PART II

- Some applications of fluid mechanics

Fluid Mechanics – Pressure

- Pressure = F/A
- Units: Newton's per square meter, Nm^{-2}, $kgm^{-1} s^{-2}$
- The same unit is also known as a Pascal, *Pa*, i.e. 1*Pa* = 1 Nm^{-2})
- English units: lbf/sqft, or inches of H_2O
- Also frequently used is the alternative SI unit the *bar,* where 1 bar = 10^5 Nm^{-2}
- Dimensions: M L^{-1} T^{-2}

Fluid Mechanics – Pressure

- Gauge pressure:

$$p_{gauge} = \_ gh$$

- Absolute Pressure:

$$p_{absolute} = \_ gh + p_{atmospheric}$$

- Head (h) is the vertical height of fluid for constant gravity (g):

$$h = p / \_ g$$

- When pressure is quoted in head, density (_) must also be given.

Fluid Mechanics – Specific Gravity

- Density (r): mass per unit volume.
- Units are M L^{-3}, (slug ft^{-3}, kg m^{-3})
- Specific weight (SW): wt per unit volume.
- Units are F L^{-3}, (lbf ft^{-3}, N m^{-3})
- sw = rg
- Specific gravity (s): ratio of a fluid's density to the density of water at 4° C

$$s = r/r_w$$

- r_w = 1.94 slug ft^{-3}, 1000 kg m^{-3}

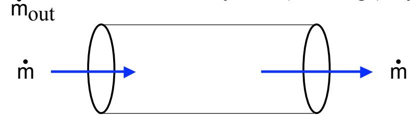
Fluid Mechanics – Continuity and Conservation of Matter

- Mass flow rate ($\dot{m}$) = Mass of fluid flowing through a control surface per unity time (kg s^{-1})
- Volume flow rate, or Q = volume of fluid flowing through a control surface per unit time (m^3 s^{-1})
- Mean flow velocity (V_m):

$$V_m = Q/A$$

Continuity and Conservation of Mass

- Flow through a pipe:
- Conservation of mass for steady state (no storage) says $\dot{m}_{in} = \dot{m}_{out}$



$$\_{1}A_1V_{m1} = \_{2}A_2V_{m2}$$

- For incompressible fluids, density does not changes ($\_{1} = \_{2}$) so $A_1V_{m1} = A_2V_{m2} = Q$

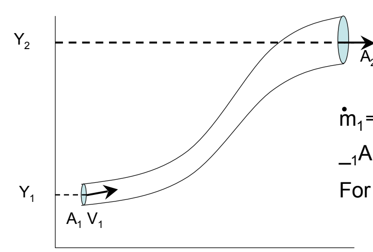
Fluid Mechanics – Continuity Equation

- The equation of continuity states that for an incompressible fluid flowing in a tube of varying cross-sectional area (A), the mass flow rate is the same everywhere in the tube:

$$\_{1}A_1V_1 = \_{2}A_2V_2$$

- Generally, the density stays constant and then it's simply the flow rate (Av) that is constant.

Bernoulli’s equation



$\dot{m}_1 = \dot{m}_2$

$\_{1}A_1V_1 = \_{2}A_2V_2$

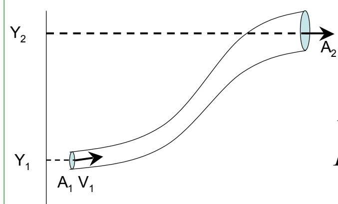
For incompressible flow

$A_1V_1 = A_2V_2$

Assume steady flow, V parallel to streamlines & no viscosity

Bernoulli Equation – energy

- Consider energy terms for steady flow:
- We write terms for KE and PE at each point



$E_i = KE_i + PE_i$

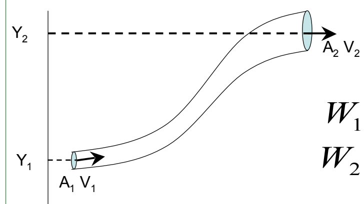
$$E_1 = \tfrac{1}{2}\dot{m}_1 V_1^2 + g\dot{m}_1 y_1$$

$$E_2 = \tfrac{1}{2}\dot{m}_2 V_2^2 + g\dot{m}_2 y_2$$

As the fluid moves, work is being done by the external forces to keep the flow moving. For steady flow, the work done must equal the change in mechanical energy.

Bernoulli Equation – work

- Consider work done on the system is Force x distance
- We write terms for force in terms of Pressure and area



$W_i = F_i V_i$ dt $= P_i V_i A_i$ dt

Note $V_i A_i$ dt = $\dot{m}_i$/_i

$$W_1 = P_1 \dot{m}_1 / \rho_1$$

$$W_2 = -P_2 \dot{m}_2 / \rho_2$$

Now we set up an energy balance on the system. Conservation of energy requires that the change in energy equals the work done on the system.

Bernoulli equation- energy balance

Energy accumulation = _Energy – Total work

$0 = (E_2 - E_1) - (W_1 + W_2)$ i.e. no accumulation at steady state

Or $W_1 + W_2 = E_2 - E_1$ Subs terms gives:

$$\frac{P_1 \dot{m}_1}{\rho_1} - \frac{P_2 \dot{m}_2}{\rho_2} = (\tfrac{1}{2}\dot{m}_2 V_2^2 + g\dot{m}_2 y_2) - (\tfrac{1}{2}\dot{m}_1 V_1^2 + g\dot{m}_1 y_1)$$

$$\frac{P_1 \dot{m}_1}{\rho_1} + \tfrac{1}{2}\dot{m}_1 V_1^2 + g\dot{m}_1 y_1 = \frac{P_2 \dot{m}_2}{\rho_2} + \tfrac{1}{2}\dot{m}_2 V_2^2 + g\dot{m}_2 y_2$$

For incompressible steady flow $\dot{m}_1 = \dot{m}_2$ and $\rho_1 = \rho_2 = \rho$

$$P_1 + \tfrac{1}{2}\rho V_1^2 + g\rho y_1 = P_2 + \tfrac{1}{2}\rho V_2^2 + g\rho y_2$$

Forms of the Bernoulli equation

- Most common forms:

$$P_1 + \tfrac{1}{2}\rho V_1^2 = P_2 + \tfrac{1}{2}\rho V_2^2 + g\rho\Delta h$$

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + \Delta P_{ht}$$

The above forms assume no losses within the volume…

If losses occur we can write:

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + losses + \Delta P_{ht}$$

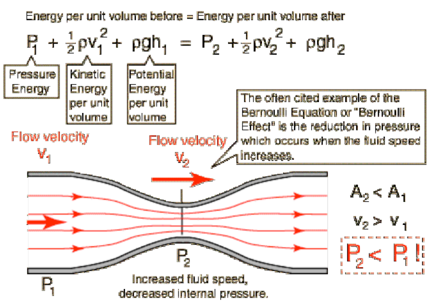
And if we can ignore changes in height:

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + losses$$

Key eqn

Application of Bernoulli Equation

- Daniel Bernoulli developed the most important equation in fluid hydraulics in 1738. this equation assumes constant density, irrotational flow, and velocity is derived from velocity potential:



Bernoulli Equation for a venturi

- A venturi measures flow rate in a duct using a pressure difference. Starting with the Bernoulli eqn from before:

$$P_{S1} + P_{V1} = P_{S2} + P_{V2} + losses + \Delta P_{ht}$$

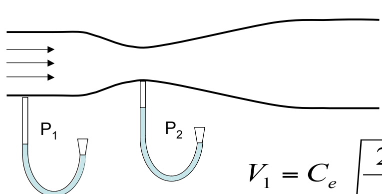
- Because there is no change in height and a well designed venturi will have small losses (<~2%) We can simplify this to:

$$P_{S1} + P_{S2} = P_{V2} + P_{V1} \quad or - \Delta P_S = \Delta P_V$$

- Applying the continuity condition (incompressible flow) to get:

$$V_1 = \sqrt{\frac{2(P_{S1} - P_{S2})}{\rho\left(1 - A_2^2 / A_1^2\right)}}$$

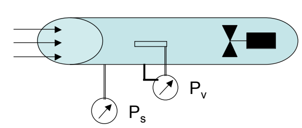
Venturi Meter



$$V_1 = C_e \sqrt{\frac{2(P_{S1} - P_{S2})}{\rho\left(1 - A_2^2 / A_1^2\right)}}$$

- Discharge Coefficient C_e corrects for losses = $f(R_e)$

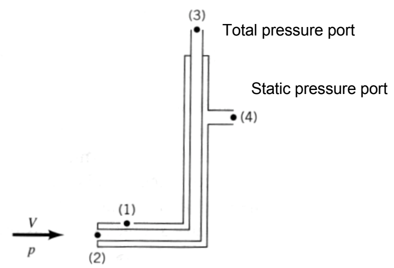
Duct pressures



- $P_T = P_S + P_V$ ← i.e. an energy balance
- Static pressure – potential energy
- Velocity pressure – KE
- Total pressure – total energy
- Recall KE is _ mV^2 so KE term is proportional to V^2
- At NTP, $P_V = (V/4005)^2$ for V in ft/min, P_V in inches H_2O
  - Or $V = 4005\sqrt{P_V}$ {at non std conditions, $V = 4005\sqrt{(P_V/d)}$}

Pitot tube

The static and Pitot tube are often combined into the one-piece Pitot-static tube.



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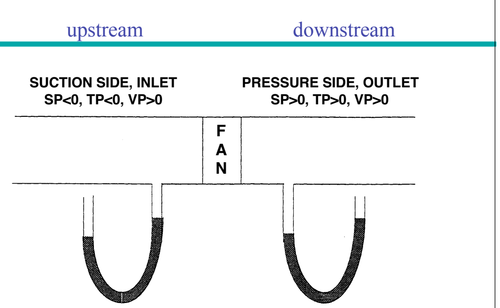
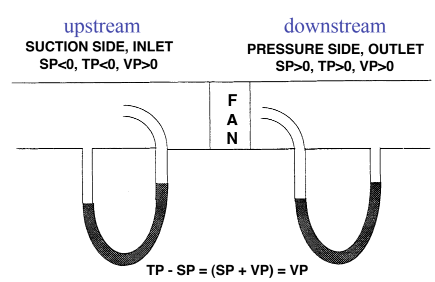
Pressure, velocities & flow rates

- Units of pressure = “wg
  - The height of a liquid column that would be supported by that pressure
  - 406”wg = 1 atm
  - Traditional units not used since small pressure differentials are involved

Static pressure measurement



VP Measurement



Velocity & VP

- Velocity pressure can also be determined by measuring velocity at a given point and using the following relationship:

$$V = 4005\sqrt{VP}$$

V = velocity in fpm
VP = velocity pressure in “wg

Density correction

- Density of standard air = 0.075 lb/ft3
  - Air density affected by: moisture, temperature & altitude above sea level
  - Roughly, density corrections are needed, when:
    - Moisture exceeds 0.02 lbs water/lb of air
    - Air temp outside of 40 – 100F range
    - Altitude exceeds +1000 ft relative to sea level

Density correction

$$VP = \left(\frac{V}{4005}\right)^2 \left(\frac{Density_{actual}}{0.075 lb/ft^3}\right)$$

$$Density_{actual} = \frac{0.075 lb}{ft^3} x \frac{530F}{(460+t)F} x \frac{Bar.pressure}{29.92}$$

$Density_{actual}$ = air density in lb/ft3
T = temperature in ^oF
Bar. Pressure = pressure in inches of mercury

Pressures in LEV

- Total pressure (TP)
  - sum of static pressure & velocity pressure
  - - upstream of fan
  - + downstream of fan

TP = SP + VP