Glacier Thermodynamics: Ice Temperature and Heat Transfer Processes

ESS431: Principles of Glaciology
ESS505: The Cryosphere

Wednesday, 10/23 – Ben Hills & Knut Christianson
Today’s Objectives:

• Why do we care about ice temperature?

• Cover the fundamentals of heat transfer. What are the important time/length scales for heat transfer in ice?

• What is the general thermal structure of a glacier or an ice sheet? What heat transfer processes are important in which regions?
Importance of Ice Temperature

Background on Heat Transfer

Ice Sheet Thermal Structure
Why Ice Temperature?

1. Ice Deformation (Viscosity)

\[ \dot{e} = A(T)\tau^n \]
\[ A(T) = A_0 e^{-\frac{Q_a}{RT}} \]

Joughin et al. (2010)
Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding

Credit: Paul D. Bons, Ilka Weikusat
Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding
3. Paleoclimate

[Graph: Comparison of Greenland and Antarctica Ice Core Temperature Records]

Data from www.climatedata.info
Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding
3. Paleoclimate
4. Drilling
Why Ice Temperature?

1. Ice Deformation (Viscosity)
2. Basal Sliding
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5. Geophysics
   (Radar and Seismic)
Why Ice Temperature?

1. Ice Deformation (Viscosity)
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5. Geophysics
   (Radar and Seismic)
6. Mass Balance

van den Broeke et al. (2011)
Importance of Ice Temperature

Background on Heat Transfer

Ice Sheet Thermal Structure
Laws of Thermodynamics

1. Energy Conservation:
   • Energy can neither be created nor destroyed; energy can only be transferred or changed from one form to another.
   • \( \Delta E = Q - W \)

2. Entropy
   • The entropy of any isolated system always increases. Isolated systems spontaneously evolve towards thermal equilibrium—the state of maximum entropy of the system.
   • \( \Delta S \geq 0 \)
Modes of Heat Transfer

1. Conduction
   - Transfer of heat via direct molecular collision.
Background on Heat Transfer

Modes of Heat Transfer
1. Conduction
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*Fourier’s Law:*

\[ q = -k \nabla T \]

- \( k \) is the thermal conductivity of the material
Background on Heat Transfer

Modes of Heat Transfer

1. Conduction
   - Transfer of heat via direct molecular collision.

2. Convection
   - Thermal energy is carried by a moving fluid.

\[
\frac{dT}{dt} = -u \nabla T
\]
Background on Heat Transfer

Modes of Heat Transfer

1. Conduction
   - Transfer of heat via direct molecular collision.

2. Convection
   - Thermal energy is carried by a moving fluid.

3. Radiation
   - The emission of electromagnetic waves carries energy away from a thermal body.
Background on Heat Transfer

3. Radiation
   - The emission of electromagnetic waves carries energy away from a thermal body.

**Stefan-Boltzmann Law:**

\[ E = \sigma T^4 \]
The Heat Equation

Heat Capacity Definition

\[ e(T) = C_p T \]

Total Derivative

\[ \frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} \frac{dx}{dt} = \frac{\partial T}{\partial t} + u \cdot \nabla T \]

Incompressibility and Total Derivative

\[ \frac{d(\rho e)}{dt} = q \bigg|_x - q \bigg|_{x+dx} + \phi \]

\[ \frac{d(\rho e)}{dt} = -\frac{\partial q}{\partial x} + \phi \]

\[ \frac{d\rho}{dt} C_p T + \rho C_p \frac{dT}{dt} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \phi = 0 \]

\[ \rho C_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = -k \frac{\partial^2 T}{\partial x^2} + \phi \]

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} - u \frac{\partial T}{\partial x} + \frac{\phi}{\rho C_p} \]
The Heat Equation for Ice

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\varepsilon} \tau + L_f M_f) \]

- Diffusion (mostly vertical)
- Advection (all 3 directions)
- Sources (see figure)
Characteristic Diffusion Length

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\varepsilon} T + L_f M_f) \]

\[ l = 2\sqrt{\alpha t} \]

Copper \quad 1.11 \times 10^{-4} \text{ m}^2/\text{s}
Iron \quad 2.3 \times 10^{-5} \text{ m}^2/\text{s}
Air \quad 1.9 \times 10^{-5} \text{ m}^2/\text{s}
Stainless Steel \quad 4.2 \times 10^{-6} \text{ m}^2/\text{s}
**Ice** \quad 1.09 \times 10^{-6} \text{ m}^2/\text{s}
Water \quad 1.43 \times 10^{-7} \text{ m}^2/\text{s}
Peclet Number

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\varepsilon}T + L_f M_f) \]

Gulf Stream \ Pe \approx 10^{12}

Ice Sheet (horizontal) \ Pe \approx 10^5

Solid earth (mantle convection) \ Pe \approx 10^3

Ice Sheet (vertical) \ Pe \approx 10^0 - 10^1

Near-surface

Ice Sheet \ Pe \approx 10^{-1}

Granite Outcrop \ Pe \ll 1
Importance of Ice Temperature

Background on Heat Transfer

Ice Sheet Thermal Structure
Thermal Classifications of Glaciers

Temperate
- High Accumulation
- High Melt
- Fast Moving

Franz Josef Glacier
New Zealand

Cold
- Low Accumulation
- Low Melt
- Slow Moving

McMurdo Dry Valleys
Antarctica

Polythermal
- Possibly bridges climate zones (i.e. cold at top and warm at bottom)
- Includes both ice sheets

Grand Pacific Glacier
Glacier Bay, Alaska
Regions

1. Surface
2. Bed
3. Ice Divide
4. Downstream of Divide

Boundary Conditions

5. Fast-Flowing Ice
6. Ablation Zone

Frictional Heating

Latent Heating

Advection
1) Surface

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\varepsilon} T + L_f M_f)
\]

- Dominated by conduction with the overlying atmosphere.
- Melting becomes important in the ablation and percolation zones.
1) Surface
1) Surface

Skin Depth is period dependent
1) Surface

Hills et al. (2018)
1) Surface

Hills et al. (2018)
2) Bed

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\epsilon} + L_f M_f)
\]

- Heat sources from the friction of ice sliding over bedrock and from geothermal heating.
- Basal melting can add to the total mass balance, but is mostly important for its influence on sliding.
2) Bed

MacGregor et al. (2016)
3) Ice Divide

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p}(\dot{\varepsilon} + L_f M_f) \]

- Ice flow is purely vertical.
- The only source term here is firn compaction which is small compared to shear strain or latent heating.
3) Ice Divide

Air temperature variations through time

Geothermal flux
4) Downstream of Divide

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{e}_T + L_f M_f)
\]

- Ice starts moving downhill, so the surface boundary warms at the lapse rate.
- Horizontal advection is typically faster than diffusion.
4) Downstream of Divide

Luthi et al. (2002), J.Glac.
Iken et al. (1993), J.Glac.
5) Fast-Flowing Ice

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T - u \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\varepsilon} \tau + L_f M_f)
\]

- Ice streams represent a sharp boundary in ice velocity, so the ‘shear margins’ at their boundary have very high strain rates.
- Some argue that the strain rate is high enough that a significant portion of the ice column is temperate.
5) Fast-Flowing Ice

\[ \dot{\epsilon} \tau = \left[ \frac{m}{m_s} \right] \left[ \frac{N}{m^2} \right] = \left[ \frac{J}{m^3 s} \right] \]
5) Fast-Flowing Ice

Harrison et al. (1998), *JGlac.*
6) Ablation Zone

\[
\frac{\partial T}{\partial t} = \alpha \nabla^2 T - \mathbf{u} \cdot \nabla T + \frac{1}{\rho C_p} (\dot{\varepsilon} \tau + L_f M_f)
\]

- Much liquid water available at the surface.
- If water finds a pathway into the ice (through fractures) it can refreeze and release the latent heat to warm the ice.
6) Ablation Zone

Latent Heat

Breaking Bonds
\[ \delta = 2\sqrt{at} \]
Summary

\[ \frac{\partial T}{\partial t} = \alpha \nabla^2 T - \mathbf{u} \cdot \nabla T + \frac{\phi}{\rho C} \]
More Reading

• Cuffey and Paterson (2010) – Chapter 9
• van der Veen (2013) – Chapter 6
• Hooke (2005) – Chapter 6
• Zotikov (1986)
• Thermodynamics of Glaciers (Aschwanden, 2014; McCarthy Summer School Notes)
Hypothesis:
Differential advection creates horizontal temperature gradients.