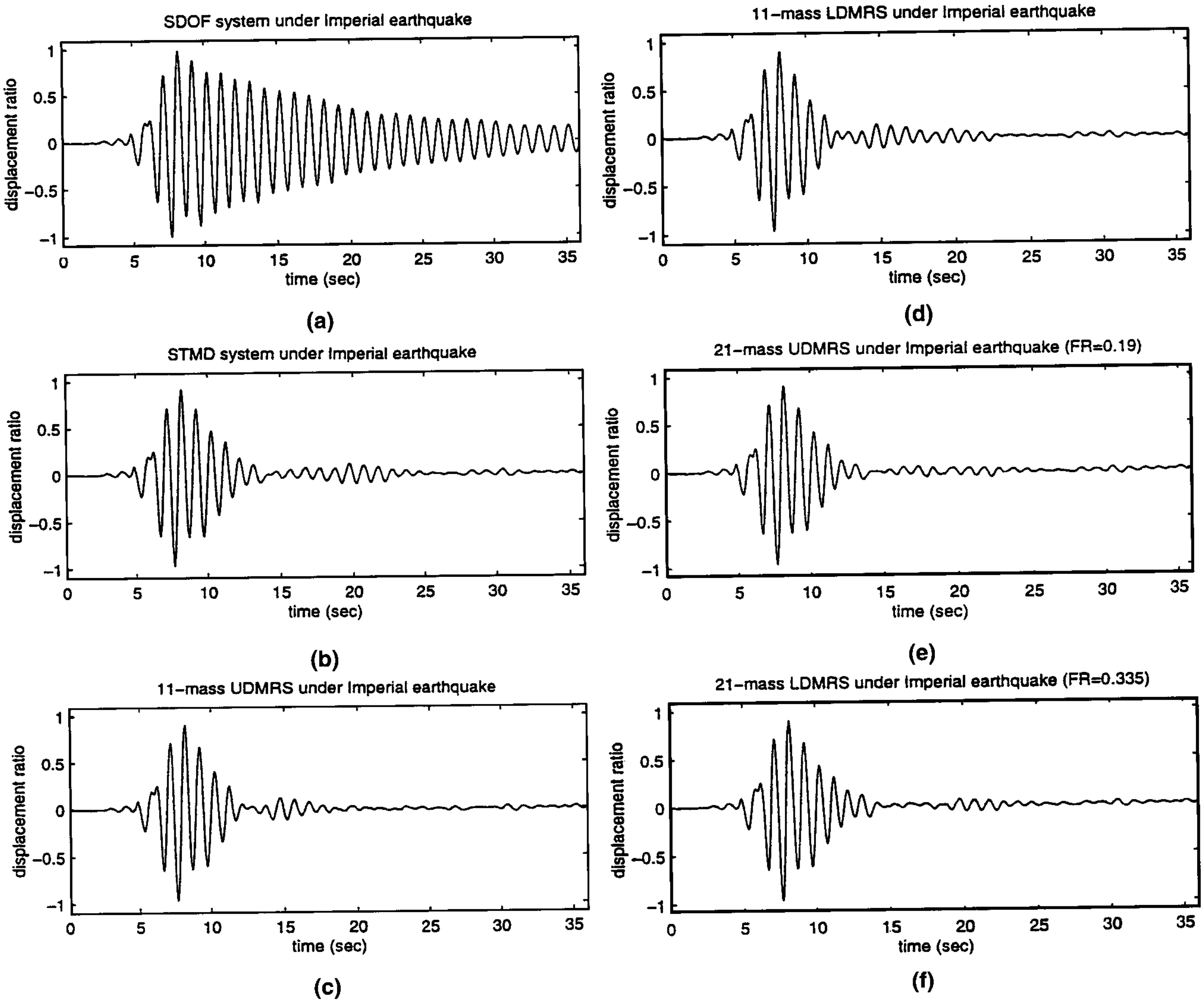

Fig. 18. The response of the single degree of freedom (SDOF) structure without dampers attached to the Imperial Valley ground excitation. (b) Response of the SDOF system with a single tuned mass damper. (c) Response of the SDOF system with an 11 uniformly distributed mass system. (d) Response of the SDOF system with an 11 linearly distributed mass system. (e) Response of the SDOF system with an 21 uniformly distributed mass system. (f) Response of the SDOF system with an 21 linearly distributed mass system.

peak DMF than the LDMRS for loss of individual dampers, but the amount is not very large. This analysis is not exhaustive, but it provides some indication of the reliability of each system when individual dampers fail.

6. Consideration of earthquake loadings

We also studied the effect of the two systems for earthquake loading conditions. Consistent with the statements made by Kaynia and Veneziano [7], Sladek and Klingner [8] and Abe [9] for the passive TMD systems, for this loading type, the major influence of these dampers is to increase the rate at which the vibration is reduced and active systems are preferred for reduction of peak values. The El Centro and Imperial Valley earthquake records were used in this analysis. They were chosen as representative of two types of records: El Centro generates relatively continuous excitation while the Imperial Valley is more of an impulse type excitation. Even though Imperial Valley strong motion lasts for twice as long as the El Centro, the significant excitation is observed only for the first 16 s.

The constraints applied to the model in the earthquake excitation analysis are the same as those used for the sinusoidal excitation study: the total mass ratio is 0.01 and the structural damping ratio is 0.01. The structure is modeled as a single degree of freedom system with mass dampers attached. The total mass of the structure was chosen to be 100 kg with a period of 1 s. The mass can be an arbitrary number since the analysis is undertaken using the ratio of the parameter values. The relative displacements of the structure were divided by the maximum displacement of the single degree of freedom structure without any mass dampers. In this manner, comparison was made based on the displacement ratio of various cases, not absolute displacement.

For El Centro, Fig. 17(a–d) show the vibration of a single degree of freedom system without artificial damping; the vibration when a single mass damper is employed; the vibration when an 11-mass uniformly distributed system is employed, and the 11-mass linearly distributed system, respectively. The effectiveness of the 11-mass systems was comparable to the single mass damper. The parameters for the uniformly and linearly distributed systems were derived from the optimization under harmonic forcing. Tweaking the number of dampers, the frequency ranges and individual damping values, yielded slightly better effectiveness from both systems as can be seen in Fig. 17(e,f) for the 21-mass uniformly and linearly distributed systems, respectively. The relevant parameters for the systems are provided in Table 3.

For Imperial Valley, Fig. 18(a–d) show the vibration of a single degree of freedom system without artificial damping; the vibration when a single mass damper is employed; the vibration when an 11-mass uniformly distributed system is employed, and the 11-mass linearly distributed system, respectively. The influence of the dampers on the decay rate is much more pronounced with the Imperial loading. If the number of dampers is increased to 21, as with the El Centro analysis, the resulting vibration decay is only slightly better than the 11 mass systems, as shown in Fig. 18(e,f) for the uniformly and linearly distributed systems, respectively. Table 3 contains the details of the parameters employed in the study.

This limited earthquake analysis shows the effectiveness of the multiple mass systems for earthquake loadings in terms of increasing the decay rate of vibration. The 21-mass system is only slightly more effective than the 11-mass system. Significant differences in the effectiveness between the two damper systems are not apparent under the earthquake loading conditions.

7. Conclusions

For harmonic excitation, we show that the uniformly distributed mass system is more effective in reducing the peak dynamic magnification factor. The linearly distributed system is slightly more robust to mistuning. It is also more robust to damping variation for low damping, but the effectiveness of the two systems converges as damping increases. The uniformly distributed system is more reliable when an individual damper fails. The eleven mass system is optimum for both configurations under harmonic excitation. A limited study was undertaken for vibration control under the El Centro and Imperial Valley earthquake excitation records. For both cases, the performance of the 11-mass systems was comparable to a single mass damper; and, the 21-mass systems appeared to be slightly more effective in vibration decay over the entire loading period. Neither system favored the other under this loading condition.

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