EARTH AND SPACE SCIENCE **Autumn 2017**

431 *PRINCIPLES OF GLACIOLOGY* 4 Credits, SLN 14841

505 *THE CRYOSPHERE* 4 Credits, SLN 14862

Homework Week 8 – Isotope Fractionation

On Bean World, *Bean Ocean* contains N_G=1000 Garbanzo beans (analog of ^{18}O on Earth) and N_P =1000 Pinto beans (analog of ^{16}O on Earth) in every cubic meter. The ratio of Garbanzos to Pintos in Bean Ocean is

$$R_{ocean} = \frac{N_G}{N_P} = 1 \qquad (1)$$

Climate processes on Bean World can lift some of these beans out of the Ocean (analog of evaporation on Earth). The *Bean Vapor* moves poleward, cooling as it goes. As they cool, beans have a progressively harder time staying aloft, and some of them fall back to the surface (analog of condensation/precipitation) for every cooling step.

The Garbanzo beans are harder to lift out of the ocean (evaporate) and they tend to fall back to the surface more easily (condense/precipitate), compared to the Pinto beans (because they are larger and heavier).

Because of these differences, the ratio R_{sample} of Garbanzo beans to Pinto beans in samples of Beans from different places on Bean World can vary, and Bean Chemists on Bean World have found it convenient to describe the composition of a sample by a *delta value* δB defined by

$$\delta B = 1000 \times \frac{R_{sample} - R_{ref}}{R_{ref}} \qquad (2)$$

δB measures the composition of a sample relative to a standard *Reference Sample* of Beans. For convenience, the reference ratio R_{ref} is taken to be the initial ratio of Garbanzos to Pintos (R_{ocean} in Bean Ocean). Negative δB means that a sample is depleted in Garbanzos relative to the initial Bean Ocean. Positive δB means that a sample is enriched in Garbanzos relative to the initial Bean Ocean.

In the order in which you will encounter them, here are the key points of our current scientific understanding of science on Bean World.

1. **Fractionation on Evaporation**

Pintos are preferentially evaporated from Bean Ocean. Garbanzos prefer to stay in Bean Ocean. R_{vapor} is the ratio of Garbanzos to Pintos in the Bean Vapor that forms by evaporation from any Bean Liquid that has a Garbanzo/Pinto ratio of R_{liq}. As a special case, the liquid could be Bean Ocean. Then,

$$R_{liq} = \alpha R_{vapor}, \text{ and } R_{vap} = \frac{1}{\alpha} R_{ocean}$$

where α is the *fractionation coefficient*. On Bean World, $\alpha = 2$ (On Earth, the fractionation coefficient for isotopes of water is more like 1.01. But "2" is an easier number to work with, so it is fortunate for us that $\alpha = 2$ on Bean World.)

2. **Weather on Bean World**

For each 10ºC that the Bean Vapor cools, a fraction γ (gamma) of its beans "condense" and fall back to the surface of Bean World. γ describes the physics of rain formation on Bean World. On Bean World, $\gamma = 0.2$.

3. Fractionation on Condensation

Garbanzos are preferentially condensed and precipitated out of Bean Vapors on Bean World, i.e. the ratio of Garbanzos to Pintos in the "rain" is always greater than the ratio in the "vapor" from which it condensed. On condensation:

$$R_{precip} = \alpha R_{vapor}$$

where the fractionation coefficient α is the same as in Rule (1). This *Bean Rain* immediately falls to the surface of Bean World, leaving fewer Garbanzos and Pintos in the Vapor.

BEAN WORLD Exercise

(a) What is the initial δB value of Bean Ocean?

(b) In a *Bean Age Cycle*, 300 beans are "evaporated" from Bean Ocean. i.e. N_{vapor}= 300.

- What is the ratio R_{vapor} of Garbanzos to Pintos in this vapor?
- Translate this into actual numbers of each type of bean in the vapor using Rule 1.
- Enter these numbers in columns 3, 4 and 5 in the Table below.

(c) Follow the Bean Vapor through several cooling steps of 10ºC each. At each step:

- Find the number N_{precip} of beans in this batch of *Bean Rain* (column 7) based on the number of beans N_{vapor} (column 2) currently in the *Bean Vapor*, using Rule 2.
- Find the ratio R_{precip} of Garbanzos to Pintos in this precipitation batch (column 8), based on the current ratio R_{vapor} in the Bean Vapor (column 5) using Rule 3.
- Find the number of Garbanzos and the number of Pintos (9 & 10) that come closest to the required ratio R_{precip}(8), and when added together, give the correct sum of N_{precip} beans (7).

- Remove these Garbanzos and Pintos from the vapor as Bean Rain, and save them separately.
- Calculate the δB value (11) for the Bean Rain that just fell.
- Update the number of Garbanzos and Pintos and total number of Beans (2, 3 & 4) remaining in the vapor for the next step.
- Update the ratio R_{vapor} (5) of Garbanzos to Pintos remaining in the Bean Vapor.
- Calculate the new δB value (6) of the remaining vapor.

1	2	3	4	5	6	7	8	9	10	11
		VAPOR						PRECIP		
T_{vapor} (°C)	N_{vapor} beans	Garbanzo beans	Pinto beans	R_{vapor}	δB vapor	N_{precip} beans	R_{precip}	Garbanzo beans	Pinto beans	δB precip
10	300	100	200	0.5		60	1.0			
0	240									
-10										
-20										
-30										
-40										
-50										

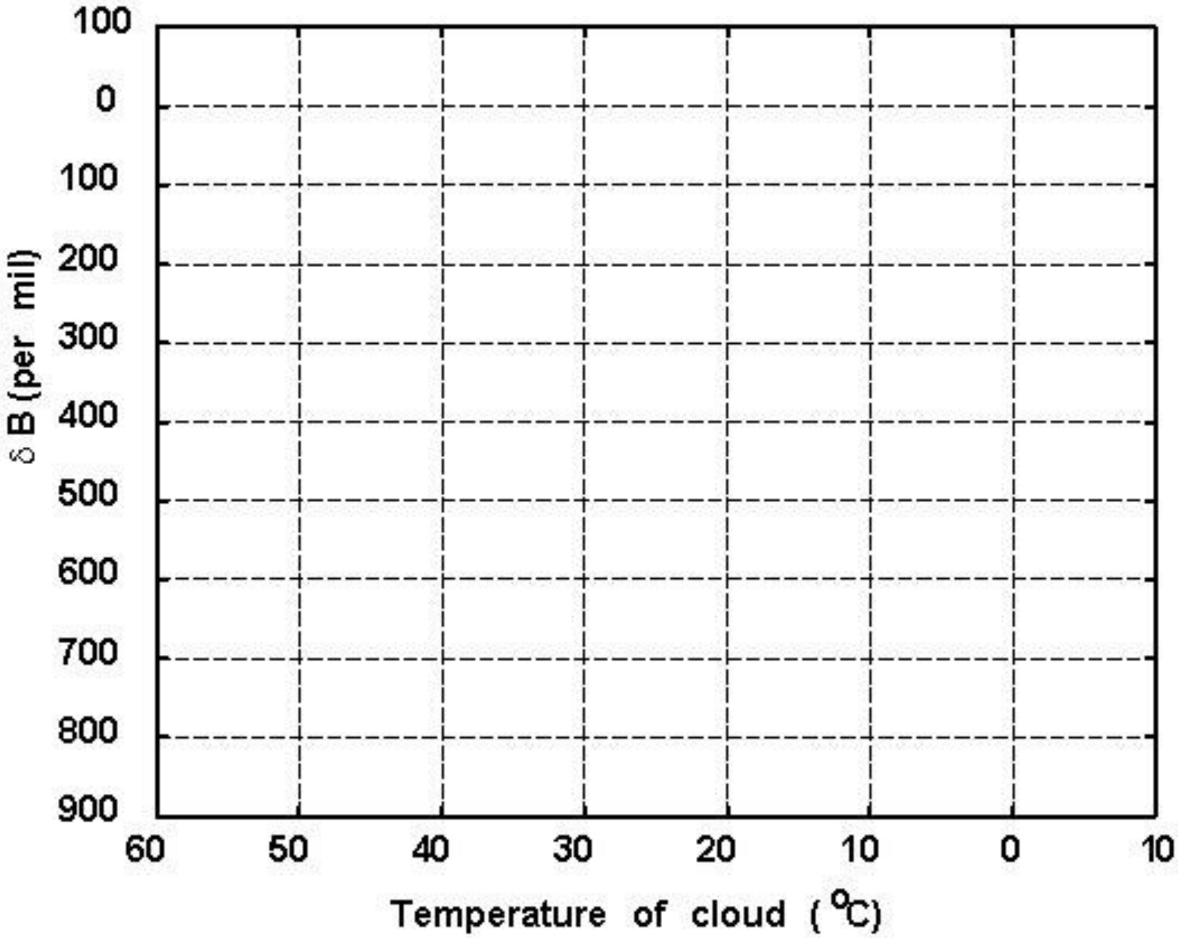
(d) Draw graphs showing the δB Bean composition of the Bean Vapor and the δB Bean composition of the Bean Rain as a function of the temperature in the Bean Vapor. Does the δB value of the Bean Rain appear to be related to the temperature in the Bean Vapor when the Bean Rain formed?

(e) Now suppose that all the beans that precipitated at temperatures above -30ºC ended up back in Bean Ocean, while those beans that precipitated at colder temperatures (-30ºC to -59ºC) stayed as *Ice Beans* to form a resurgent *Bean Glacier* that did not "melt".
Meanwhile, after reaching -59ºC, the remaining Bean Vapor continued out over Bean Ocean, all the remaining beans precipitated, and eventually returned to Bean Ocean.

Because some of the Beans are now sequestered on the land, this *Bean Cycle* also illustrates the transition to a *Bean Age* in Bean World.

- What is the δB value of Bean Glacier in this Bean Age? To find this you need to find the ratio of Garbanzos to Pintos in the glacier.
- What is the δB value of Bean Ocean in this Bean Age? To find this, you need to find the total number of Garbanzos and Pintos that are still in Bean Ocean after accounting for the "missing" Beans that are now in Bean Glacier.

(f) Mr. Bean was last reported to be outdoors eating plates of beans with $\delta B = -200$ per mil. What sort of clothing do you think he was wearing at the time?


100
0
100
200
300
400
500
600
700
800
900
δ B (per mil)
60
50
40
30
20
10
0
10
Temperature of cloud (°C)