

# ME 230 Kinematics and Dynamics

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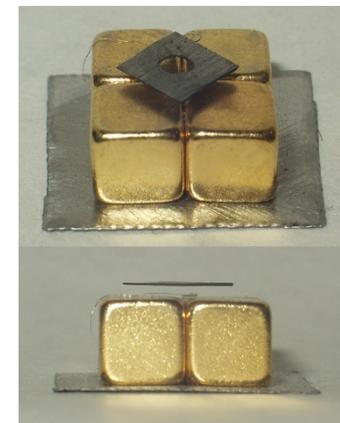
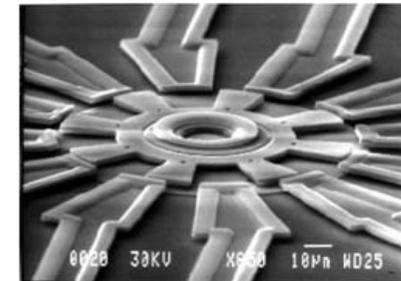
## Extra Credit Projects (up to 10%)

(Magnetic propulsion or electromagnetic propulsion device)

- Design a magnetic propulsion or electromagnetic propulsion device.

Thing to include in the write-up:

- Briefly describe your proposed design.
- What is the function of your machine?
- List and describe the key components of the machine and show the concept or calculation using kinetic or kinematic equations.
- more credit will be given to those who actually build the device.



# Magnetic Levitation

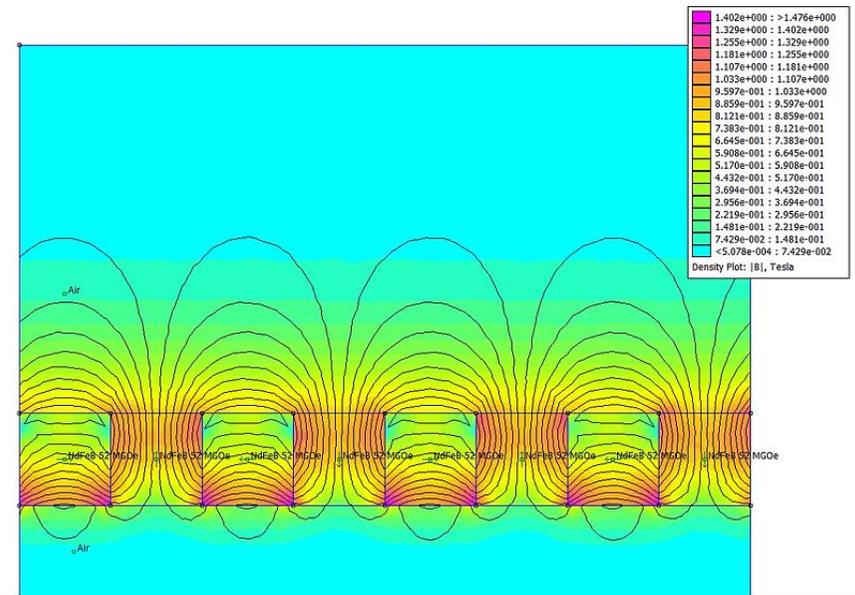
1. Superconductor
2. Diamagnetic Levitation
3. Magnetic Levitation (Meg Lev)
  - Electrodynamic
  - Electromegntic
  - Inductrack (Eddy Current Induction)

# Magnetic Propulsion

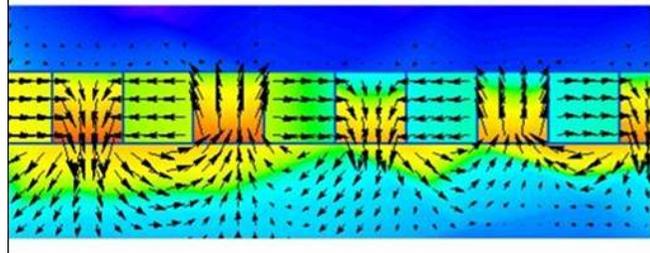
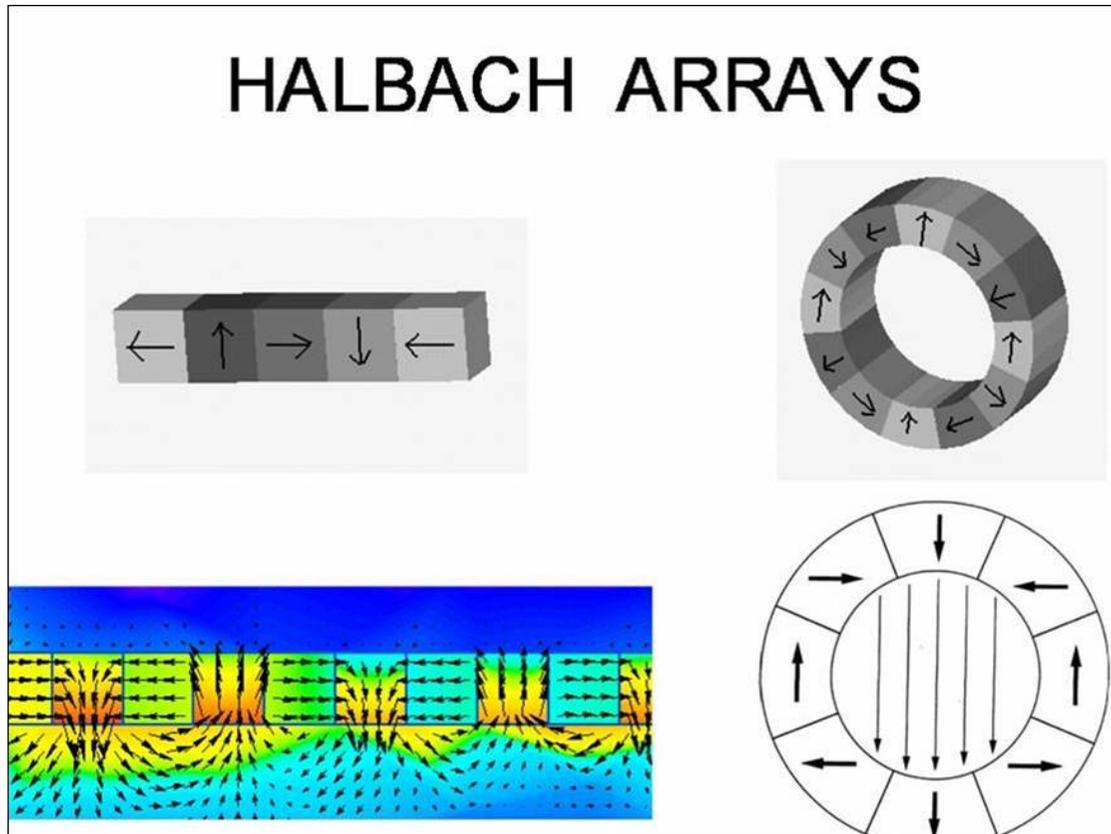
- Lorentz Force ( $I\mathbf{l} \times \mathbf{B}$ )
- Magnetic Force (magnet repulsion and attraction) ( $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$ )

# Halbach Array

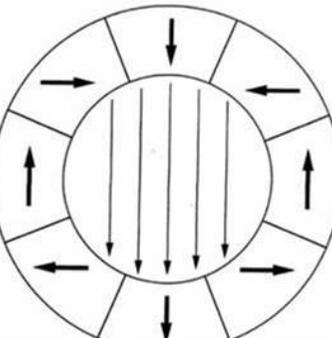
- **Monopole like magnetic field. Magnetic field on one side is increased and the other side is decreased.**
- This is achieved by having a spatially rotating pattern of magnetization.
- The effect was discovered by John C. Mallinson in 1973, and these "one-sided flux" structures were initially described by him as a "curiosity", although he recognized at the time the potential for significant improvements in magnetic tape technology.[2]
- In the 1980s, Lawrence Berkeley National Laboratory physicist Klaus Halbach, independently invented the Halbach array to focus accelerator particle beams.[3]



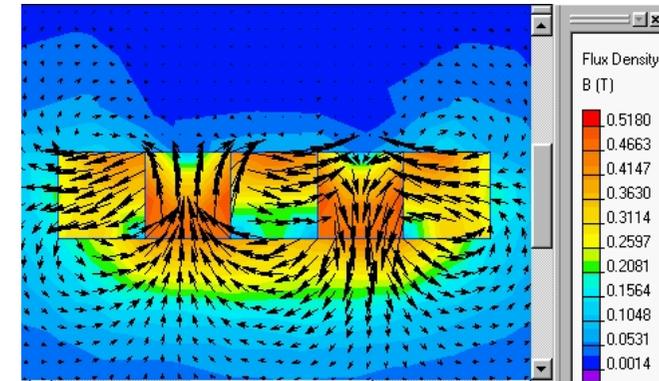
# Two Configurations of Halbach Array



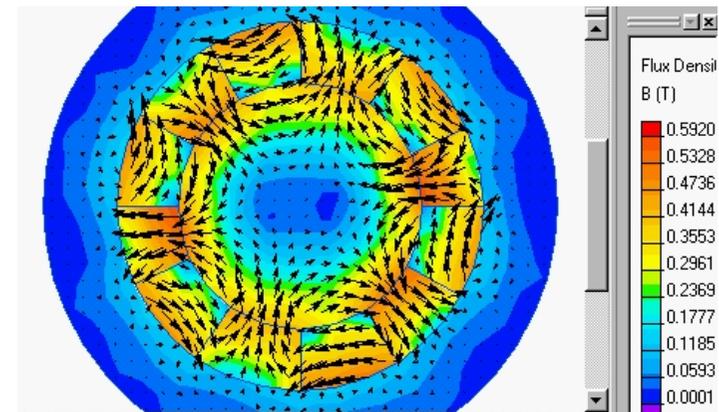
Monopole fields



Uniform fields



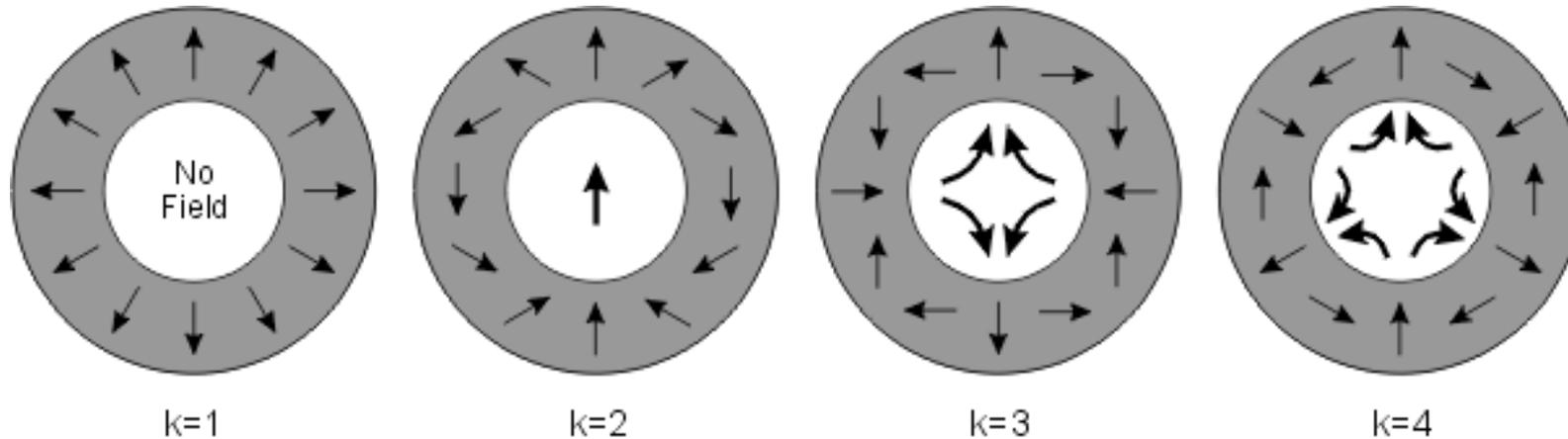
Monopole fields



Halbach cylinder

Illustrations used by  
permission from  
[DMBoss1021@aol.com](mailto:DMBoss1021@aol.com)--  
thanks!

# Halbach Cylinder



An intense magnetic field confined entirely within the cylinder with zero field outside

From Wikipedia

The direction of magnetization within the ferromagnetic material, in plane perpendicular to the axis of the cylinder, is given by

$$M = M_r \left[ \sin(k\phi) \hat{\rho} - \cos(k\phi) \hat{\phi} \right]$$

where  $M_r$  is the ferromagnetic [remanence](#) (A/m). A choice of  $+k$  gives an internal magnetic field and  $-k$  gives an external magnetic field.

# Application

These cylindrical structures are used in devices such as brushless AC motors, magnetic couplings and high field cylinders. Both brushless motors and coupling devices use multipole field arrangements:

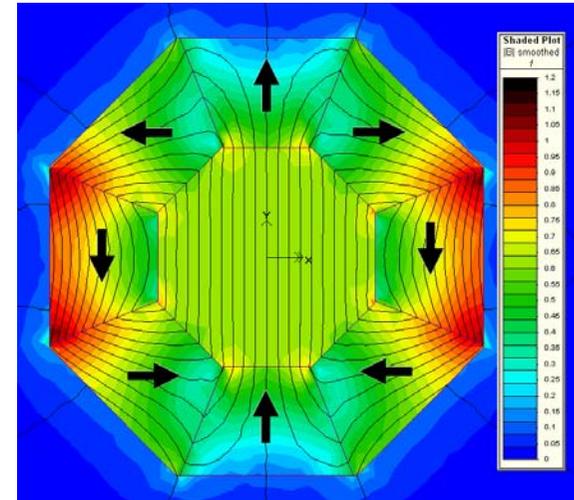
- Brushless motors typically use cylindrical designs in which all the flux is confined to the centre of the bore (such as  $k = 4$  above, a six pole rotor) with the AC coils also contained within the bore. Such self-shielding motors designs are more efficient and produce higher torque than conventional motor designs.
- Magnetic coupling devices transmit torque through magnetically transparent barriers (that is, the barrier is non-magnetic or is magnetic but is not affected by an applied magnetic field), for instance between sealed containers or pressurised vessels. The optimal torque couplings consists of a pair of coaxially nested cylinders with opposite  $+k$  and  $-k$  flux magnetization patterns, as  $-k$  magnetization patterns produce fields entirely external to the cylinder. In the lowest energy state, the outer flux of the inner cylinder exactly matches the internal flux of the outer cylinder. Rotating one cylinder relative to the other from this state results in a restoring torque.

# Halbach Cylinder: Uniform Field

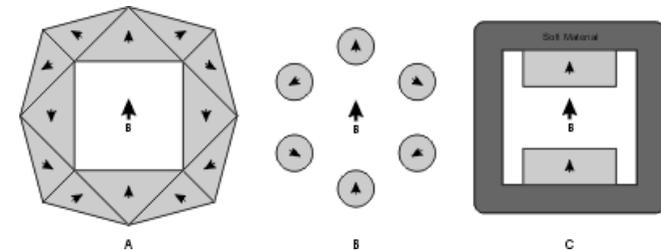
For the special case of  $k = 2$ , the field inside the bore is uniform, and is given by:

$$H = M_r \ln \left( \frac{R_o}{R_i} \right) \hat{y}$$

where the inner and outer cylinder radii are  $R_i$  and  $R_o$ , respectively.  $H$  is in the  $y$  direction. This is the simplest form of the Halbach cylinder, and it can be seen that if the ratio of outer to inner radii is greater than e the flux inside the bore actually exceeds the remanence of the magnetic material used to create the cylinder.



These Halbach dipoles have been used to conduct low-field NMR experiments.[8] Compared to commercially available (Bruker Minispec) standard plate geometries (C) of permanent magnets, they, as explained above, offer a huge bore diameter, while still having a reasonably homogeneous field. (NMR=Nuclear magnetic resonance)

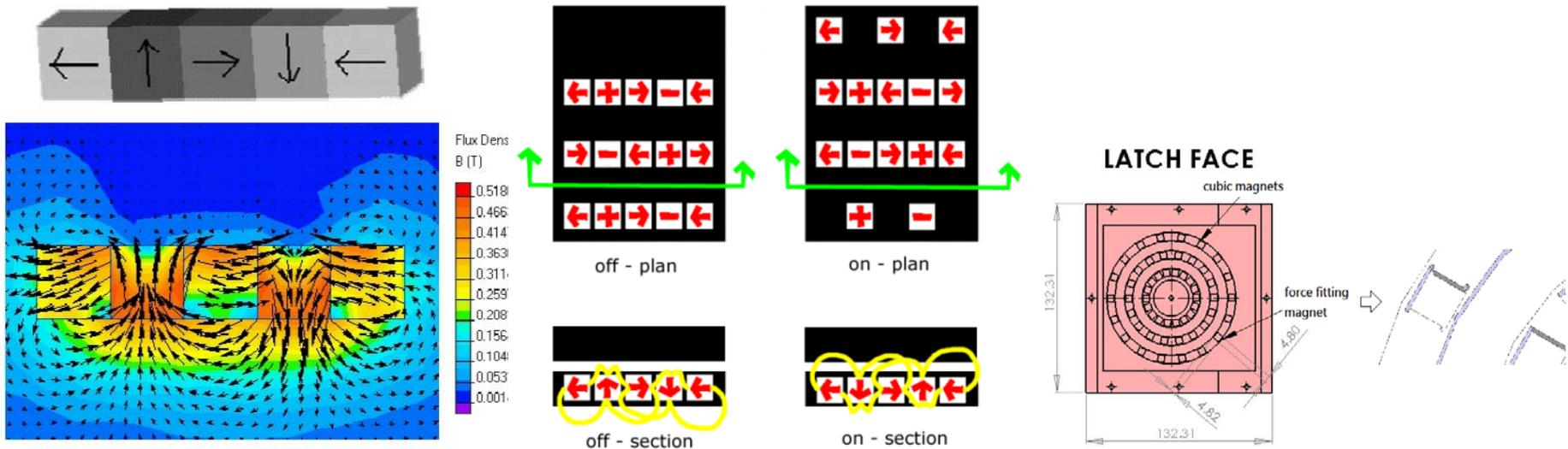


Three designs producing uniform magnetic fields within their central air gap

# Halbach array latch

# Halbach array latch

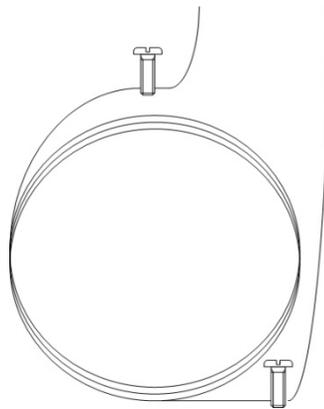
- Electromagnets or permanent magnets?
- Shift the arrangement arrays for different augmentation of the overall magnetic field, allowing a magnetic ON/OFF state
- No power consumption
- Shape Memory Alloy to drive the rotation of array augmentation



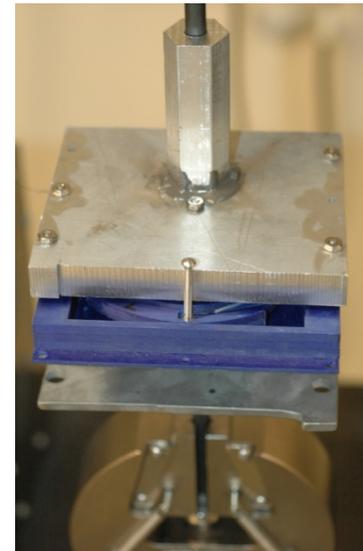
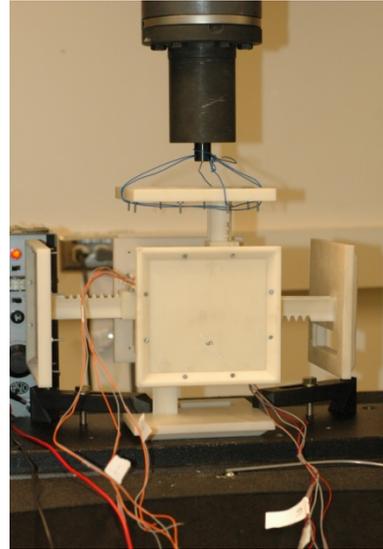
# Latch System

- Rotate 20°
- The total force ~2.05 lb (930g)  
(~ equal to the number on SMA manual.)
- Total power consumption is about 21.28W to 22.95W

SMA loops  
and anchors



# Testing data



- ON/OFF configuration test is also conducted.

ON lb (N)	OFF lb (N)
11.81 (52.4)	0.85 (3.79)

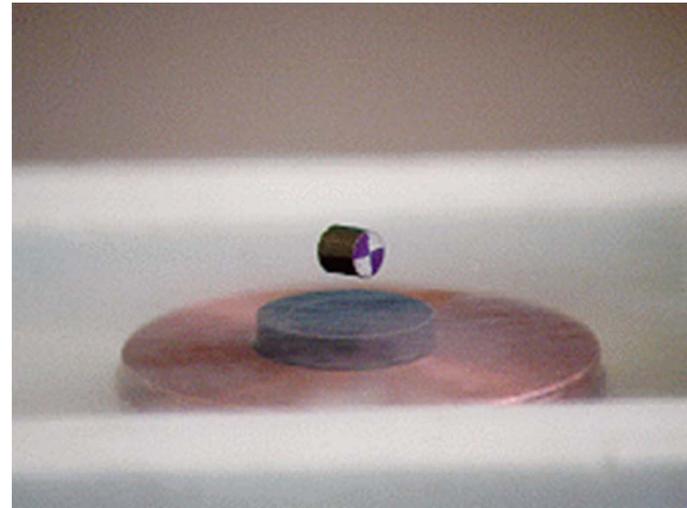


# Magnetic Levitation

1. Superconductor
2. Diamagnetic Levitation
3. Magnetic Levitation (Meg Lev)
  - Electrodynamic
  - Electromegntic
  - Inductrack (Eddy Current Induction)

# 1. Superconductivity

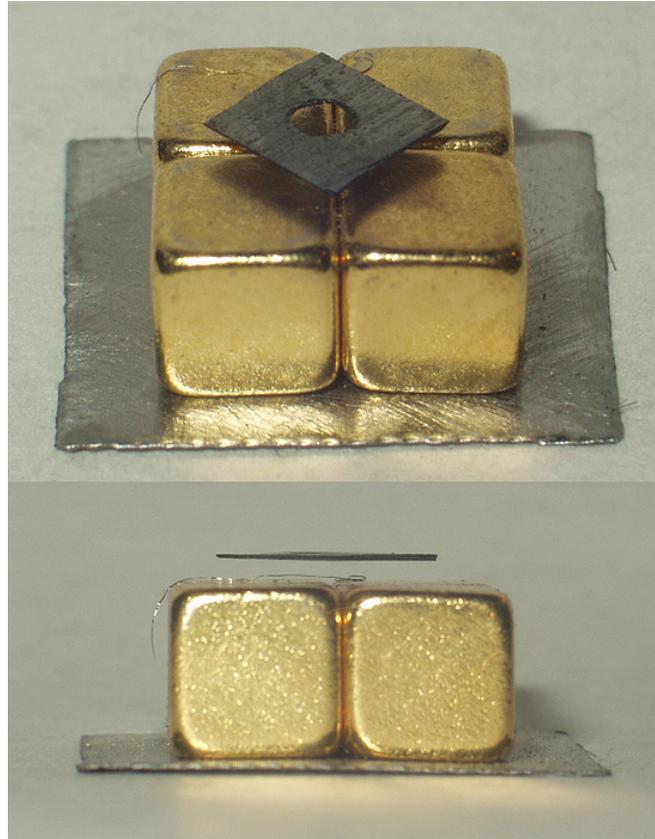
High Field  
Magnet Laboratory  
[University of Nijmegen](http://www.uva.nl/en/physics/high-field-magnet-laboratory)



Yttrium(1)Barium(2)Copper(3)Oxygen(6.95). Zero resistance- perfect conductor, perfect diamagnets

Actually, to be correct, superconductors are more interesting than perfect conductors. In fact, superconductors are really **perfect diamagnets**.

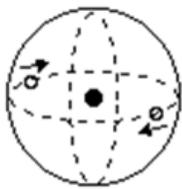
## 2. Diamagnetic Levitation



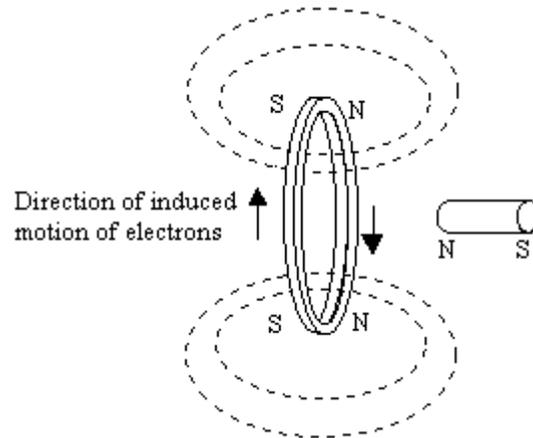
# Diamagnetic Materials

When an external magnetic field is applied to a material, these current loops will tend to align in such a way as to oppose the applied field. This may be viewed as an atomic version of Lenz's law.

*Diamagnetism* is a natural consequence of Lenz's law, according to which the electric current resulting from an applied field will be in the direction that opposes the applied field. In other words, the induced current will flow in the direction that creates a field opposite to the applied field



**Diamagnetism:**  
Lenz's Law applied to orbiting electrons causes atoms to be repelled from a magnetic field.



Lenz's Law: "The Induced current is such as to OPPOSE the CHANGE in applied field."

**It's like Lenz's Law, but in atomic level, where the orbiting electrons can create a B field to oppose the applied field.**

# Diamagnetic levitation



**H**igh **F**ield  
**M**agnet **L**aboratory  
[University of Nijmegen](http://www.hfm.unijmegen.nl)

# Example: Three Phase Diamagnetic Levitation Motor



Magnetic restoring force  $\sim$  spring

UWMicroTech

### 3. Magnetic Levitation (Meg Lev)

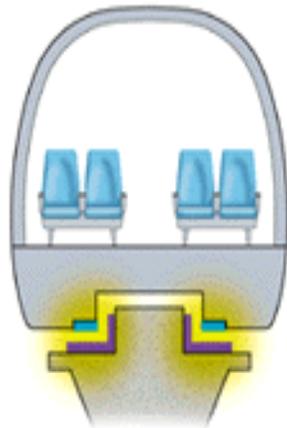


**A Transrapid train at the Emsland, Germany test facility**

# Magnetic Levitation (Mag Lev)

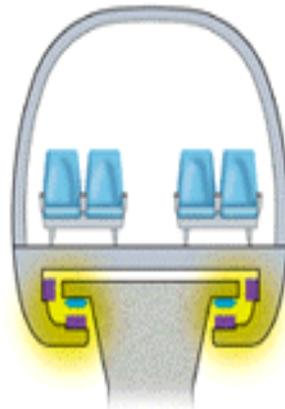
## Levitation Techniques

**ELECTRODYNAMIC**



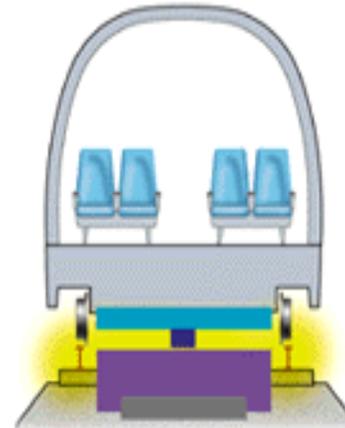
Electromagnets on the guideway levitate the car.

**ELECTROMAGNETIC**



Electromagnets on the cars lift the cars.

**INDUCTRACK**



Permanent magnets levitate over passive coils.

 Permanent magnets

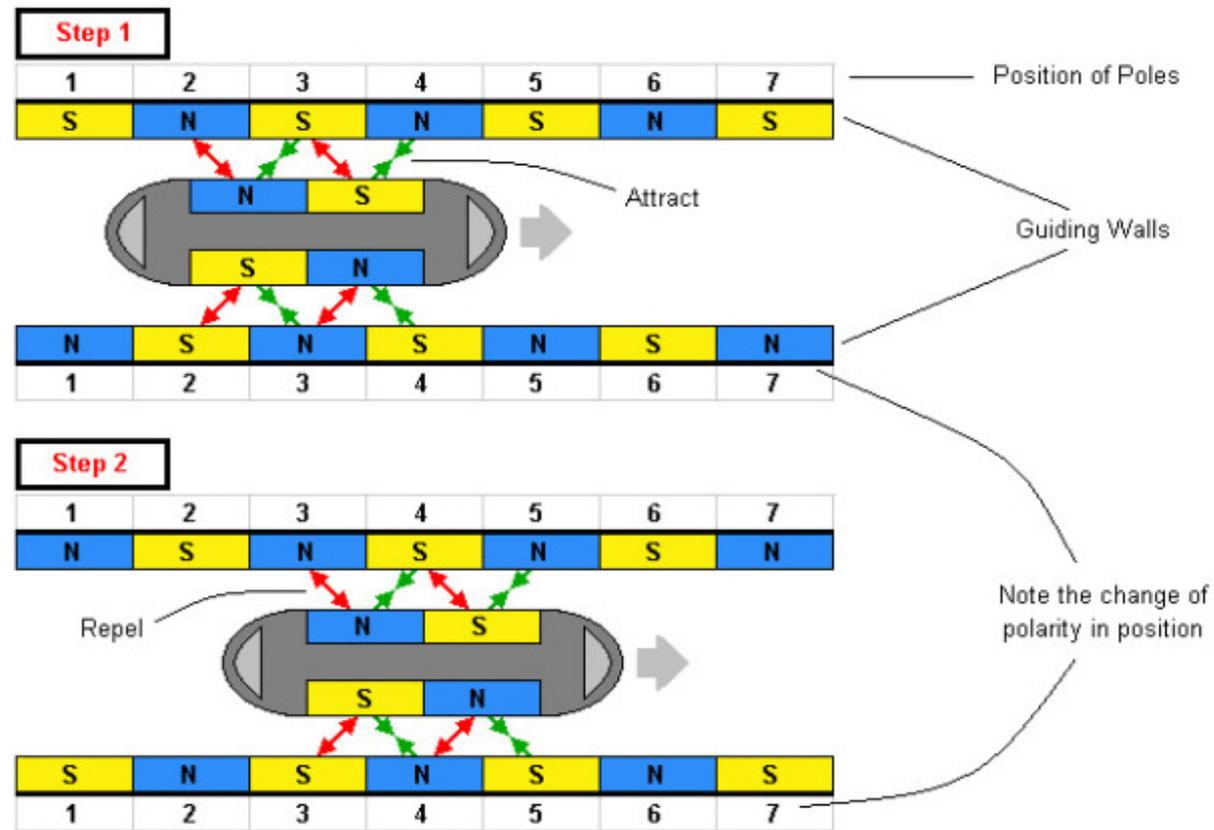
 Electromagnets

Eddy current  
Newton 3<sup>rd</sup> Law

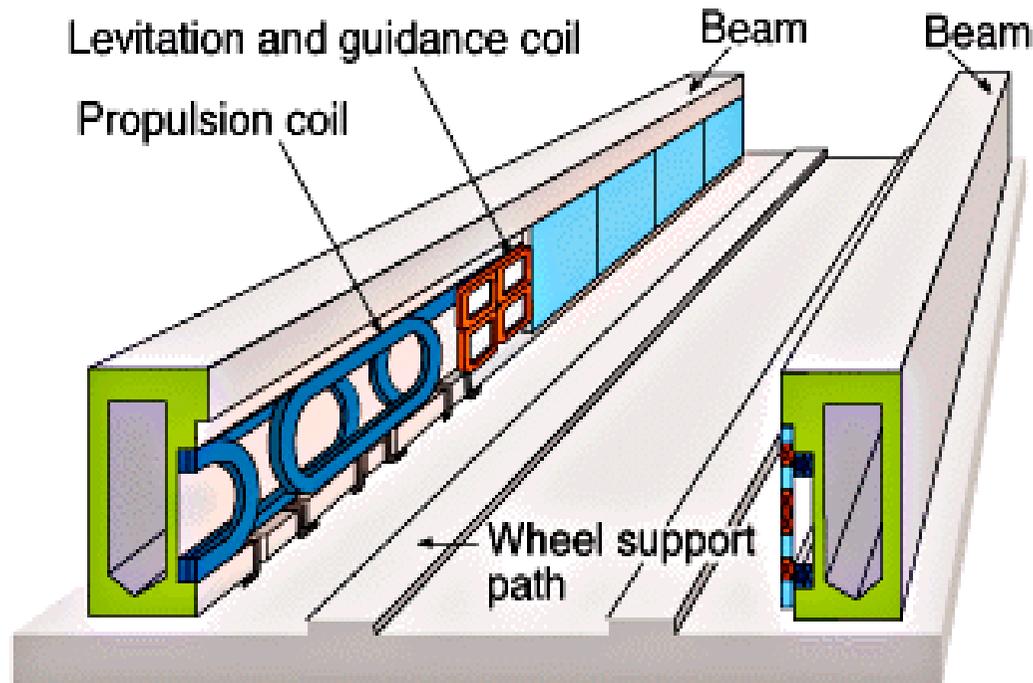
# Magnetic Propulsion

- Lorentz Force ( $I\ell \times B$ )
- Magnetic Force (magnet repulsion and attraction)

# Alternating Electromagnet Propulsion

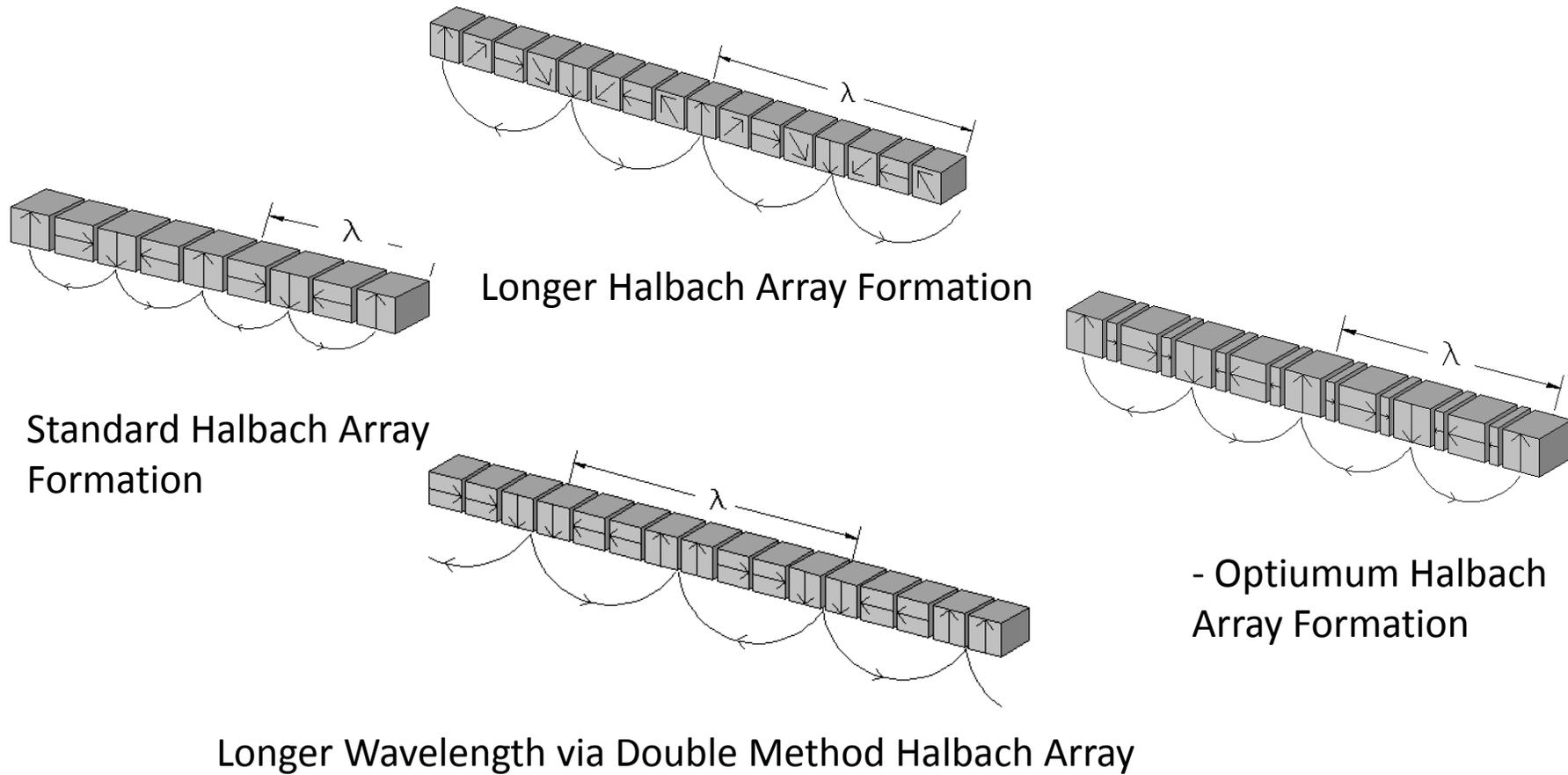


# Example of Maglev Track



by [Kevin Bonsor](#)

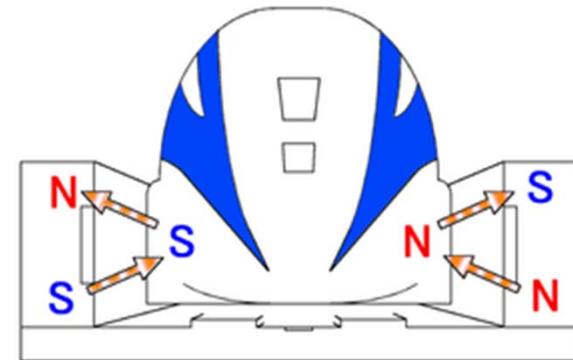
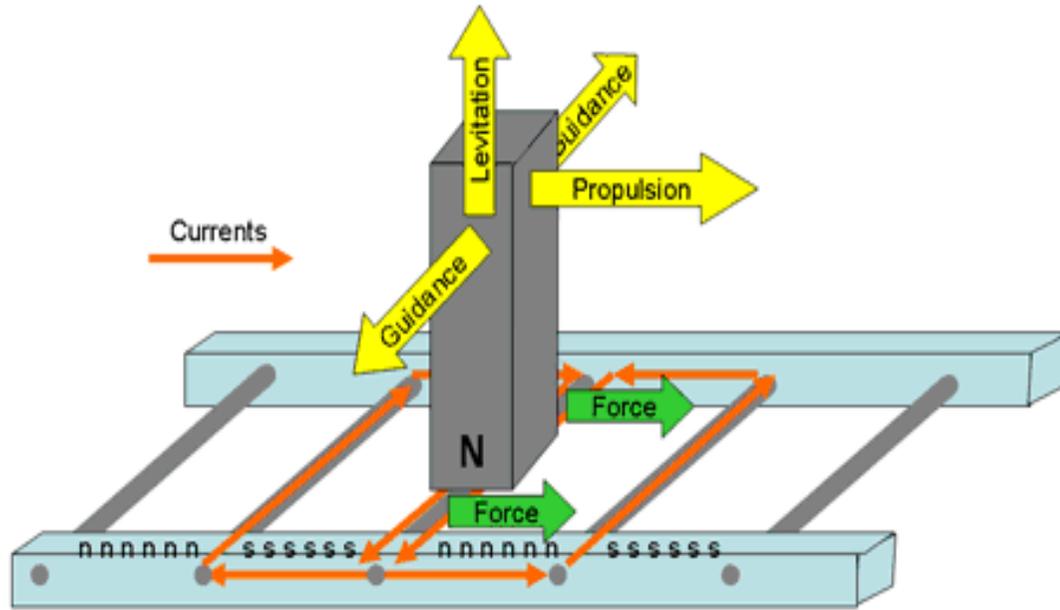
# Examples of Halbach Array



# Magnetic Propulsion

- Lorentz Force ( $I\ell \times B$ )
- Magnetic Force (magnet repulsion and attraction)

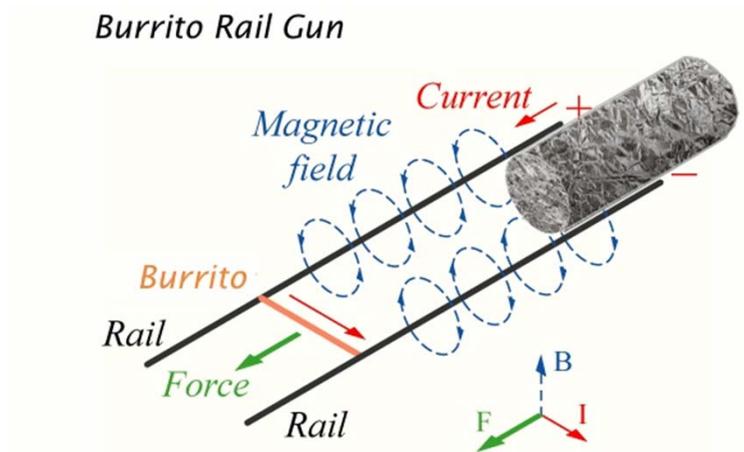
# Lorentz force Propulsion



**Linear Motion and Magnetic Levitation**

The train has no onboard motor. Electromagnets in the track are excited in sequence creating a linear rather than a rotating field. By transformer action, the track coils induce currents in coils on board the train which are used to energize powerful electromagnets. The Lorentz force between the track currents and the onboard electromagnets causes the magnets to be propelled along by the moving field.

# Rail Gun



A **railgun** is an [electrically powered](#) electromagnetic projectile launcher based on similar principles to the [homopolar motor](#). A railgun comprises a pair of parallel conducting rails, along which a sliding armature is accelerated by the electromagnetic effects of a current that flows down one rail, into the armature and then back along the other rail

Naval Surface Warfare Center test firing in January 2008; a plume of flame is produced behind the projectile

Lorentz force law:

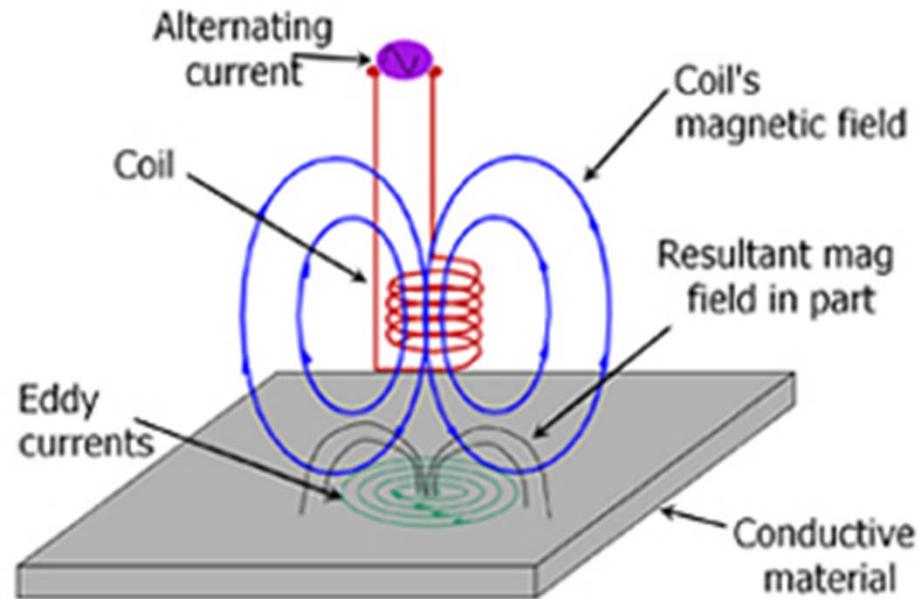
$$F = I l \times B$$

# Magnetic levitation by Eddy currents

Electrodynamic levitation constitutes an interesting research area in the fields of vehicle transportation (MagLev trains), electromagnetic launchers and flywheel energy storage techniques. A first approximation to the problem is achieved observing the damped fall of a magnet through a nonmagnetic conductive tube, the slow sliding of a magnet over an aluminum slab or the damped oscillations of a nonmagnetic conductor when submitted to a magnetic field.



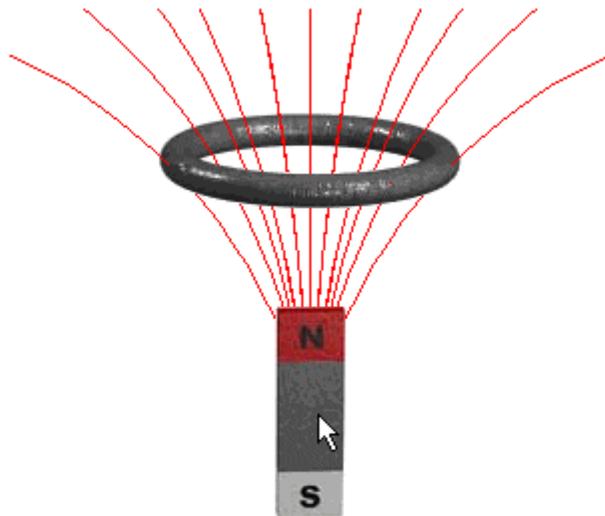
# Eddy Current



Eddy current is based on Lenz's Law, where if we have a changing B field cutting a conductor, then as soon as the B field changes, we will get an induced current which has a direction decided by Lenz's Law (Usually opposite direction of incoming B field). The induction current on the metal conductor will create a B field trying to oppose the B field in the coil and thus create a lift. Larger the rate of changing magnetic flux, the higher it will lift. In addition to the lift, there is a drag force due to  $F = qv \times B$ , the B field on the coil will cause a drag force proportional to that equation that is trying to slow down the metal if metal is moving.

# Electrodynamical magnetic levitation

Eddy current is based on Lenz's Law, where if we have a changing B field cutting a conductor, then as soon as the B field changes, we will get an induced current which has a direction decided by Lenz's Law.

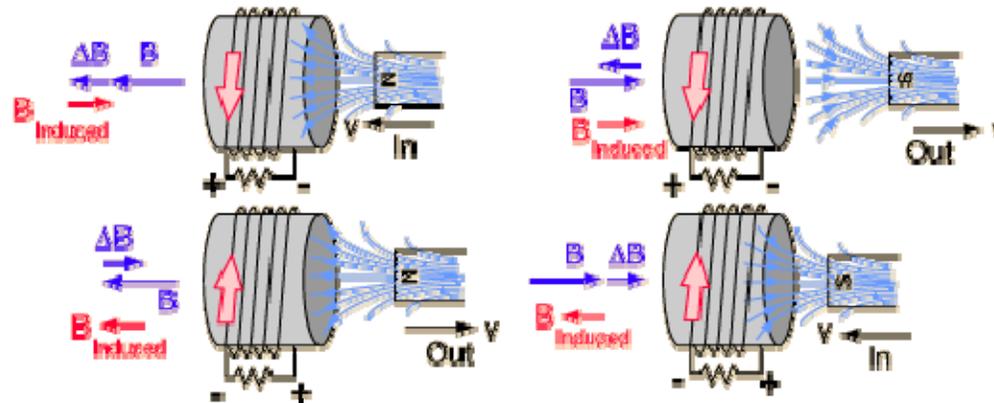


Lenz's law describes the direction of the current induced by Faraday's Law. It states that the induced current in a loop is in the direction such that the resultant magnetic field opposes the change in flux which produced it (the current attempts to keep the original magnetic flux by opposing changes to it)

"The Induced current is such as to OPPOSE the CHANGE in applied field."

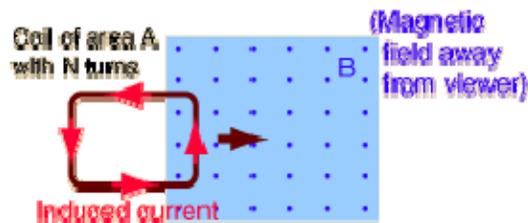
# Faraday's Law

When an emf is generated by a change in magnetic flux according to [Faraday's Law](#), the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



# Faraday's law

Faraday's law is a fundamental relationship which comes from [Maxwell's equations](#). It serves as a succinct summary of the ways a [voltage](#) (or emf) may be generated by a changing magnetic environment. The induced emf in a coil is equal to the negative of the rate of change of [magnetic flux](#) times the number of turns in the coil. It involves the interaction of charge with magnetic field.



A coil of wire moving into a magnetic field is one example of an emf generated according to Faraday's Law. The current induced will create a magnetic field which opposes the buildup of magnetic field in the coil.

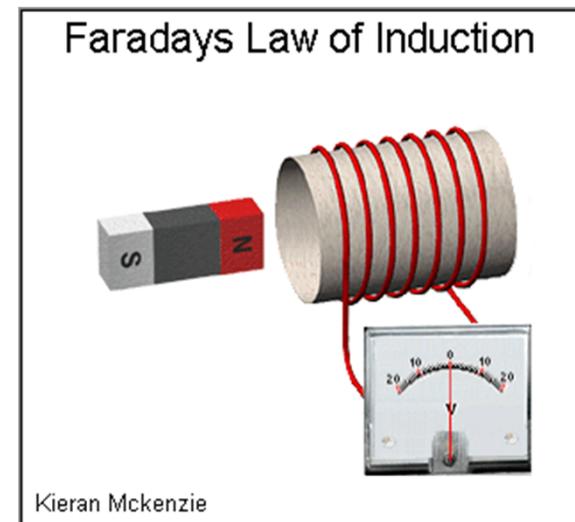
Faraday's Law

$$\text{Emf} = -N \frac{\Delta\Phi}{\Delta t} = IR$$

↑  
Lenz's Law

where  $N$  = number of turns  
 $\Phi = BA$  = magnetic flux  
 $B$  = external magnetic field  
 $A$  = area of coil

The minus sign denotes Lenz's Law.  
Emf is the term for generated or induced voltage.



$I$  = current induced in the loop  
 $R$  = electrical resistance in the loop

# Ampere's Law

The [magnetic field](#) in space around an [electric current](#) is proportional to the electric current which serves as its source, just as the [electric field](#) in space is proportional to the [charge](#) which serves as its source. Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the [permeability](#) times the electric current enclosed in the loop.



In the electric case, the relation of field to source is quantified in [Gauss's Law](#) which is a very powerful tool for calculating electric fields.

# Solenoid

If we assume the solenoid to be long, then we can assume the magnetic field inside to be uniform. We can apply Ampere's Law to the loop  $l$

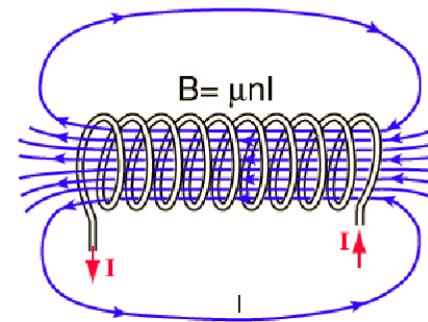
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad (1)$$

If there are  $n$  turns of wire per metre of the solenoid then the loop  $l$  will enclose  $nL$  turns of wire, each carrying a current  $I$ , so will enclose a total current of  $nLI$ . The right hand side of (1) is then  $\mu_0 nLI$ .

As for the left hand side, outside the coil the magnetic field is zero so this part of the path contributes nothing to the integral. Inside the coil, the magnetic field is a uniform  $\vec{B}$ , so for the coil of length  $L$  the left hand integral is  $BL$

Hence  $BL = \mu_0 nLI$

$L$  cancels from both sides to give  $B = \mu_0 nI$

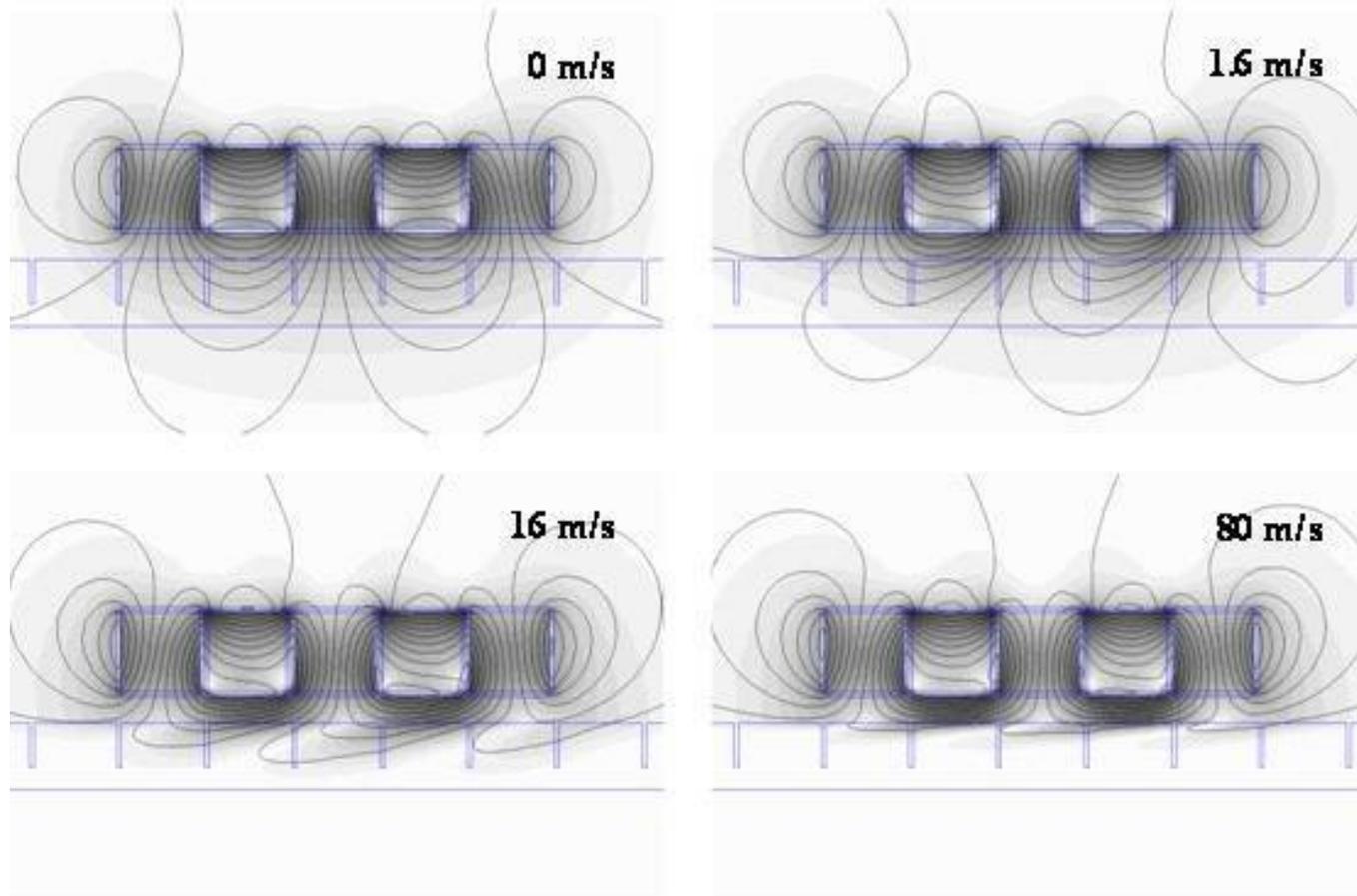


The magnetic field is concentrated into a nearly uniform field in the center of a long solenoid. The field outside is weak and divergent.

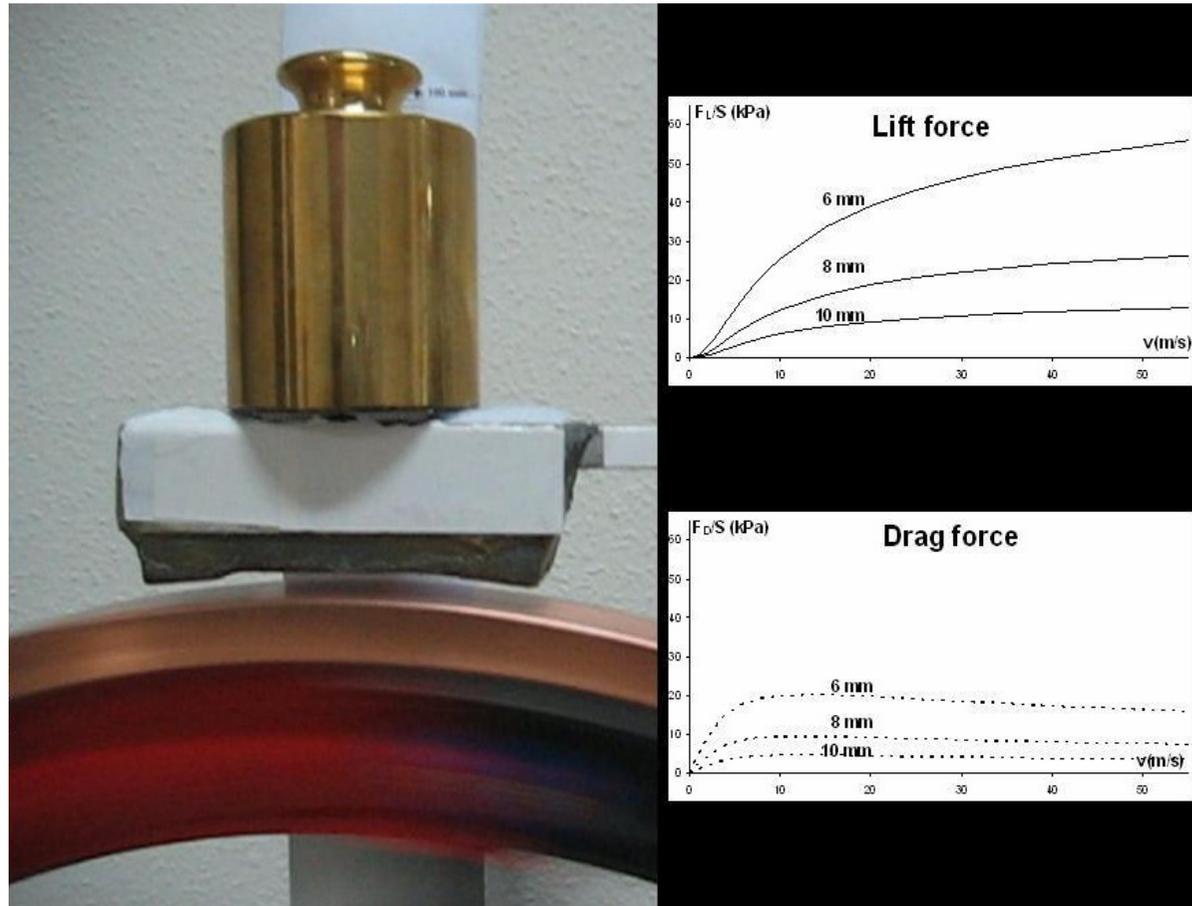
# Halbach Array Magnetic Levitation System

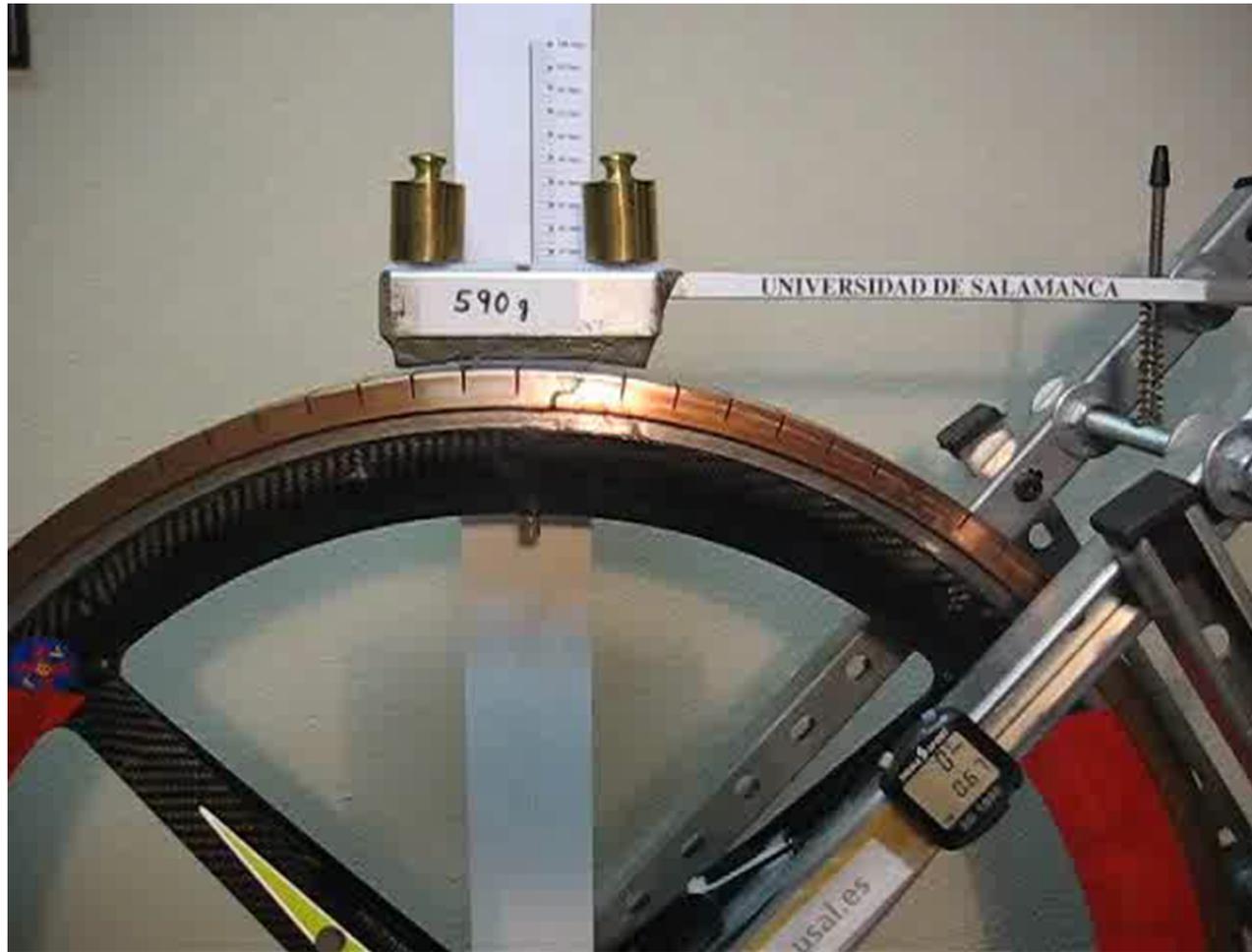
Using adequate configurations of magnet arrays (Halbach type) it results possible to consider interesting applications in the above-quoted fields. Here you can see the drag and lift forces versus speed in a small laboratory prototype, the magnetic lines of force and detailed videos showing how it works.

# Magnetic field lines versus Speed



# Lift and drag forces vs speed of a Halbach array





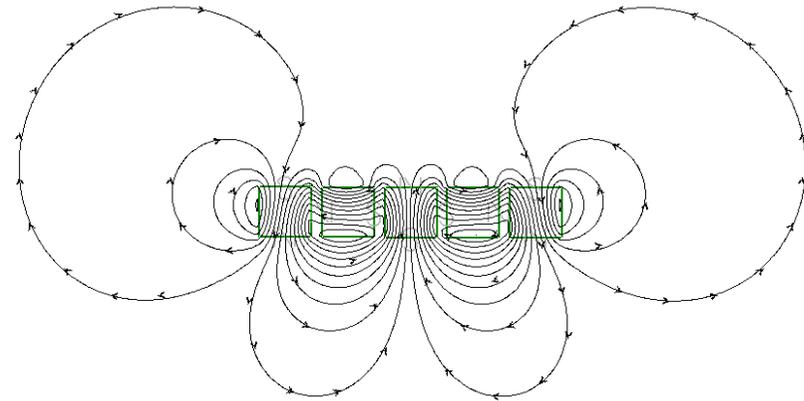
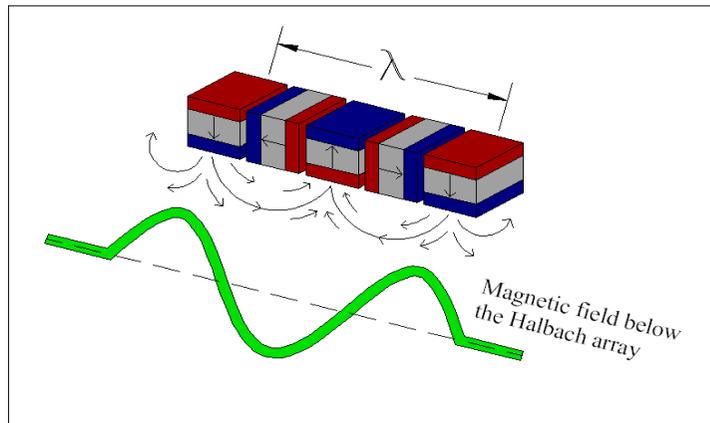
- Levitation of 1 kg

# Inductrack

**Inductrack** is a [passive](#), [fail-safe electrodynamic magnetic levitation](#) system, using only unpowered loops of wire in the track and permanent magnets (arranged into [Halbach arrays](#)) on the vehicle to achieve [magnetic levitation](#). The track can be in one of two configurations, a "ladder track" and a "laminated track". The ladder track is made of unpowered [Litz wire](#) cables, and the laminated track is made out of stacked copper or aluminium sheets.

Inductrack (or Inductrak) was invented by a team of scientists at [Lawrence Livermore National Laboratory](#) in [California](#), headed by physicist [Richard F. Post](#), for use in [maglev trains](#), based on technology used to levitate flywheels. [\[1\]\[2\]\[dead link\]\[3\]](#) At constant velocity, power is only required to push the train forward against air and electromagnetic [drag](#). Above a minimum speed, as the velocity of the train increases, the levitation gap, lift force and power used are largely constant. The system can lift 50 times the magnet weight.

# Physics of Halbach Array Magnets



- Designed by Klaus Halbach
- Creates a strong, enhanced magnetic field on one side, while almost cancelling the field on the opposite side
- Peak strength of the array:

$$B_0 = B_r (1 - e^{-kd}) \sin(\pi/M) / (\pi/M) \text{ Tesla}$$

$$k = 2\pi/\lambda, M = \# \text{ of magnets,}$$

$$B_r = \text{magnet strength, } d = \text{thickness of each magnet}$$

$$\lambda = \text{Halbach array wavelength}$$

# Physics of the Inductrack

- Halbach array moving at velocity  $v$  m/sec over inductrack generates flux

$\phi_0 \sin(\omega t)$ ,  $\phi_0$  Tesla-m<sup>2</sup>, linking the circuit

$$\omega = (2\pi/\lambda)v \text{ rad/sec}$$

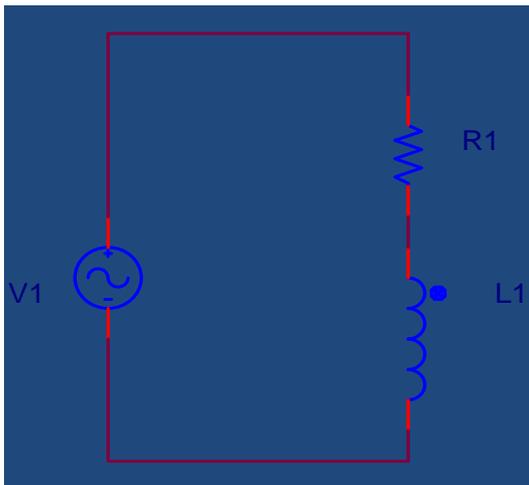
- Voltage induced in inductrack circuit:

$$V(t) = \omega \phi_0 \cos(\omega t)$$

- Inductrack R-L circuit current equation:

$$V(t) = L \cdot di(t)/dt + R \cdot i(t)$$

# Physics of the Inductrack Cont



- Close-packed conductors, made utilizing thin aluminum or copper sheets
- Allows for levitation at low speeds
- Can be modeled as an RL circuit
- Transfer function has pole at  $-R/L$

# Physics of the Inductrack Cont.

- Dr. Post used the induced current and magnetic field to derive
  - Lift force:
    - $\langle F_y \rangle = B_0^2 w^2 / 2kL * (1 / (1 + (R/\omega L)^2)) * e^{-ky_1}$
  - Drag force:
    - $\langle F_x \rangle = B_0^2 w^2 / 2kL * (R/\omega L) / (1 + (R/\omega L)^2) * e^{-ky_1}$

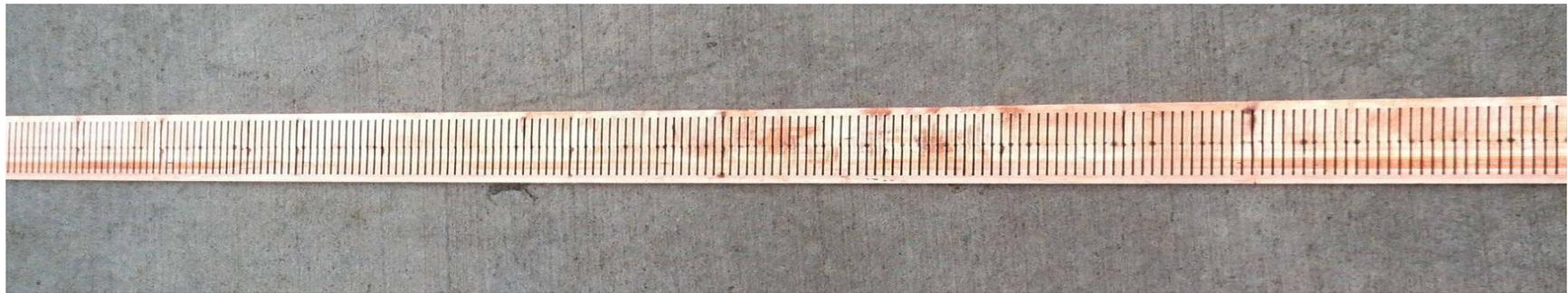
Where  $y_1$  is the levitation height in meters

# Physics of the Inductrack Cont.

- Phase shift relates to drag and levitation forces
- Lift/Drag =  $\omega * L/R$
- $L = \mu_0 w / (2kd_c)$  , where  $d_c$  is the center to center spacing of conducting strips and  $w$  is the track width

# Lab Work - Design

- Designed wheel and copper track to be built
- Wheel and track were machined by Tri-City Machining

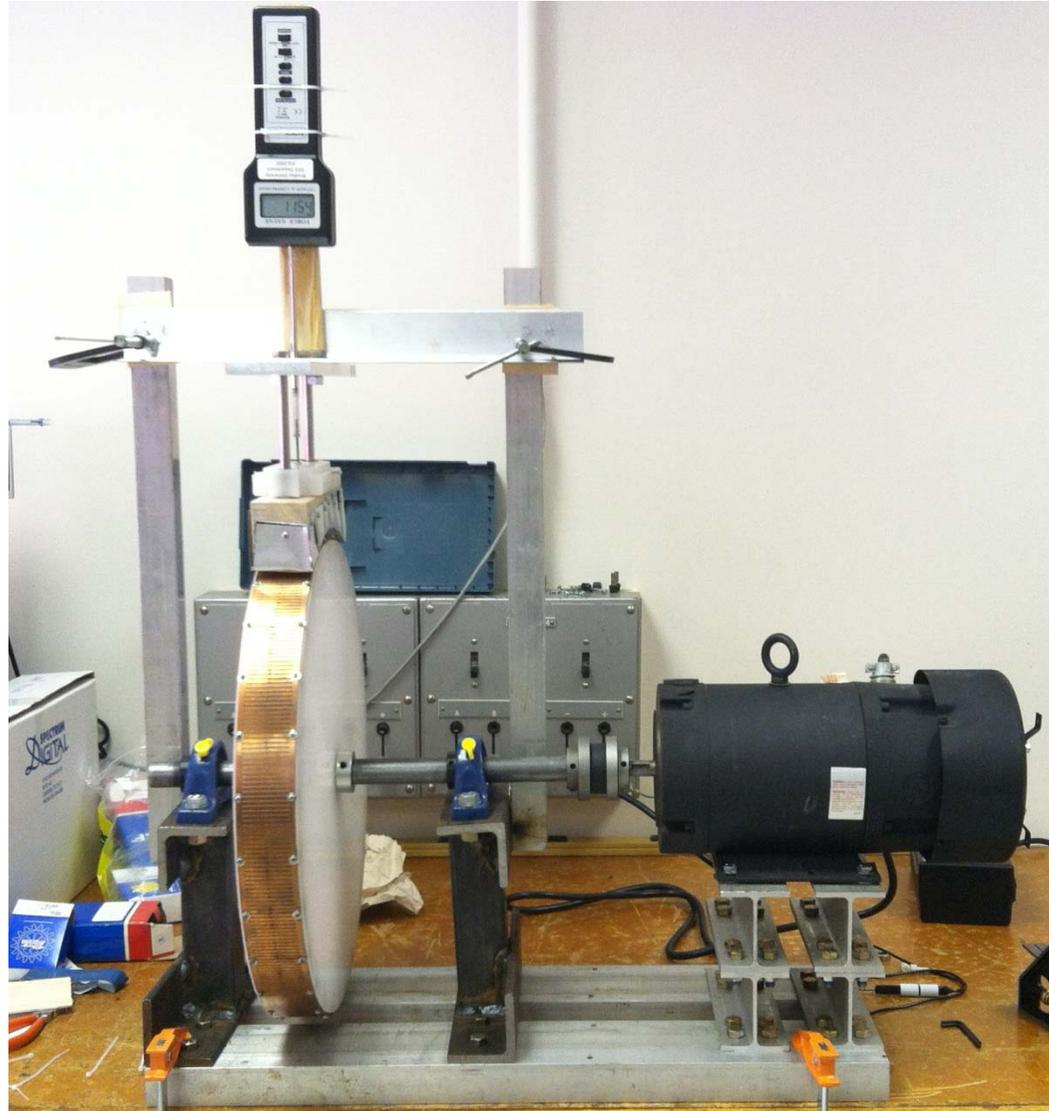


# Lab Work – Halbach Array Device

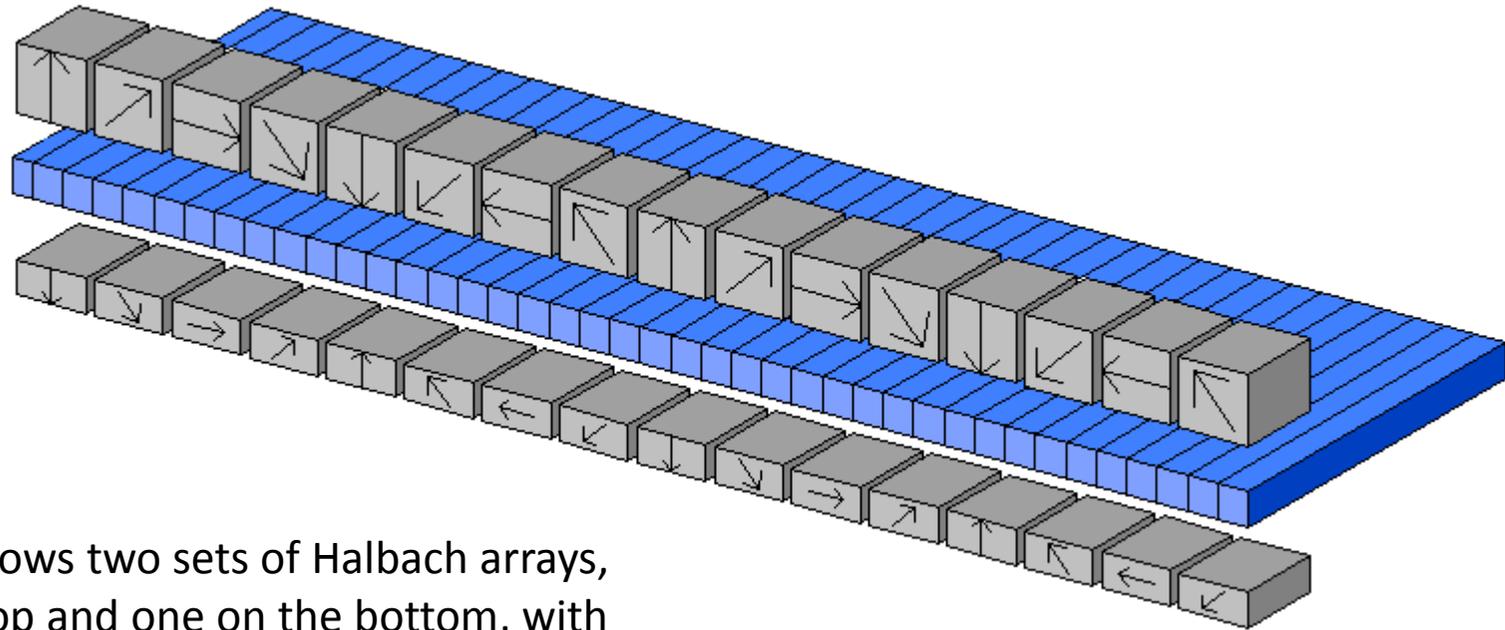
- Balsa wood structure built
- Magnets glued into balsa wood
  - Used shrink wrap and epoxy
- Aluminum covering built to ensure magnets do not eject from balsa wood



# Lab Work – Set up



# Double Halbach Array System of Inductrack II



The figure shows two sets of Halbach arrays, one on the top and one on the bottom, with the track between them. Both Halbach arrays have the same wavelength, but the upper array has a greater strength to provide a greater force upwards.

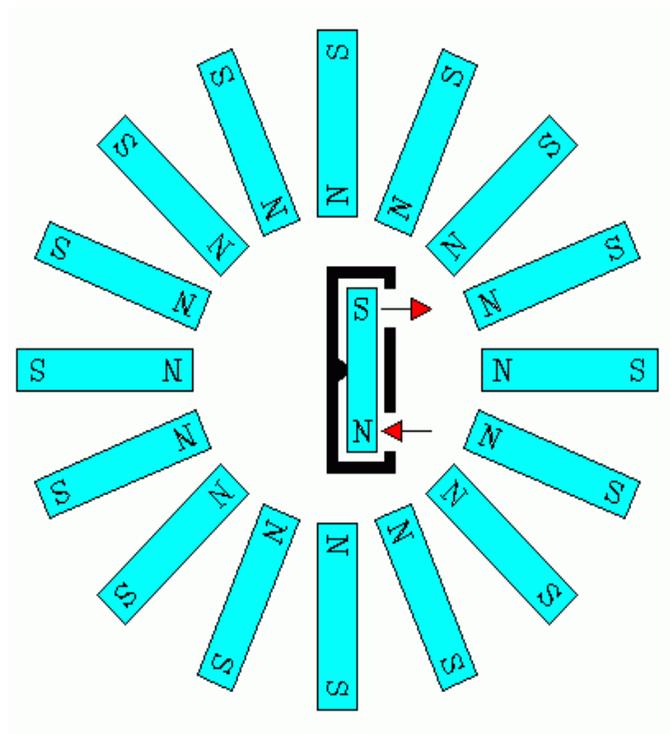
Bradley University, Department of  
Electrical and Computer Engineering

# Magnetic Propulsion

- Lorentz Force ( $I\mathbf{l} \times \mathbf{B}$ )
- Magnetic Force (magnet repulsion and attraction) ( $\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$ )

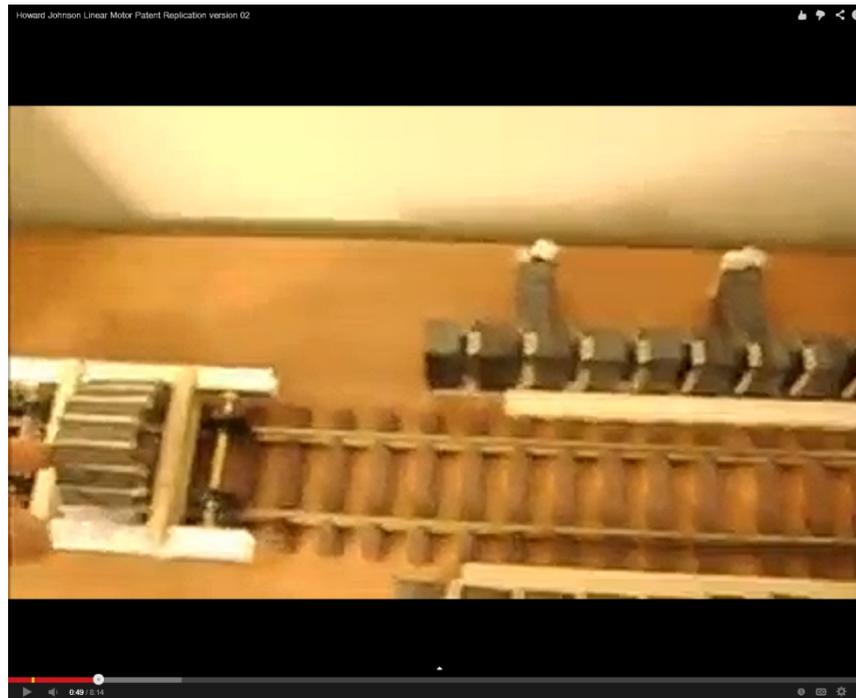
# Magnetic Force

# Magnet Motor



# Howard Johnson Linear Motor Patent Replication

- <http://www.youtube.com/watch?v=OZ-mGJaipT0>



# Rotating Magnetic Motor Design

- <http://www.youtube.com/watch?v=9hInwyQiG24>



# Magnetic Rail Gun - Halbach Array

- <http://www.youtube.com/watch?v=Ak06g3GVHuc>

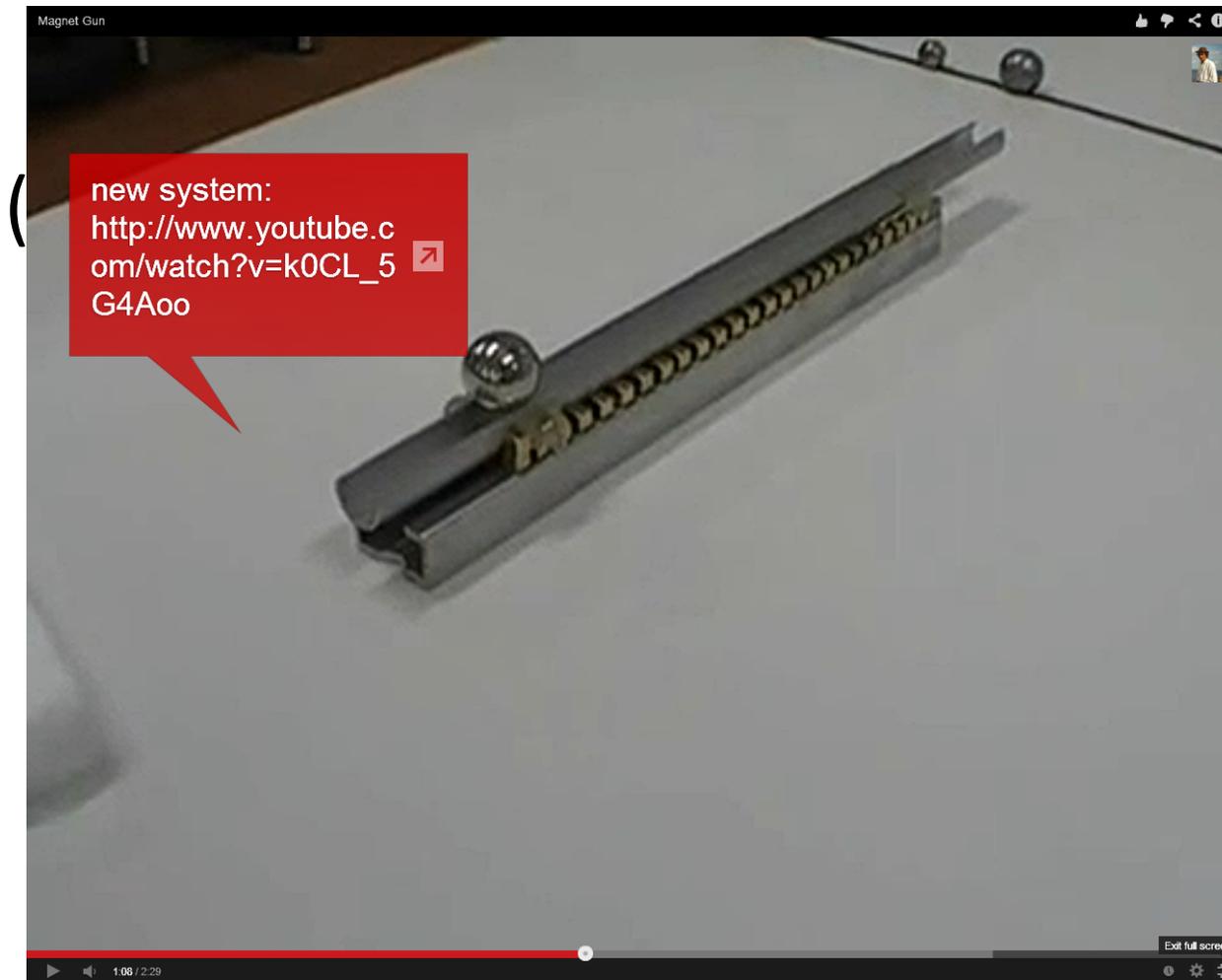


# Cool Magnet Gun - How it Works

- <http://www.youtube.com/watch?v=6um4if2KUgl>



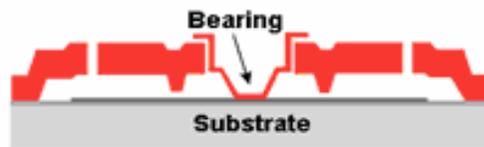
# Magnet Gun



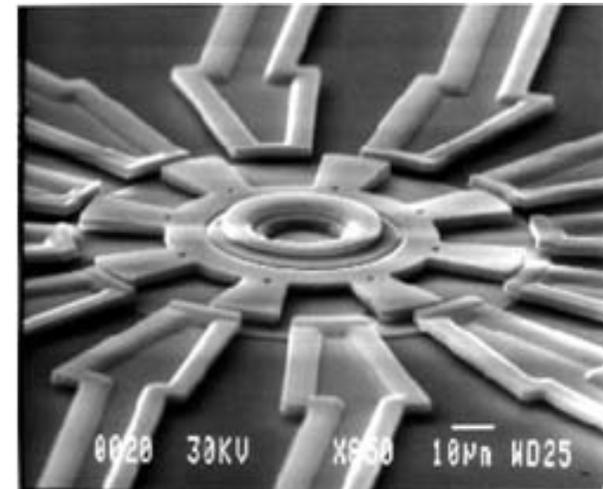
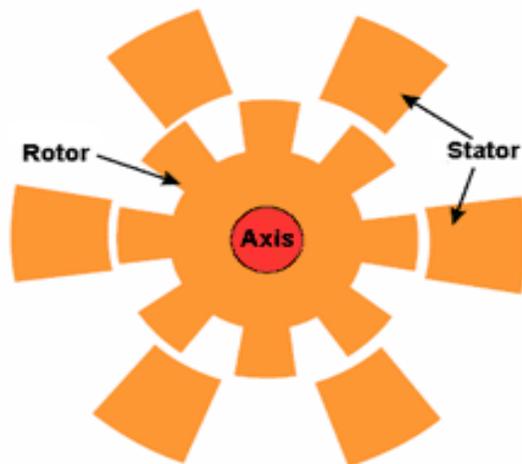
# MK2 Coil Gun - 10-Stage, 720 Rounds per Minute - Fully Automatic



# Micro-motors (Micro-ElectroMechanical Systems - MEMS)



electrostatic attraction

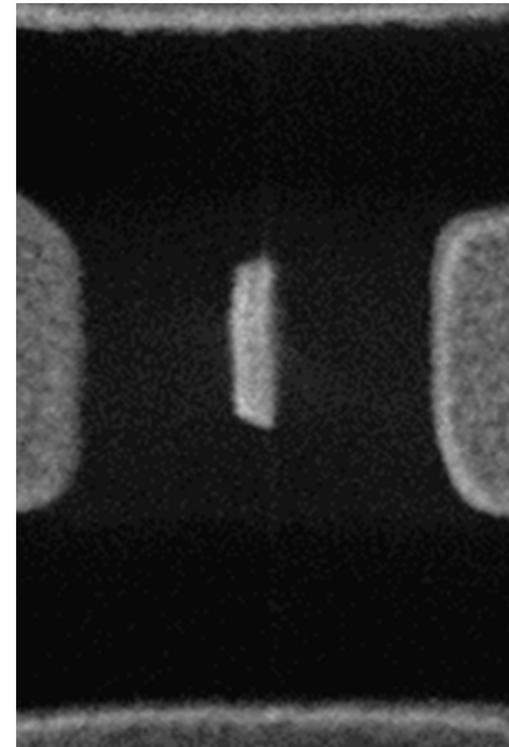


Fan L-S, Tai Y-C and R S Muller 1988  
Integrated moveable micromechanical  
structures for sensors and actuators  
IEEE Trans. Electron Devices

Fan Long-Shen, Tai Yu-Chong and Richard S.  
Muller 1989  
IC-processed electrostatic micromotors  
***Sensors Actuators 20 41-7***

# Nano-motors (Nano-ElectroMechanical Systems - NEMS)

An example is shown below. It consists of a tiny gold slab rotor, about 100 nm square, mounted on concentric carbon [nanotubes](#). The outer tube carries the rotor, driven by electrostatic electrodes, rotating around an inner tube which acts as a supporting shaft. By applying voltage pulses of up to 5 Volts between the rotor plate and stators, the position, speed and direction of rotation of the rotor can be controlled. It measures about 500 nanometers across, 300 times smaller than the diameter of a human hair.



**Electrostatic Motor**

multiwalled nanotubes

# Magnetic Force

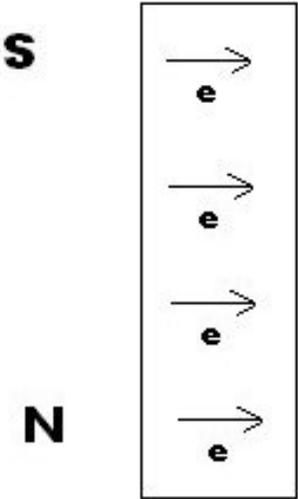
In the physically correct Ampère model, there is also a force on a magnetic dipole due to a non-uniform magnetic field, but this is due to [Lorentz forces](#) on the current loop that makes up the magnetic dipole. The force obtained in the case of a current loop model is

$$\mathbf{F} = \nabla (\mathbf{m} \cdot \mathbf{B})$$

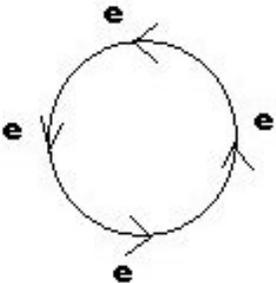
where the [gradient](#)  $\nabla$  is the change of the quantity  $\mathbf{m} \cdot \mathbf{B}$  per unit distance, and the direction is that of maximum increase of  $\mathbf{m} \cdot \mathbf{B}$ . To understand this equation, note that the [dot product](#)  $\mathbf{m} \cdot \mathbf{B} = mB\cos(\theta)$ , where  $m$  and  $B$  represent the [magnitude](#) of the  $\mathbf{m}$  and  $\mathbf{B}$  vectors and  $\theta$  is the angle between them. If  $\mathbf{m}$  is in the same direction as  $\mathbf{B}$  then the dot product is positive and the gradient points 'uphill' pulling the magnet into regions of higher B-field (more strictly larger  $\mathbf{m} \cdot \mathbf{B}$ ). This equation is strictly only valid for magnets of zero size, but is often a good approximation for not too large magnets. The magnetic force on larger magnets is determined by dividing them into smaller regions having their own  $\mathbf{m}$  then summing up the forces on each of these regions.

# Explaining Magnet using Ampere's Law

## Magnets



From top

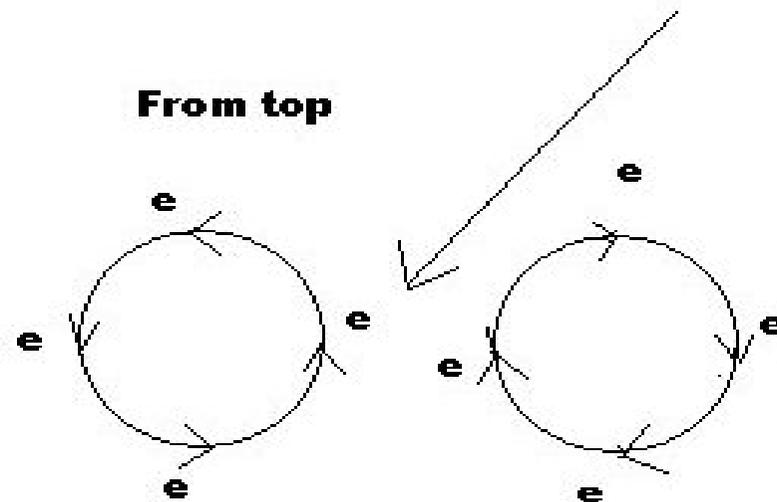
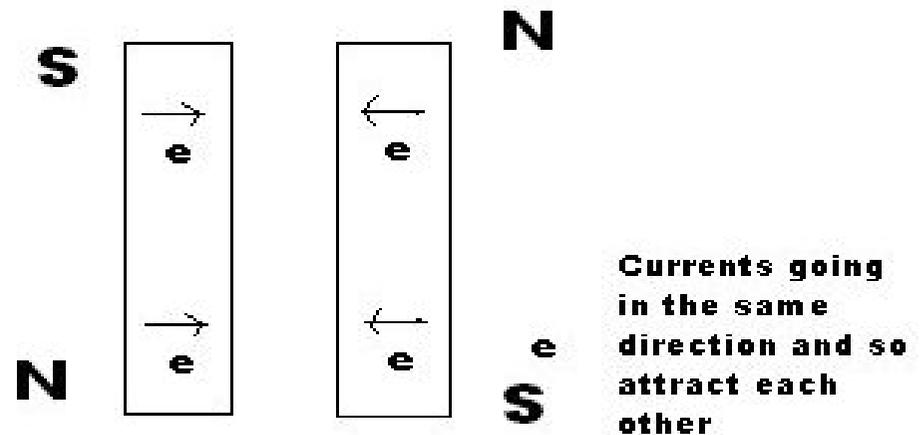


**The equivalent current is the same from the top to the bottom of the magnet**

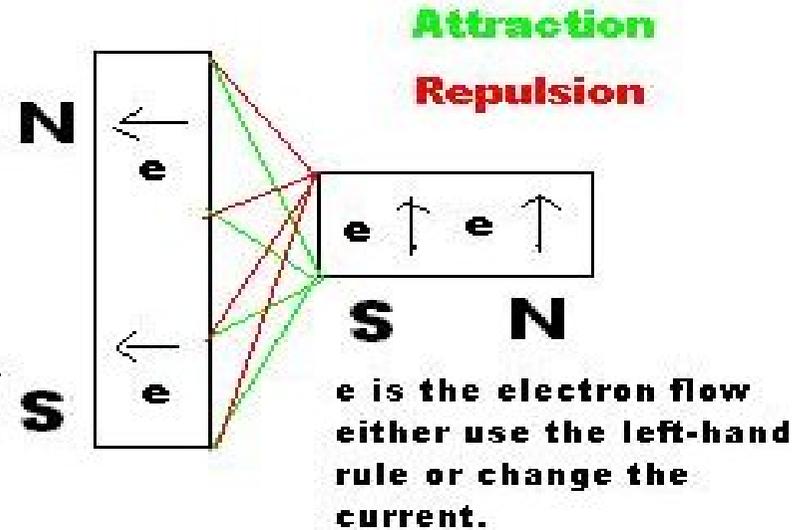
# The Attraction and Repulsion of two Magnets

## Magnets Attraction

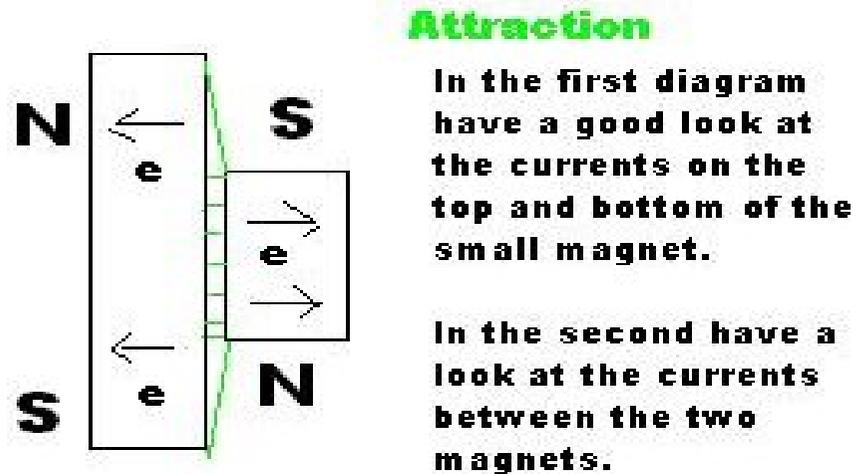
Lorentz's Law:  $F = E * q + B * v * q$



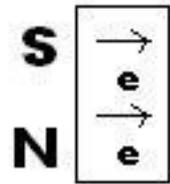
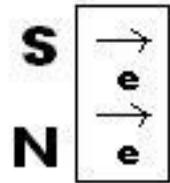
# Magnets Attraction



## Most likely outcome



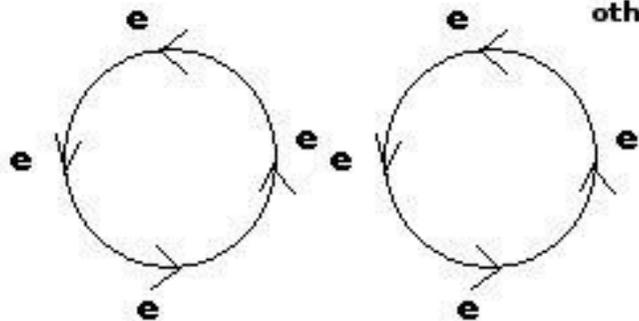
# Magnets Attraction



Currents going in the same direction and so attract each other

The two circles are one directly over the other

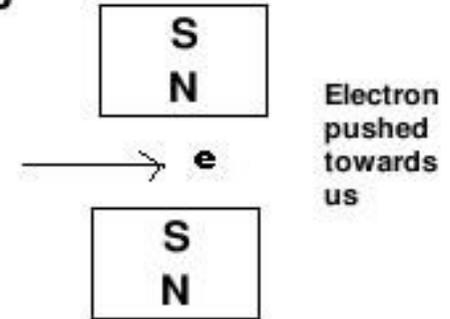
From top



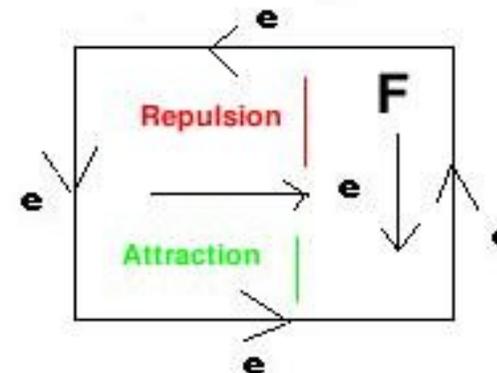
# Magnets Attraction

$$F = q \cdot E + q \cdot v \times B$$

The basis of magnetism is relativity. This gives Ampere's Law which explains magnetic fields.



From top



The magnets are simplified as squares. Both the magnets currents flow in the same direction.

# Go Hawks!

