EARTH AND SPACE SCIENCES **Autumn 2018**

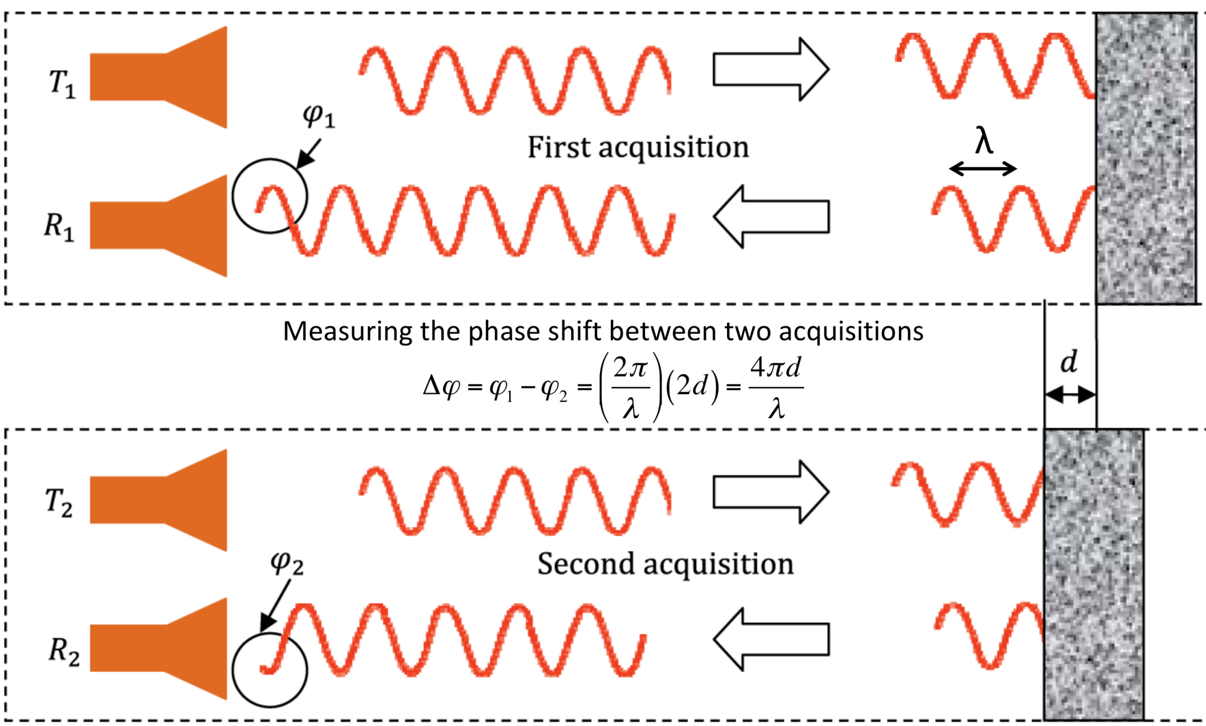
431 *PRINCIPLES OF GLACIOLOGY* 4 Credits, SLN 14855

505 *THE CRYOSPHERE* 4 Credits, SLN 14871

Measuring Changes in Ice Flow Speeds

Ice flow speeds are commonly measured using a technique called "Interferometric Synthetic Aperture Radar" (InSAR). This is an "active" imaging technique – the instrument generates and transmits a wave, and records information about the wave once it returns after being reflected off of the Earth's surface. InSAR systems are typically "FMCW Radar" (Frequency Modulated Continuous Wave Radar), which transmit waves over a range of frequencies, precisely measuring the phase of the wave as it departs and returns to the radar. Using frequency information from the wave, these instruments are pretty good at measuring distance (often referred to as range), but using phase information, they are incredibly good at measuring *change* in distance. Based on this description, sketch two scenes – one with a radar imaging a target sitting still, and one imaging the same target after it has moved slightly away from the radar. Pay specific attention to the phase of your returning wave:

An example is shown here for moving towards the target. Not that the phase has changed as indicated by the equation for a shift of an arbitrary distance d.



Instrument measures Δφ (in radians). We need to solve for d to infer change detection.

Instrument Characteristics – Terrasar X:
Center Frequency: 9.65 GHz (10^9 [1/s])
Bandwidth: 150 MHz (10^6 [1/s])

The "range resolution" (R) of FMCW radar, defining how precisely it can measure distance, is a function of the instrument Bandwidth (BW) according to the following equation:

$$R = \frac{c}{2 * BW_{Hz}}$$

In this case, c is the speed of light (~3×10^8 m/s). When a target is moving, the change in distance between the radar and the target (Δz) results in a change in phase ($\Delta\varphi$). That change in phase can be converted back to a distance using the wavelength (λ in m; remember $c = f\lambda$, where f is the frequency of the wave and c is the speed of the wave) of the instrument:

$$\Delta z = \frac{\lambda\Delta\varphi}{4\pi}$$

Measuring the change in phase for a reflection allows you to compute the change in distance. Assuming Terrasar-X has an ability to measure changes in phase down to 1°, show that it can detect changes in position that are below the range resolution.

Here, we simply plug and chug:

$$R = \frac{c}{2 * BW_{Hz}} = \frac{3 \times 10^8 \text{ m/s}}{2 \times 150 \times 10^6 \text{ 1/s}} = 1 \text{ m}$$

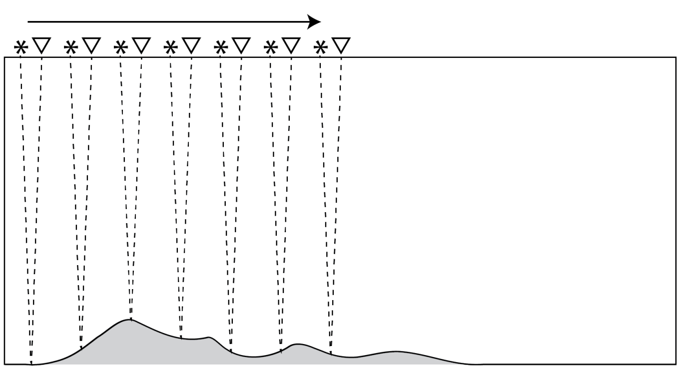
From interferometric phase, we can resolve a phase difference of 1°, or (keeping in mind we can relate frequency to wavelength using $c = f\lambda$:

$$\Delta z = \frac{\lambda\Delta\varphi}{4\pi} = \frac{\left(\frac{3 \times 10^8 \text{ m/s}}{9.65 \times 10^9 \text{ 1/s}}\right) \times 1° \times \frac{\pi}{180°}}{4\pi} = 0.043 \text{ mm}$$

A difference in resolution of a few orders of magnitude is theoretically possible; in reality, we are probably talking about resolution on the order of millimeters once confounding effects of atmosphere and topography are considered.

Measuring Ice Thickness and Subglacial Material Properties

Ice thickness is most easily measured using wave reflection techniques, either electromagnetic waves or acoustic waves. Ice is mostly transparent to light waves in the radio-frequency range, so plane-based radar systems are most common for measuring ice thickness. These are typically "pulse radar" systems – they transmit a very brief pulse of energy at a specific frequency, and wait for the pulse to reflect and return to the system. These instruments are designed to measure time very precisely – with knowledge of the speed of the wave in different media, and the amount of time it took for the reflection to return to the instrument, you can compute the distance. Sketch the physical set-up for a radar transmitting a wave from as airplane, through the air and into an ice sheet. Now, assume it has emitted a pulse with a single cycle of a sine wave - plot a qualitative graph of detected wave energy versus time after transmission for your physical set up:



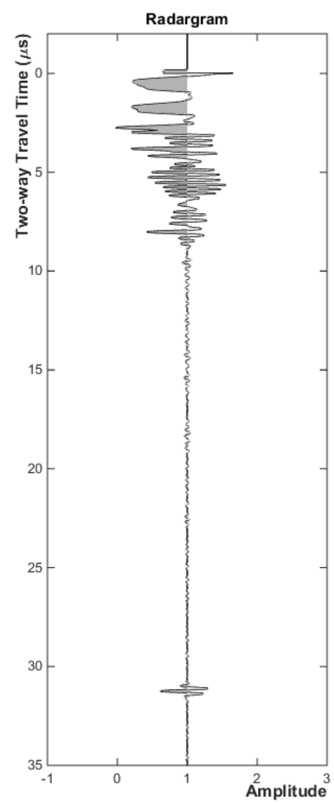


Figure: (a) A plane or snow mobile would travel near or at the ice surface and the wave would be transmitted and received at near normal incidence as shown above (here source is the star and receiver is the triangle). (b) The return pulse would look like that shown in the right (b) with many reflections near the surface that become attenuated and then some time later a large reflection at the ice-bed interface due to the large difference in electrical properties between ice and rock.

The reflection of an electromagnetic wave depends on the *contrast* in material properties at the interface. For electromagnetic waves, a simple equation for the reflection coefficient is:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}$$

Where ε_2 is the dielectric permittivity of the underlying material and ε_1 is the dielectric permittivity for the overlying material. For acoustic waves, the reflection coefficient is:

$$R = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}$$

Where rho is the density and c is the wave speed. Given the following property values, do you think radar or acoustic waves are better at detecting water? Why?

	ρ	c	ε
Ice	917 kg/m^3	3810 m/s	3.2
Water	1000 kg/m^3	1498 m/s	80
Rock	2450 kg/m^3	3750 m/s	12

For radar (electromagnetic) waves for water:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} = \frac{80 - 3.2}{80 + 3.2} = 0.92$$

As these properties often vary over orders of magnitude, we frequently use a log scale to describe the power (square of the amplitude) reflectivity. Here that gives

$$P_R = 10 \times \log_{10}(R^2) = -0.7 \text{ dB}$$

Now for seismic (sound) waves:

$$R = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} = \frac{(1000 \times 1498) - (917 \times 3810)}{(1000 \times 1498) + (917 \times 3810)} = -0.4$$

And power reflectivity would be:

$$P_R = 10 \times \log_{10}(R^2) = -8 \text{ dB}$$

For an ice-rock interface, we would have for radar waves:

$$R = \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1} = \frac{12 - 3.2}{12 + 3.2} = 0.57$$
$$P_R = 10 \times \log_{10}(R^2) = -5 \text{ dB}$$

And for seismic waves:

$$R = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1} = \frac{(2450 \times 3750) - (917 \times 3810)}{(2450 \times 3750) + (917 \times 3810)} = 0.45$$

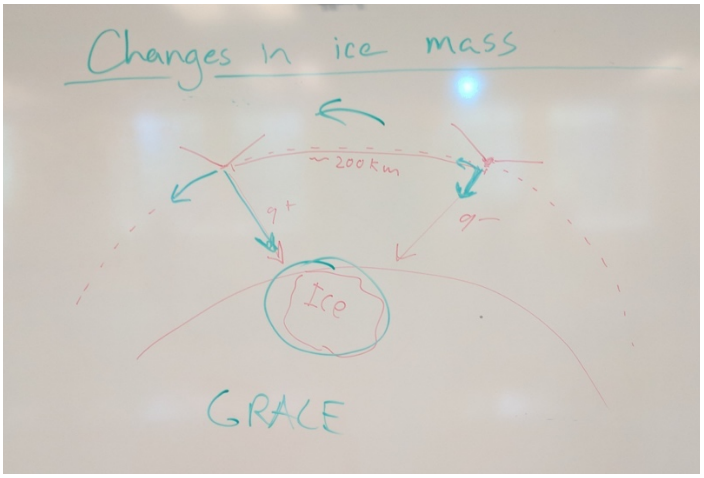
$$P_R = 10 \times \log_{10}(R^2) = -7 \text{ dB}$$

Thus, the difference in power-returned is greater for radar waves, but the switch in polarity (note the reflection changes from + to -) is extremely diagnostic in seismic data of the presence of water. No other change in likely subglacial materials can cause this.

Measuring Changes in Ice Mass

Earth is not a perfect sphere of constant density, which gives Earth a complex gravity field. One way of measuring the mass distribution of the Earth is by measuring (or estimating) the force due to gravity experienced at different locations in space. By examining changes in this force through time, it is possible to map out changes in the mass distribution on the surface of the Earth.

In 2002, NASA launched a pair of satellites called the Gravity Recovery and Climate Experiment (GRACE), which were designed to map out changes in the gravity field. These satellites have a shared orbit, with one travelling ~200km behind the other. Sketch the physical setup of this satellite constellation over Earth's surface. At Earth's north pole, add an ice sheet (represented by a sphere on Earth's surface):



The gravitational force acting on the satellites is the fundamental control on their orbital speed – stronger forces cause the satellites to speed up, weaker forces cause the satellite to slow down. Plot (qualitatively) the gravitational force of the ice sheet on the satellite as a function of position in Earth's orbit. With that in mind, knowing that one satellite is behind the other in orbit, what might the two GRACE satellites measure to infer the mass distribution of Earth's surface?

The two satellites measure the distance between them at a resolution of 10 micrometers over an average separation of 220 kilometers between the satellites. As the distance between them will change as one satellite is attracted to a positive mass anomaly at the earth's surface, this change in distance can be used to infer a change in the local gravitational field.

How does the satellite's orbit elevation affect its ability to resolve gravity anomalies in the near surface? To answer this question, compute the gravitational forces for two different orbits (100 km and 500 km) given the following values:

Universal Gravitation Constant	6.674×10^{11} m^3 kg^{-1} s^{-2}
Mass of the Earth	6×10^{24} kg
Mass of the Ice Sheet	3×10^{16} kg
Radius of the Earth	6371 km

Although this is an oversimplification, for this exercise, we will treat the combined mass of the earth and the ice sheet as if it's at the center of the earth.

For the 100 km orbit we have:

$$g = \frac{Gm_1}{R^2} = \frac{6.674 \times 10^{-11} \times (6 \times 10^{24} + 3 \times 10^{16})}{(6371000 + 100000)^2} = 9.56\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

And for the 500 km orbit

$$g = \frac{Gm_1}{R^2} = \frac{6.674 \times 10^{-11} \times (6 \times 10^{24} + 3 \times 10^{16})}{(6371000 + 500000)^2} = 8.48\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Clearly the gravitational force and its variability will be large when objects are closer (R^2 dependence). A closer orbit would be advantageous for scientific purposes but may be less advantageous for practical ones.

Additional discussions:

- The ice sheets exert a substantial gravitational force at a distance. What do you think this does to the sea-surface near the ice sheets? What happens as the ice sheets lose mass?

  As ice sheet lose mass they attract the ocean less a mass, as according to Newton's law

  $$F_g = \frac{Gm_1m_2}{R^2}$$

  so if m_2, the mass of ice decreases, the gravitational attraction of the ocean nearby to the ice sheet is less, which results in a local sea-level fall, which can act to stabilize the ice sheet against farther retreat.

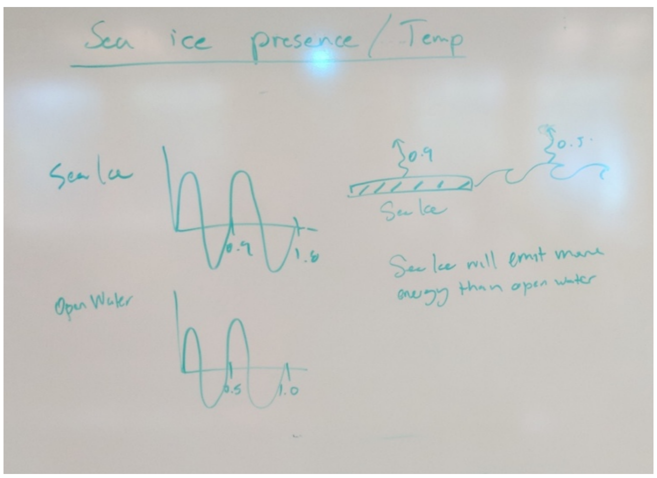
- What other observable mass changes do you think this satellite constellation has observed?

  Change in crustal depression seasonally with monsoons, groundwater depletion, changes in ocean currents (ocean surface topography)

Measuring Sea Ice Extent and Skin Temperature

In order to stay in radiative balance, everything must emit energy. The amount of energy and its frequency content vary as a function of temperature and the material's properties. Some objects emit less energy than true black bodies, due to differences in electrical properties, and can be described by their "emissivity" (the ratio of energy emitted to energy that would be emitted by a black body radiator). Passive microwave detectors measure the energy emitted at Earth's surface and use the observed energy to diagnose the surface emissivity (providing information about the material properties of Earth's surface) and the "skin temperature"– the temperature of the near surface.

For the purposes of this exercise, sea ice has an emissivity of 0.9 and open water has an emissivity of 0.5. Sketch a physical system with sea-ice next to open ocean – showing the energy emitted by the two surfaces as a sine wave with a particular amplitude. Be careful to show which material emits more energy.



Electromagnetic theory predicts that the outgoing energy in the microwave frequencies can be described by the following relation:

$L_f \approx \frac{2kTf^2}{c^2} * \varepsilon$ $\quad k \cong 1 \times 10^{-23}$, $c = 3 \times 10^8$, $f = 10 \times 10^9$, T = Temperature in Kelvin.

What do you expect the energy difference to be between 273K sea ice and 277K open water? What about between 277K open water and 290K open water? Would the sea-ice / open water stand out better or worse than a cold water versus warm water transition?

For sea-ice (at 273):

$$L_f \approx \frac{2kTf^2}{c^2} * \varepsilon = \frac{2 \times 1 \times 10^{-23} \times 273 \times (10 \times 10^9)^2}{(3 \times 10^8)^2} \times 0.9 = 5.5 \times 10^{-18}\ \mathrm{W\ Hz^{-1} sr^{-1}\ m^{-2}}$$

For open water at 277 K:

$$L_f \approx \frac{2kTf^2}{c^2} * \varepsilon = \frac{2 \times 1 \times 10^{-23} \times 277 \times (10 \times 10^9)^2}{(3 \times 10^8)^2} \times 0.5 = 3.1 \times 10^{-18}\ \mathrm{W\ Hz^{-1} sr^{-1}\ m^{-2}}$$

For open water at 290 K:

$$L_f \approx \frac{2kTf^2}{c^2} * \varepsilon = \frac{2 \times 1 \times 10^{-23} \times 290 \times (10 \times 10^9)^2}{(3 \times 10^8)^2} \times 0.5 = 3.2 \times 10^{-18} \mathrm{W\ Hz^{-1} sr^{-1}\ m^{-2}}$$

So definitely more of difference in emitted radiation between sea ice and open water than between open water at different temperature.