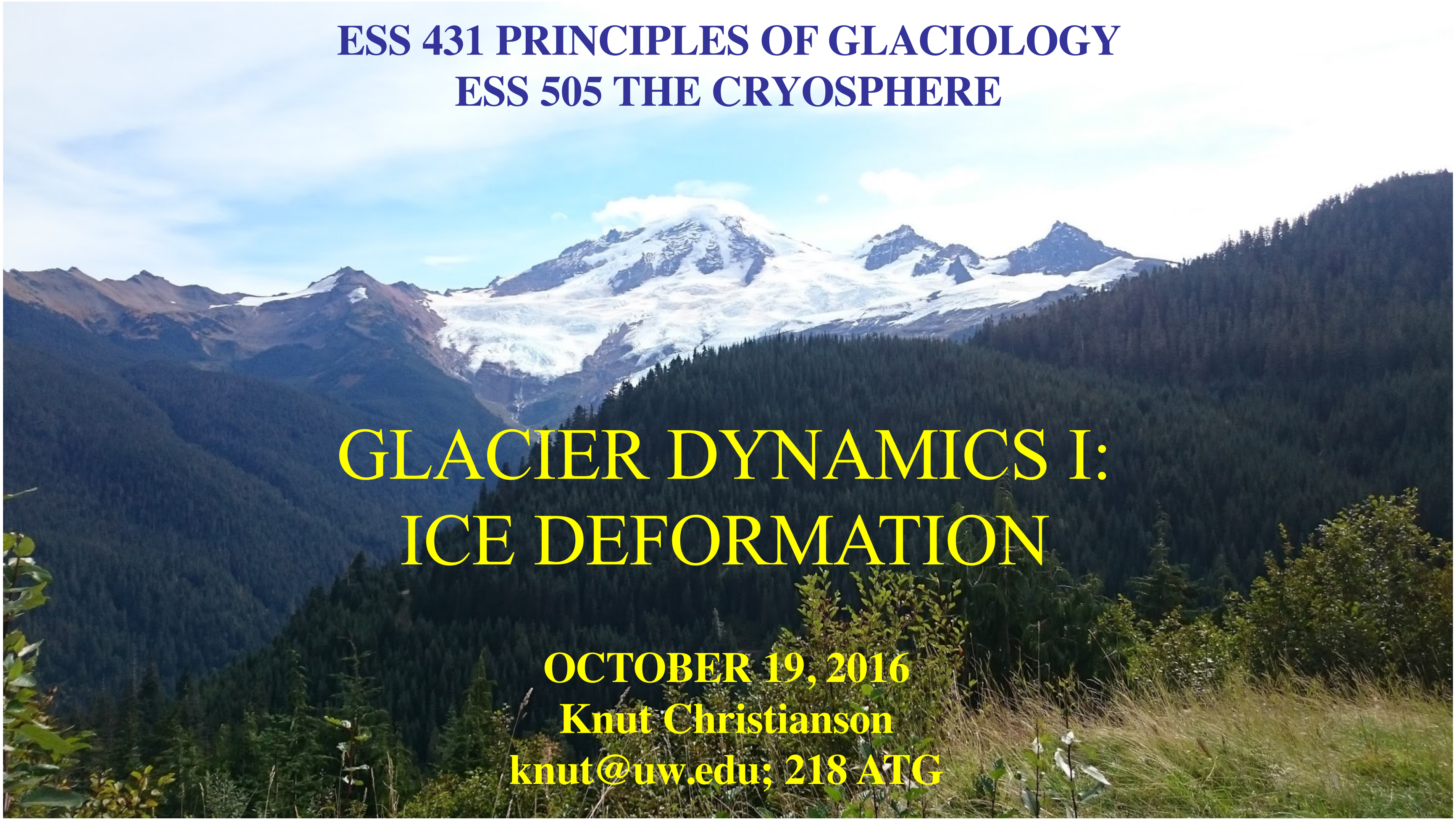


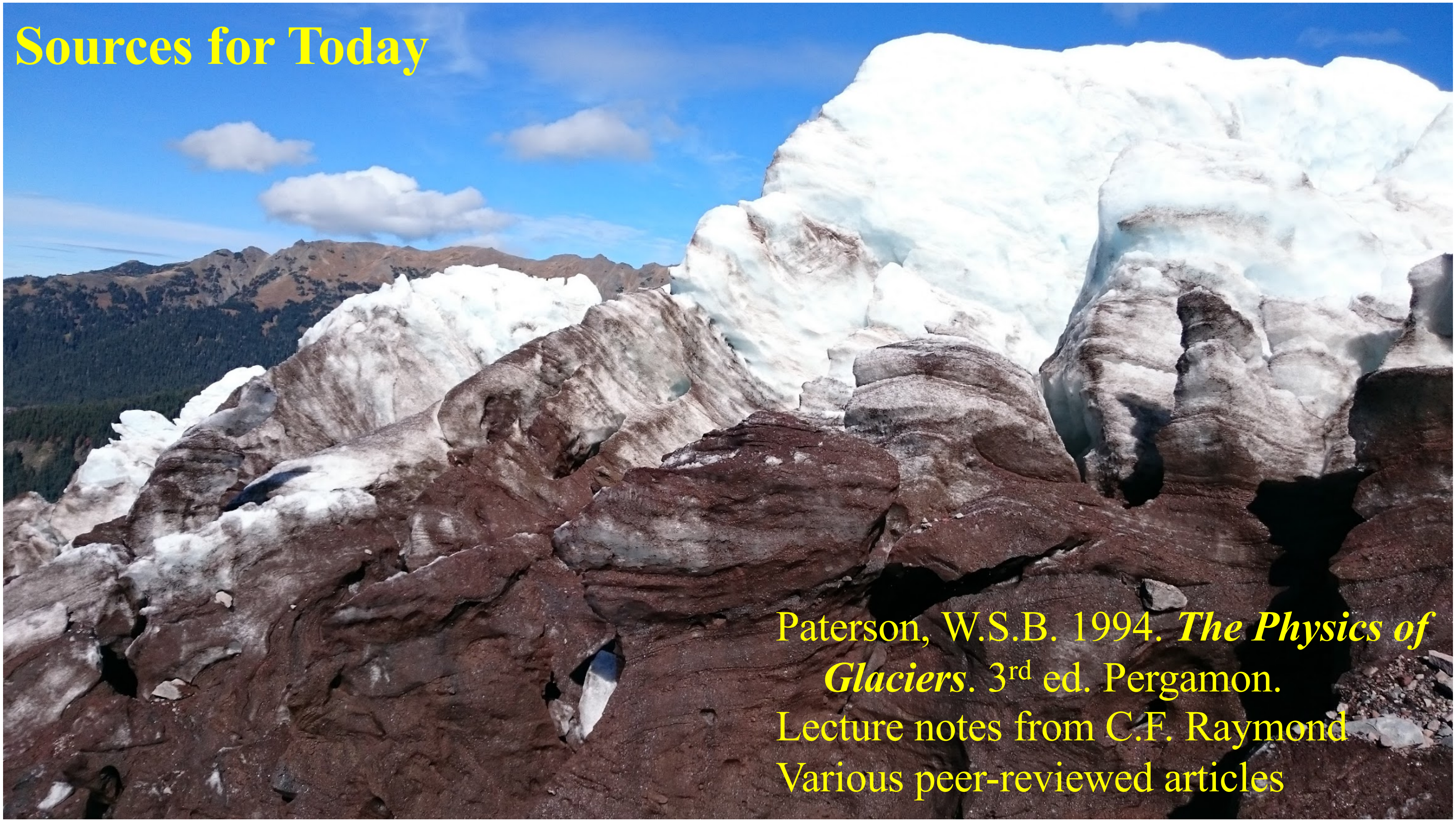
ESS 431 PRINCIPLES OF GLACIOLOGY
ESS 505 THE CRYOSPHERE

GLACIER DYNAMICS I: ICE DEFORMATION

OCTOBER 19, 2016

Knut Christianson
knut@uw.edu; 218 ATG


Homework
• This week – Harry and Gwen estimate the speed of a glacier.
Friday Discussion Session
• Supercooling
• Avalanches
• Glacier flow exercises


Sources for Today
Paterson, W.S.B. 1994. The Physics of Glaciers. 3rd ed. Pergamon.
Lecture notes from C.F. Raymond
Various peer-reviewed articles

Some Important Questions:

- How big are glaciers? (Area and volume, sea-level equivalent)

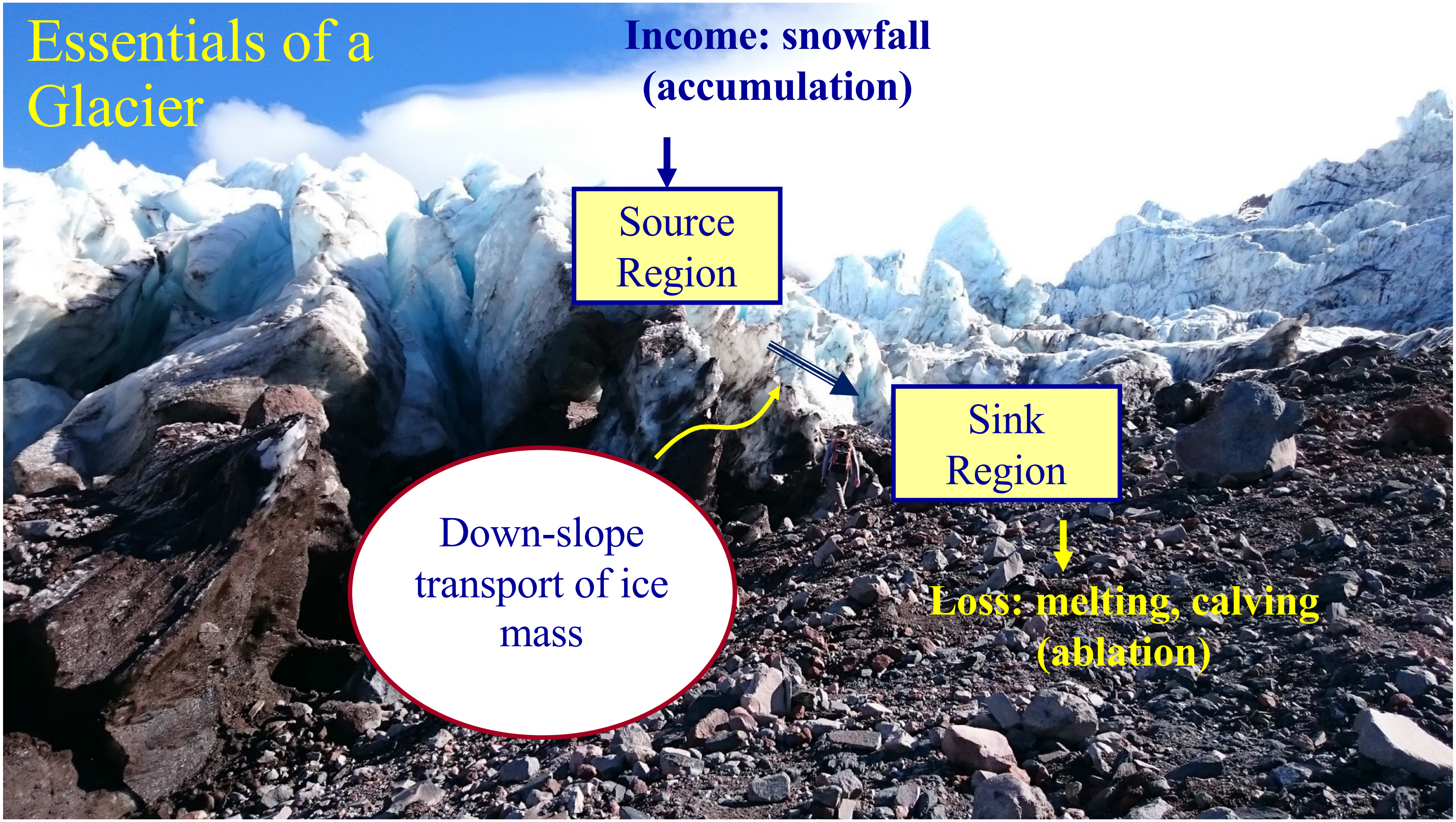
- How (fast) do glaciers move? (TODAY)

- How water runs off/ is stored/ influences flow? (Hydrology)

- How do glaciers erode old landscapes and build up new landscapes? (Geomorphology)

- How do glaciers change with climate? (Sensitivity to climate change and response times)

Glacier “Flow” Chart


Essentials of a Glacier
Income: snowfall
(accumulation)
Source Region
Sink Region
Down-slope transport of ice mass
Loss: melting, calving
(ablation)

GLACIER MOTION?

Historic glacier footage shot by UW glaciologists:

- ***Blue Glacier (Mt. Olympus)***
- Nisqually Glacier (Mt. Rainer)
- South Cascade Glacier (North Cascades)

More-recent footage from ***Paulabreen (Svalbard, Norway)*** and ***Helheim Glacier, Greenland***

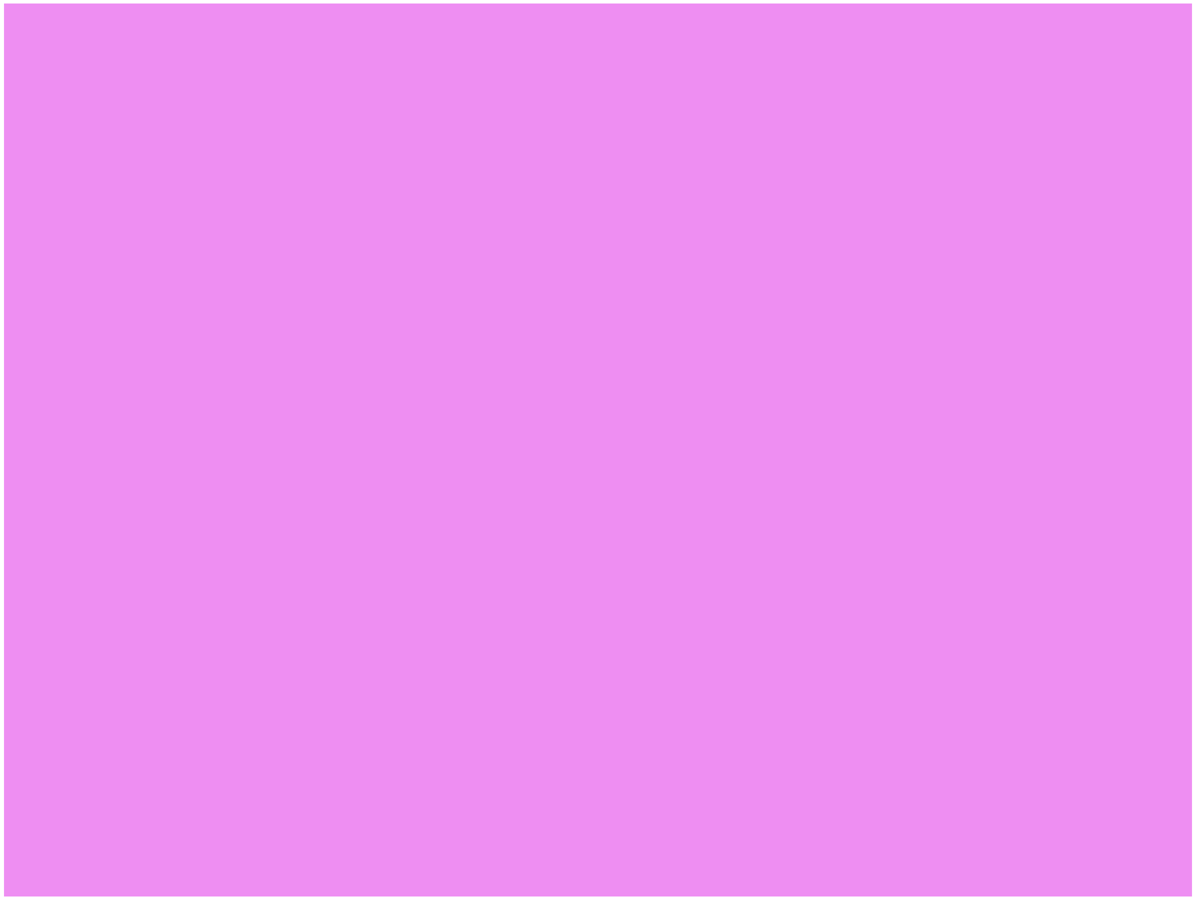
How do glaciers flow?

HOW DOES A GLACIER FLOW/MOVE?

- Ice can deform as a viscous fluid (Today/Friday)
- Ice can slide over its substrate (Monday)
- Ice can fracture (traditionally neglected, but possibly important for especially rapid changes??)

Historic Videos of Ice Deformation: Blue Glacier

SLIDING: Paulabreen Glacier




FRACTURE: Helheim Glacier

OUTLINE

- How is flow manifested in glaciers? How do we measure it?
- What determines a glacier's shape and motion?
  - Kinematic vs. dynamic descriptions
  - Mass balance
- How does a glacier flow?
  - What are the controls on ice transport?
  - How does ice deform?
    - Mechanics of ice creep
    - Forces, pressure, stress, strain, strain rate
    - Constitutive relation for ice (relates stress to strain rate)
  - Uniting the kinematic and dynamic description??
- Summary

Measuring the Motion of a Glacier

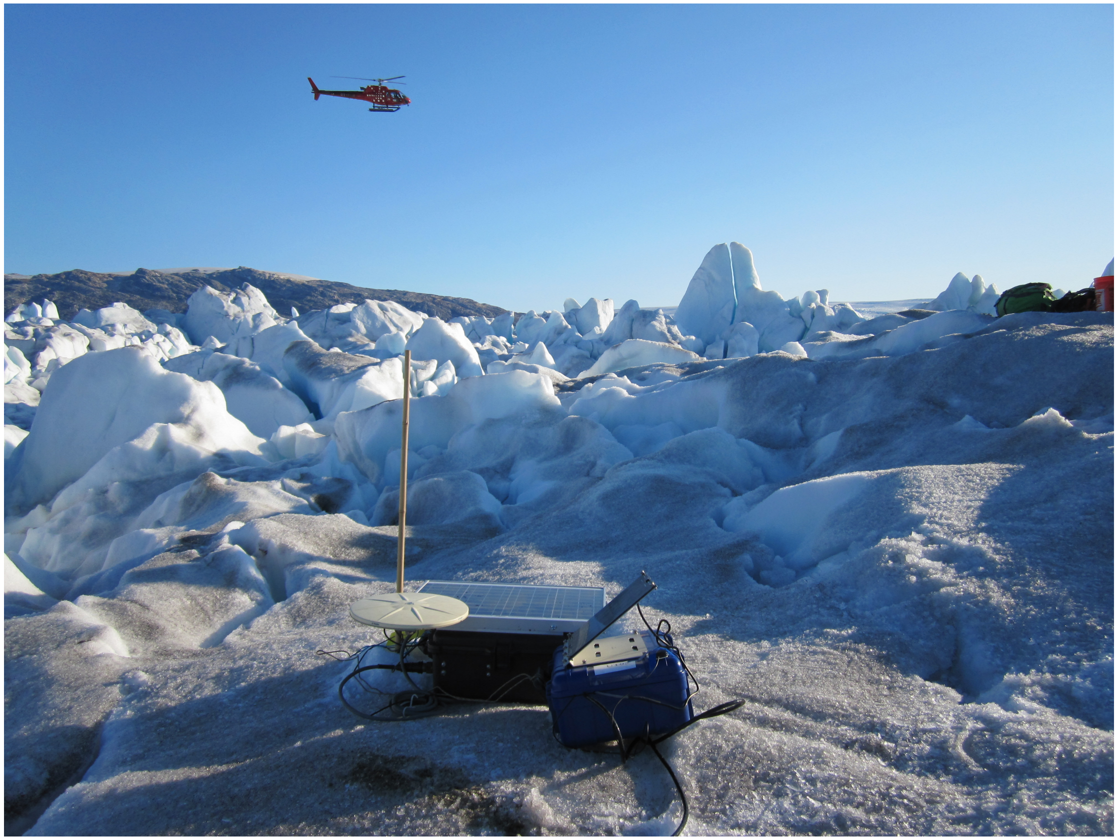
Traditional Surveying

On surface:

- Measuring angles (with theodolite) and distances (with electronic distance meter or EDM) from fixed stations on glacier margin

At depth:

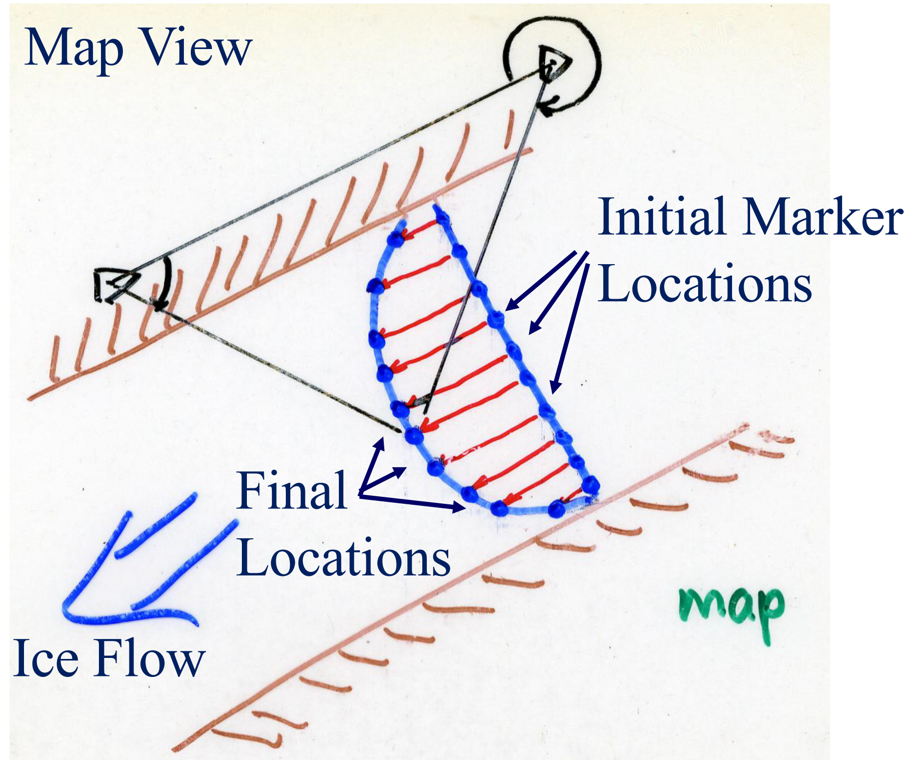
- Measuring tilt from boreholes



Traditional Surveys

Angles and distances with theodolite and EDM ("total station") measure locations of markers on/in the ice.

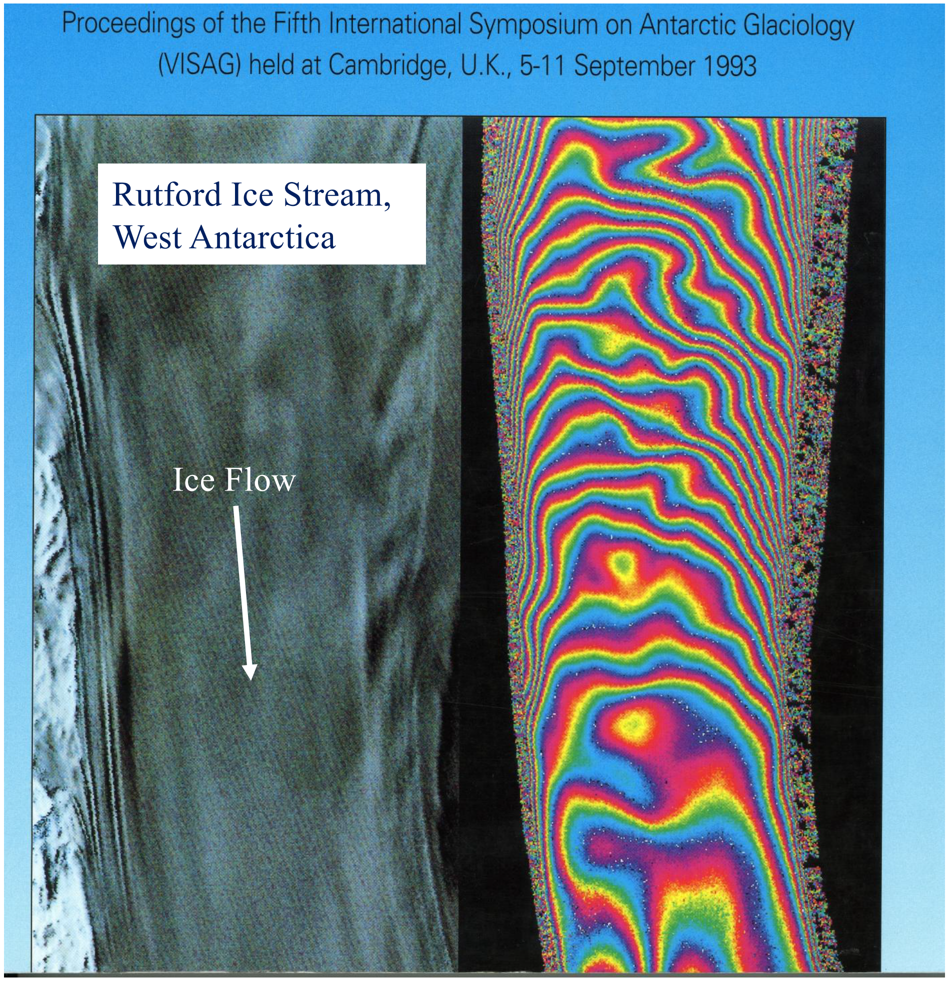
Now GPS receivers measure displacements of markers between surveys.



Measuring the Motion of a Glacier

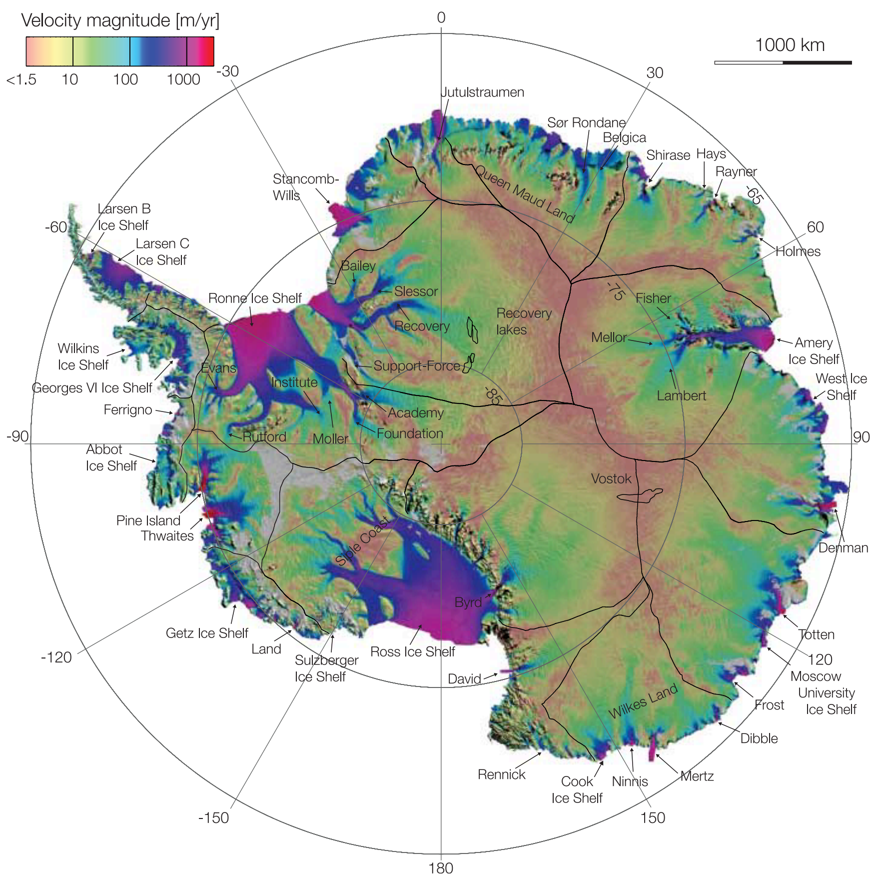
- GPS (Global Positioning System)
- Feature tracking in repeated satellite images or aerial photographs
- Interferometric Synthetic Aperture Radar (InSAR)

InSAR: Interferometric Synthetic Aperture Radar

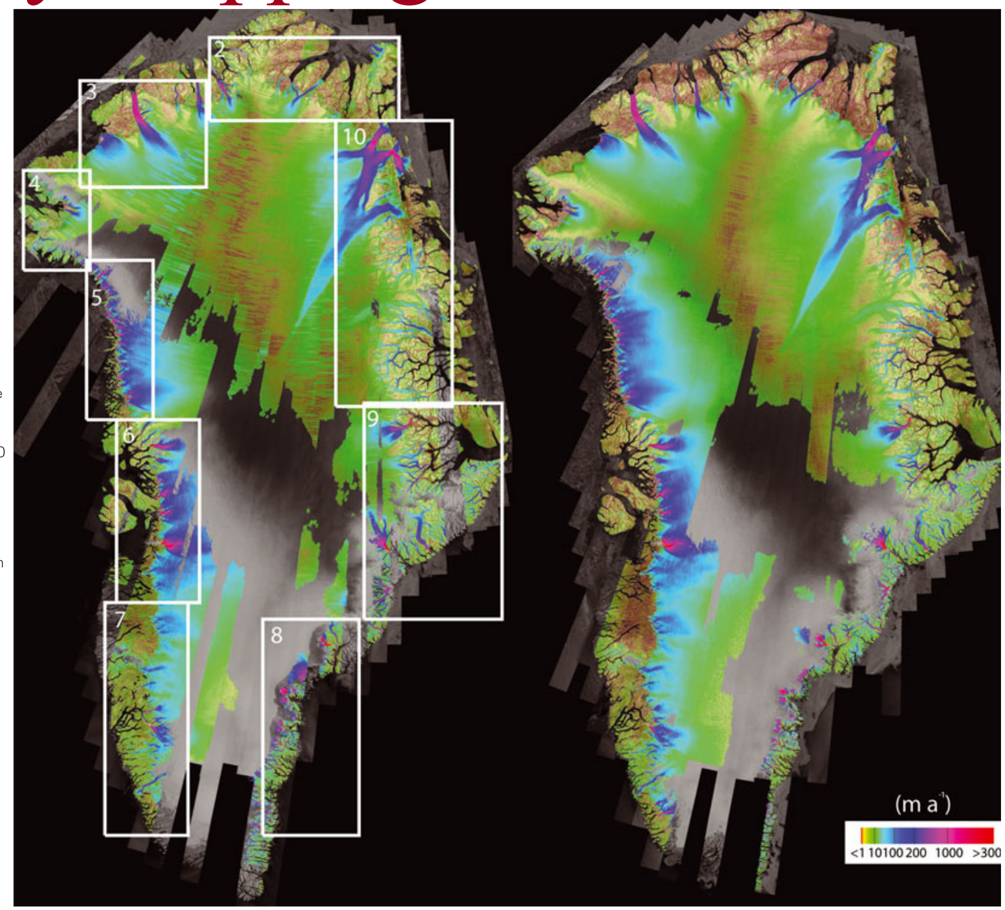


- To get speed, count fringes from a stationary point on bedrock
- Exciting in 1993
- Today we can do better

Comprehensive Velocity Mapping of Ice Sheets



(Rignot et al. (2011), *Science, 333*(6048), 1427-1430)



(Joughin et al. (2010), *J. Glaciol, 58*(197), 415-430)

Flow Variation Across a Glacier

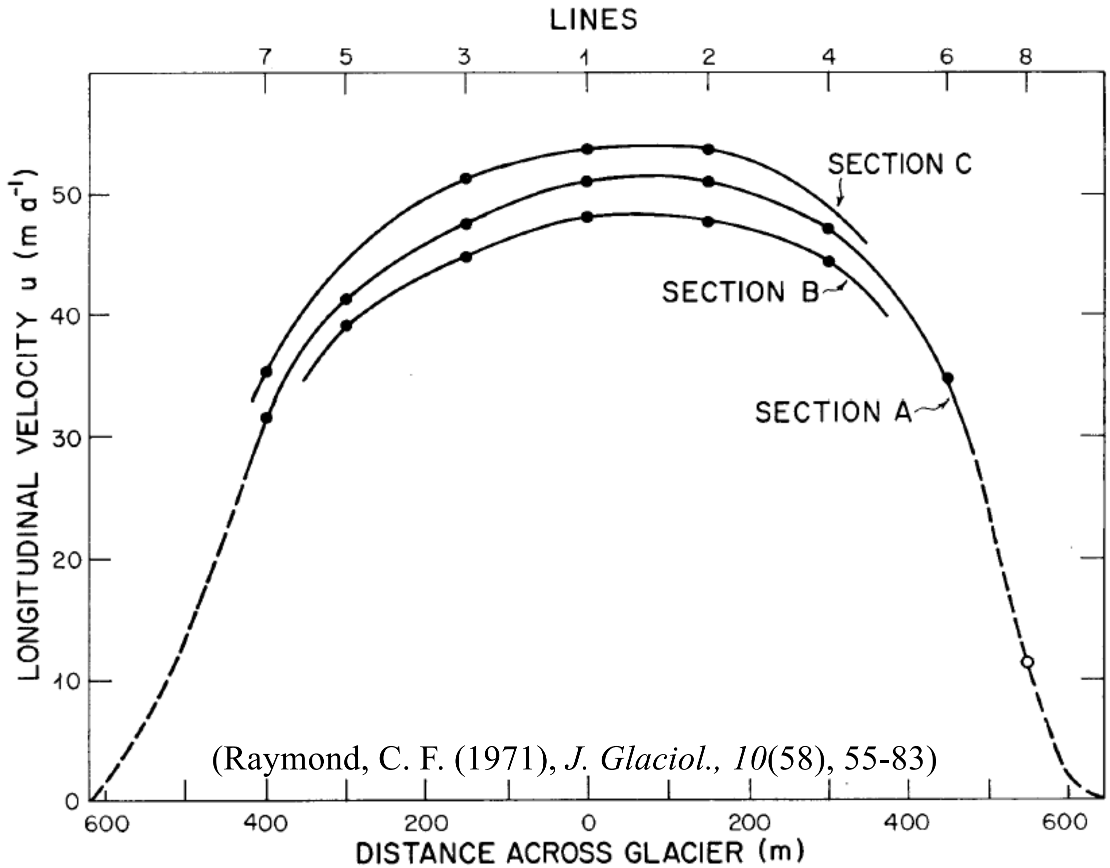
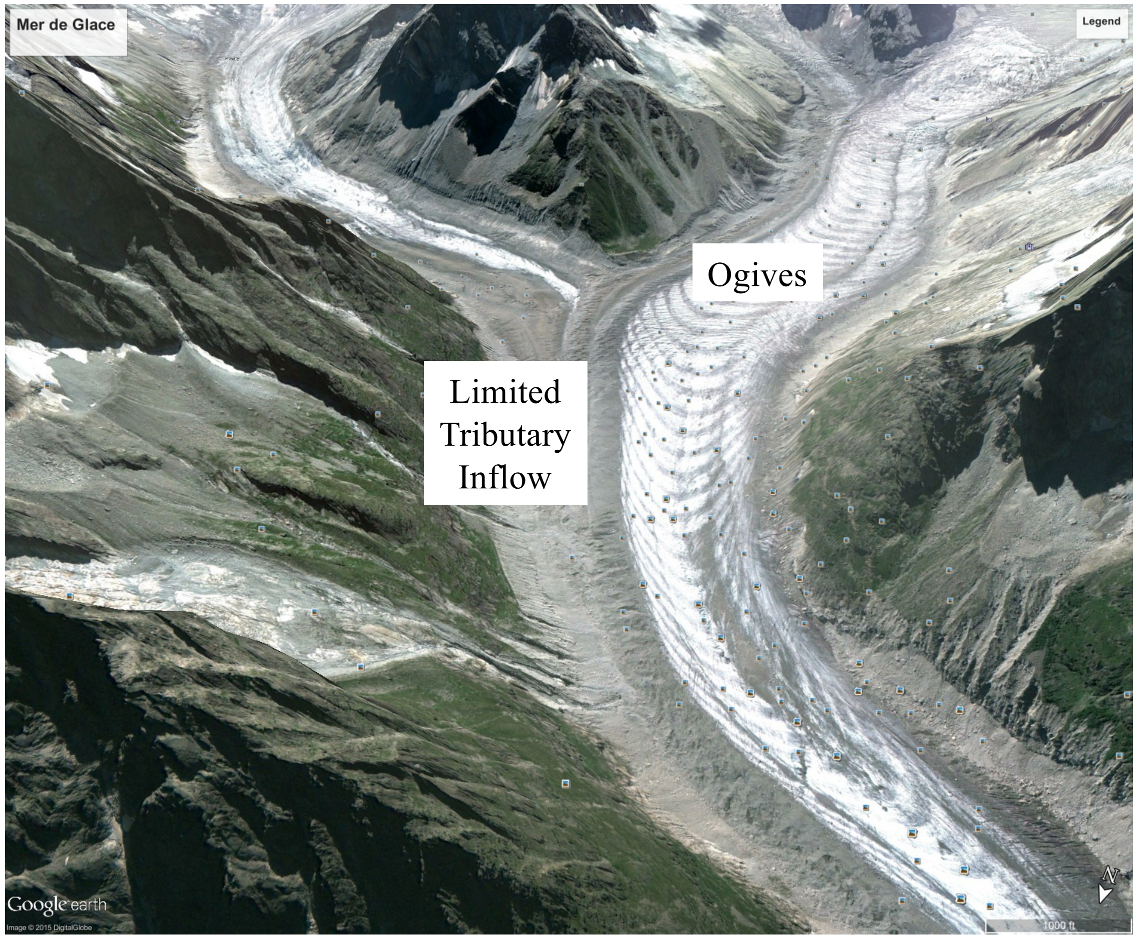


Fig. 5. Lateral variation of surface velocity. Solid circles give velocity of markers determined directly by triangulation. The open circle gives velocity as estimated from measured velocities at adjacent stakes and tape measure.

Flow Variation Across a Glacier: Mer de Glace

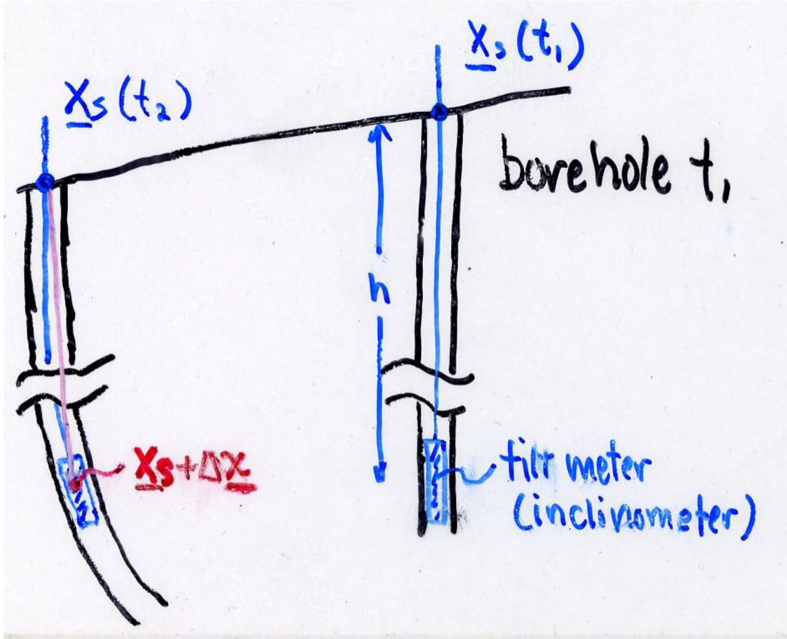


Borehole Tilting – The Third Dimension

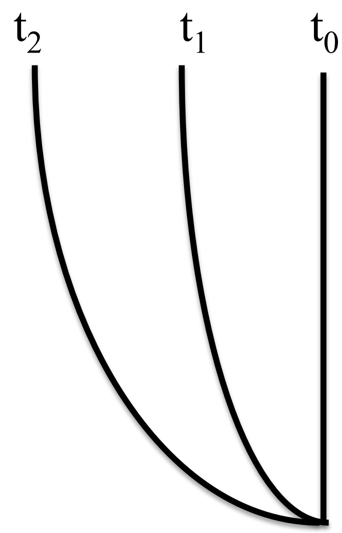
We want velocity as a function of depth.
What do you need to observe?

- $x_s(t)$ positions measured by any standard survey method

- $\Delta x(z)$ from angle of tiltmeter lowered down borehole

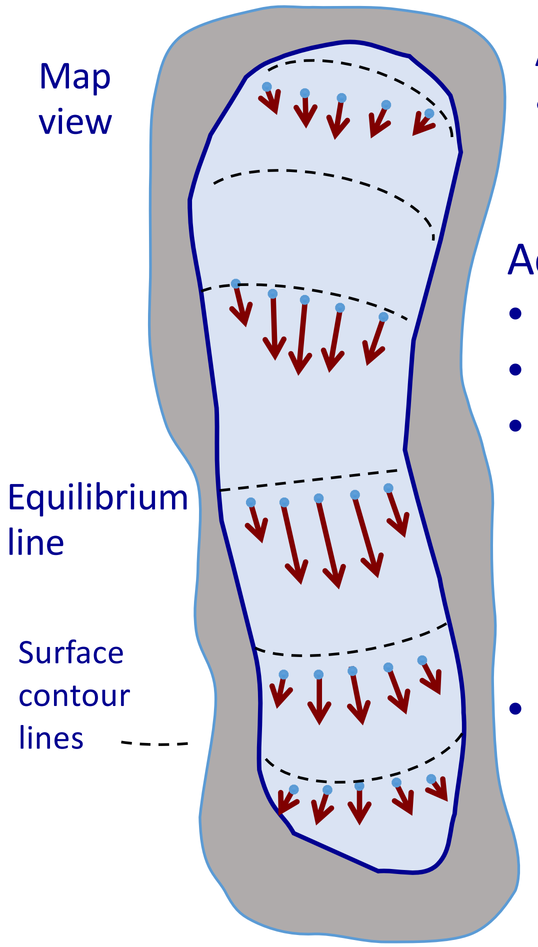


Flow Variation with Depth



Borehole tilting

Flow pattern on a steady-state glacier surface



Along center line

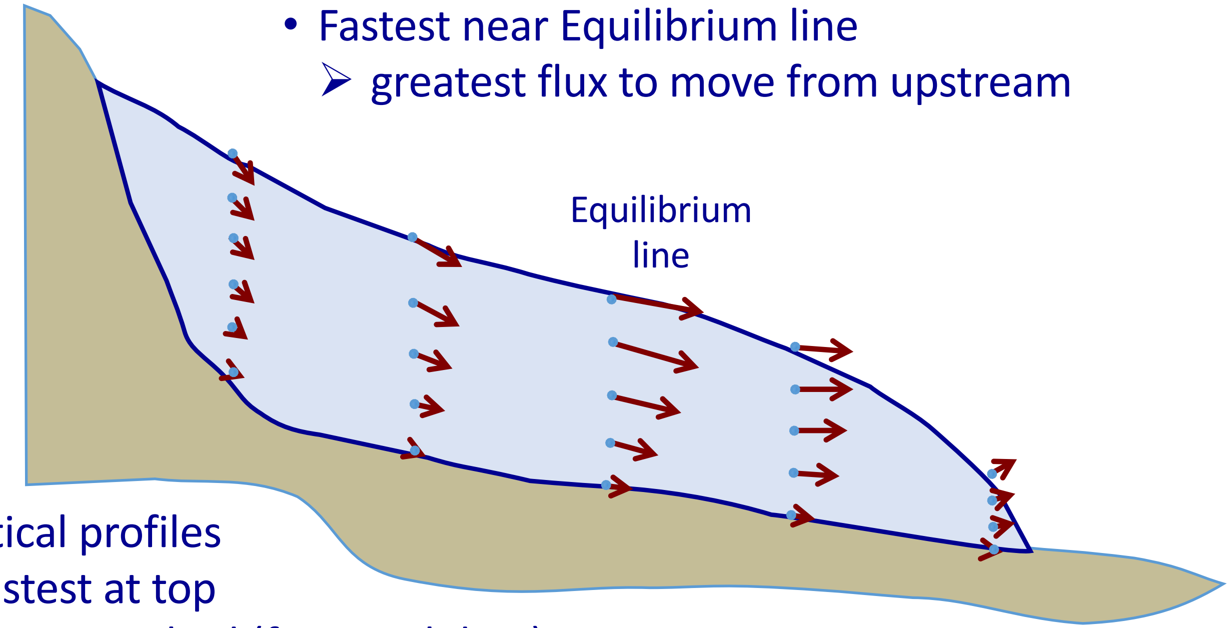
- Fastest near Equilibrium Line
  - ➢ greatest flux to move from upstream

Across channel

- Fastest near center line (deepest ice)
- Slowest near margins (frictional drag)
- Above ELA, flow in toward center
  - ➢ evacuate high accumulation from margins
  - ➢ thicken center to compensate for downstream stretching
- Below ELA, flow out toward margins
  - ➢ replace ice melted at moraines
  - ➢ stretch center to compensate for downstream slowdown/pile up

Flow pattern in a longitudinal section

- Fastest near Equilibrium line
  - ➢ greatest flux to move from upstream



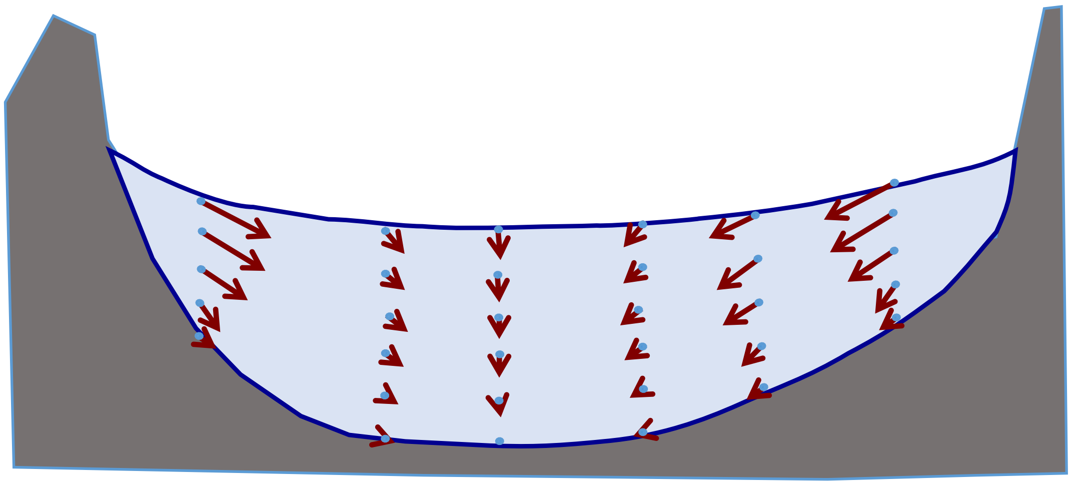
Vertical profiles

- Fastest at top
- Slowest at bed (frictional drag)
- Above ELA, flow downward into the glacier
  - ➢Leaves space for accumulation
- Below ELA, upward out of glacier
  - ➢ replace ice that is melted by surface ablation

Transverse flow profile in accumulation area

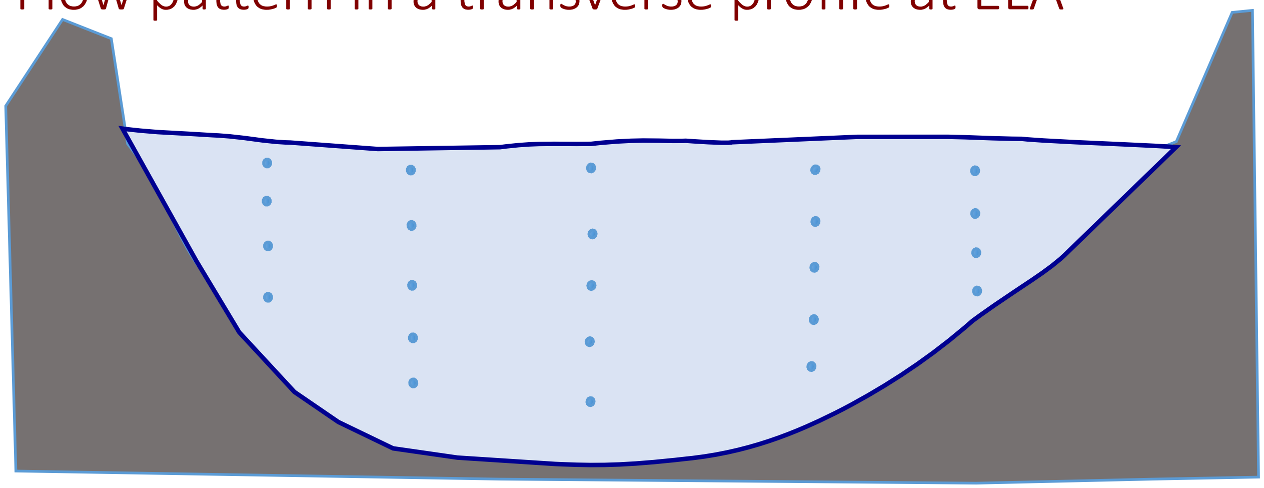
Downslope stretching and dynamic thinning along center line.

- Ice must flow toward center line to maintain thickness there.
- Surface must be concave to drive flow toward center line.



- Flow fastest at top, slowest at bed (frictional drag)
- Flow downward into the glacier
  - Leaves space for new accumulation

Flow pattern in a transverse profile at ELA



No net accumulation or ablation at ELA

- No vertical velocity up or down relative to ice surface

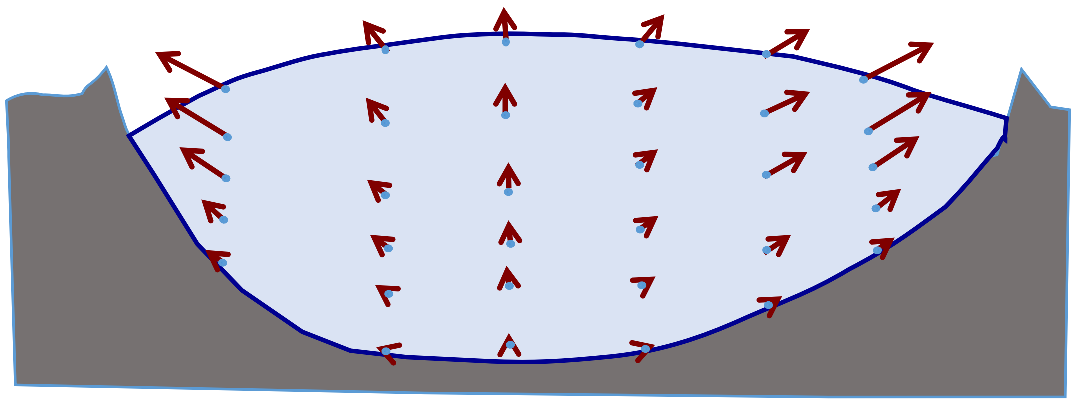
No longitudinal flux gradient at Equilibrium Line.
(that's by definition in Steady State).

- Little or no gradient in flow speed at Equilibrium Line
- Little or no dynamic thinning or thickening on centerline
    - ➢ No need for flow toward or away from center line

Transverse flow profile in ablation area

Downslope compression and dynamic thickening along center line.

- ➢ Ice flows away from center line to prevent thickening.
- ➢ Surface must be convex to drive flow toward margins.

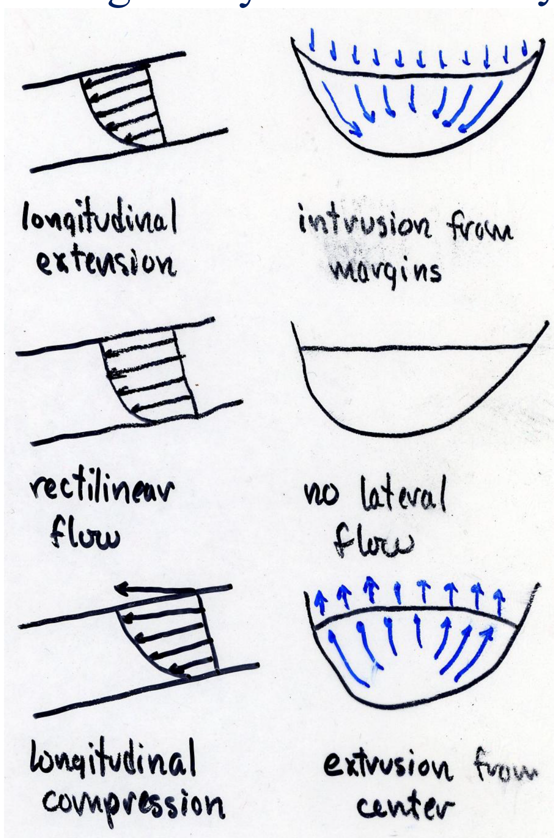


- Flow fastest at top, slowest at bed (frictional drag)
- Flow upward out of the glacier
  - ➢ Brings up ice for ablation in steady state.

Idealized Flow Patterns in a Valley Glacier

Vertical Sections

Along Valley Across Valley

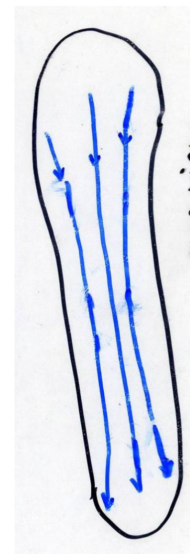


Accumulation Area

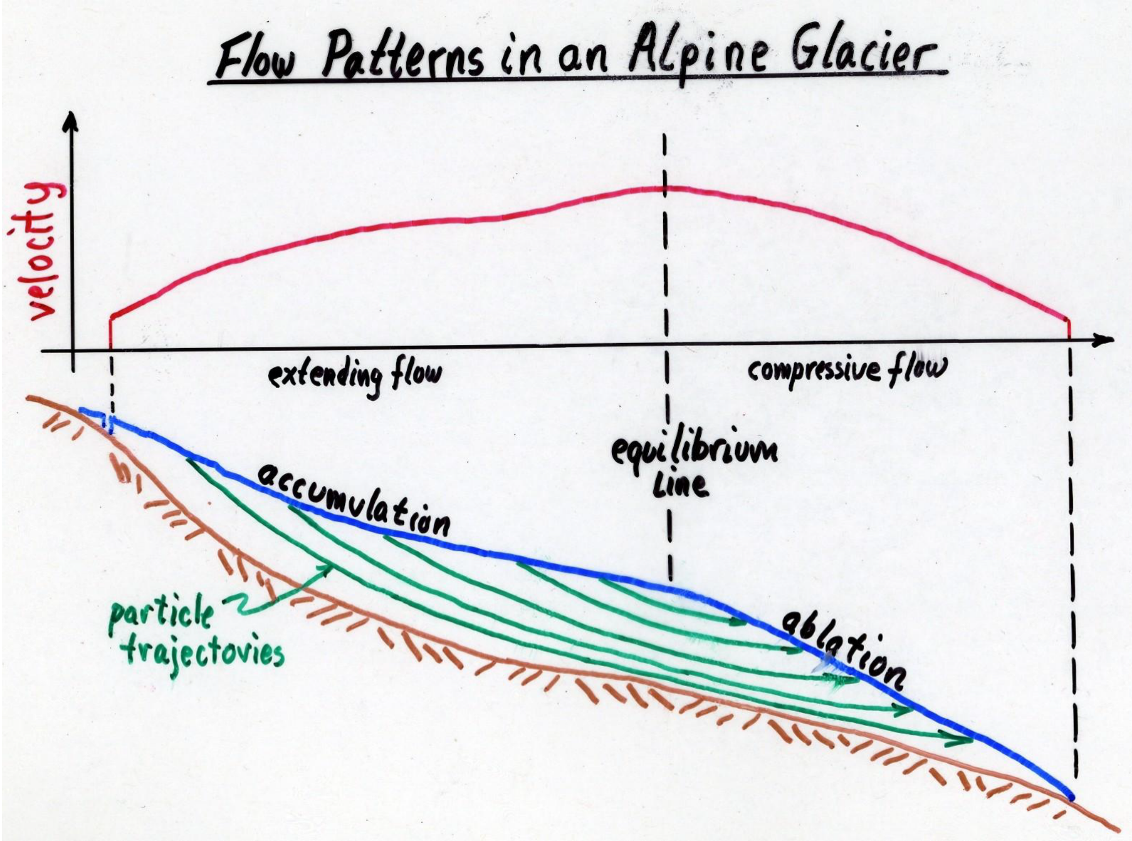
Equilibrium Line

Ablation Area

Map View



Longitudinal Flow in a Mountain Glacier



Kinematics vs Dynamics

Kinematic description of flow

In a steady state,

•Flow is what it needs to be, to carry away upstream accumulation.

•Glacier has adjusted its shape to make this flow happen.

•Glacier will grow or shrink if adjustment has not happened (yet)

Dynamic description

• Ice is a material with certain rheological properties (stiffness).

• Flow is determined by forces (stresses) applied to it.

Rheological properties don't figure in kinematic description. Accumulation and ablation don't figure in dynamic description. To figure out how a glacier changes over time, we need to use both descriptions.

What determines a glacier mass balance?

Accumulation

- ***Snow deposition***
  - Air mass characteristics
  - Topography
  - Wind deposition/avalanche
- Internal accumulation
- Superimposed accumulation

Ablation (*melt*, sublimation)

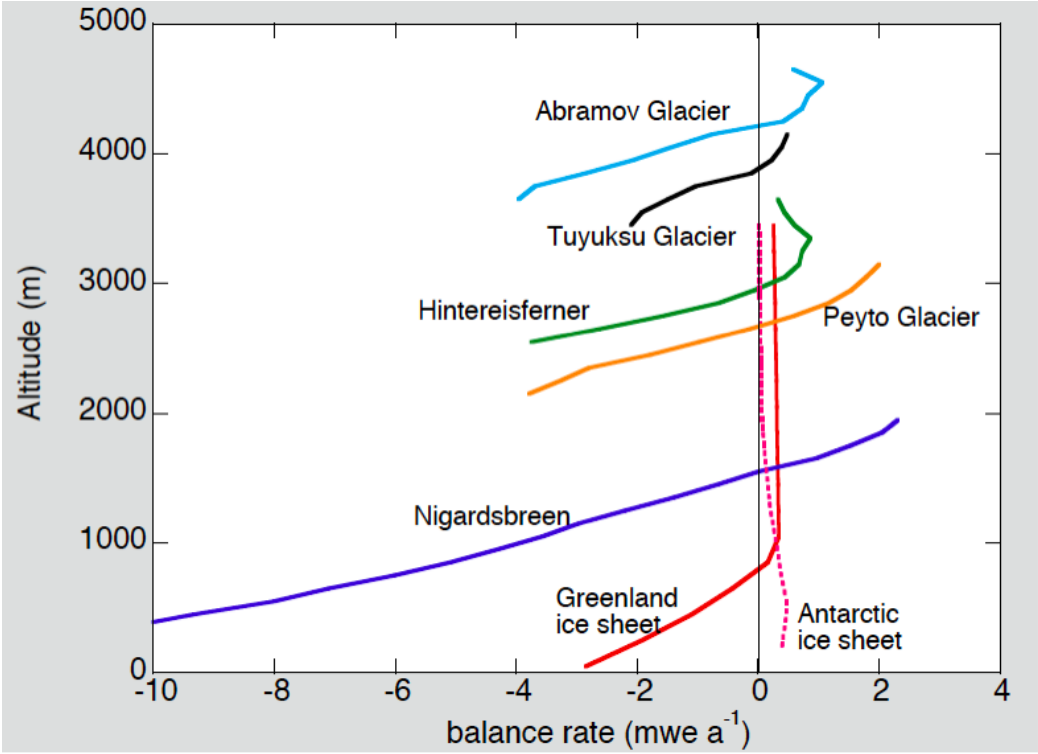
- Solar input (shortwave radiation)
- Surface reflectivity (albedo)
- Clouds (longwave radiation)
- Meteorology (wind, temperature, humidity)

For ice sheets (mainly): fracture and basal melt

Net Mass Balance for Glaciers

•Why do the curves have similar shapes?

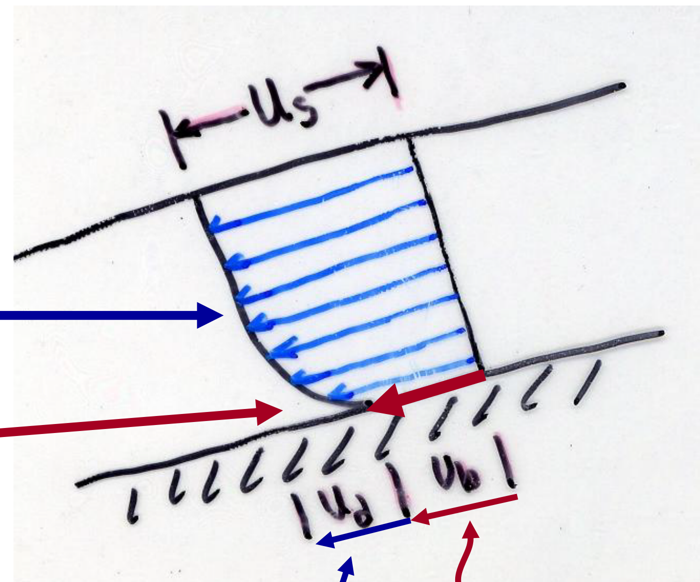
•Why are they separated so much in elevation?



(Oerlemans, 2011)

Transport Processes

- **Internal deformation**
- **Basal motion**
- **Fracture**



Speed at surface due to internal deformation

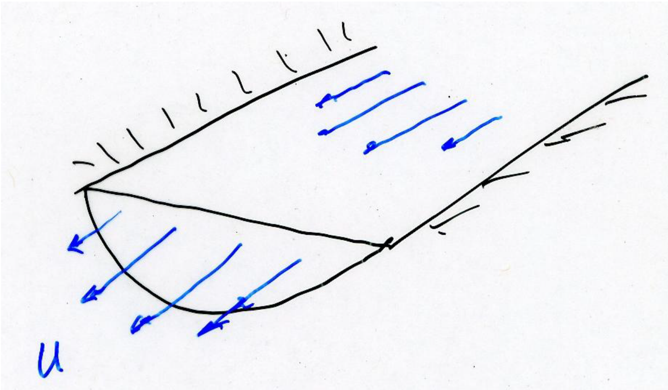
Basal motion

What Determines Ice Transport?

Q is ice flux (m^3 a^{-1}) through a cross section

$Q \propto$ thickness (m) × width (m) × speed (m^3 a^{-1})

$Q(x) =$ ice velocity $u(x, y, z)$ integrated over the cross section

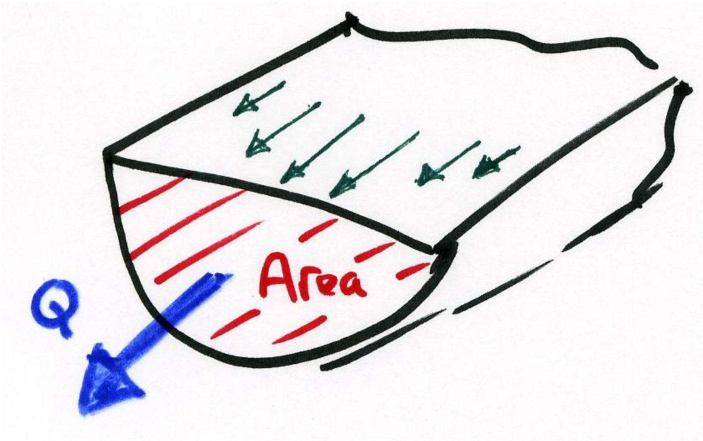


What controls ice flow speed u?

- Ice thickness
- Surface slope
- Ice properties (temperature →viscosity)
- Bed properties (temperature, slipperiness)

Ice Flux in a Temperate Glacier

Flux Q is total discharge rate across the channel in volume per unit time ($m^3\ a^{-1}$)



- We need to average velocity over both depth and width

$$\overline{\overline{u(x)}} = \frac{\int_0^{h(x,y)} \int_0^{W(x)} u(x,y,z)dydz}{Area}$$

$$Q(x) = \overline{\overline{u(x)}} \times Area$$

Ice Flux in a wide Glacier

When the depth and velocity do not vary much laterally (e.g. on a very wide glacier, or on an ice sheet), we may sometimes talk about

- Ice flux per unit width of channel $q(x)$ (units of $m^3\ a^{-1}$)

$$q(x) = \overline{u(x)} \times h(x)$$

What determines glacier motion?

Ice Deformation

- Ice "flow law"
  - Temperature
  - Fabric
  - Grain size
  - Chemistry (impurities)

Motion over Bed

Monday

- Sliding (depends on?)
- Bed deformation (depends on?)

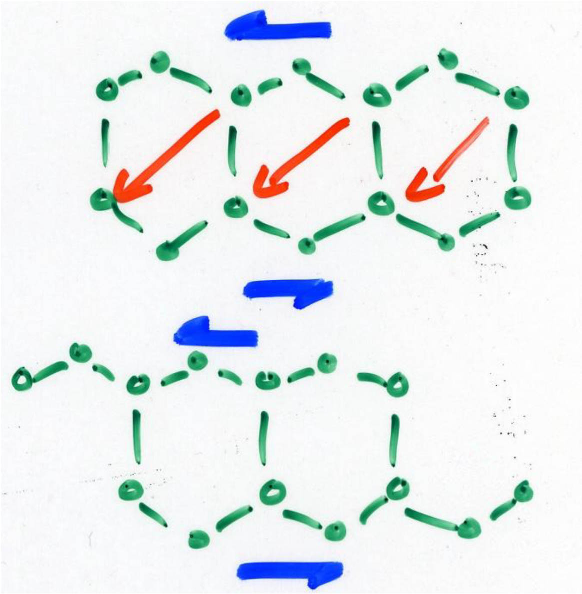
Fracture/Failure

- Crevassing
- Calving
- Avalanches (serac falls)

How can a Crystalline Solid Flow?

Basal planes are held together by hydrogen bonds

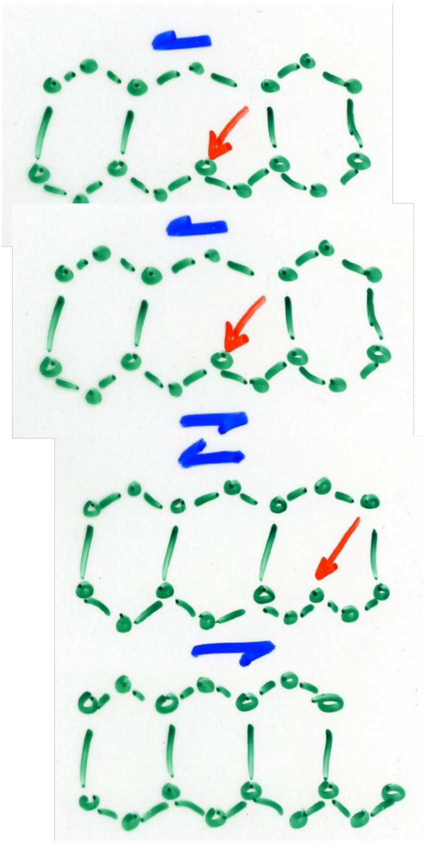
- Ice can deform along basal planes like a deck of cards
- Bonds break, shift and reconnect



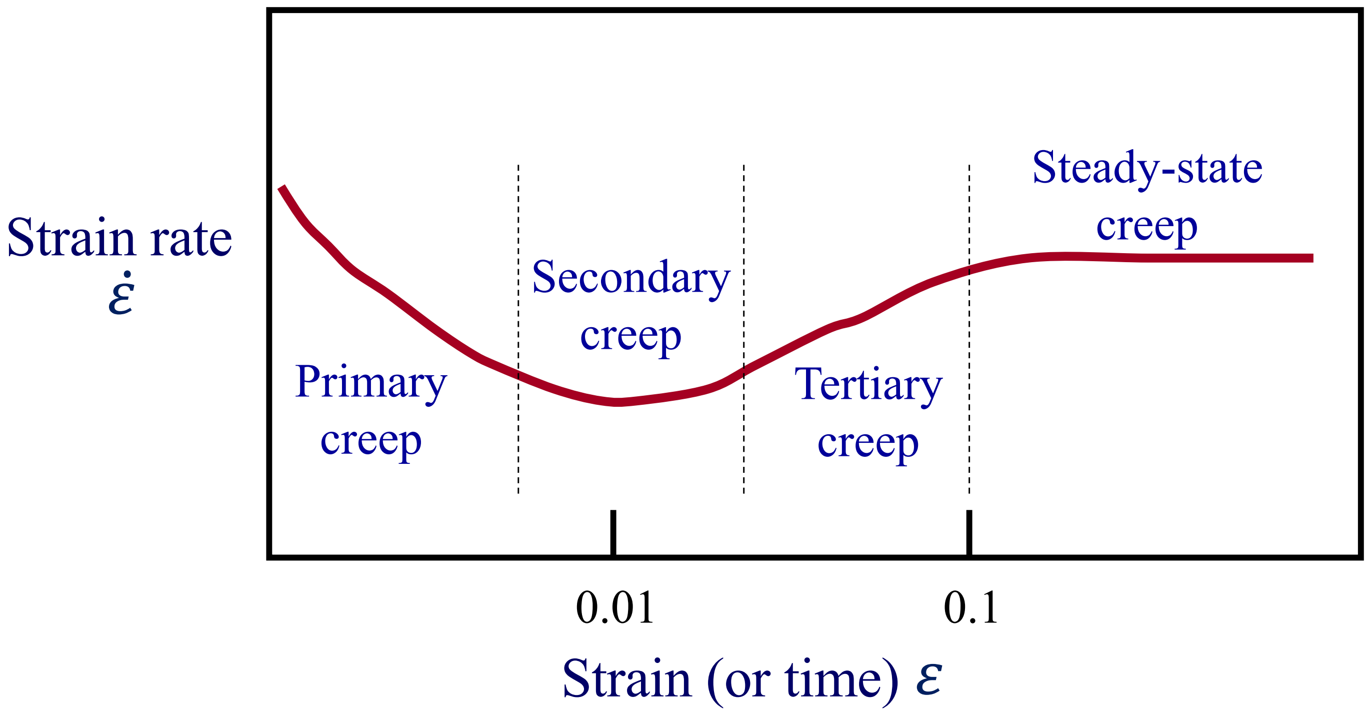
Dislocations Help Deformation

Do we need to break all the bonds at once right across the crystal?

- No. All crystals have defects or imperfections called dislocations.
- Need to break only a few bonds at a time
- We recover energy when bonds connect at new sites
- Dislocation has moved
- Dislocations act as catalyst for easy deformation



Creep Behavior of Ice

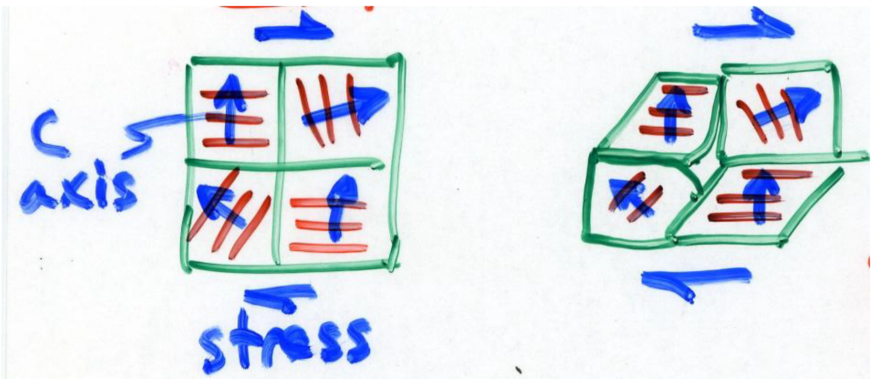


Primary Creep

Some crystals are hard and others are soft due to the orientation of their basal planes relative to the applied stress

- Soft grains deform easily at first
- After some deformation has occurred, hard grains start to stop deformation of soft grains

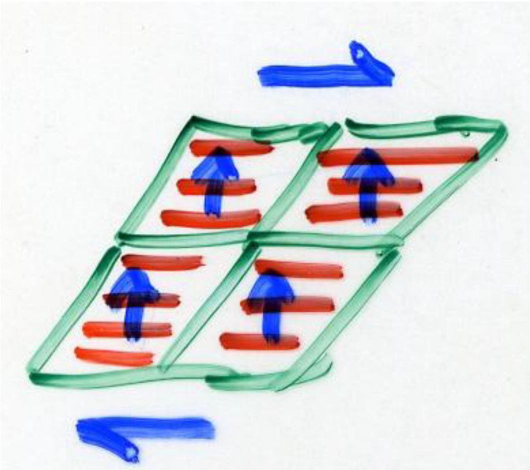
Ice initially gets harder with time



Secondary to Tertiary Creep

- Hard crystals get bent elastically, acquiring energy like a stretched spring
- This is an energetically unfavorable state
- They lose mass to neighbors that are able to creep without picking up strain energy

As hard grains shrink and disappear, ice becomes softer again



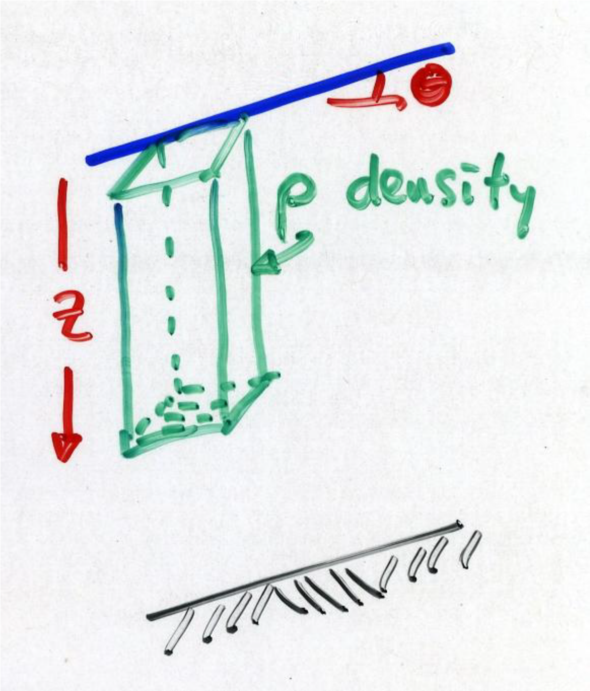
Forces

- **Forces exist inside continuous bodies (like a glacier)**
- **These forces can cause a material to deform**
- **We must understand how this works to understand glacier motion**

Stress: Are you stressed?

Ice inside a glacier is subjected to forces due to:

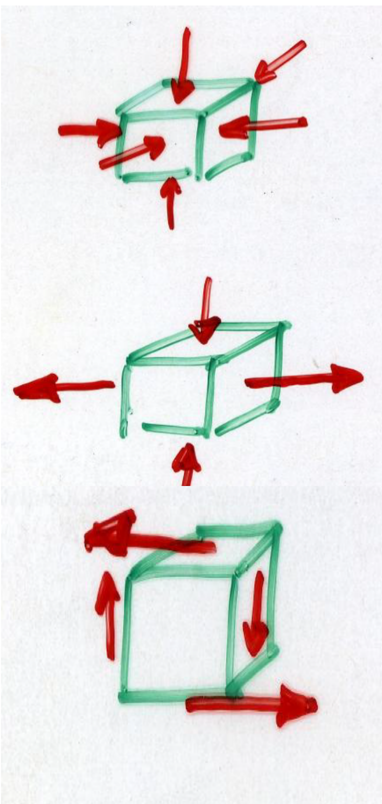
- weight of the overlying ice
- the shape of the glacier surface



Stress = force per unit area

Types of Stress

As a force/unit area, stress also has direction



Force can be directed normal to the area:

- Result is pressure if force is the same on all faces of a cube
- Result is normal stress if forces are different on different faces of a cube

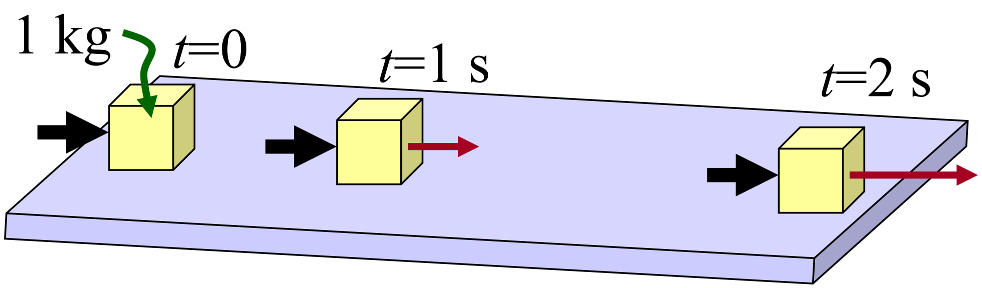
Forces can be directed parallel to the area:

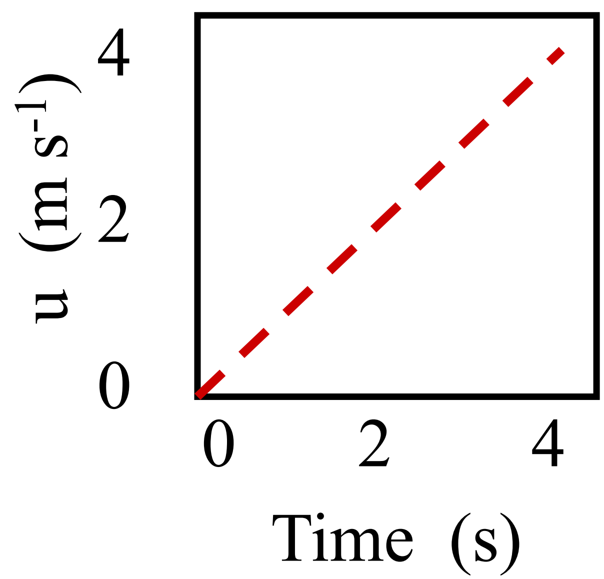
- Result is shear stress. Shear stress on internal areas parallel to the sloping glacier surface at depth z drives ice flow: $\tau = \rho\, g\, z\, \sin\theta$

Magnitude of Stress

Stress = force/unit area. Units are N/m^2 or Pascals (Pa)

N = Force = $M\ a$ = 1 kg x 1 m s^{-2}





This is the "push" that must be continually applied to a 1 kg mass to make it accelerate at 1 m s^{-2} on a frictionless tables

Opps…! Just dropped your water bottle

Bottle is in free fall…

- Volumes ~ 1 liter, What is the mass M?
- Gravitational acceleration $g = 9.8$ m s^{-2} (for back-of-the-envelope estimations $g = 10$ m s^{-2})
- $F = M\,g = 1$ kg x 10 m s^{-2} = 10 N

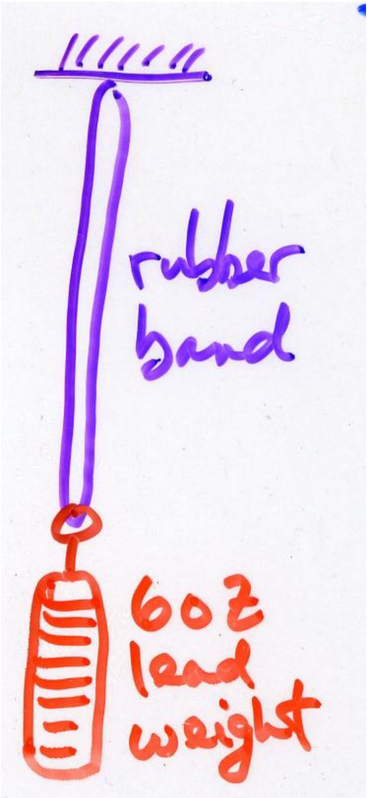
Stretching a rubber band

- Mass of lead weight: $M = 0.2$ kg
- Cross-sectional area of rubber band
  ~1 mm x 5 mm ~ 5 x 10^{-6} m^2
- Force stretching rubber band:

$F = M\,g = 0.2$ kg x 10 m s^{-2} = 2 N

- Stress in a rubber band:

$\tau = \frac{F}{Area} = \frac{2\text{ N}}{5 \times 10^{-6}\text{m}^2}$ = 4 x 10^5 Pa = 4 bar



- For comparison, note that 1 atmosphere ~ 10^5 Pa (1 bar)

Pressure in a Glacier

Mass $M = \rho V$

- ρ = ice density = 900 kg m^{-3}
- V = volume = Area x depth (z)

How deep do we have to drill into a glacier to before ice pressure is 1 atmosphere?

So pressure at depth z is:

- $P = \frac{F}{Area} = \frac{Mg}{Area} = \frac{\rho \times Area \times z \times g}{Area} = \rho\, g\, z$

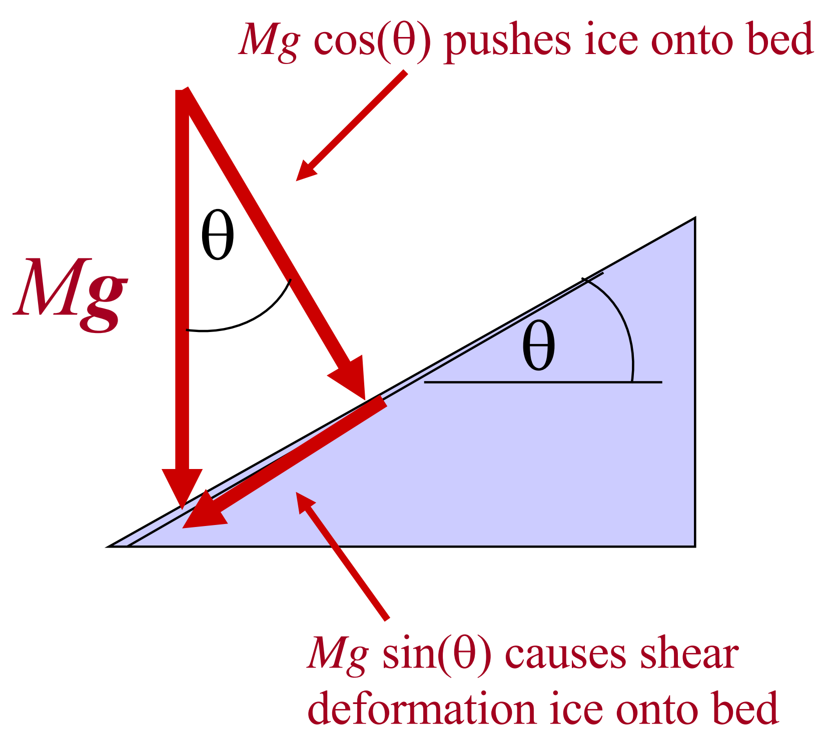
  900 kg/m^3 x 10 m s^{-2} x 11 m= 10^5 Pa = 1 bar

Shear Stress τ

Total force from ice column:

$F = M\,g = \rho V g = \rho\, g \times Area \times h$

How much of this weight will contribute to shear deformation?



$$\tau = \frac{F}{Area}$$

$$\tau = \frac{Mg\,\sin(\theta)}{Area} = \frac{\rho V g\,\sin(\theta)}{Area} = \frac{\rho(Area \times h)\,g\,\sin(\theta)}{Area} = \rho g h \sin(\theta)$$

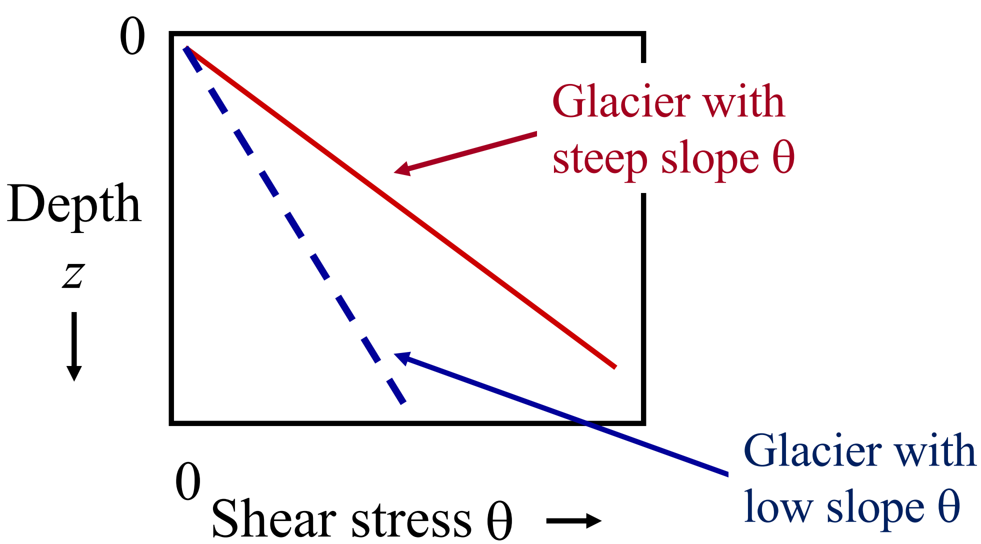
Units are Pa, just like pressure

How is Shear Stress related to Depth?

Shear stress increases with:

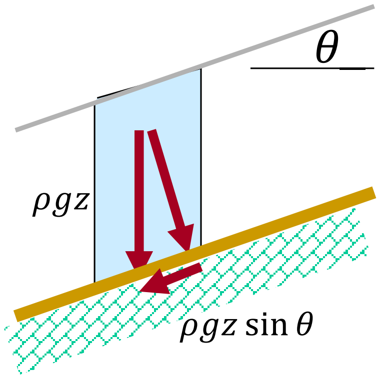
- depth z
- glacier slope θ

$$\tau = \rho g z \sin\theta$$



Shear Stress in a Glacier

$$\tau = \rho g z \sin(\theta)$$



A Mountain Glacier

If $z = h = 130$ m
$\theta = 5°$

$$\tau_b \approx 900\,\frac{\text{kg}}{\text{m}^3}\times 10\,\frac{\text{m}}{\text{s}^2}\times 130\text{m}\times\sin(5°) \approx 10^5 \text{ Pa=1 bar}$$

An Ice Sheet

If $z = h = 1000$ m
$\theta = 0.001°$

$$\tau_b \approx 900\,\frac{\text{kg}}{\text{m}^3}\times 10\,\frac{\text{m}}{\text{s}^2}\times 1500\text{m}\times\sin(0.005) \approx 10^5 \text{ Pa=1 bar}$$

This is a typical value for basal shear stress under a glacier

Are Glacier Thickness and Slope Related?

Suppose a glacier has become thicker or steeper (due to mass imbalance)→

- It flows faster
- It quickly reduces thickness h or slope θ, until $\tau_b \sim 1$ bar again

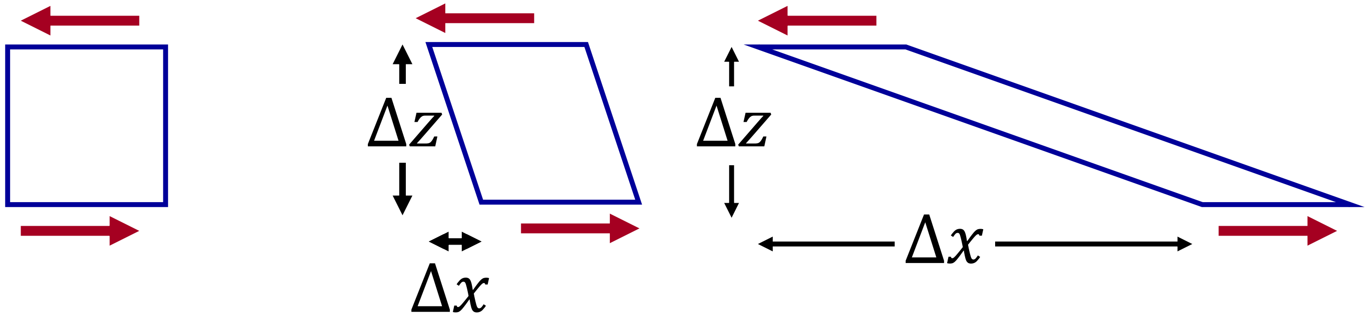
Can we estimate glacier thickness (z = h) from its (known) slope, if we know $\tau_b \sim 1$ bar?

$$\tau = \rho g z \sin(\theta)$$

$$h \sim \tau_b / [\rho g \sin(\theta)]$$

Strain Rate $\dot{\varepsilon}$

At start … after a short time … and later …



Shear strain
(deformation)

$$\varepsilon = {}^{1}\!/_{2}\left(\frac{\Delta x}{\Delta z}\right)$$

(What are its units?)

Shear-strain RATE
(rate of deformation)

(Units are time^{-1})

$$\dot{\varepsilon} = \left(\frac{\Delta \varepsilon}{\Delta t}\right) = {}^{1}\!/_{2}\left(\frac{\Delta x/_{\Delta z}}{\Delta t}\right) = {}^{1}\!/_{2}\left(\frac{\Delta x}{\Delta z \Delta t}\right) = {}^{1}\!/_{2}\left(\frac{\Delta x/_{\Delta t}}{\Delta z}\right) = {}^{1}\!/_{2}\left(\frac{\Delta u}{\Delta z}\right)$$

Constitutive Relations

Different materials respond differently to applied forces and stresses

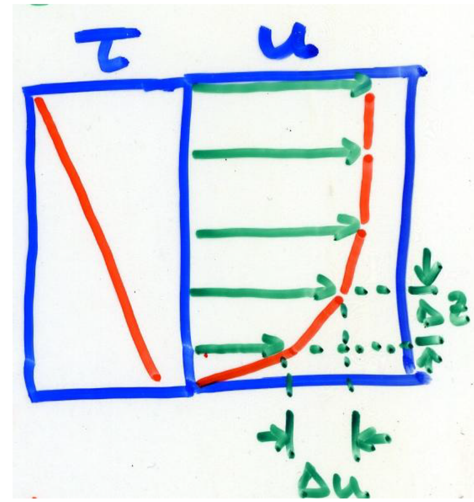
A **constitutive relation** is a relationship between stress and strain rate for a given material

Constitutive Behavior of Ice

Glen's Law (constitutive relationship for ice)

$$\frac{\Delta u}{\Delta z} = 2\dot{\varepsilon} = 2A\tau^n$$

z



A and *n* are characteristics of the material (*A* is not the area, but is the constitutive parameter describing softness of ice)

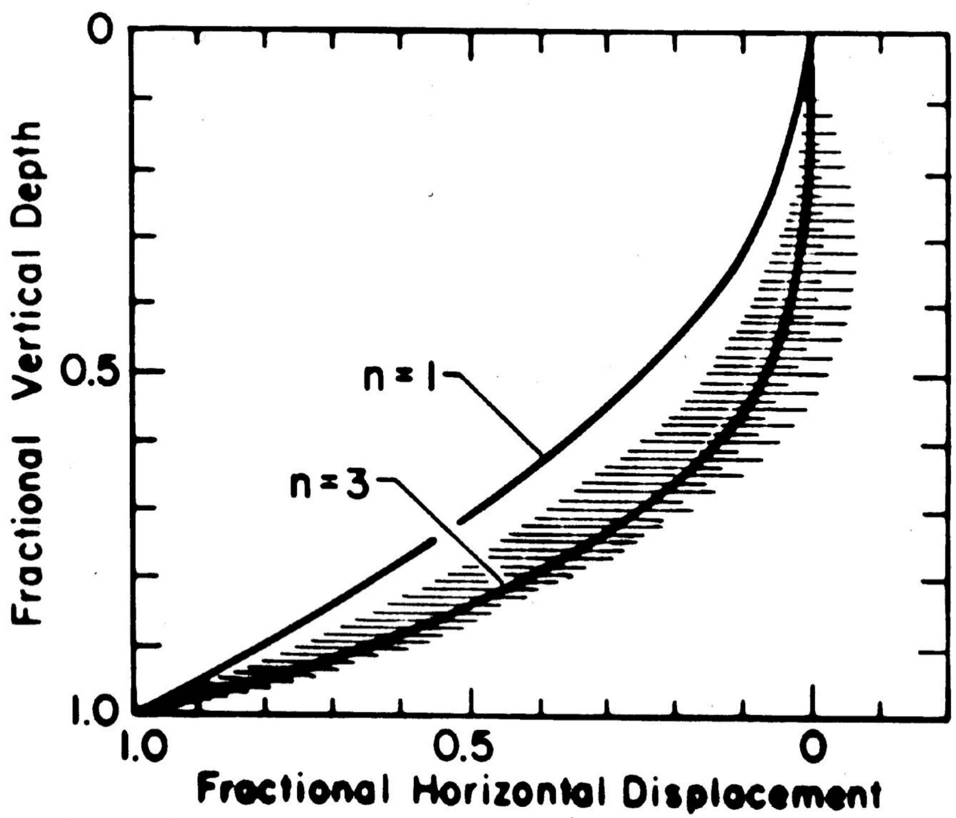
For ice:

- $n \sim 3$
- $A \sim 2\times10^{-16}$ Pa^{-3} yr^{-1} at 0ºC
- $A \sim 6\times10^{-18}$ Pa^{-3} yr^{-1} at -20ºC

These numbers distinguish is from motor oil, silly putty, molasses

Velocity Profiles in a Glacier: Does $n = 1$ or does $n = 3$?

- Does Viscous fluid: $n = 1$
- Ice (generally): $n = 3$



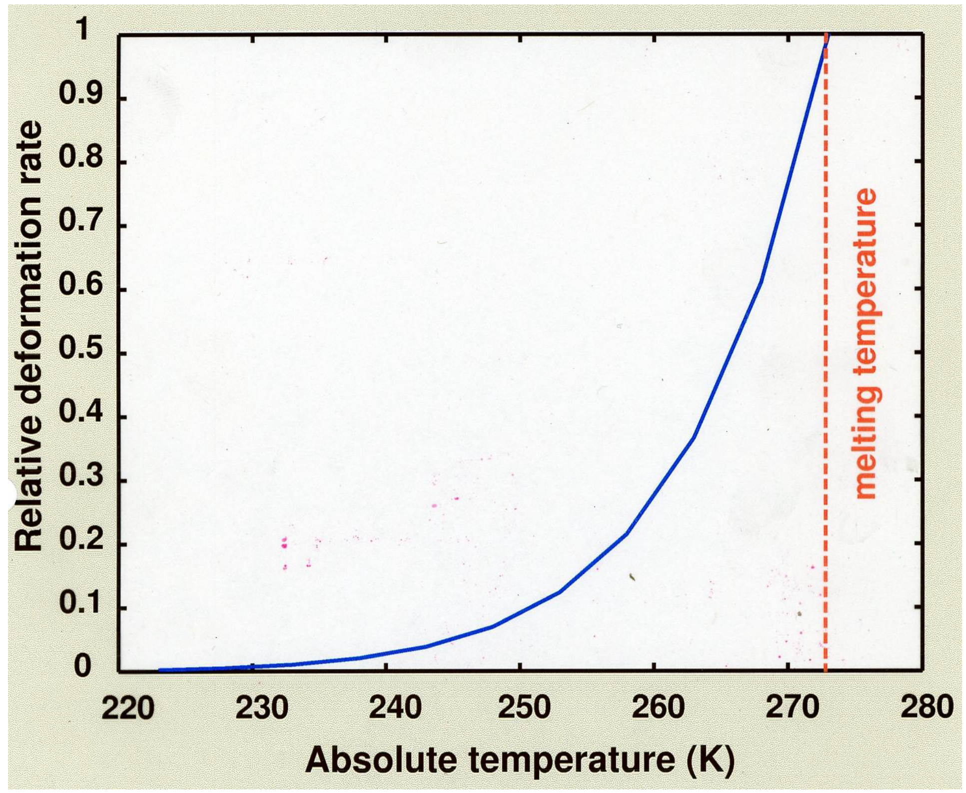
Temperature Dependence of Ice Flow

Deformation rate is proportional to $\exp(-\frac{Q}{RT})$

$Q = 60$ kJ mol^{-1} activation energy for creep

$R = 8.3142$ J mol^{-1} K^{-1} gas constant

T K temperature



For Comparison: A Viscous Fluid ($n = 1$)

$$\frac{\Delta u}{\Delta z} = 2\dot{\varepsilon} = \frac{\tau}{\eta}$$

η = viscosity, which is different for:

- Water
- Cold engine oil

Effect of $n = 3$ in ice:

The higher the shear stress τ, the softer ice becomes

$$\frac{\Delta u}{\Delta z} = 2\dot{\varepsilon} = 2A\tau^{n} = \frac{\tau}{{}^{1}/_{(2A\tau^{n-1})}}$$

As stress gets larger, viscosity gets smaller

Velocity Profile in a Temperate Glacier

If you do the calculus:

$$u(z) = \frac{A}{2}(\rho g \sin\theta)^3(h^4 - z^4)$$

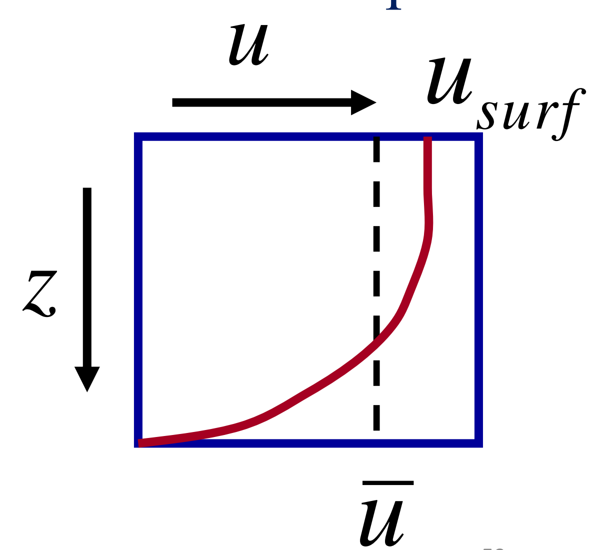
At the surface (z = 0):

$$u(z) = \frac{A}{2}h\ \ (\rho g h \sin\theta)^3 = \frac{A}{2}h\tau_b^3$$

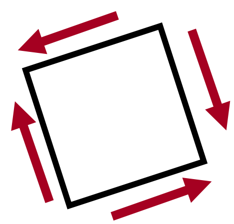
Averaged over depth h:

$$\bar{u} = \left(\frac{n+1}{n+2}\right)u_{surf} = {}^{4}/_{5}\,u_{surf}$$

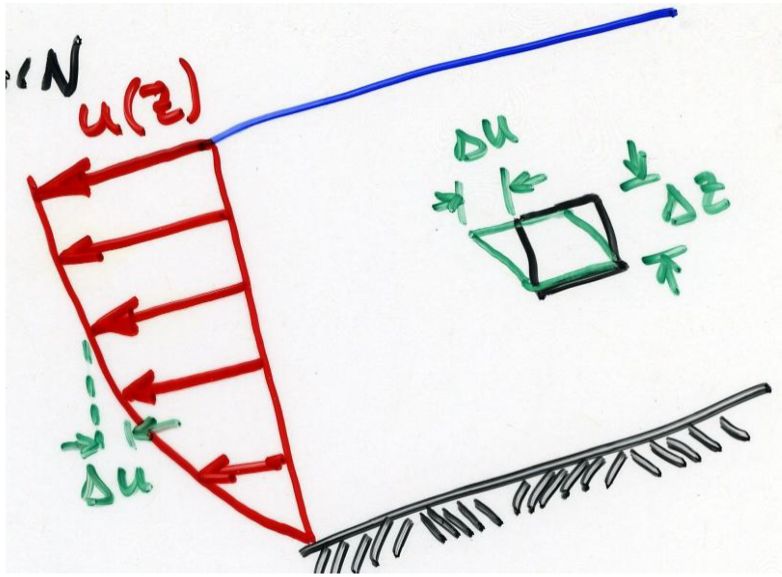
- A = softness (parameter)
- n = 3 stress exponent
- h = ice thickness
- z = depth
- θ = surface slope



Summary: Stress and Strain Rate



Shear stress τ
(Force/Area)



Shear strain rate (time^{-1})

Summary: Mechanical Behavior of Ice

Linear (Newtonian) fluid

- η = viscosity

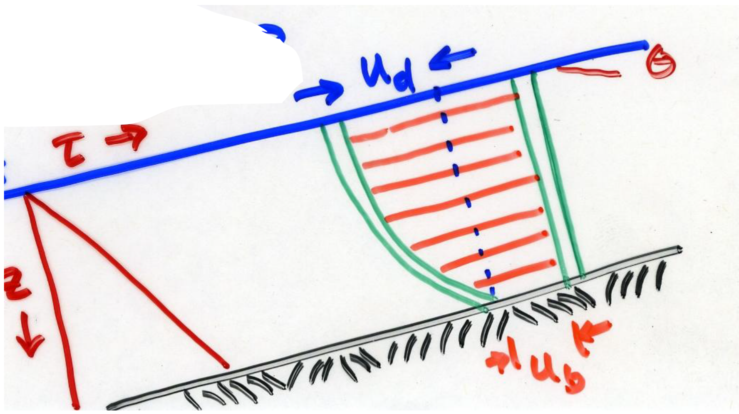
$$\frac{\Delta u}{\Delta z} = 2\dot{\varepsilon} = \frac{\tau}{\eta}$$

Ice is a nonlinear fluid

- A = softness (rheological parameter)
- $n = 3$ stress exponent

$$\frac{\Delta u}{\Delta z} = 2\dot{\varepsilon} = 2A\tau^{n}$$

Both τ and $^{du}/_{dz}$ increase with depth z



Summary: Glacier Motion

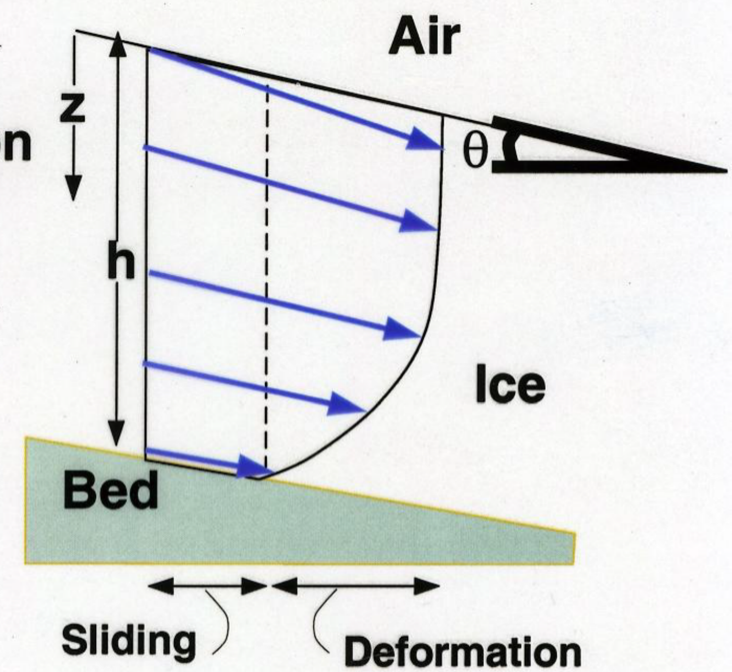
- glaciers move by a combination of sliding and internal deformation

- internal deformation is driven by shear stress $\tau = \rho g z \sin(\theta)$

- strain rate depends on stress and temperature T

$$\dot{\varepsilon} = 1/2\ \partial u/\partial z = A(T)\,\tau^{n} \quad (n=3)$$

- $$u(z) = \frac{2A(\rho g \sin(\theta))^{n}(h^{n+1} - z^{n+1})}{n+1}$$



Summary: Deformation Velocity

$$u(z) = \frac{2A}{(n+1)}(\rho g \sin\theta)^n (h^{n+1} - z^{n+1})$$

$$= \frac{2A}{(n+1)}(\rho g \sin\theta)^n h^{n+1}\left(1 - \left(\frac{z}{h}\right)^{n+1}\right)$$

$$= \frac{2A}{(n+1)}(\rho g h \sin\theta)^n h\left(1 - \left(\frac{z}{h}\right)^{n+1}\right)$$

$$= \frac{2A}{(n+1)}\tau_b^n h\left(1 - \left(\frac{z}{h}\right)^{n+1}\right)$$

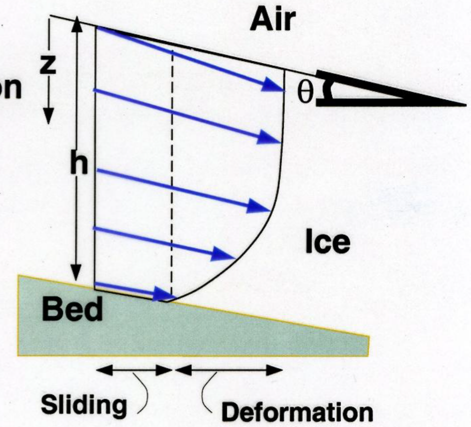
Some Typical Glacier Speeds

- glaciers move by a combination of sliding and internal deformation

- internal deformation is driven by shear stress $\tau = \rho g z \sin(\theta)$

- strain rate depends on stress and temperature T

$$\dot{\varepsilon} = 1/2\ \partial u/\partial z = A(T)\ \tau^n \quad (n=3)$$



- $$u(z) = \frac{2A(\rho g \sin(\theta))^n (h^{n+1} - z^{n+1})}{n+1}$$

h (m)	θ (deg)	T (°C)	u_{def} ($m\ a^{-1}$)
100	5	0	4.7
100	10	0	38
200	5	0	75
200	5	-10	5.4
200	5	-20	1.8