Earth’s Magnetic Field

Magnetic Potential for a dipole field pointing South

\[ V(\mathbf{r}) = \mathbf{m} \cdot \mathbf{r} / (4 \pi r^3) = -m \cos \theta / (4 \pi r^2) \] = scalar magnetic potential of dipole field. Field is expanded in spherical harmonics. First term (above) is the dipole term.

\[ m = 8 \times 10^{22} \text{ Am}^2 \] is dipole moment at center of Earth point south

\[ r = \text{distance from dipole} \]

\[ \theta = \text{colatitude} \]

\[ \mathbf{B}(\mathbf{r}) = -\mu_0 \nabla V \] = vector magnetic field

where \( \mu_0 = 4\pi \times 10^{-7} \text{ kg m A}^{-2} \text{ s}^{-2} \) = magnetic permeability in free space (A=amps)

\[ B_r = -\mu_0 dV/dr = -\mu_0 m 2 \cos \theta / (4 \pi r^3) \] radial component of magnetic field (up)

\[ B_\theta = -\mu_0 r^{-1} dV/d\theta = -\mu_0 m \sin \theta / (4 \pi r^3) \] tangential component of field (south)

\[ |\mathbf{B}| = \sqrt{B_r^2 + B_\theta^2} = \frac{\mu_0 m}{(4 \pi R^3)} \sqrt{\sin^2 \theta + 4 \cos^2 \theta} = B_0 \sqrt{1 + 3 \cos^2 \theta} \]

size of magnetic field at Earth’s surface

at \( r = R = 6371 \times 10^3 \text{ m (surface of earth)} \) define:

\[ B_0 = \frac{\mu_0 m}{(4 \pi R^3)} \]

\[ B_0 = 4\pi \times 10^{-7} \text{ kg m A}^{-2} \text{ s}^{-2} \times 8 \times 10^{22} \text{ Am}^2 / (4 \pi (6.371 \times 10^6 \text{ m})^3) \]

\[ = 10^{-22+22-18} \times 8 \times 6.371^{-3} \text{ kg A}^{-1} \text{ s}^{-2} = 3 \times 10^{-5} \text{ kg A}^{-1} \text{ s}^{-2} = 3 \times 10^{-7} \text{ Tesla (T)} = 3 \times 10^4 \text{ nanoTesla} \]

So the strength of the magnetic field at Earth’s surface varies from 30,000nT at the equator (\( \theta = 90; \cos^2 \theta = 0 \)) to 60,000nT at the poles (\( \theta = 0 \) or \( 180; \cos^2 \theta = 1 \)).

The inclination angle (I) is defined to be the dip of magnetic field: horizontal=0, down is positive) so:

\[ \tan(I) = -B_r / |B_\theta| = (B_0 2 \cos \theta) / (B_0 \sin \theta) = 2/\tan \theta = 2 \tan \lambda \]

negative B_r because we measure dip pointing down but positive r is up

\( \theta \) is colatitude; \( \lambda = 90 - \theta \) is latitude

Time variation of magnetic field

Secular variation

Intensity

Direction

Westward drift of non-dipole field

Reversals
Properties of Earth’s magnetic field

- Approximated with a centered dipole tilted by 11.5°. (explains 90% of field at surface)
- Dipole moment of $8 \times 10^{22}$ Am$^2$ (60,000 nT at poles, 30,000 nT at equator)
  - Discuss non-dipole in terms of multipoles (quadrapole etc)
- Field tilted down in northern hemisphere.
- Define “inclination” and “declination” (geomagnetic poles and equator are where $I = \pm 90°$ and $0°$ respectively). Magnetic poles and equator are for best fitting dipole.
- Has internal and external sources. External sources associated with magnetosphere-sun interactions.

Variable properties of field:
  - External (order 100 nT)
    - Micropulses (ms-minutes) 1 nT
    - Magnetospheric substorms (hours) 10 nT
    - Solar daily variations 20 nT
    - Solar storms (4-60 hrs) 40 nT
    - Sun spot cycle (11 and 22 years)
  - Internal sources (greater than 4 years)
Recent observation of rapid movement of the magnetic pole:

This has resulted in a change in declination at Johnson Hall ( -122.30861, 47.65453) from 22.3° E in 1950 to 18.7 in 2000 to 16.6 in 2012, currently dropping at a rate of 1.8° per decade. http://www.ngdc.noaa.gov/geomag-web/#ushistoric
Variation of the dipole axis as represented by the change in position of North Geomagnetic Pole. After Fraser-Smith (1987).

Geometry of Earth Magnetic field
Dipole field
Non-dipole field
Inclination
Declination
equation for dipole magnetic field
equation relating inclination to latitude
GMP, VGP, PaleoMagnetic Pole
Geocentric Axial dipole hypothesis
Fig. 2.3c. Isodynamic chart for 1990 showing the variation of total intensity over the Earth's surface. Contours are labeled in nT.

Fig. 2.3b. Isoclinic chart for 1990 showing the variation of inclination in degrees over the Earth's surface.

1930

Fig. 2.10. The vertical component of the nondipole field for 1930 and 1990. Contours are labeled in units of 1000 nT.
History of field as recorded in rocks
  Ferromagnetic minerals
  Magnetic Hysteresis
  Curie temperature
  Blocking temperature
  Remanent magnetization
  TRM
  IRM
  DRM
  CRM
  VRM

Paleomagnetic data from continents
magnetic data from oceans

Dynamo theory
  MHD
  Dynamo equations
  “Frozen-in” field
  Free decay
  numerical simulations
    how close are simulations?
    Examples
  Role of inner core

\[
\begin{align*}
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \cdot \mathbf{D} &= \rho_e \\
\mathbf{J} &= \sigma \mathbf{E} + \sigma (\nabla \times \mathbf{B}) \\
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{v} \right) &= 2 \rho (\nabla \times \mathbf{v}) \\
&= -\nabla P + \eta \nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) - \rho \nabla \phi_g + \mathbf{J} \times \mathbf{B} \\
\nabla \cdot (\rho \mathbf{v}) + \frac{\partial \rho}{\partial t} &= 0 \\
\nabla^2 \phi_g &= -4\pi G \rho \\
\frac{\partial T}{\partial t} &= k_f \nabla^2 T + (\nabla k_f \cdot \nabla T) - \nabla \cdot \mathbf{v} + \epsilon \\
\rho &= \text{Function}(P, T, H)
\end{align*}
\]

Notation:
\[\begin{align*}
\mathbf{H} &= \text{magnetic field} & \rho &= \text{material density} \\
\mathbf{B} &= \text{magnetic induction} & \sigma &= \text{conductivity} \\
\mathbf{J} &= \text{electric current} & T &= \text{temperature} \\
\mathbf{E} &= \text{electric field} & P &= \text{pressure} \\
\mathbf{D} &= \text{electric displacement vector} & G &= \text{gravitational constant} \\
\mathbf{v} &= \text{velocity} & \phi_g &= \text{gravitational potential} \\
\eta &= \text{viscosity} & \epsilon &= \text{heat source term} \\
\rho_e &= \text{electric charge density} & k_f &= \text{thermal diffusivity} \\
\Omega &= \text{angular velocity of rotation}
\end{align*}\]
The magnetohydrodynamic (MHD) assumption that \( \frac{dD}{dt} = 0 \) is almost always made, so:

\[
\nabla \times \mathbf{H} = \sigma \mathbf{E} + \sigma (\mathbf{v} \times \mathbf{B}) \quad (8.2.1)
\]

Then taking the curl of both sides of this equation and using (8.1.12), (8.1.13) and \( \mathbf{B} = \mu_0 \mathbf{H} \), one obtains the magnetic induction equation:

\[
\frac{\partial \mathbf{H}}{\partial t} = \frac{1}{\sigma \mu_0} \nabla^2 \mathbf{H} + \nabla \times (\mathbf{v} \times \mathbf{H}) \quad (8.2.2)
\]

This and \( \nabla \cdot \mathbf{H} = 0 \) replaces equations 8.1.9-8.1.13 and removes \( \mathbf{E} \), \( \mathbf{B} \), \( \mathbf{J} \) and \( \mathbf{D} \) from the equations. Also \( \mathbf{J} \times \mathbf{B} \) in eq 8.1.14 is replaced by \( \mu_0 (\nabla \times \mathbf{H}) \times \mathbf{H} \).

Now, if \( \mathbf{H} \) is a solution to the MHD equations, so is \( -\mathbf{H} \), so regardless of the flow field, initial conditions, or boundary conditions, there is no preference for “normal” versus “reversed” field polarity.
Fig. 8.8. Production of a poloidal magnetic field. A region of fluid upwelling, illustrated by dotted lines on the left, interacts with a toroidal magnetic field (solid line). Because of the Coriolis effect (northern hemisphere) the fluid exhibits helicity, rotating as it moves upward (thin lines, center). The magnetic field line is carried with the conducting liquid and is twisted to produce a poloidal loop as on the right. After Parker (1955a.)

Fig. 8.9. Convergence of two poloidal loops as produced in Fig. 8.8 results in a larger poloidal loop. After Parker (1955a.)