Earth's Magnetic Field
Magnetic Potential for a dipole field pointing South

$V(\mathbf{r}) = \mathbf{m} \bullet \mathbf{r} / (4 \pi r^3) = - m \cos\theta / (4 \pi r^2)$ = scalar magnetic potential of dipole field. Field is expanded in spherical harmonics. First term (above) is the dipole term.

$m = 8 \times 10^{22}$ Am^2 is dipole moment at center of Earth point south
r = distance from dipole
θ = colatitude

$\mathbf{B}(\mathbf{r}) = - \mu_0 \text{ grad } V$ = vector magnetic field

where $\mu_0 = 4\pi \times 10^{-7}$ kg m A^{-2} s^{-2} = magnetic permeability in free space (A=amps)

$B_r = - \mu_0 \, dV/dr = - \mu_0 \, m \, 2 \cos\theta / (4 \pi r^3)$ radial component of magnetic field (up)
$B_\theta = - \mu_0 \, r^{-1} \, dV/d\theta = - \mu_0 \, m \sin\theta / (4 \pi r^3)$ tangential component of field (south)

$|\mathbf{B}| = \text{sqrt}(B_r^2 + B_\theta^2) = \mu_0 m / (4 \pi R^3) \text{ sqrt}(\sin^2\theta + 4\cos^2\theta) = B_0 \text{ sqrt}(1+3\cos^2\theta)$
size of magnetic field at Earths surface

at $r = R = 6371*10^3$ m (surface of earth) define:
$B_0 = \mu_0 m / (4 \pi R^3)$
$B_0 = 4\pi \times 10^{-7}$ kg m A^{-2} s^{-2} $* 8 \times 10^{22}$ Am^2 $/ (4 \pi (6.371*10^6)^3 m^3)$
$= 10^{-7+22-18} *8*6.371^{-3}$ kg A^{-1} s^{-2} $= 3 * 10^{-5}$ kg A^{-1} s^{-2} =
$3 * 10^{-5}$ Tesla (T) $= 3 * 10^4$ nanoTesla
So the strength of the magnetic field at Earth's surface varies from 30,000nT at the equator ($\theta = 90$; $\cos^2\theta=0$) to 60,000nT at the poles ($\theta = 0$ or 180; $\cos^2\theta=1$).

The inclination angle (I) is defined to be the dip of magnetic field: horizontal=0, down is positive) so:

$\tan(I) = -B_r / | B_\theta | = (B_0 2 \cos\theta) / (B_0 \sin\theta) = 2/\tan\theta = 2 \tan\lambda$
negative B_r because we measure dip pointing down but positive r is up
θ is colatitude; $\lambda = 90 - \theta$ is latitude

Time variation of magnetic field

- Secular variation
  - Intensity
  - Direction
- Westward drift of non-dipole field
- Reversals

Properties of Earth's magnetic field

- Approximated with a centered dipole tilted by 11.5°. (explains 90% of field at surface)
- dipole moment of $8x10^{22}$ Am^2 (60,000 nT at poles, 30,000 nT at equator)
    - discuss non-dipole in terms of multipoles (quadrapole etc)
- Field tilted down in northern hemisphere.
- Define "inclination" and "declination" (geomagnetic poles and equator are where I=±90° and 0° respectively). Magnetic poles and equator are for best fitting dipole.
- Has internal and external sources. External sources associated with magnetosphere-sun interactions

Variable properties of field:

external (order 100 nT)

micropulses (ms-minutes) 1 nT

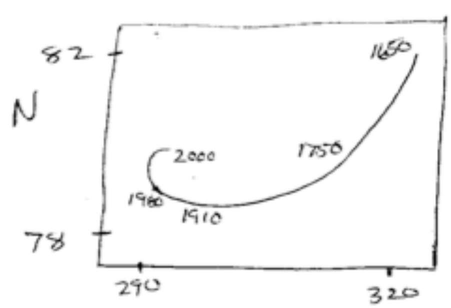
magnetospheric substorms (hours) 10 nT

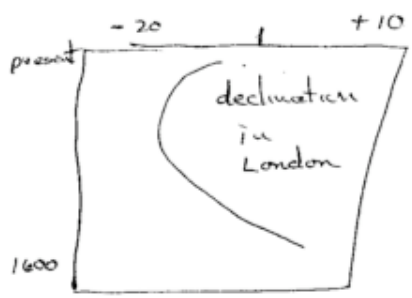
solar daily variations 20 nT

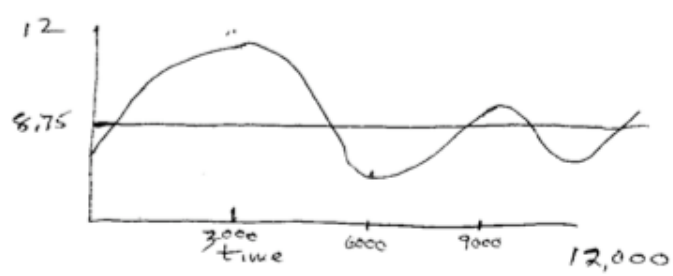
solar storms (4-60 hrs) 40 nT

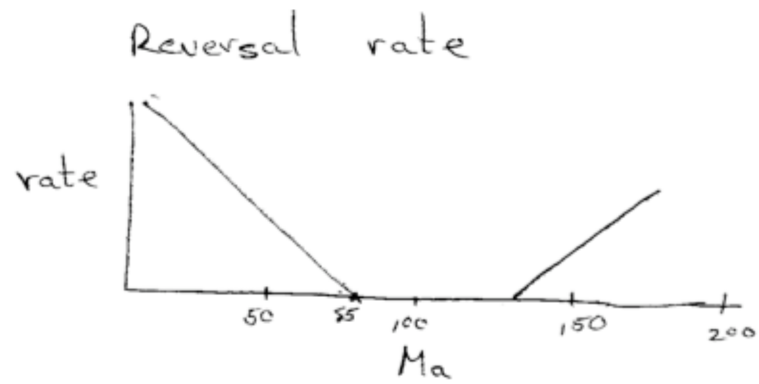
sun spot cycle (11 and 22 years)

internal sources (greater than 4 years)

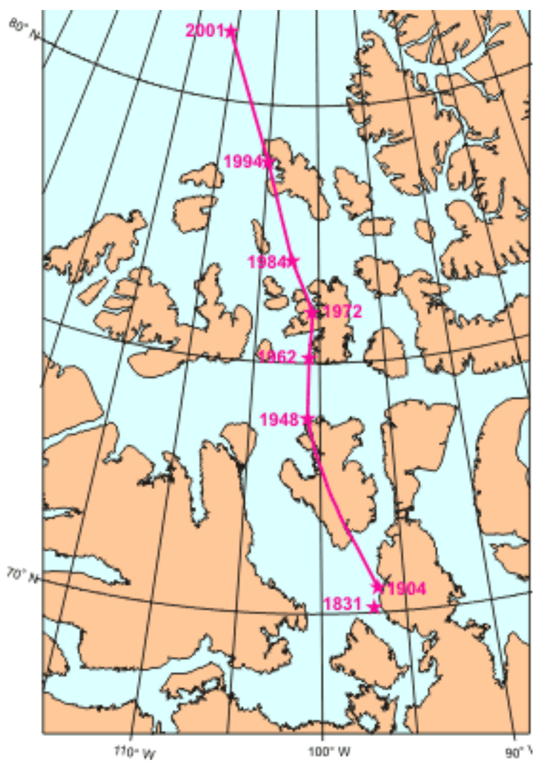


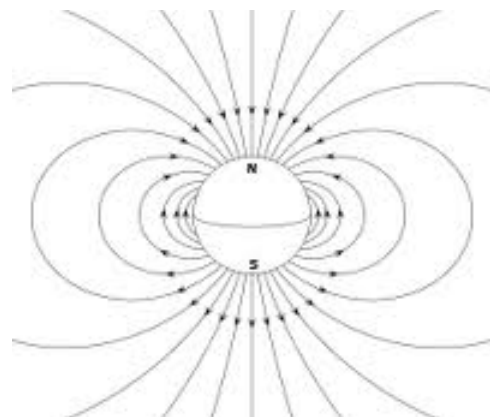




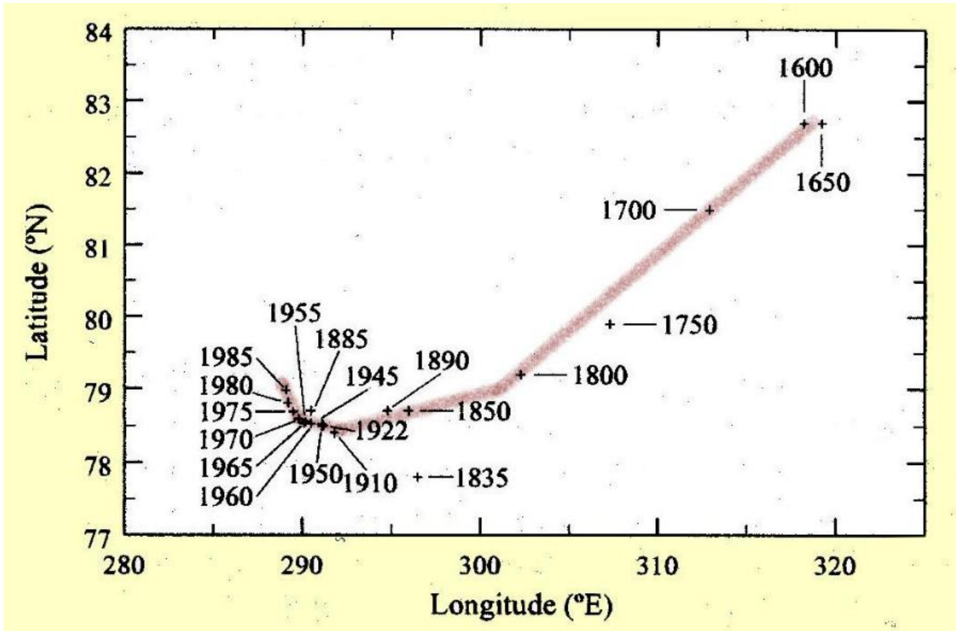


Recent observation of rapid movement of the magnetic pole:
(from http://science.nasa.gov/science-news/science-at-nasa/2003/29dec_magneticfield/)

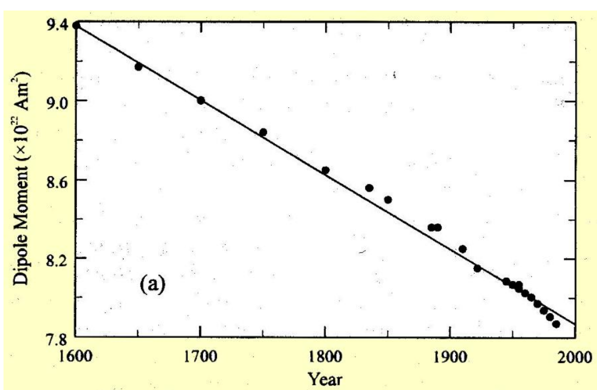




This has resulted in a change in declination at Johnson Hall (-122.30861, 47.65453) from 22.3° E in 1950 to 18.7 in 2000 to 16.6 in 2012, currently dropping at a rate of 1.8° per decade. http://www.ngdc.noaa.gov/geomag-web/#ushistoric



Variation of the dipole axis as represented by the change in position of North Geomagnetic Pole. After Fraser-Smith(1987).



Variation of dipole moment from spherical harmonic analysis.

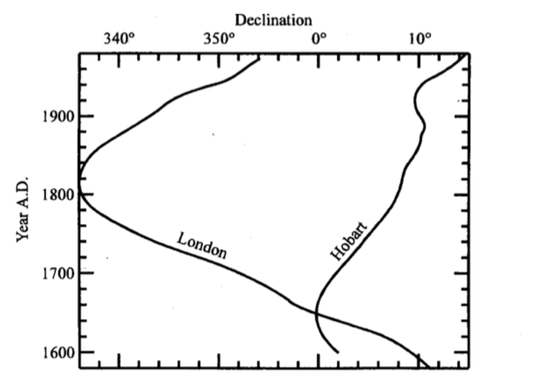


Fig. 2.8. Variation in declination at London, England (51.5°N), and at Hobart, Tasmania (42.9°S), from observatory measurements. The earliest measurement in the Tasmanian region was made by Abel Tasman at sea in 1642 in the vicinity of the present location of Hobart. Preobservatory data have been derived also by interpolation from isogonic charts.

Geometry of Earth Magnetic field

- Dipole field
- Non-dipole field
- Inclination
- Declination
- equation for dipole magnetic field
- equation relating inclination to latitude
- GMP, VGP, Paleomagnetic Pole
- Geocentric Axial dipole hypothesis

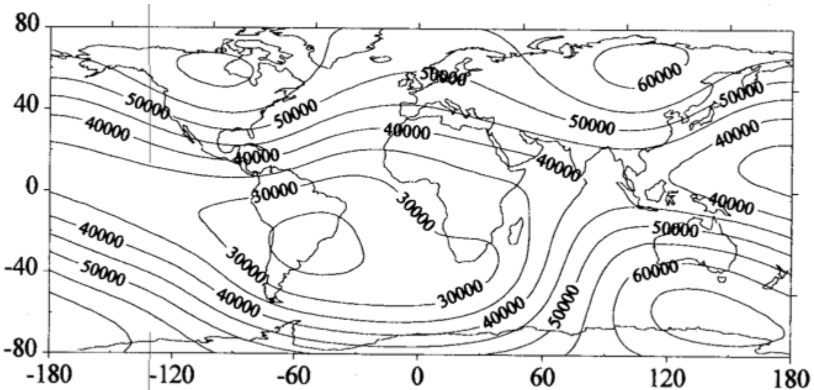


Fig. 2.3c. Isodynamic chart for 1990 showing the variation of total intensity over the Earth's surface. Contours are labeled in nT.

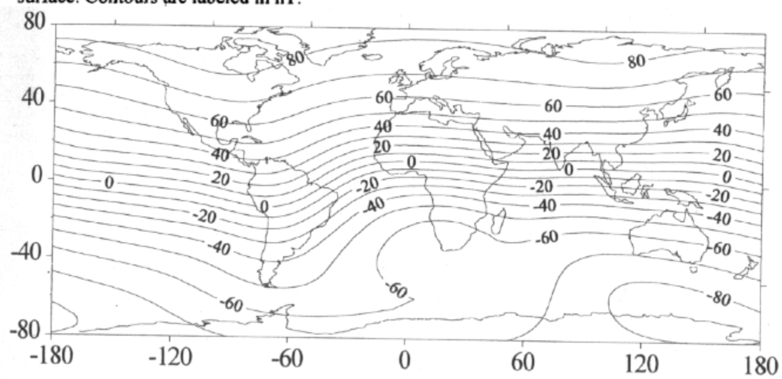
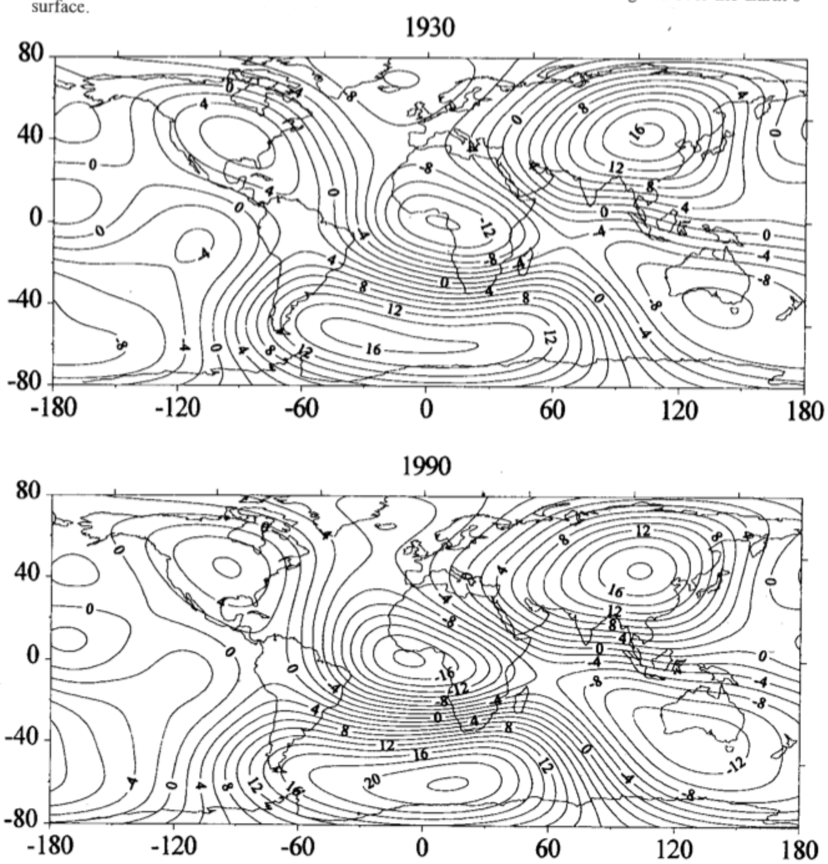


Fig. 2.3b. Isoclinic chart for 1990 showing the variation of inclination in degrees over the Earth's surface.



I **Fig. 2.10.** The vertical component of the nondipole field for 1930 and 1990. Contours are labeled in units of 1000 nT.

History of field as recorded in rocks

- Ferromagnetic minerals
- Magnetic Hysteresis
- Curie temperature
- Blocking temperature
- Remanent magnetization
    - TRM
    - IRM
    - DRM
    - CRM
    - VRM

Paleomagnetic data from continents

magnetic data from oceans

Dynamo theory

- MHD
- Dynamo equations
- "Frozen-in" field
- Free decay
- numerical simulations
    - how close are simulations?
    - Examples
    - Role of inner core

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (8.1.9)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (8.1.10)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (8.1.11)$$

$$\nabla \cdot \mathbf{D} = \rho_e \quad (8.1.12)$$

(8.1.9)–(8.1.12): Maxwell's equations

Ohm's Law (8.1.13)

$$\mathbf{J} = \sigma \mathbf{E} + \sigma(\mathbf{v} \times \mathbf{B}) \quad (8.1.13)$$

Navier–Stokes' equation (8.1.14)

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\mathbf{v} + 2\rho(\mathbf{\Omega} \times \mathbf{v}) = -\nabla P + \eta \nabla^2 \mathbf{v} + \tfrac{1}{3}\eta \nabla(\nabla \cdot \mathbf{v}) - \rho \nabla \phi_g + \mathbf{J} \times \mathbf{B} \quad (8.1.14)$$

Continuity equation (8.1.15)

$$\nabla \cdot (\rho \mathbf{v}) + \frac{\partial \rho}{\partial t} = 0 \quad (8.1.15)$$

Poisson's equation (8.1.16)

$$\nabla^2 \phi_g = -4\pi G \rho \quad (8.1.16)$$

Generalized heat equation (8.1.17)

$$\frac{\partial T}{\partial t} = k_T \nabla^2 T + (\nabla k_T \cdot \nabla T) - \mathbf{v} \cdot \nabla T + \varepsilon \quad (8.1.17)$$

Equation of state (8.1.18)

$$\rho = \text{Function}(P, T, H) \quad (8.1.18)$$

Notation:

- $\mathbf{H} \equiv$ magnetic field
- $\mathbf{B} \equiv$ magnetic induction
- $\mathbf{J} \equiv$ electric current
- $\mathbf{E} \equiv$ electric field
- $\mathbf{D} \equiv$ electric displacement vector
- $\mathbf{v} \equiv$ velocity
- $\eta \equiv$ viscosity
- $\rho_e \equiv$ electric charge density
- $\mathbf{\Omega} \equiv$ angular velocity of rotation
- $\rho \equiv$ material density
- $\sigma \equiv$ conductivity
- $T \equiv$ temperature
- $P \equiv$ pressure
- $G \equiv$ gravitational constant
- $\phi_g \equiv$ gravitational potential
- $\varepsilon \equiv$ heat source term
- $k_T \equiv$ thermal diffusivity

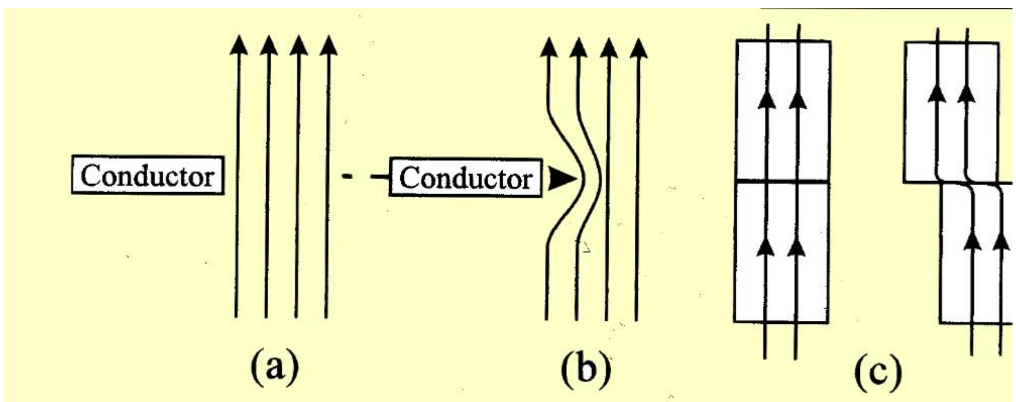
The magnetohydrodyanmic (MHD) assumption that d**D**/dt=0 is almost always made, so:

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \sigma(\mathbf{v} \times \mathbf{B}) \quad . \qquad (8.2.1)$$

Then taking the curl of both sides of this equation and using (8.1.12), (8.1.13) and $\mathbf{B}=\mu_0\mathbf{H}$, one obtains the *magnetic induction equation*:

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{1}{\sigma\mu_0}\nabla^2\mathbf{H} + \nabla \times (\mathbf{v} \times \mathbf{H}) \quad . \qquad (8.2.2)$$

This and $\nabla \bullet \mathbf{H} = \mathbf{0}$ replaces equations 8.1.9-8.1.13 and removes **E**, **B**, **J** and **D** from the equations. Also **J** x **B** in eq 8.1.14 is replaced by $\mu_0(\nabla \times \mathbf{H}) \times \mathbf{H}$.
Now, if **H** is a solution to the MHD equations, so is –**H**, so regardless of the flow field, initial conditions, or boundary conditions, there is no preference for “normal” versus “reversed” field polarity.



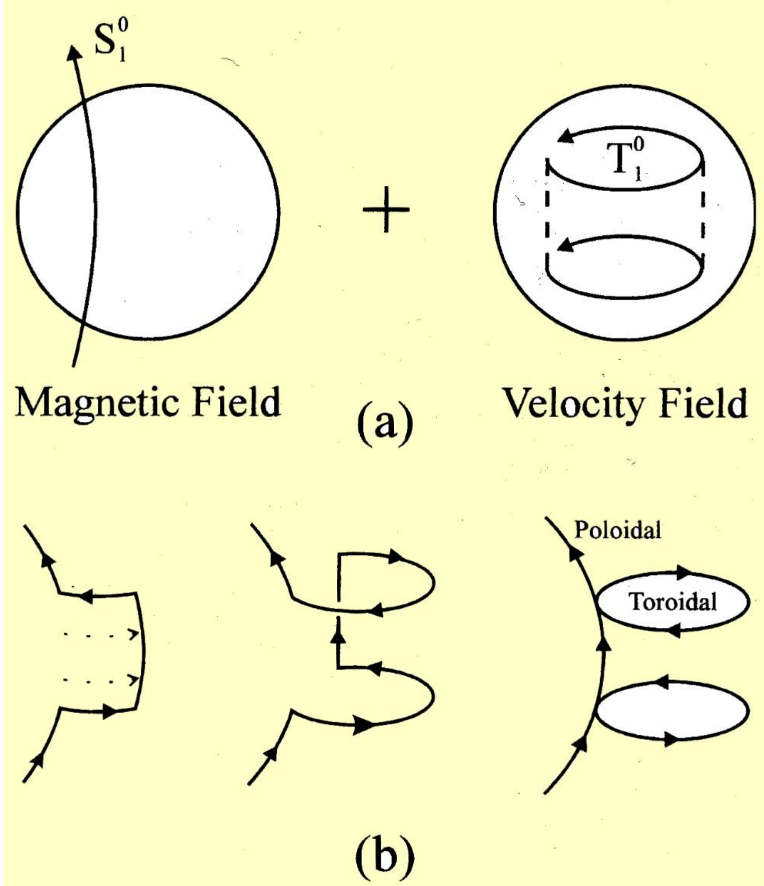


Fig. 8.8. Production of a poloidal magnetic field. A region of fluid upwelling, illustrated by dotted lines on the left, interacts with a toroidal magnetic field (solid line). Because of the Coriolis effect (northern hemisphere) the fluid exhibits helicity, rotating as it moves upward (thin lines, center). The magnetic field line is carried with the conducting liquid and is twisted to produce a poloidal loop as on the right. After Parker (1955a.)

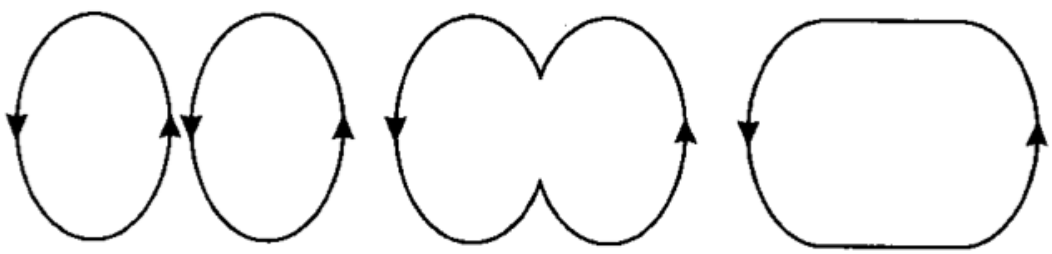
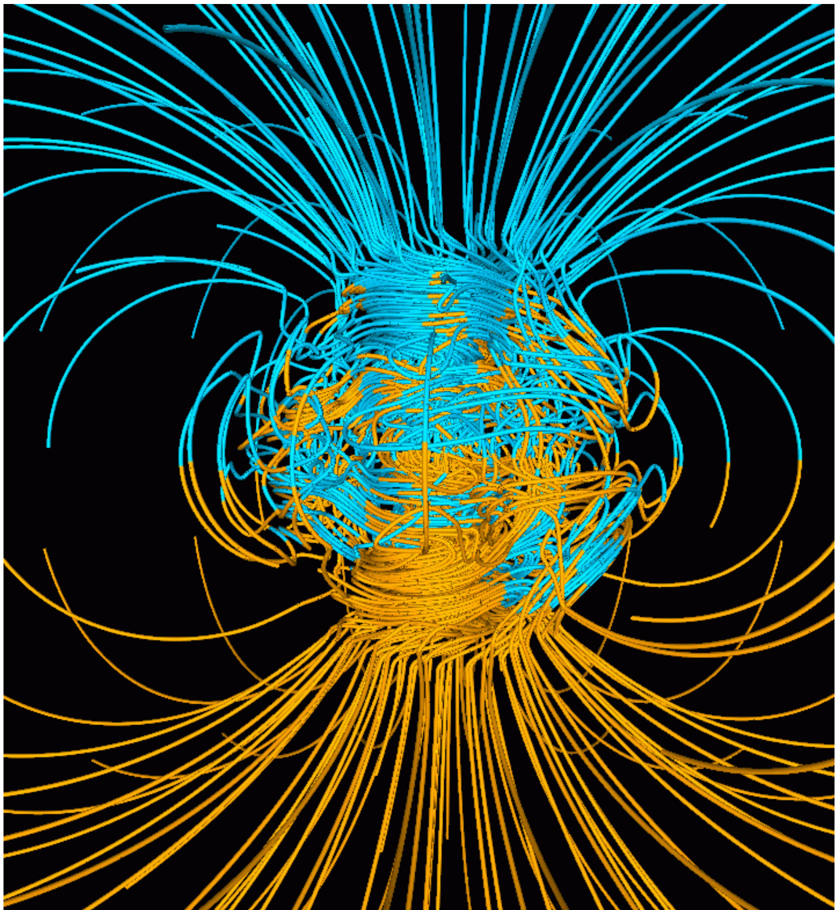
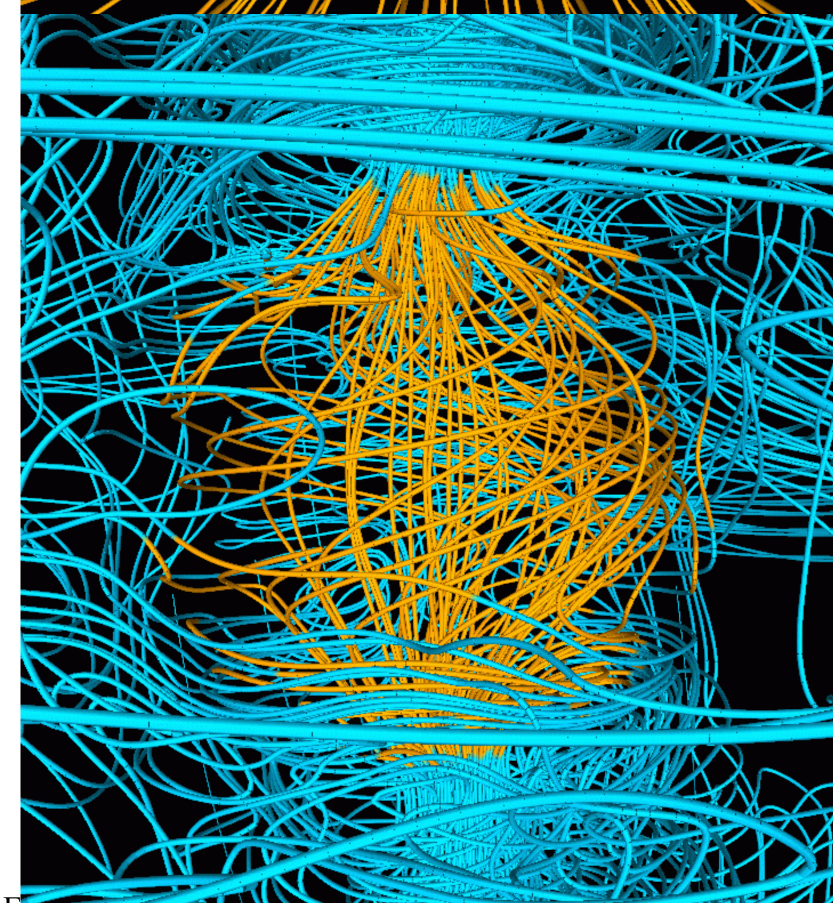
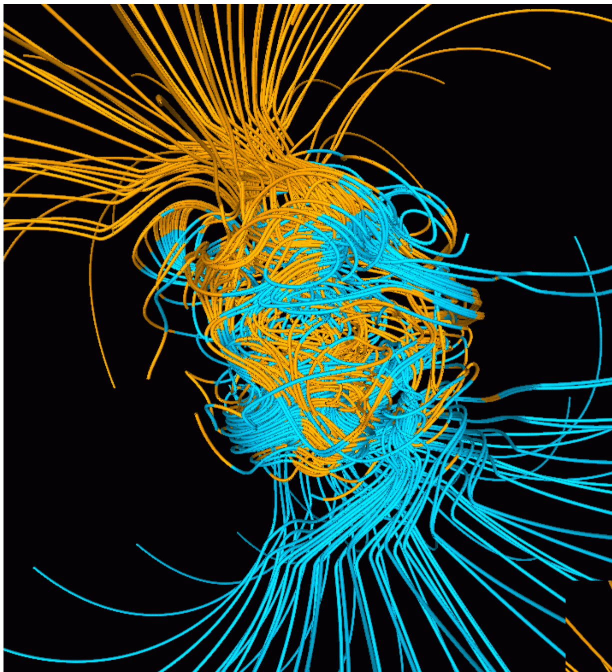
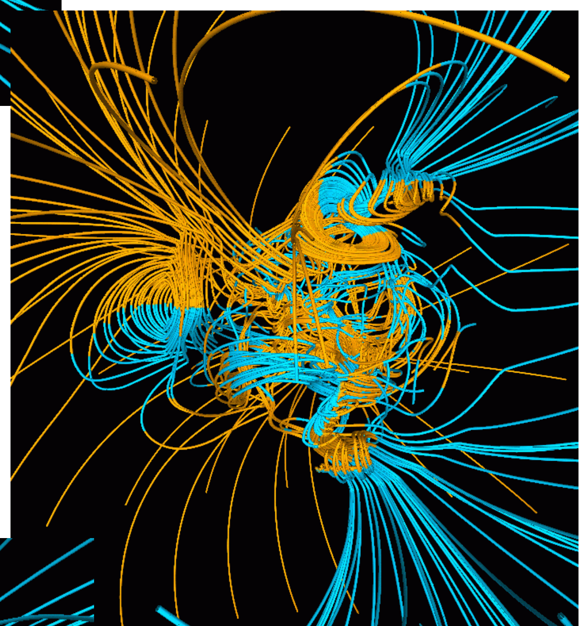


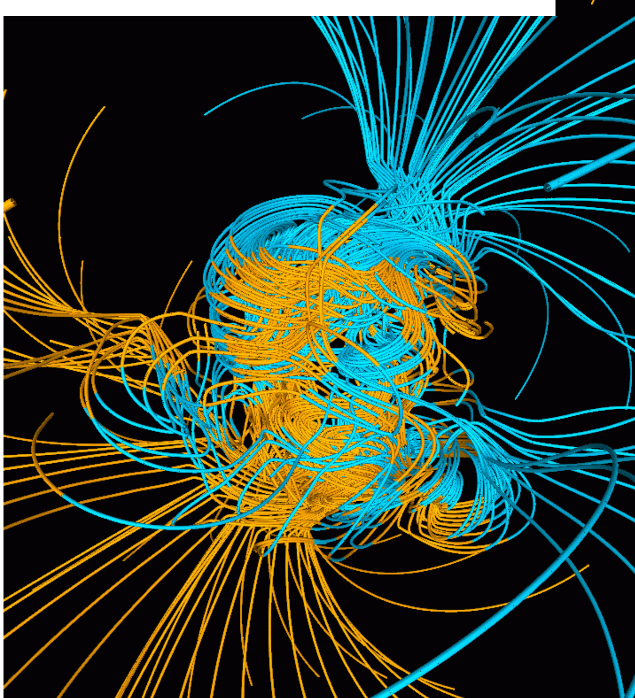
Fig. 8.9. Convergence of two poloidal loops as produced in Fig. 8.8 results in a larger poloidal loop.After Parker (1955a.)

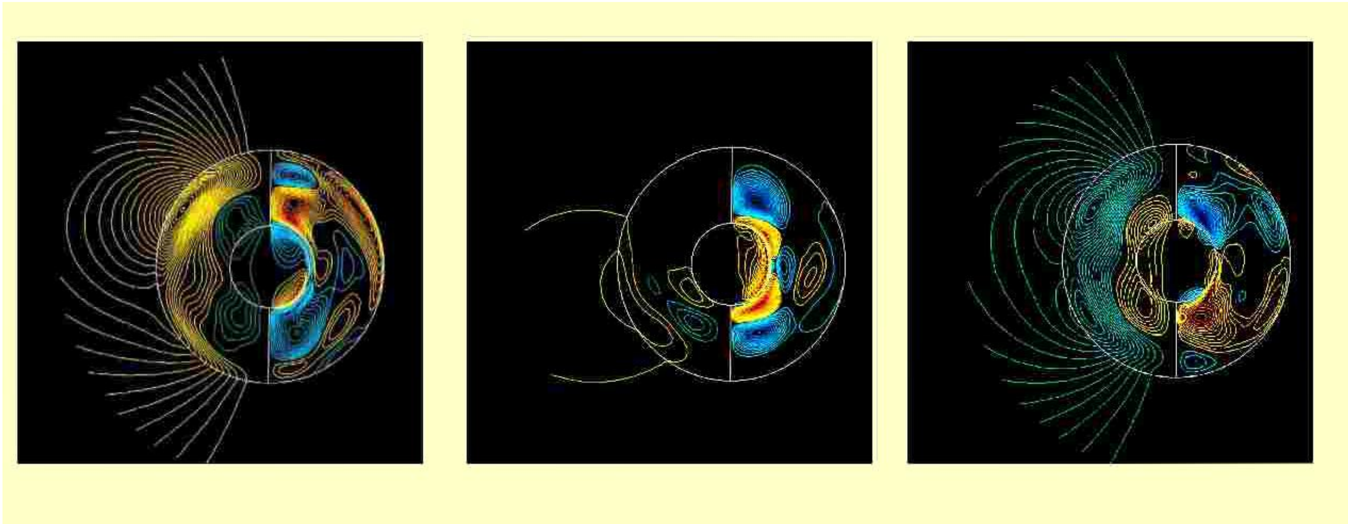


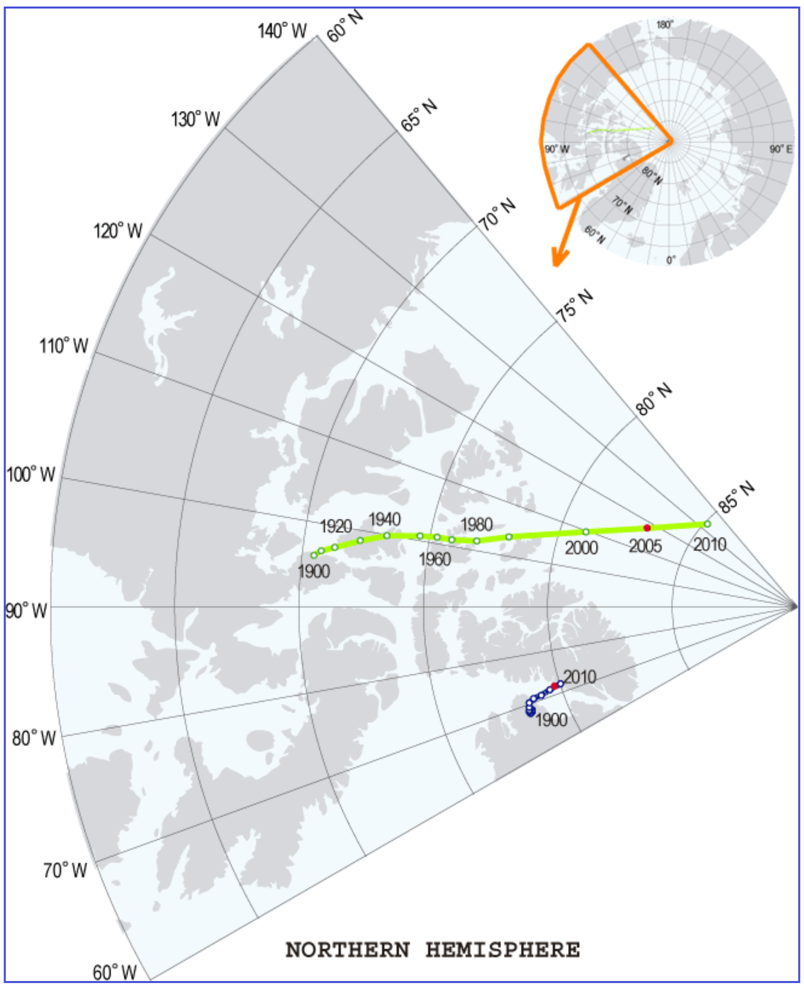












140° W
60° N
130° W
65° N
120° W
70° N
110° W
75° N
100° W
80° N
85° N
90° W
80° W
70° W
60° W
1900
1920
1940
1960
1980
2000
2005
2010
2010
1900
180°
90° W
90° E
80° N
70° N
60° N
0°
NORTHERN HEMISPHERE