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| **EARTH AND SPACE SCIENCE****431** *PRINCIPLES OF GLACIOLOGY***505** *THE CRYOSPHERE* | **Autumn 2018**4 Credits, SLN 148554 Credits, SLN 14871 |
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| **Homework Week 8 – Isotope Fractionation** |

On Bean World, *Bean Ocean* contains *NG*=1000 Garbanzo beans (analog of 18O on Earth) and *NP* =1000 Pinto beans (analog of 16O on Earth) in every cubic meter. The ratio of Garbanzos to Pintos in Bean Ocean is

  (1)

Climate processes on Bean World can lift some of these beans out of the Ocean (analog of evaporation on Earth). The *Bean Vapor* moves poleward, cooling as it goes. As they cool, beans have a progressively harder time staying aloft, and some of them fall back to the surface (analog of condensation/precipitation) for every cooling step.

The Garbanzo beans are harder to lift out of the ocean (evaporate) and they tend to fall back to the surface more easily (condense/precipitate), compared to the Pinto beans (because they are larger and heavier).

Because of these differences, the ratio *Rsample* of Garbanzo beans to Pinto beans in samples of Beans from different places on Bean World can vary, and Bean Chemists on Bean World have found it convenient to describe the composition of a sample by a *delta value*  defined by

  (2)

measures the composition of a sample relative to a standard *Reference Sample* of Beans. For convenience, the reference ratio *Rref* is taken to be the initial ratio of Garbanzos to Pintos (*Rocean* in Bean Ocean). Negative  means that a sample is depleted in Garbanzos relative to the initial Bean Ocean. Positive  means that a sample is enriched in Garbanzos relative to the initial Bean Ocean.

In the order in which you will encounter them, here are the key points of our current scientific understanding of science on Bean World.

1. **Fractionation on Evaporation**

Pintos are preferentially evaporated from Bean Ocean. Garbanzos prefer to stay in Bean Ocean. *Rvapor* is the ratio of Garbanzos to Pintos in the Bean Vapor that forms by evaporation from any Bean Liquid that has a Garbanzo/Pinto ratio of *Rliq*. As a special case, the liquid could be Bean Ocean. Then,

, and 

 where  is the *fractionation coefficient*. On Bean World,  (On Earth, the fractionation coefficient for isotopes of water is more like 1.01. But “2” is an easier number to work with, so it is fortunate for us that  on Bean World.)

1. **Weather on Bean World**

 For each 10oC that the Bean Vapor cools, a fraction  (gamma) of its beans "condense" and fall back to the surface of Bean World.  describes the physics of rain formation on Bean World. On Bean World, .

1. **Fractionation on Condensation**

 Garbanzos are preferentially condensed and precipitated out of Bean Vapors on Bean World, i.e. the ratio of Garbanzos to Pintos in the “rain” is always greater than the ratio in the “vapor” from which it condensed. On condensation:



 where the fractionation coefficient  is the same as in Rule (1). This *Bean Rain* immediately falls to the surface of Bean World, leaving fewer Garbanzos and Pintos in the Vapor.

**BEAN WORLD Exercise**

(a) [2 points] What is the initial  value of Bean Ocean?

(b) [2 points] In a *Bean Age Cycle*, 300 beans are "evaporated" from Bean Ocean. i.e. *Nvapor*= 300.

* What is the ratio *Rvapor* of Garbanzos to Pintos in this vapor?
* Translate this into actual numbers of each type of bean in the vapor using Rule 1.
* Enter these numbers in columns 3, 4 and 5 in the Table below.

(c) [6 points] Follow the Bean Vapor through several cooling steps of 10oC each. At each step:

* Find the number *Nprecip* of beans in this batch of *Bean Rain* (column 7) based on the number of beans *Nvapor* (column 2) currently in the *Bean Vapor*, using Rule 2.
* Find the ratio *Rprecip* of Garbanzos to Pintos in this precipitation batch (column 8), based on the current ratio *Rvapor* in the Bean Vapor (column 5) using Rule 3.
* Find the number of Garbanzos and the number of Pintos (9 & 10) that come closest to the required ratio *Rprecip*(8), and when added together, give the correct sum of *Nprecip* beans (7).
* Remove these Garbanzos and Pintos from the vapor as Bean Rain, and save them separately.
* Calculate the value (11) for the Bean Rain that just fell.
* Update the number of Garbanzos and Pintos and total number of Beans (2, 3 & 4) remaining in the vapor for the next step.
* Update the ratio *Rvapor* (5) of Garbanzos to Pintos remaining in the Bean Vapor.
* Calculate the new value (6) of the remaining vapor.

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| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|  |  | VAPOR |  |  |  |  |  | PRECIP |  |  |
| *Tvapor*(oC) | *Nvapor*beans | Garbanzo beans | Pintobeans | *Rvapor* | vapor | *Nprecip*beans | *Rprecip* | Garbanzo beans | Pinto beans | precip |
| 10 | 300 | 100 | 200 | 0.5 |  | 60 | 1.0 |  |  |  |
| 0 | 240 |  |  |  |  |  |  |  |  |  |
| -10 |  |  |  |  |  |  |  |  |  |  |
| -20 |  |  |  |  |  |  |  |  |  |  |
| -30 |  |  |  |  |  |  |  |  |  |  |
| -40 |  |  |  |  |  |  |  |  |  |  |
| -50 |  |  |  |  |  |  |  |  |  |  |

 (d) [2 points] Draw graphs showing the Bean composition of the Bean Vapor and the Bean composition of the Bean Rain as a function of the temperature in the Bean Vapor. Does the value of the Bean Rain appear to be related to the temperature in the Bean Vapor when the Bean Rain formed?

 (e) [4 points] Now suppose that all the beans that precipitated at temperatures above -30oC ended up back in Bean Ocean, while those beans that precipitated at colder temperatures (-30oC to -59oC) stayed as *Ice Beans* to form a resurgent *Bean Glacier* that did not “melt”.

Meanwhile, after reaching -59oC, the remaining Bean Vapor continued out over Bean Ocean, all the remaining beans precipitated, and eventually returned to Bean Ocean.

Because some of the Beans are now sequestered on the land, this *Bean Cycle* also illustrates the transition to a *Bean Age* in Bean World.

* What is thevalue of Bean Glacier in this Bean Age? To find this you need to find the ratio of Garbanzos to Pintos in the glacier.
* What is the value of Bean Ocean in this Bean Age? To find this, you need to find the total number of Garbanzos and Pintos that are still in Bean Ocean after accounting for the “missing” Beans that are now in Bean Glacier.

(f) [2 points] Mr. Bean was last reported to be outdoors eating plates of beans with  per mil. What sort of clothing do you think he was wearing at the time?

