Glacier Thermodynamics: Ice Temperature and Heat Transfer Processes

ESS431: Principles of Glaciology

ESS505: The Cryosphere

Wednesday, 10/24 – Ben Hills

Today's Objectives:

- Why do we care about ice temperature?
- Cover the fundamentals of heat transfer. What are the important time/length scales for heat transfer in ice?
- What is the general thermal structure of a glacier or an ice sheet? What heat transfer processes are important in which regions?

Importance of Ice Temperature

Background on Heat Transfer

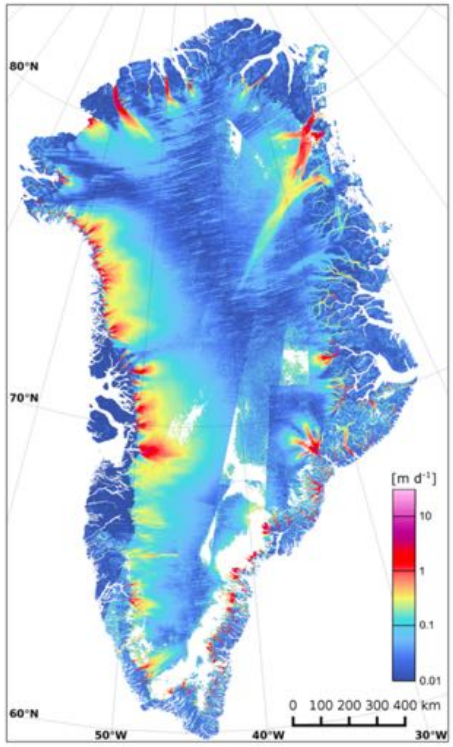
Ice Sheet Thermal Structure

Why Ice Temperature?

1. Ice Deformation (Viscosity)

$$\dot{\epsilon} = A(T)\tau^n$$

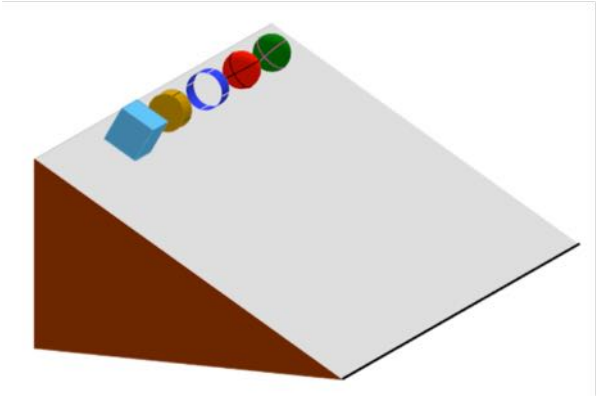
$$A(T) = A_0 e^{-\frac{Q_a}{RT}}$$

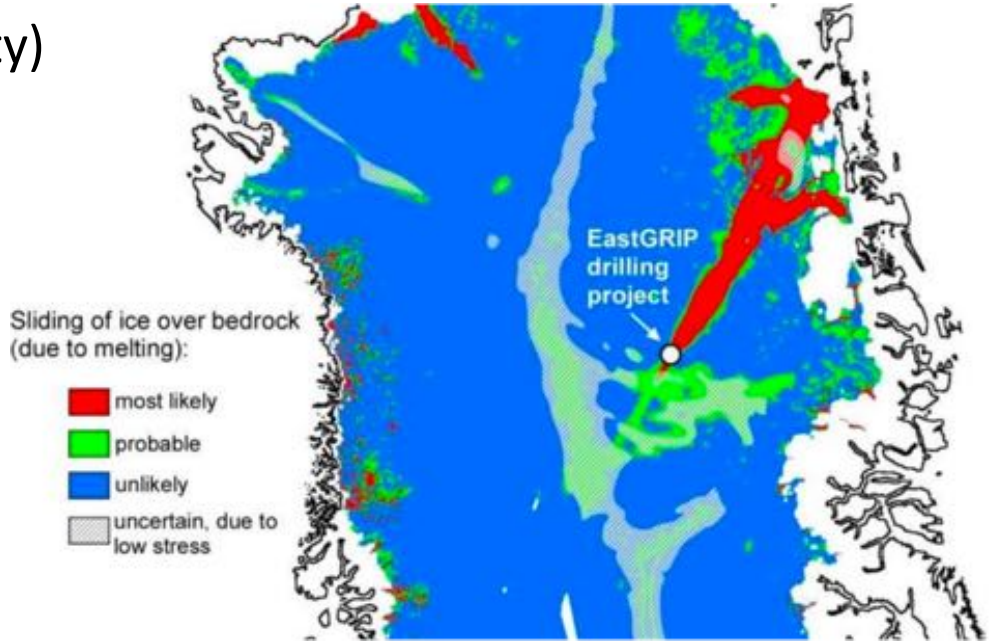


Joughin et al. (2010)

Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding

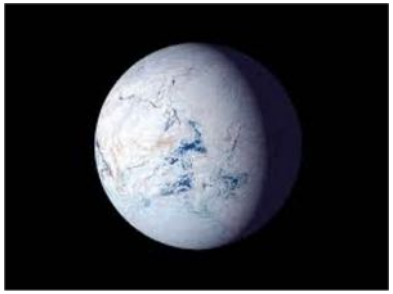


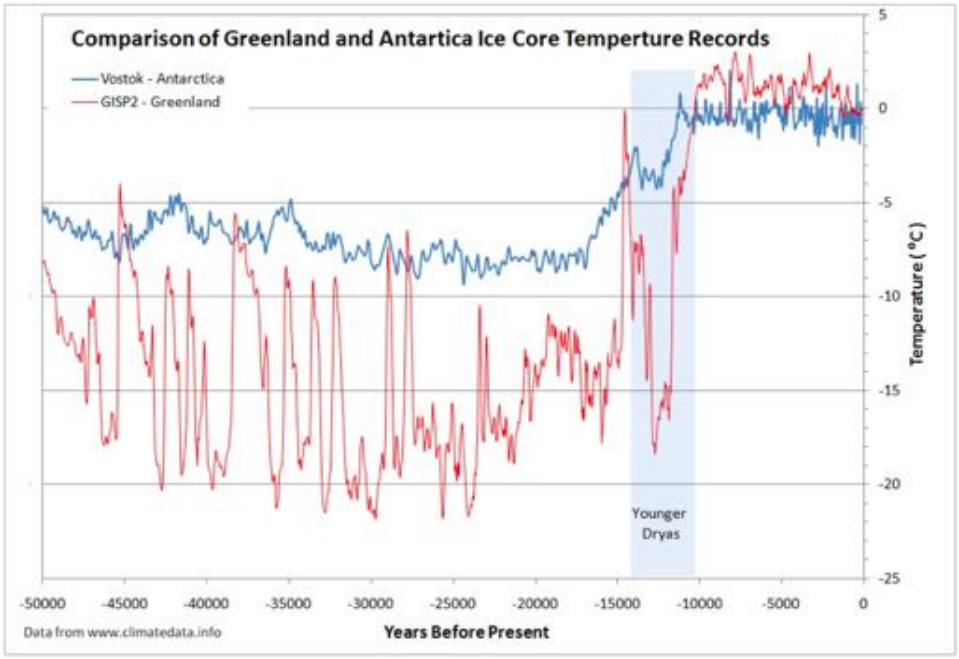


Credit: Paul D. Bons, Ilka Weikusat

Why Ice Temperature?

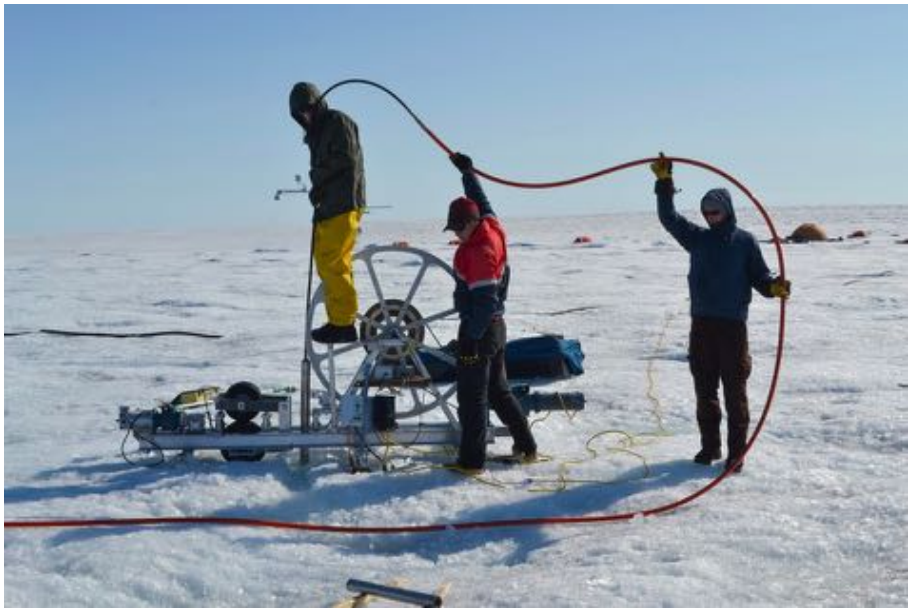
1. Ice Deformation (Viscosity)
2. Basal Sliding
3. Paleoclimate





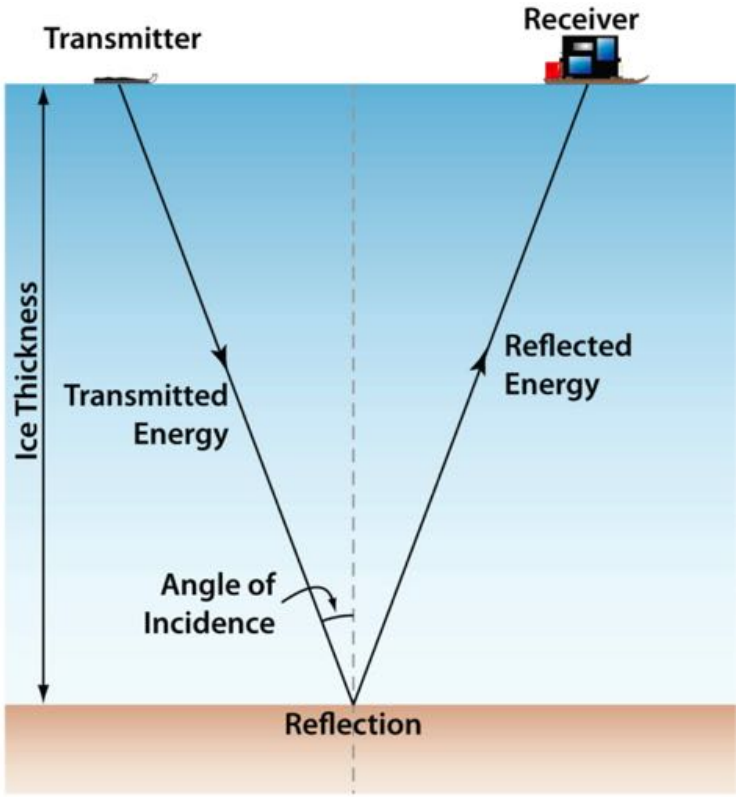
Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding
3. Paleoclimate
4. Drilling



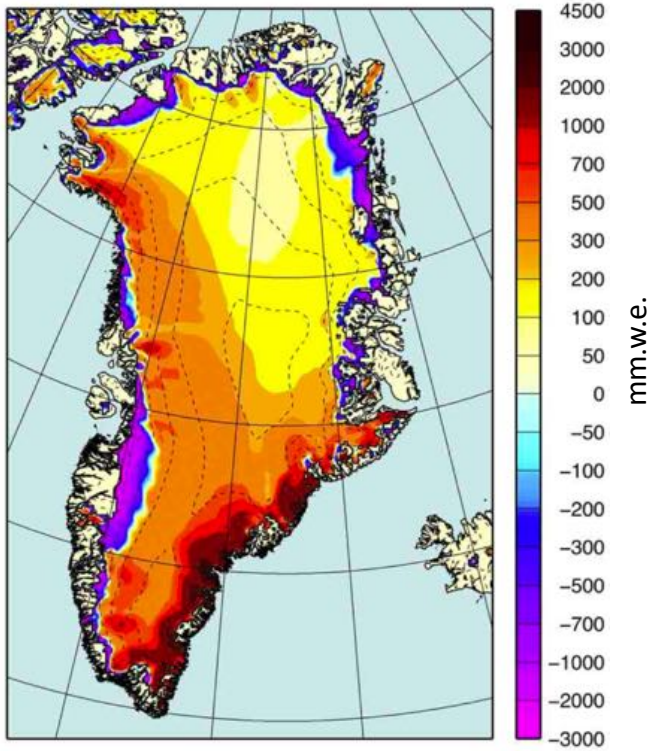
Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding
3. Paleoclimate
4. Drilling
5. Geophysics
   (Radar and Seismic)



Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding
3. Paleoclimate
4. Drilling
5. Geophysics
   (Radar and Seismic)
6. Mass Balance



van den Broeke et al. (2011)

Importance of Ice Temperature

Background on Heat Transfer

Ice Sheet Thermal Structure

Background on Heat Transfer

Laws of Thermodynamics

1. Energy Conservation:
   - Energy can neither be created nor destroyed; energy can only be transferred or changed from one form to another.
   - $\Delta E = Q - W$

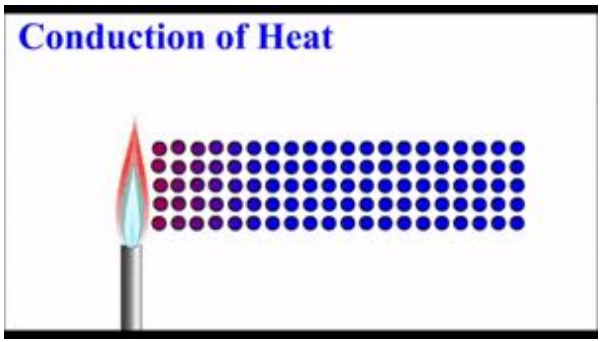
2. Entropy
   - The entropy of any isolated system always increases. Isolated systems spontaneously evolve towards thermal equilibrium—the state of maximum entropy of the system.
   - $\Delta S \geq 0$

Background on Heat Transfer

Modes of Heat Transfer

1. Conduction

   - Transfer of heat via direct molecular collision.



Background on Heat Transfer

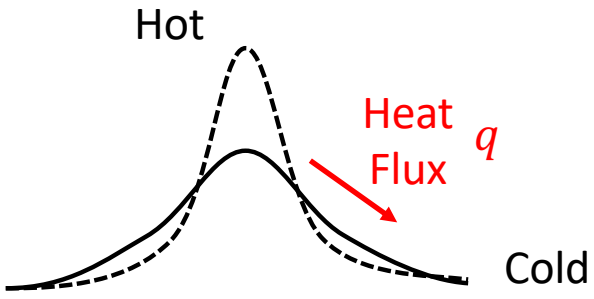
Modes of Heat Transfer

1. Conduction
   - Transfer of heat via direct molecular collision.

Fourier's Law:

$$q = -k\nabla T$$

k is the thermal conductivity of the material

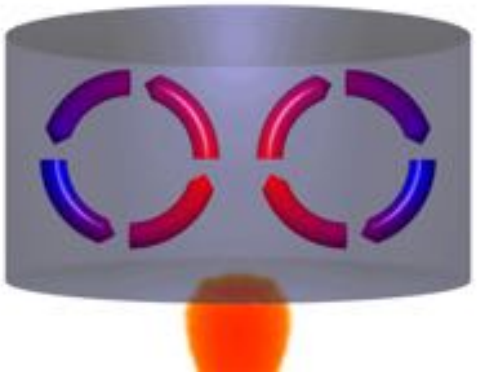


Background on Heat Transfer

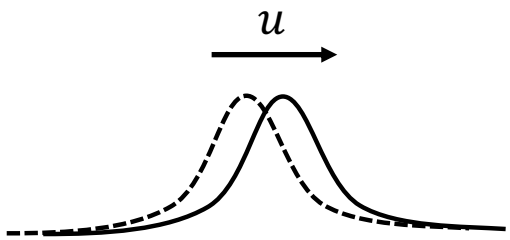
Modes of Heat Transfer

1. Conduction
   - Transfer of heat via direct molecular collision.

2. Convection
   - Thermal energy is carried by a moving fluid.



$$\frac{\mathrm{dT}}{\mathrm{dt}} = -u\nabla T$$



Background on Heat Transfer

Modes of Heat Transfer

1. Conduction

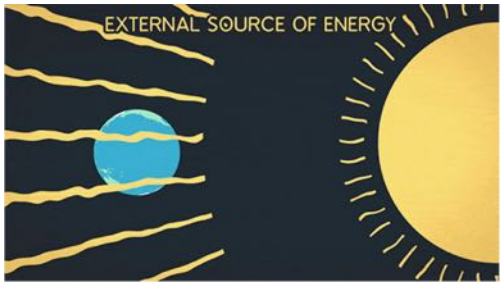
   - Transfer of heat via direct molecular collision.

2. Convection

   - Thermal energy is carried by a moving fluid.

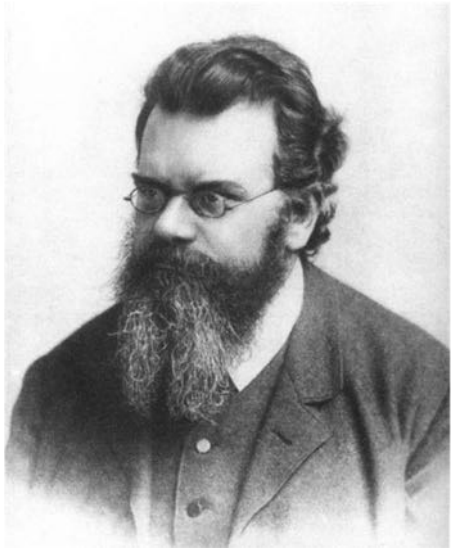
3. Radiation

   - The emission of electromagnetic waves carries energy away from a thermal body.



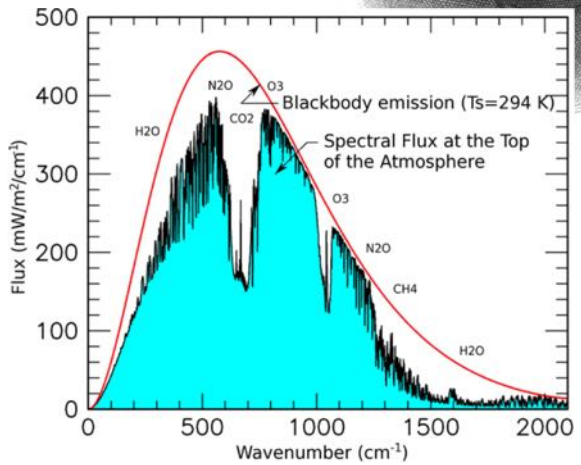
Background on Heat Transfer

3. Radiation
   - The emission of electromagnetic waves carries energy away from a thermal body.

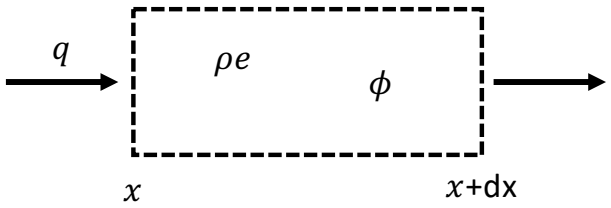


Stefan-Boltzmann Law:

$$E = \sigma T^4$$



The Heat Equation



$$\frac{d(\rho e)}{dt} = q\Big|_x - q\Big|_{x+dx} + \phi$$

$$\frac{d(\rho e)}{dt} = -\frac{\partial q}{\partial x} + \phi$$

$$\frac{d\rho}{dt} C_p T + \rho C_p \frac{dT}{dt} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \phi = 0$$

Incompressibility and Total Derivative

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathrm{u}\frac{\partial T}{\partial x}\right) = -k\frac{\partial^2 T}{\partial x^2} + \phi$$

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} - \mathrm{u}\frac{\partial T}{\partial x} + \frac{\phi}{\rho C_p}$$

Heat Capacity Definition

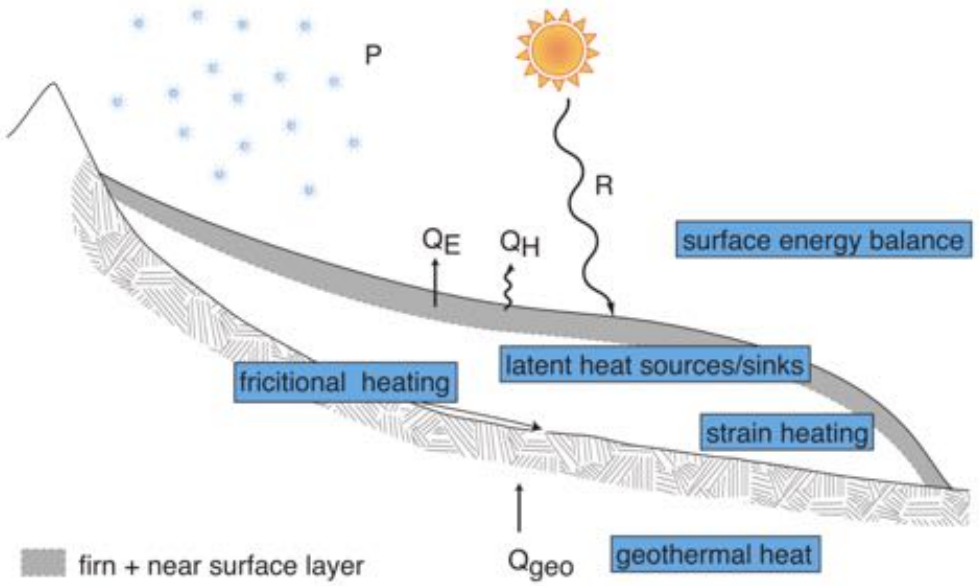
$$e(T) = C_p T$$

Total Derivative

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial T}{\partial t} + u \cdot \nabla T$$

The Heat Equation for Ice

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$



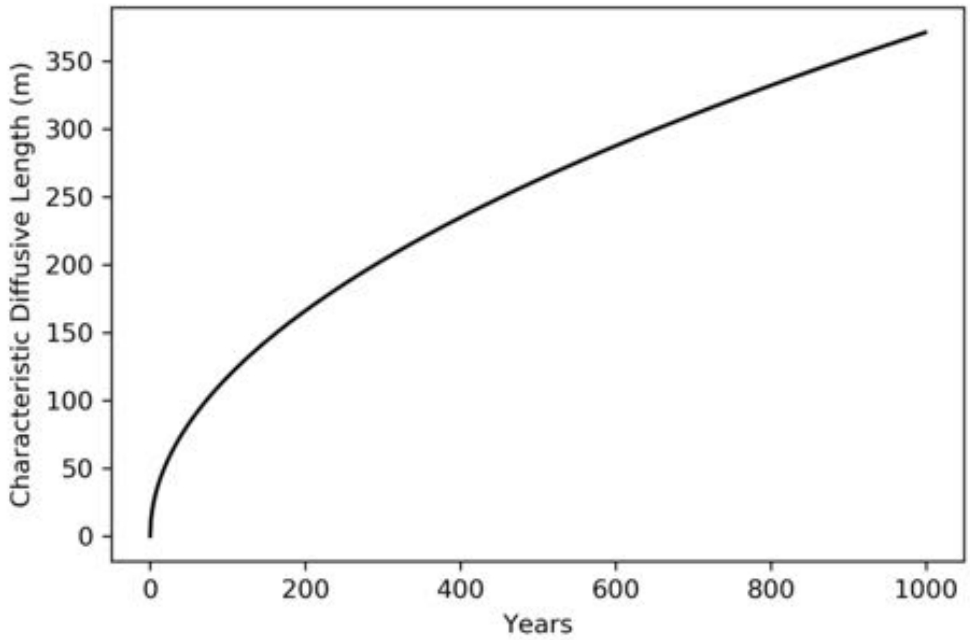
- Diffusion
  (mostly vertical)
- Advection
  (all 3 directions)
- Sources
  (see figure)

Characteristic Diffusion Length

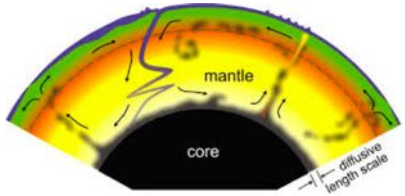
$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

$$l = 2\sqrt{\alpha t}$$

Copper	1.11×10^{-4} m²/s
Iron	2.3×10^{-5} m²/s
Air	1.9×10^{-5} m²/s
Stainless Steel	4.2×10^{-6} m²/s
Ice	**1.09×10^{-6} m²/s**
Water	1.43×10^{-7} m²/s



Peclet Number



$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

Gulf Stream Pe~10^{12}

Ice Sheet (horizontal) Pe~10^{5}

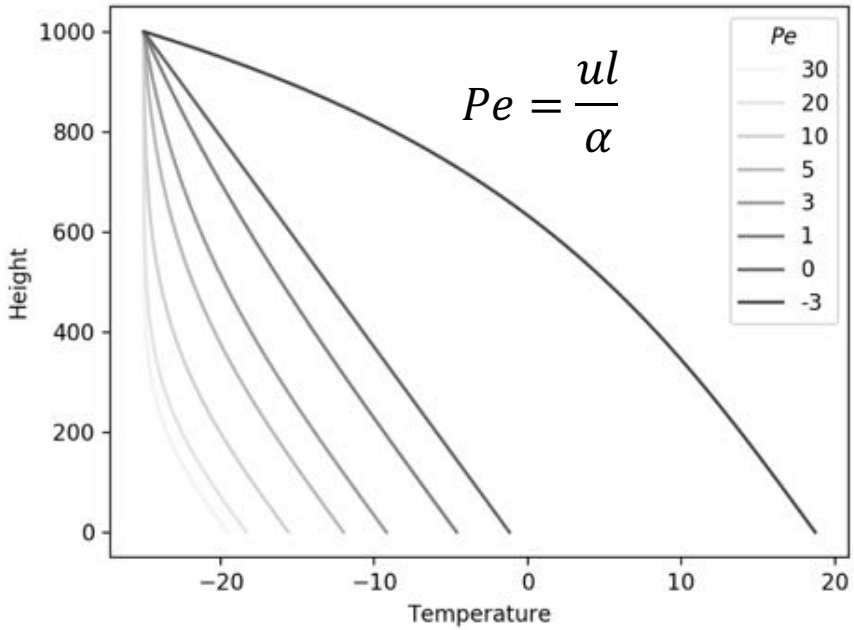
Solid earth (mantle convection) Pe~10^{3}

Ice Sheet (vertical) Pe~10^{0}-10^{1}

<u>Near-surface</u>

Ice Sheet Pe~10^{-1}

Granite Outcrop Pe<<1



Importance of Ice Temperature

Background on Heat Transfer

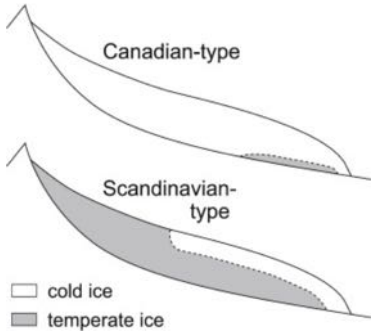
Ice Sheet Thermal Structure

Thermal Classifications of Glaciers

Temperate
- High Accumulation
- High Melt
- Fast Moving

Cold
- Low Accumulation
- Low Melt
- Slow Moving

Polythermal
- Possibly bridges climate zones (i.e. cold at top and warm at bottom)
- Includes both ice sheets



Regions

1. Surface
2. Bed

Boundary Conditions

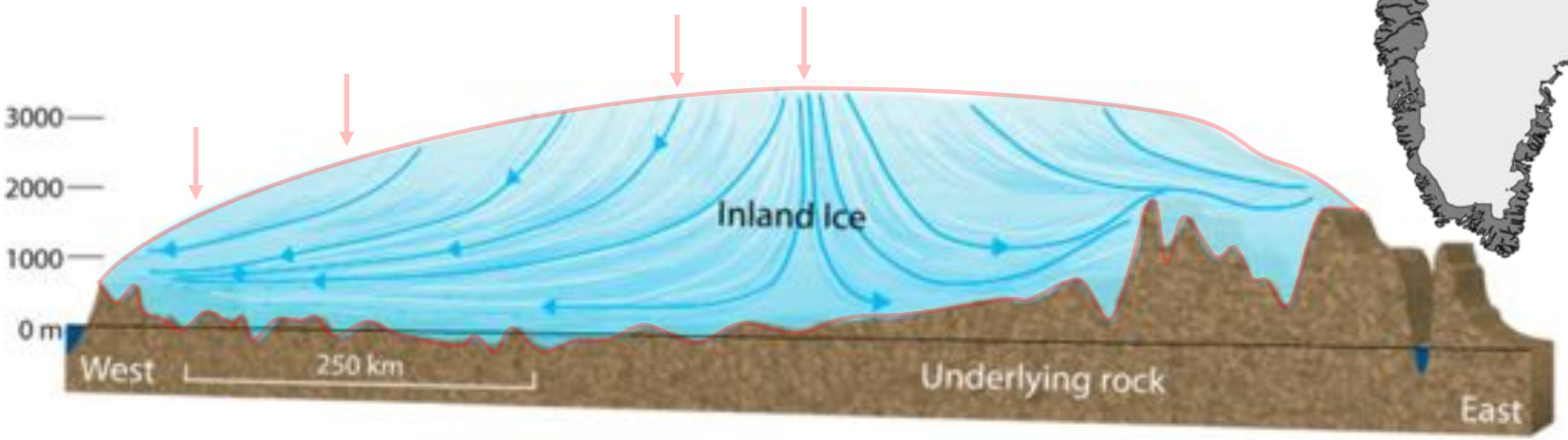
3. Ice Divide
4. Downstream of Divide

Advection

Frictional Heating

5. Fast-Flowing Ice
6. Ablation Zone

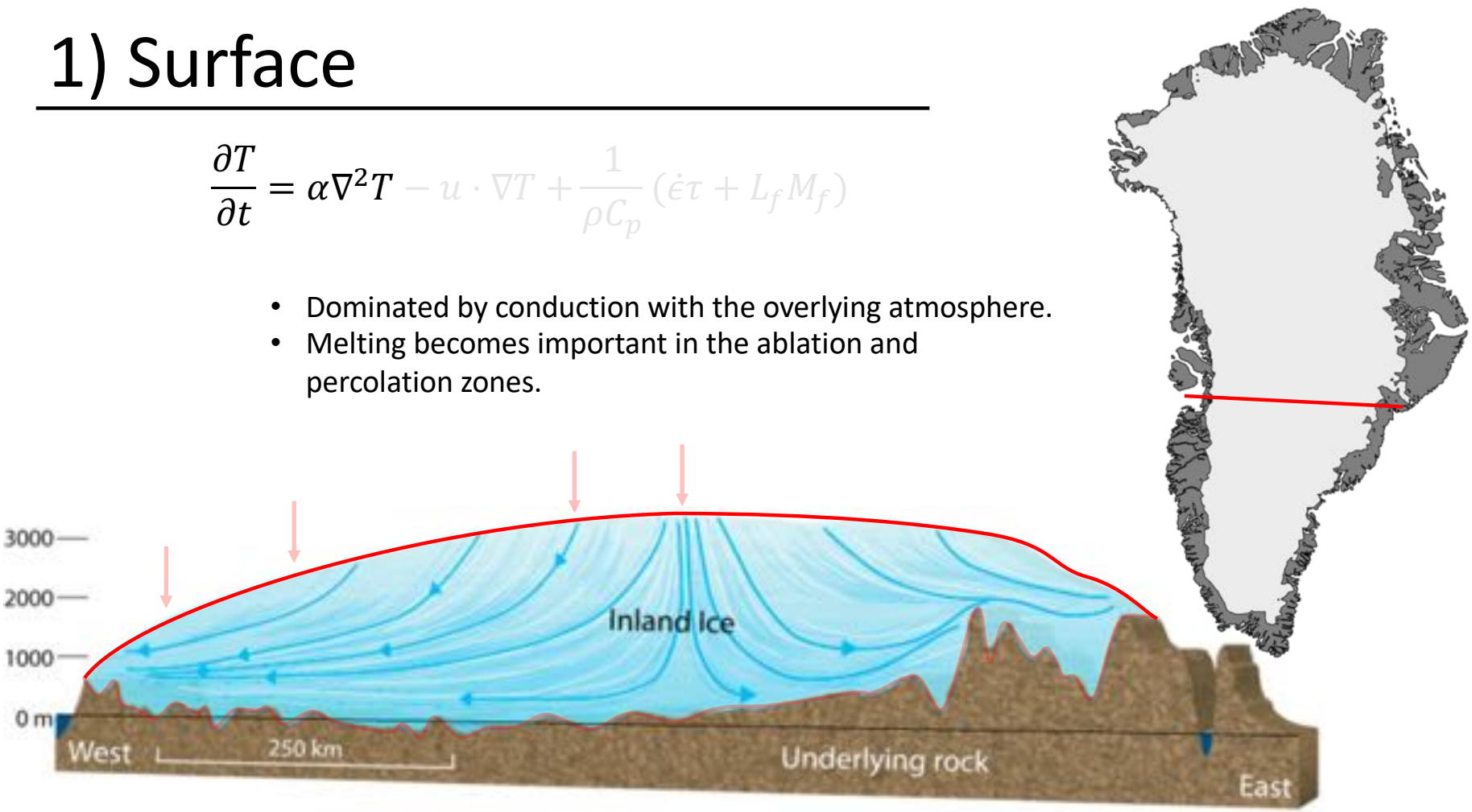
Latent Heating



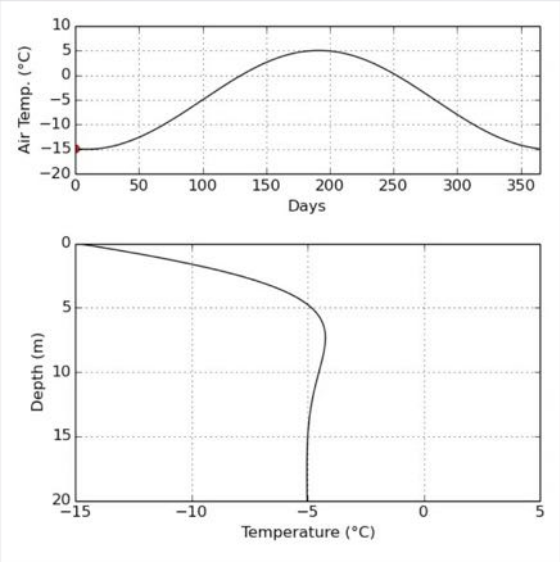
1) Surface

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

- Dominated by conduction with the overlying atmosphere.
- Melting becomes important in the ablation and percolation zones.

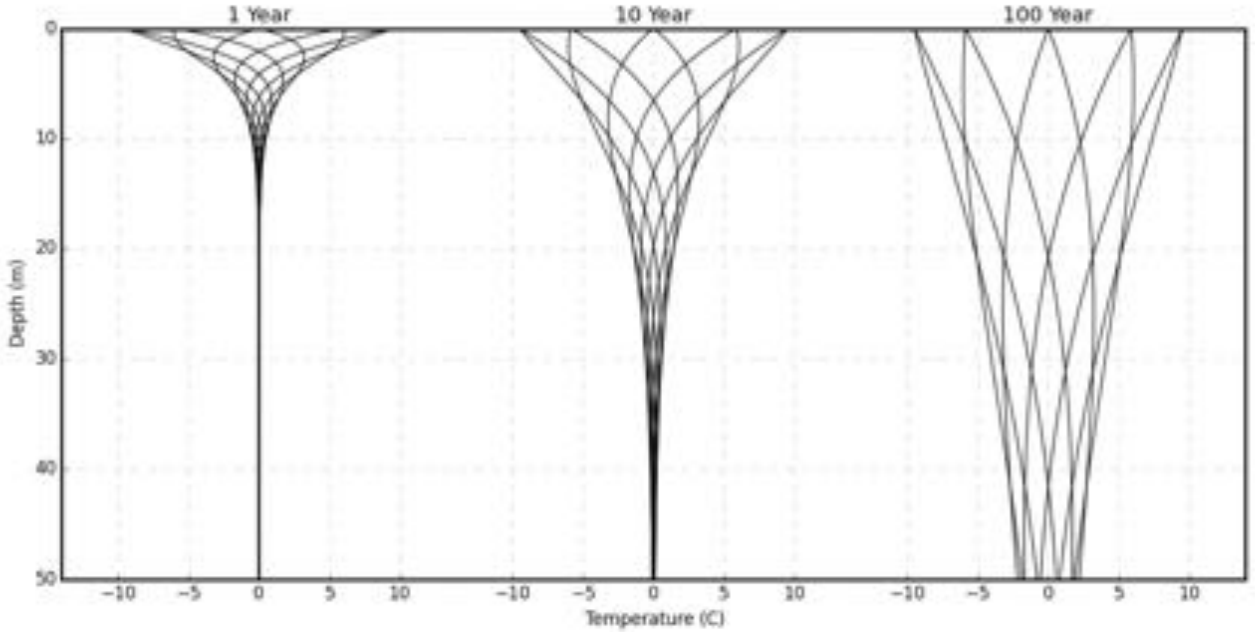


1) Surface


Air Temp. (°C)
10
5
0
−5
−10
−15
−20
0
50
100
150
200
250
300
350
Days
Depth (m)
0
5
10
15
20
−15
−10
−5
0
5
Temperature (°C)

1) Surface

Skin Depth is period dependent



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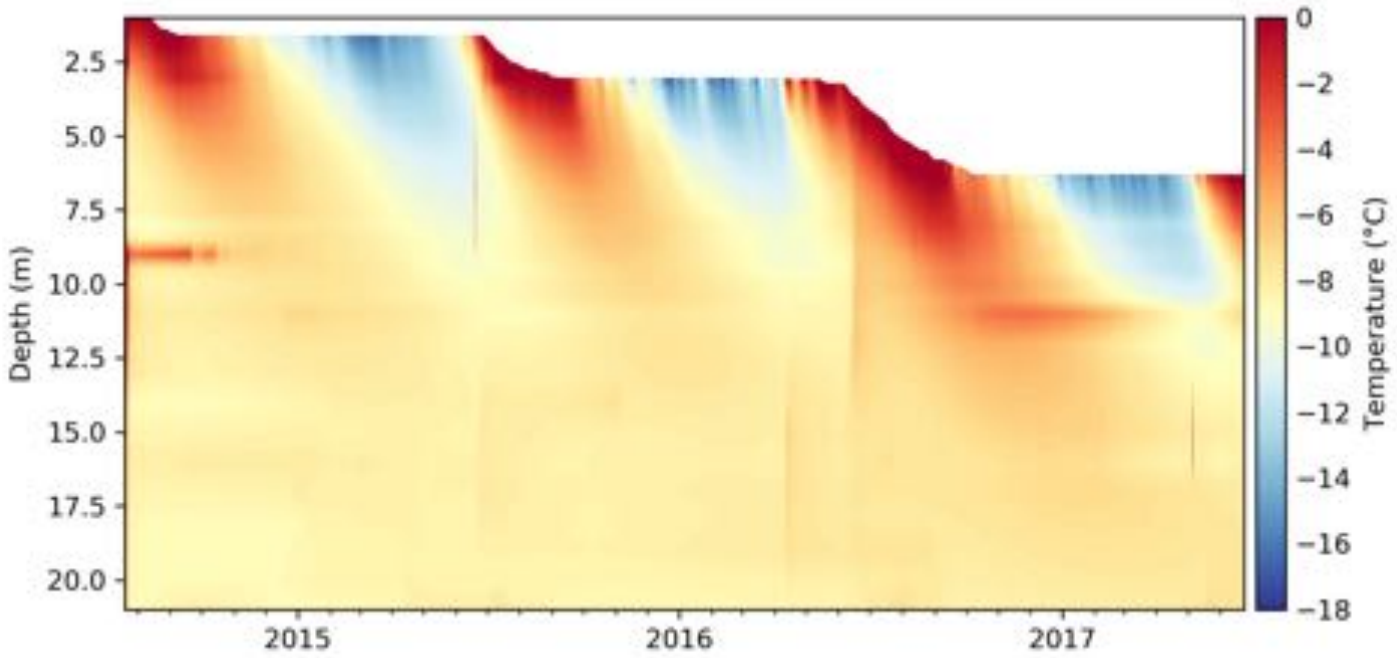
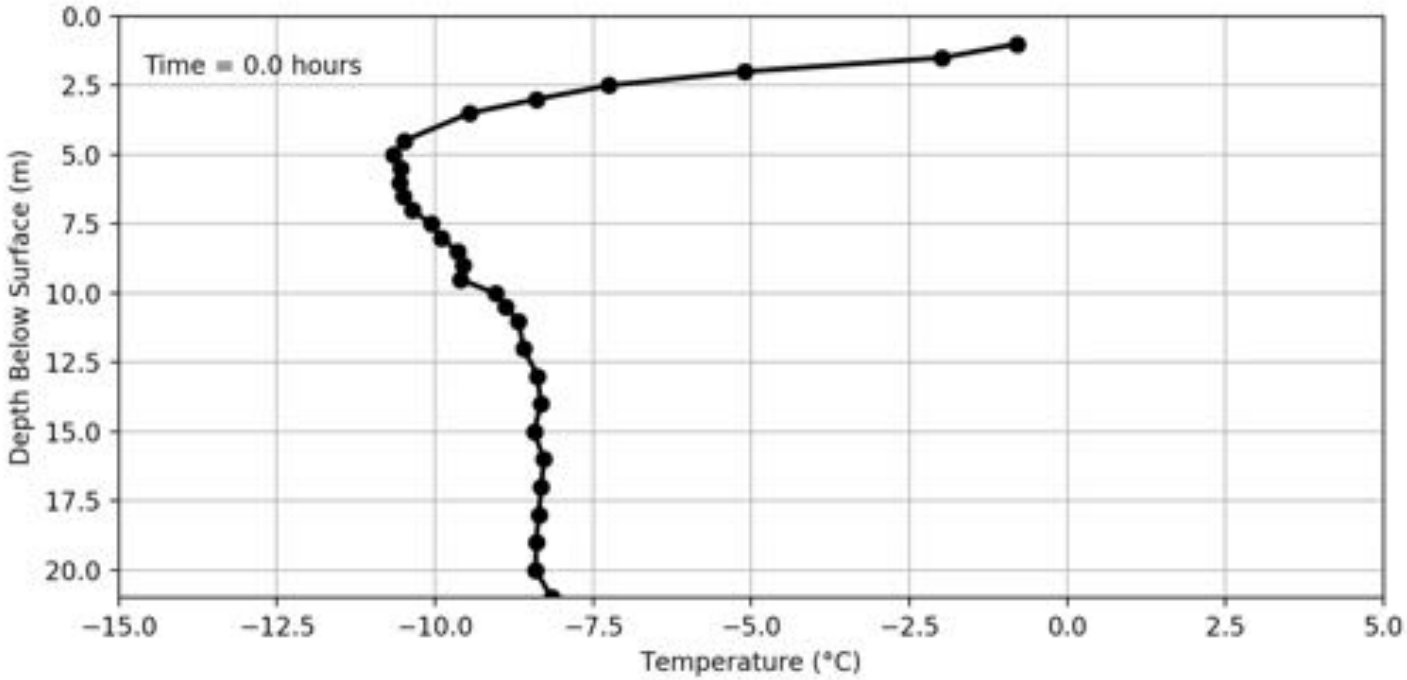


Hills et al. (2018)

1) Surface

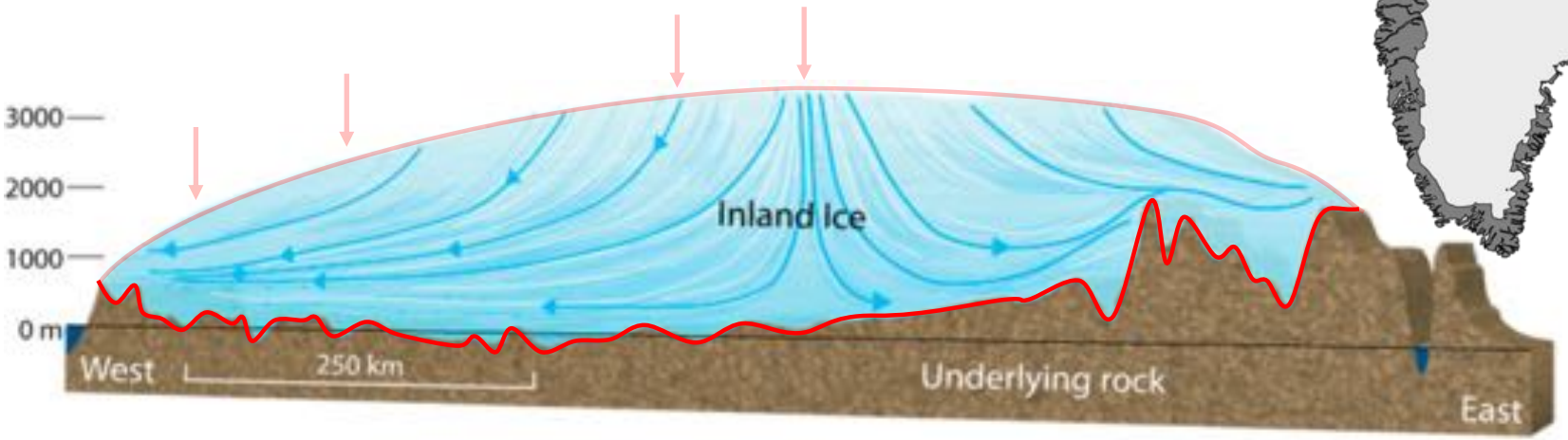


Hills et al. (2018)

2) Bed

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

- Heat sources from the friction of ice sliding over bedrock and from geothermal heating.
- Basal melting can add to the total mass balance, but is mostly important for its influence on sliding.



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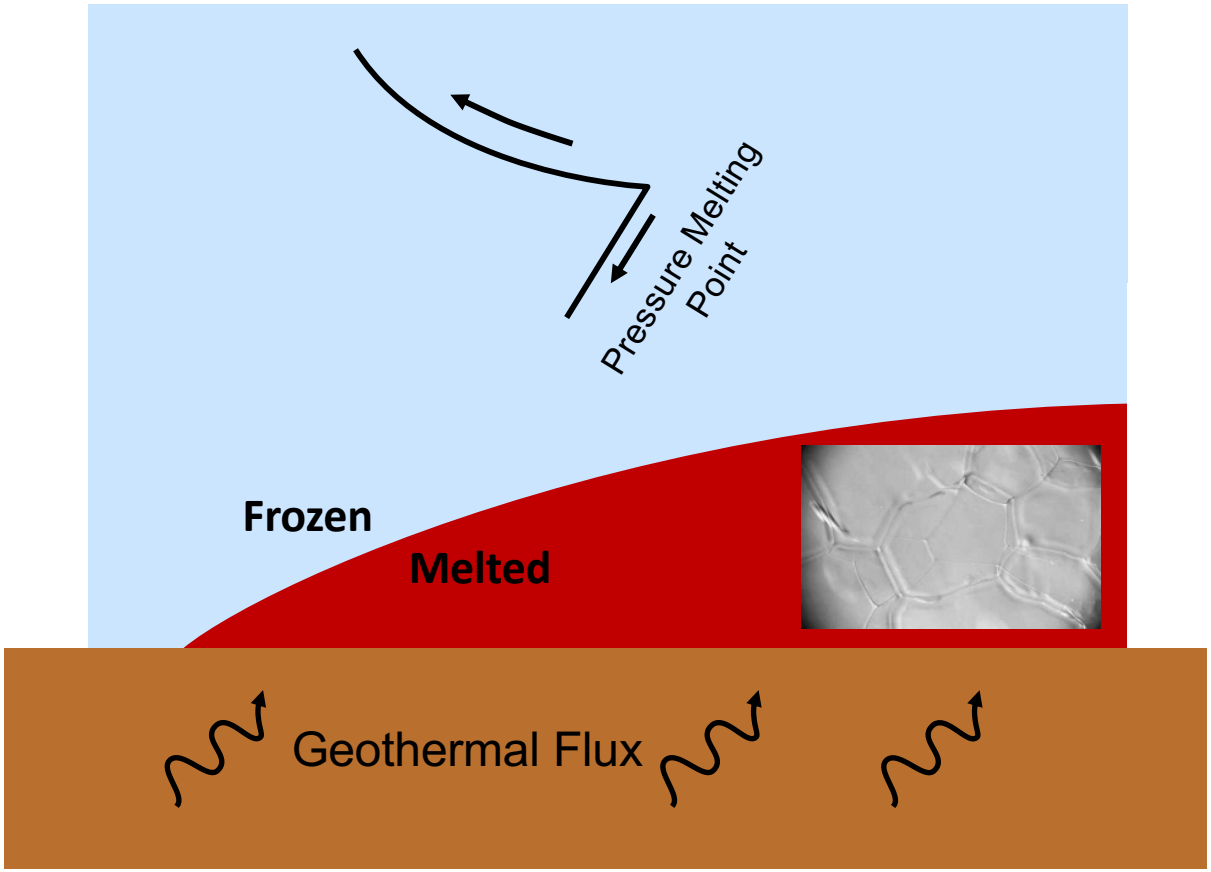
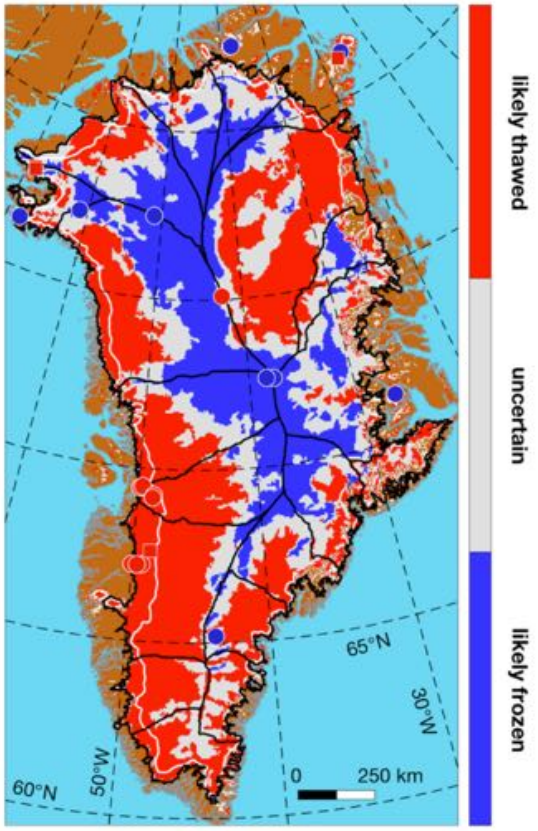


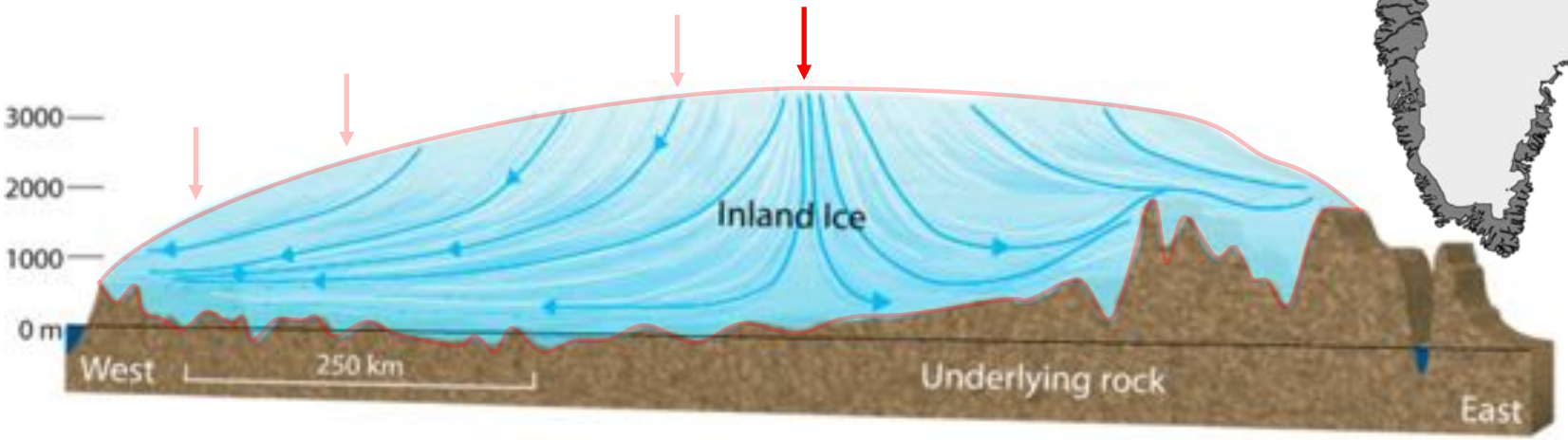


MacGregor et al. (2016)

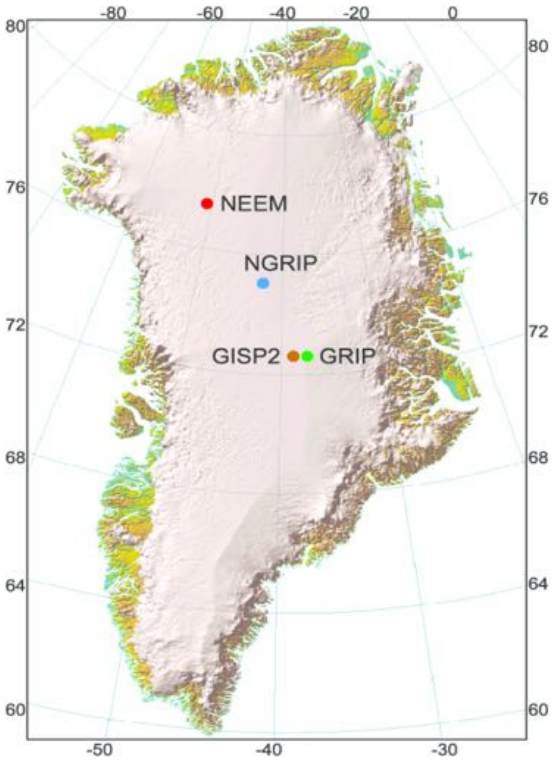
3) Ice Divide

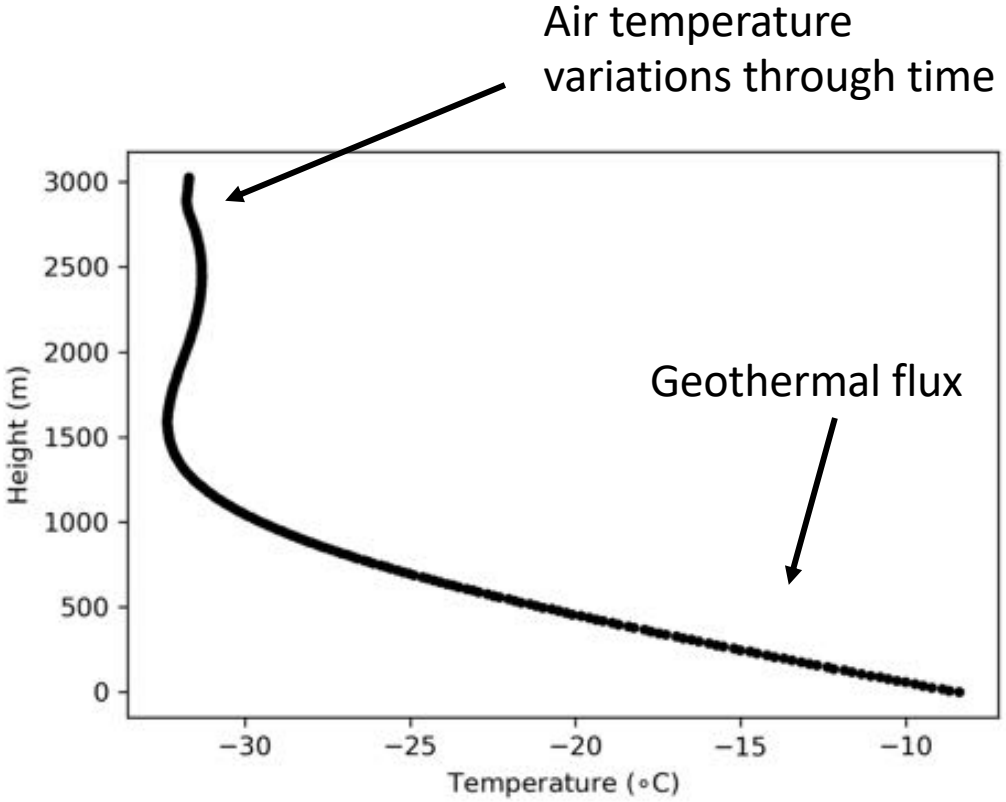
$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

- Ice flow is purely vertical.
- The only source term here is firn compaction which is small compared to shear strain or latent heating.



3) Ice Divide

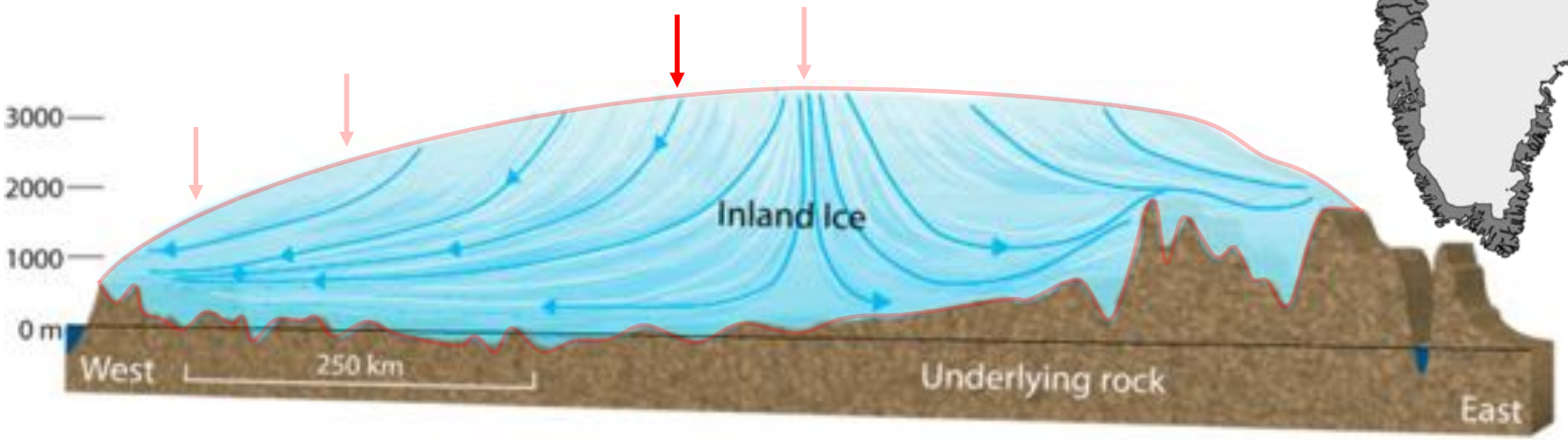

-80
-60
-40
-20
0
80
76
72
68
64
60
NEEM
NGRIP
GISP2
GRIP
-50
-40
-30


Air temperature variations through time
Geothermal flux
Height (m)
3000
2500
2000
1500
1000
500
0
−30
−25
−20
−15
−10
Temperature (∘C)

4) Downstream of Divide

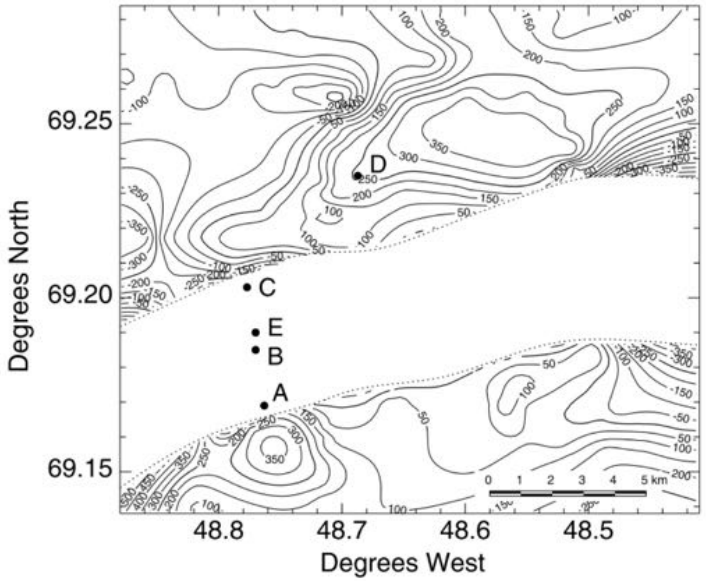
$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

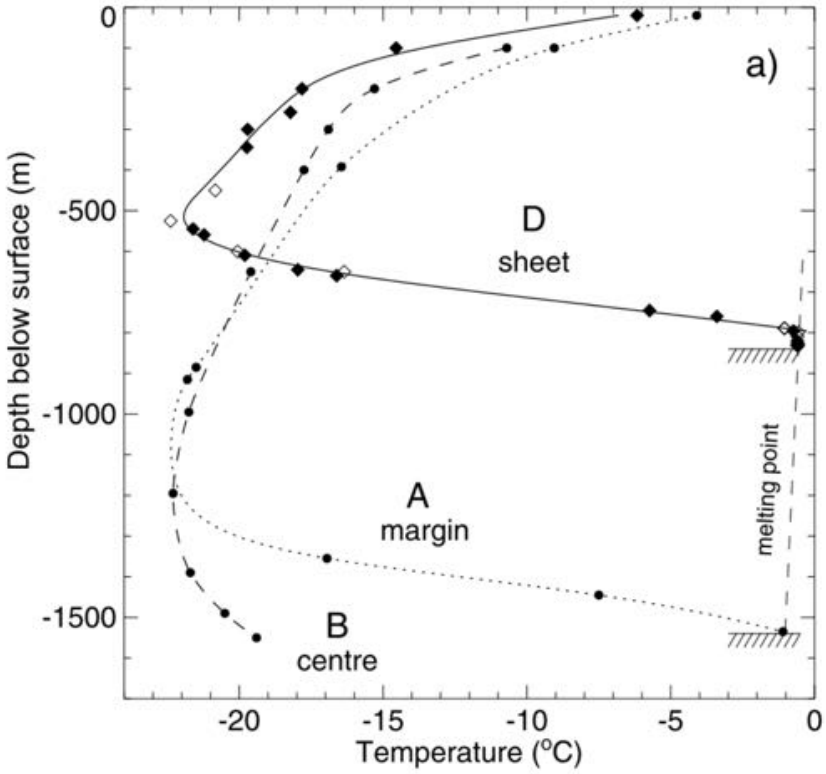
- Ice starts moving downhill, so the surface boundary warms at the lapse rate.
- Horizontal advection is typically faster than diffusion.



4) Downstream of Divide

Jakobshavn Isbræ



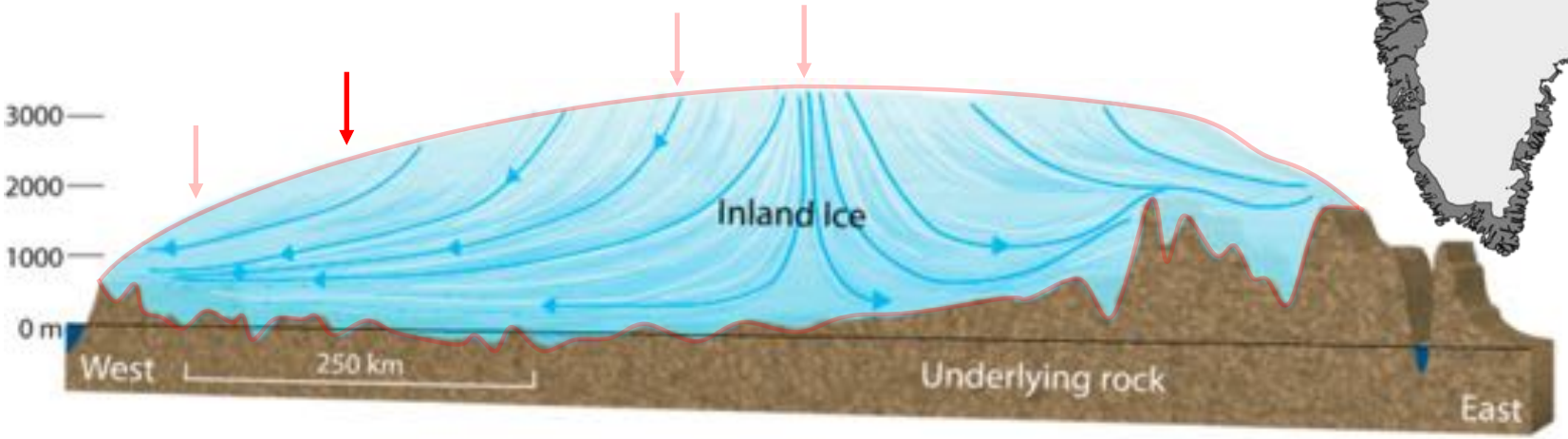


Luthi et al. (2002), *J.Glac.*
Iken et al. (1993), *J.Glac.*

5) Fast-Flowing Ice

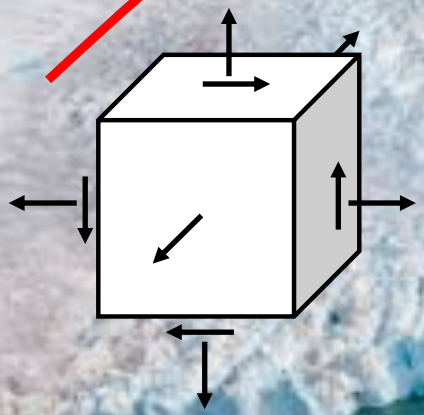
$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

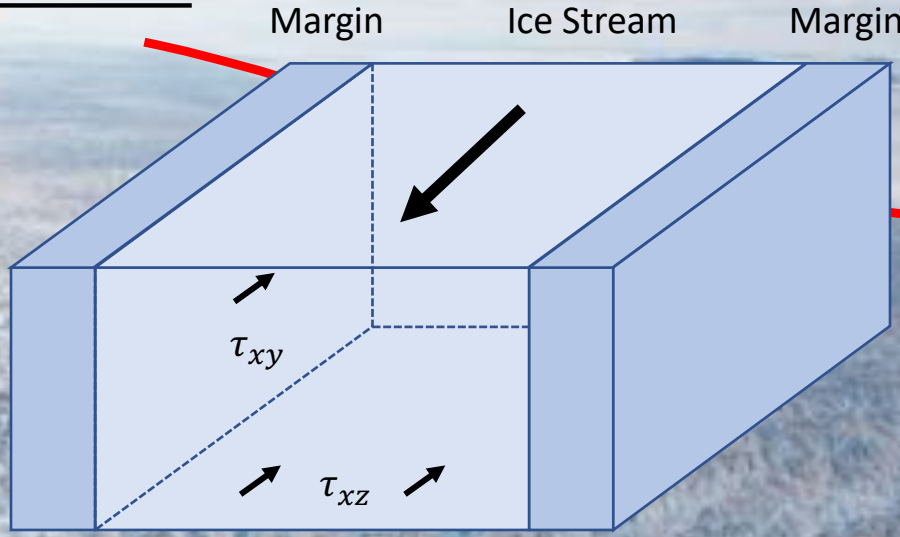
- Ice streams represent a sharp boundary in ice velocity, so the 'shear margins' at their boundary have very high strain rates.
- Some argue that the strain rate is high enough that a significant portion of the ice column is temperate.



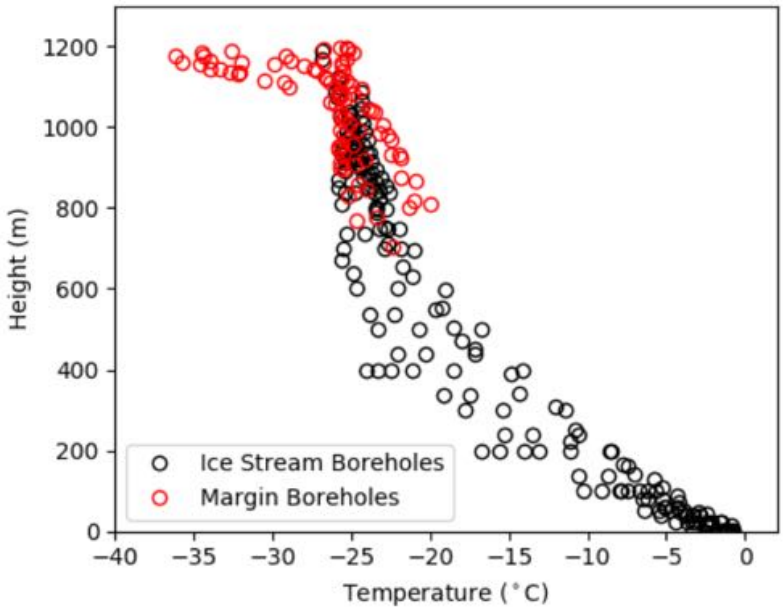
5) Fast-Flowing Ice

$$\dot{\epsilon}\tau = \left[\frac{m}{ms}\right]\left[\frac{N}{m^2}\right] = \left[\frac{J}{m^3 s}\right]$$

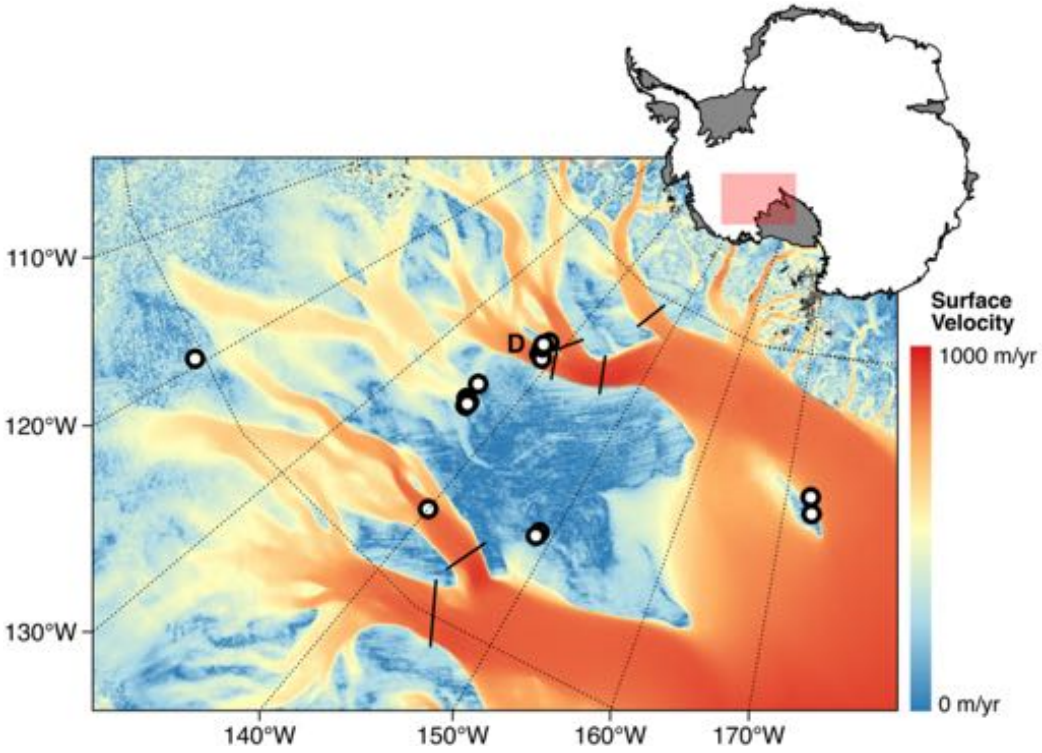




5) Fast-Flowing Ice



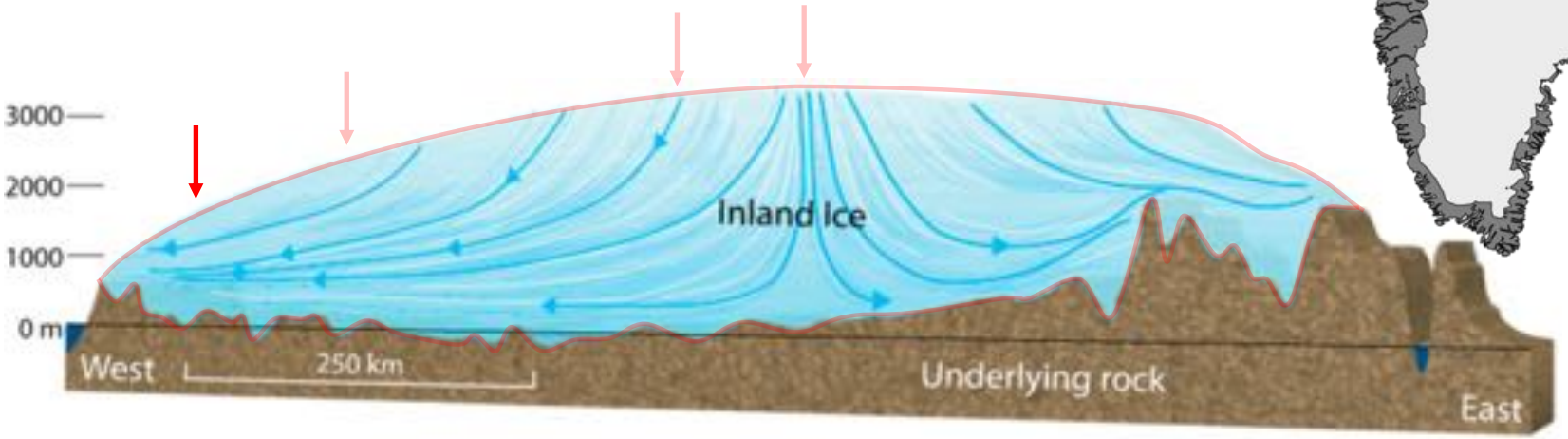
Harrison et al. (1998), *JGlac.*
Engelhardt (2004), *Ann. Glac.*



6) Ablation Zone

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\epsilon}\tau + L_f M_f)$$

- Much liquid water available at the surface.
- If water finds a pathway into the ice (through fractures) it can refreeze and release the latent heat to warm the ice.



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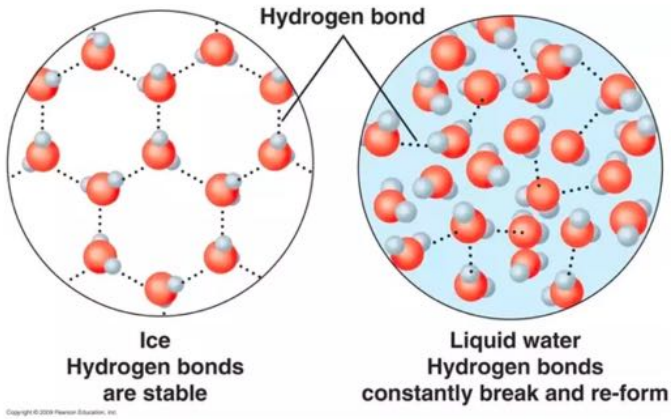
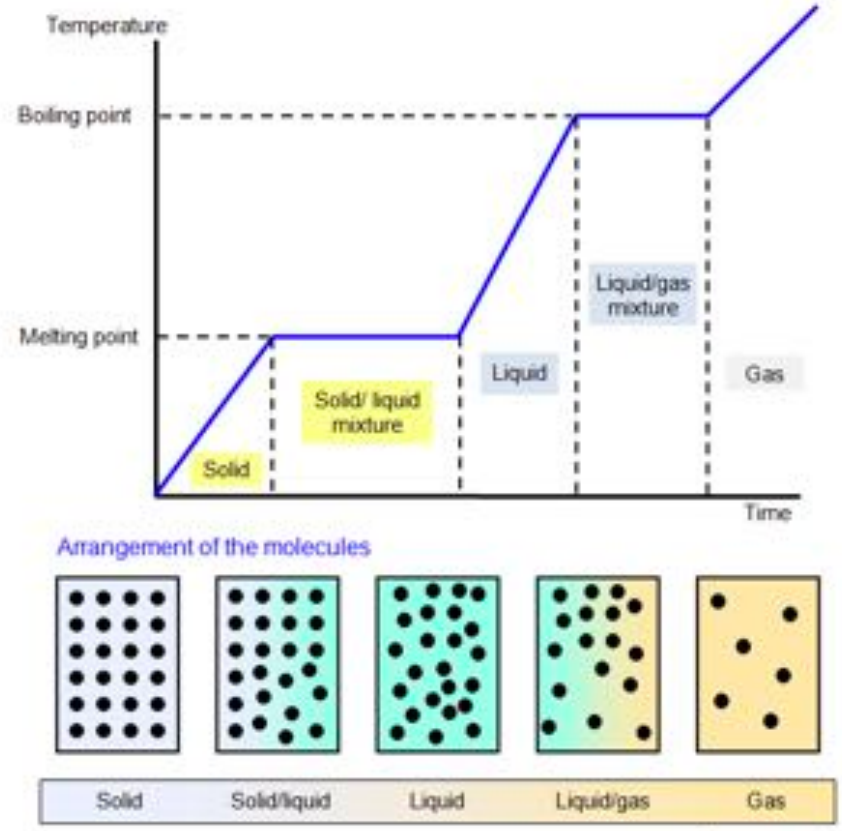
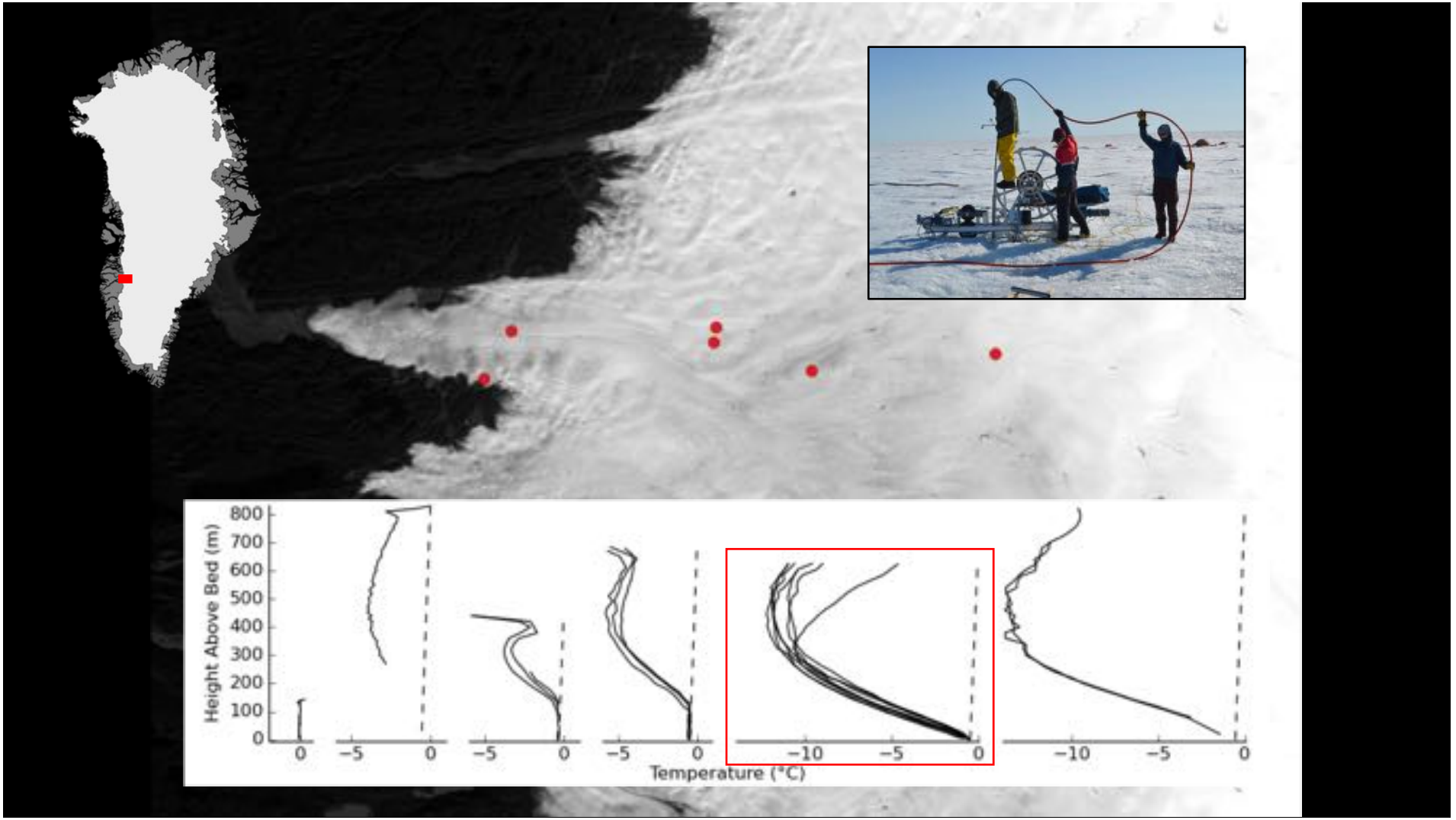
6) Ablation Zone

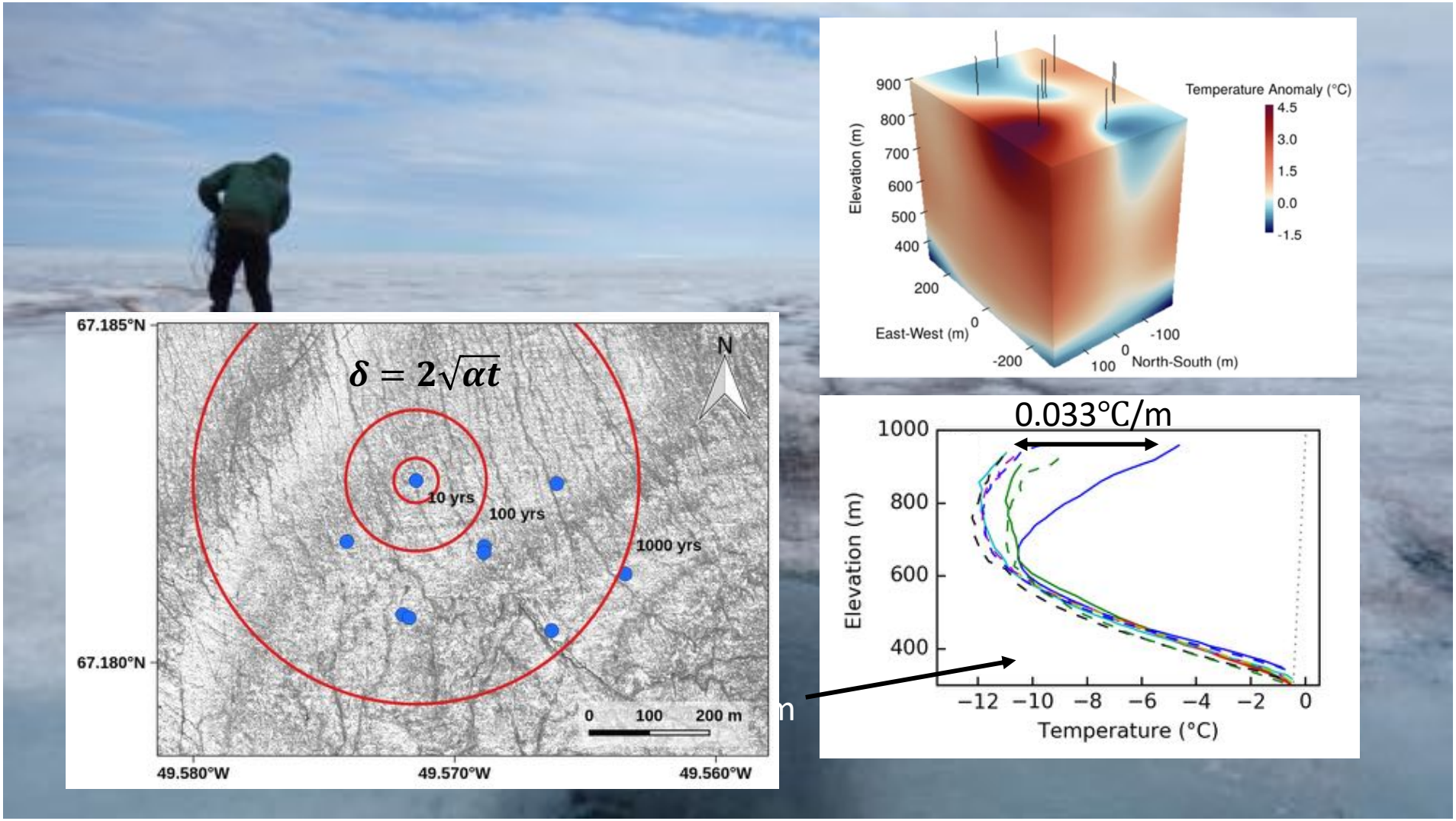
Latent Heat

Breaking Bonds



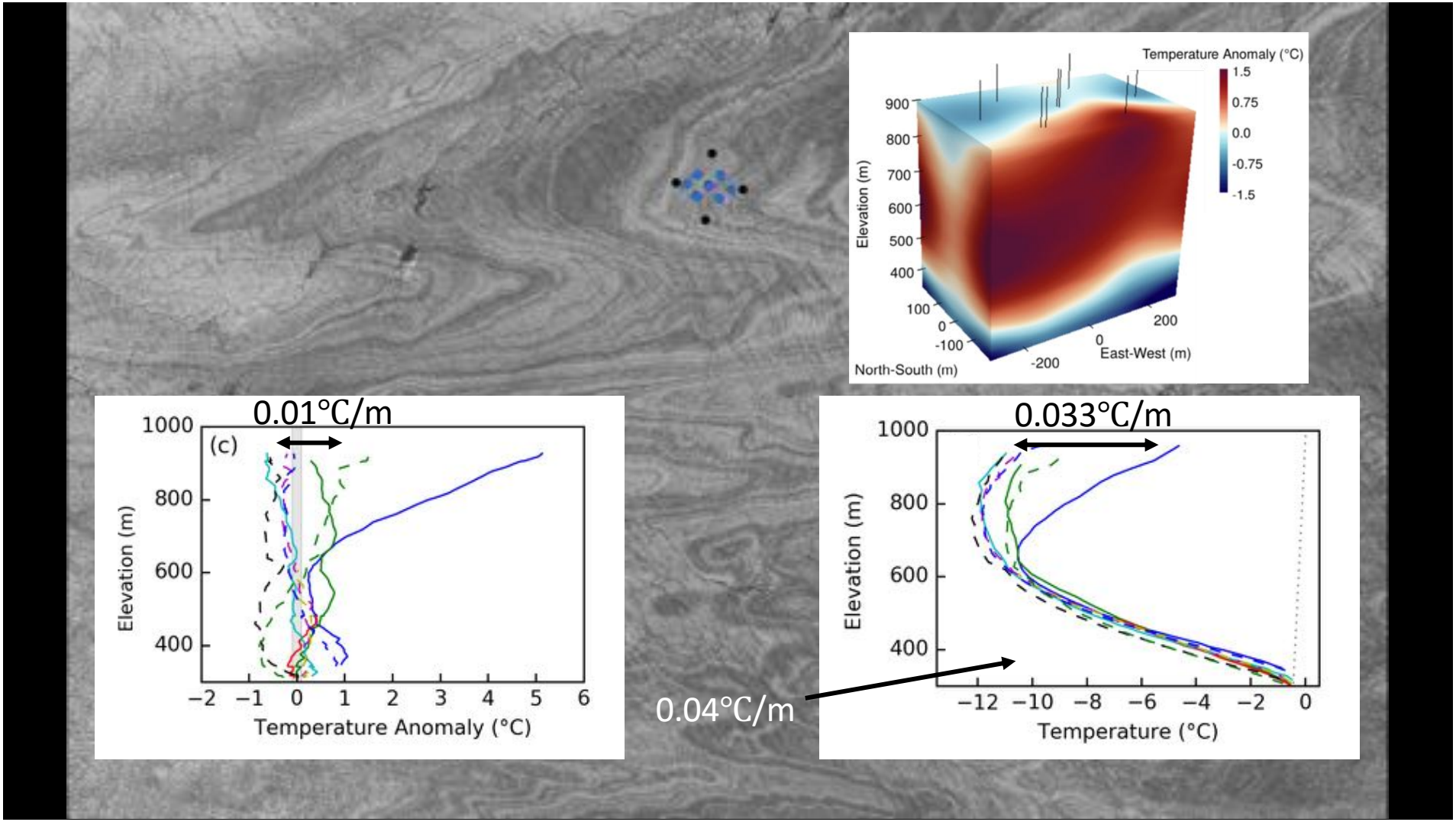


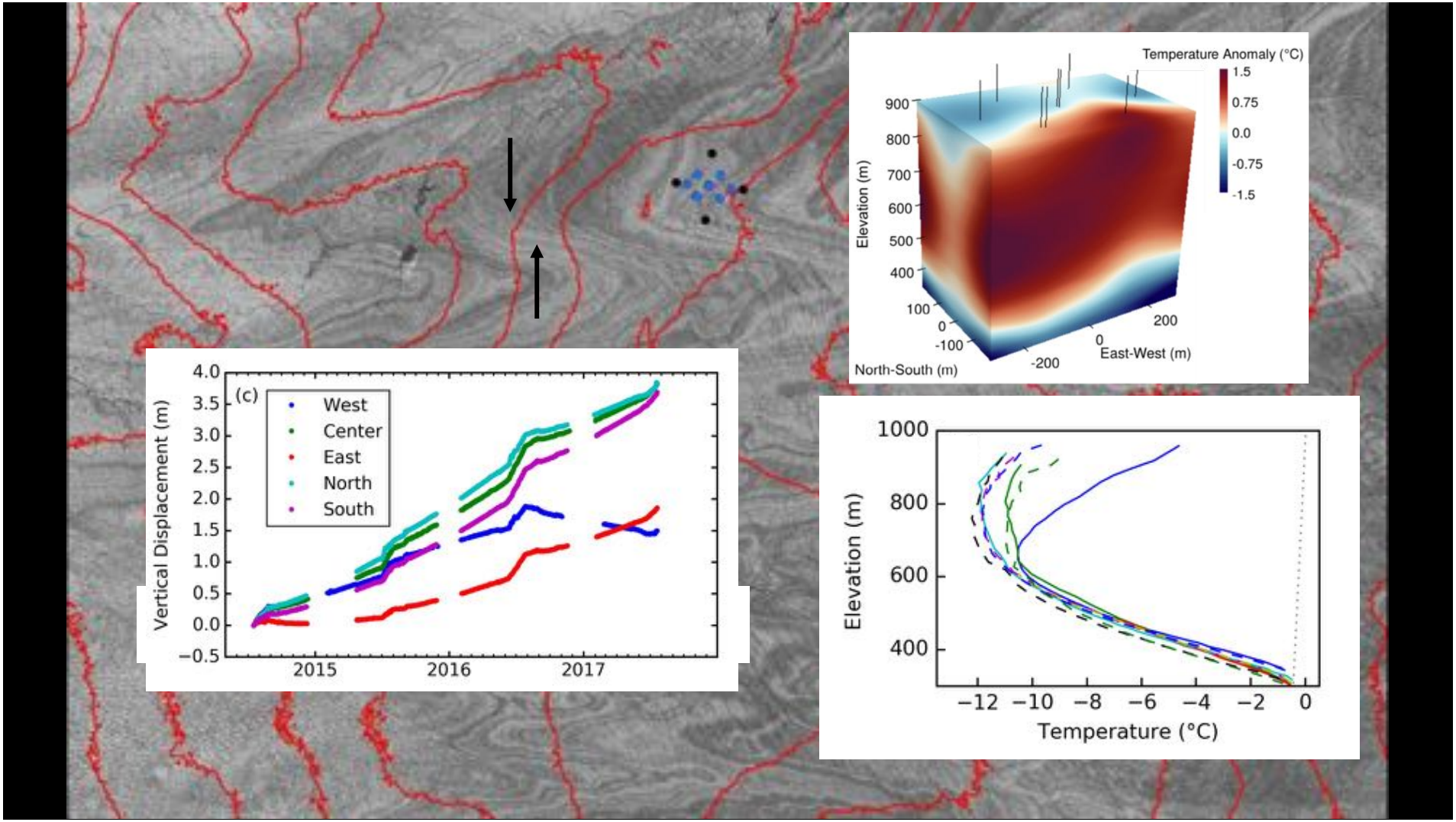

Height Above Bed (m)
800
700
600
500
400
300
200
100
0
0
−5
0
−5
0
−5
0
−10
−5
0
−10
−5
0
Temperature (°C)

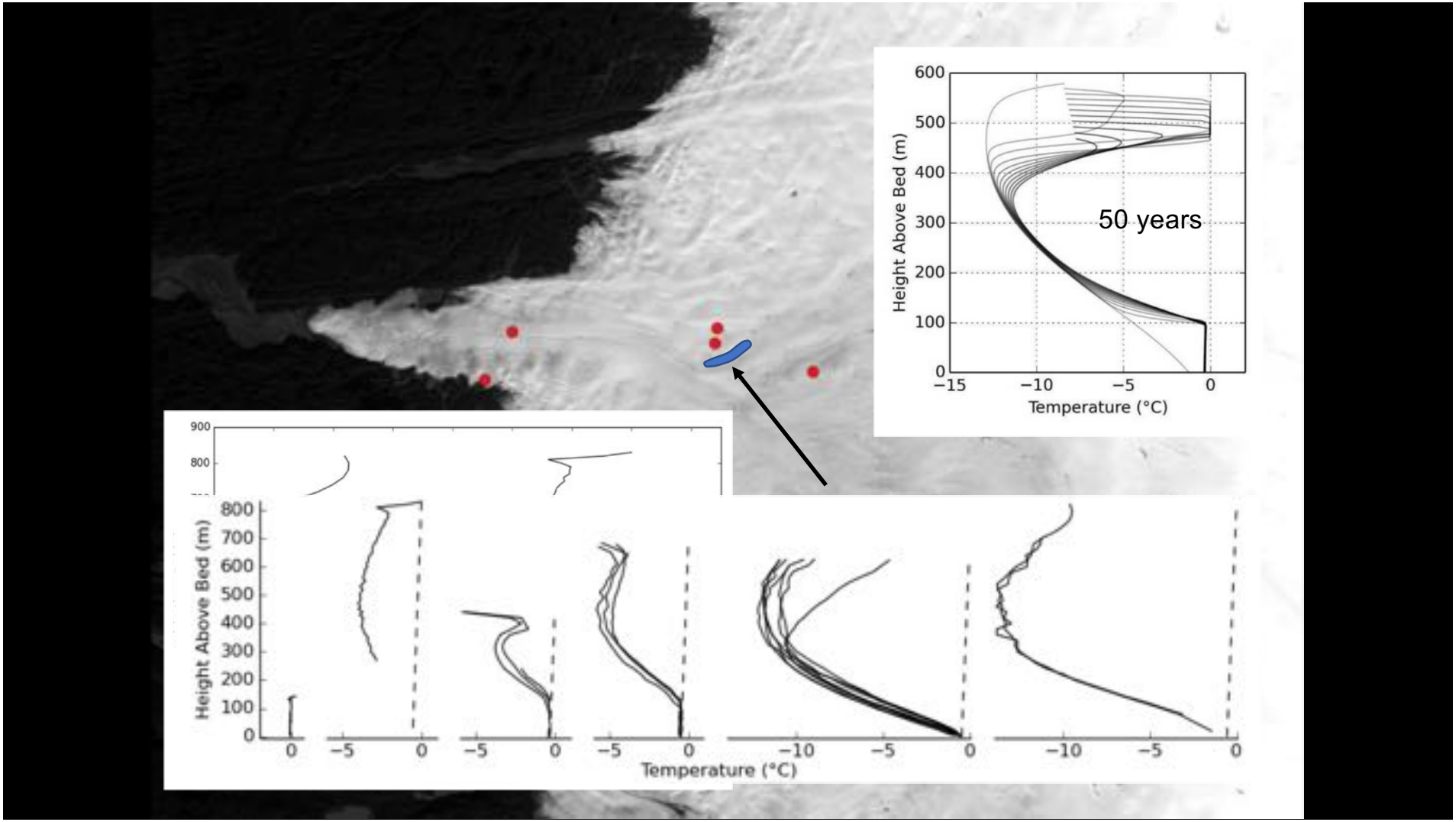

67.185°N
67.180°N
49.580°W
49.570°W
49.560°W
N
$\delta = 2\sqrt{\alpha t}$
10 yrs
100 yrs
1000 yrs
0
100
200 m

Temperature Anomaly (°C)
4.5
3.0
1.5
0.0
-1.5
Elevation (m)
900
800
700
600
500
400
200
East-West (m)
0
-200
-100
0
100
North-South (m)

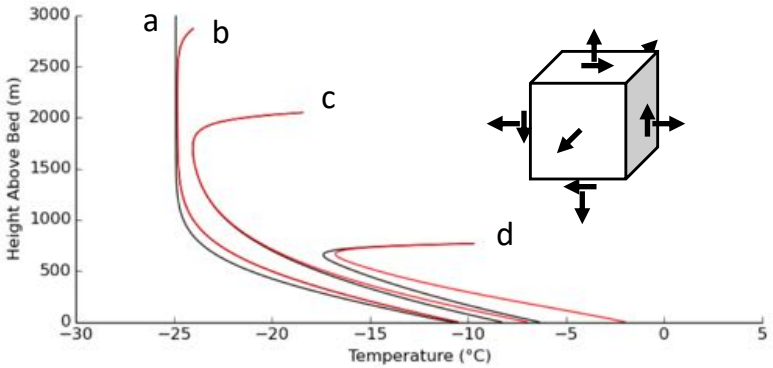
0.033°C/m
Elevation (m)
1000
800
600
400
−12
−10
−8
−6
−4
−2
0
Temperature (°C)


Temperature Anomaly (°C)
1.5
0.75
0.0
-0.75
-1.5
900
800
700
600
500
400
Elevation (m)
100
0
-100
North-South (m)
-200
0
200
East-West (m)
0.01°C/m
(c)
1000
800
600
400
Elevation (m)
−2 −1 0 1 2 3 4 5 6
Temperature Anomaly (°C)
0.033°C/m
0.04°C/m
1000
800
600
400
Elevation (m)
−12 −10 −8 −6 −4 −2 0
Temperature (°C)

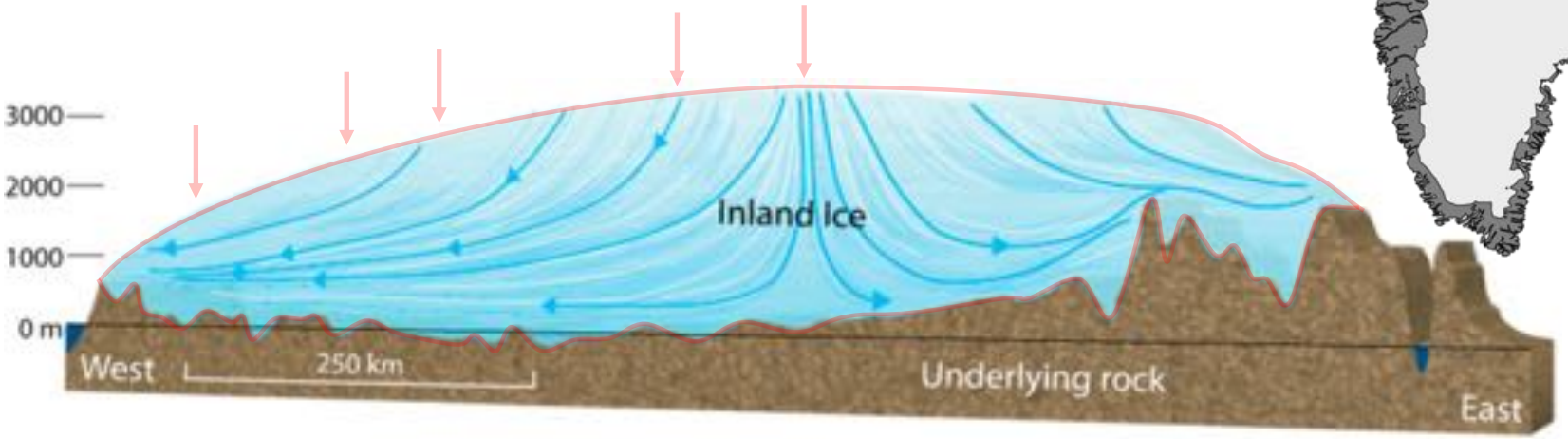

Temperature Anomaly (°C)
1.5
0.75
0.0
-0.75
-1.5
Elevation (m)
900
800
700
600
500
400
100
0
-100
-200
North-South (m)
0
200
East-West (m)
(c)
West
Center
East
North
South
Vertical Displacement (m)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
2015
2016
2017
Elevation (m)
1000
800
600
400
−12
−10
−8
−6
−4
−2
0
Temperature (°C)


600
500
400
300
200
100
0
Height Above Bed (m)
50 years
−15
−10
−5
0
Temperature (°C)
900
800
800
700
600
500
400
300
200
100
0
Height Above Bed (m)
0
−5
0
−5
0
−5
0
−10
−5
0
−10
−5
0
Temperature (°C)

Summary



$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T - \mathrm{u} \cdot \nabla T + \frac{\phi}{\rho C}$$



More Reading

- Cuffey and Paterson (2010) – Chapter 9
- van der Veen (2013) – Chapter 6
- Hooke (2005) – Chapter 6
- Zotikov (1986)
- Thermodynamics of Glaciers (Aschwanden, 2014; McCarthy Summer School Notes)