EARTH AND SPACE SCIENCE **Autumn 2018**

431 *PRINCIPLES OF GLACIOLOGY* 4 Credits, SLN 14855

505 *THE CRYOSPHERE* 4 Credits, SLN 14871

Lab Week 4 – Glacier Flow

Find 1 or 2 partners, and work together to answer 1 of the following 2 questions. You and your partners will be asked to report your results to the class before the end of lab.

(1) Dynamic ice-flow estimation

A temperate glacier is 100 meters thick, with a slope of 6.3° (0.11 radian). The glacier is not sliding. Your coordinate system has *x* along the flow direction, and z is vertical, with values increasing up from the bed. The corresponding velocity components are *u* and *w*.

- Estimate the bed-parallel shear stress $\sigma_{xz}(z)$ at the bed (*z*=0), at 20 m, 40 m, 60 m, 80 m above the bed, and at the surface (z=100 m). Plot your results on the provided axes.

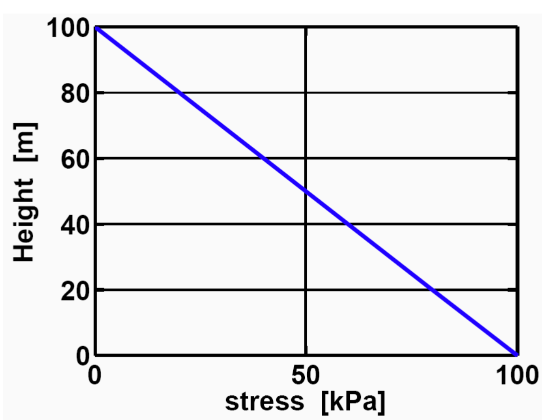
  To do this we need to use a few relations from class. First, the bed-parallel shear stress at an arbitrary height z is given by:

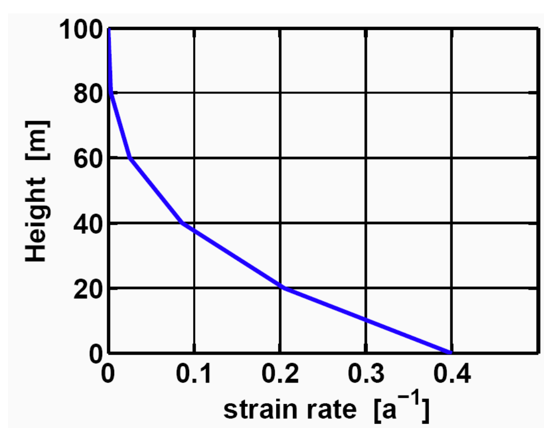
  $$\sigma_{xz}(z) = \rho g(h - z)\sin\theta$$

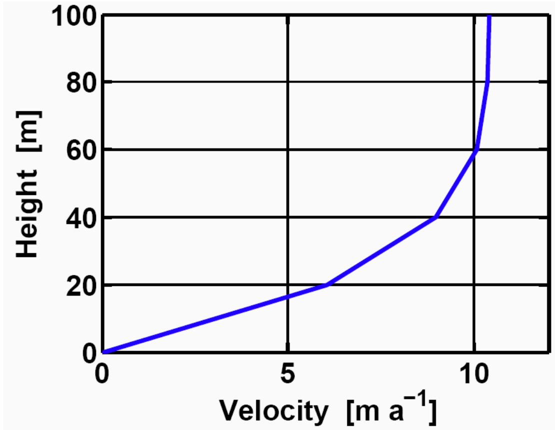
  and we can relate this to the along-flow horizontal velocity u as a function of height using the constitutive relation for ice (Glen's flow law), which is $\dot{\epsilon}_{xz} = A\tau_{xz}$, so

  $$\frac{\partial u}{\partial z} = 2A[\sigma_{xz}(z)]^3$$

- Using Glen's flow law for ice at 0°C, estimate the shear strain rate (*∂u/∂z)* at the same 6 depths. The plot below show the results of these calculations on the provided axes.







- Estimate how fast the ice might be moving at each depth by populating the table on the following page. (Note that we are not asking you to use the analytic solution for ice velocity provided in class. You are approximating a solution to $\int_0^H \frac{\partial u}{\partial z} \partial z$ by computing $\frac{\partial u}{\partial z}$ at a variety of depths, and summing the computed values). Plot your results.

	ESS 431 Principles of Glaciology				Glacier Flow Worksheet - KEY			
	Height	Depth	Shear stress	Strain rate	Interval average	Depth interval	Velocity difference	Velocity
	z	(h-z)	$\sigma_{xz}(z)$	$\partial u/\partial z$	$\partial u/\partial z$	Δz	Δu	$u(z)$
	(m)	(m)	(Pa)	(a^{-1})	(a^{-1})	(m)	(m a^{-1})	(m a^{-1})
	100	0	0	0.00E+00				10.4
					0.0016	20	0.032	
	80	20	20000	3.20E-03				10.368
					0.0144	20	0.288	
	60	40	40000	2.56E-02				10.08
					0.056	20	1.12	
	40	60	60000	8.64E-02				8.96
					0.1456	20	2.912	
	20	80	80000	2.05E-01				6.048
					0.3024	20	6.048	
	0	100	100000	4.00E-01				0
		z = height above bed						
		h = ice thickness						
		$\sigma_{xz}(z)$ = bed-parallel shear stress at height z						
		$\tau_b = \sigma_{xz}(0)$ = basal shear stress						
		$\partial u/\partial z$ = shear strain rate						
		Δz = depth interval between calculation points						
		$u(z)$ = bed-parallel velocity at height z						

Key					
h	100	m			
ρ	900	kg m^{-3}		$\sigma_{xz}(z)$	$= \rho g (h-z) \sin(\theta)$
g	10	m s^{-2}		τ_b	$= \sigma_{xz}(0)$
θ	0.111111	radians		$\partial u/\partial z$	$= 2A\,[\sigma_{xz}(z)]^n$
A	2.00E-16	Pa^{-3} a^{-1}			
n	3				
$u(0)$	0				

(2) Kinematic ice-flow estimation

A steady-state glacier has a net balance rate of 0.5 m a^{-1} (ice-equivalent) in the accumulation area upstream from a target cross-section, 1 km from the headwall. At this cross-section, the glacier happens to be 200 m across, and 100 m deep.

- Find the total volumetric ice flux through this cross-section for a glacier in balance.

$$Q = L \times W \times \dot{b} = 10^3\text{m} \times 2 \times 10^2\text{m}^2 \times 0.5 \text{ m yr}^{-1} = 10^5\text{m}^3 \text{ yr}^{-1}$$

- Find the average ice flux per unit width for this cross-section.

$$q = \frac{Q}{W} = L \times \dot{b} = 10^3\text{m} \times 0.5 \text{ m yr}^{-1} = 500 \text{ m}^2 \text{ yr}^{-1}$$

- Find the depth- and width-averaged velocity of ice flowing through this cross-section.

$$\bar{\bar{u}} = \frac{Q}{h \times W} = \frac{10^5\text{m}^3 \text{ yr}^{-1}}{10^2\text{m} \times 2 \times 10^2\text{m}^2} = 5 \text{ m yr}^{-1}$$

- Knowing that there is drag from the sides and the bottom, use your averaged velocity to make a rough estimate of the actual speed of a marker on the surface at the center line. Explain your assumptions

  This is roughly half the speed of what we calculated in problem #1. We know that in a very wide channel, the surface speed is equal $\frac{n+2}{n+1} = \frac{5}{4}$ times the depth-averaged speed. Problem #1 also made no consideration of the shape of the valley, which might act to slow the glacier due to drag along the valley walls. For a rectangular valley, the side drag has been modeled to decrease the velocity by 4/5 of what it would be for a glacier with no side drag. Thus to account for the width-and-depth averaged sense of the speed, we might consider multiplying the width-and-depth averaged speed by $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$ =1.56, i.e. u_{surf}~7.8 m yr^{-1}.

OK, after hearing the other group reports, now all groups answer Question 3.

(3) Compare the dynamic and kinematic treatments

Both glaciers are 100 m thick and temperate. What factors might account for the differences in the estimates of the speeds of the markers at the surface on the center lines?

- Glacier 1 may not be in steady state.
- Glacier 2 may have a different surface slope from Glacier 1.
- The channel cross-section for Glacier 2 may not be such a simple rectangle, so your simple correction for side and basal drag on Glacier 2 may be inaccurate.
- Glacier 1 may have normal stresses stretching it along its length as well as shear stresses acting on it from the bottom.
- Glacier 1 may also be experiencing side-wall drag.