Notes on Seismic Tomography 1/18/16

Further reading:
1) Look at the pdf tomography tutorial by Simons that is on the web.
2) Read Fowler, Section 4.1 (Waves through the Earth: pp 100-111) and 4.2.7 (The observations of earthquakes: seismic phases: pp 126-130)

Sources

Earthquakes
Nuclear Explosions
Glaciers
Atmospheric
Oceanic
Volcanic
Biologic
Anthropogenic
…

Structure

Tomography - produces blurry 3D images of shear and compressional wave speed inside the earth

Scattering - produces estimates of steep gradients or discontinuities in wave speeds and density

Body Waves travel through the body of the earth
P – Primary or first arriving waves (Compressional wave: particle motion in the direction of the ray path)
S – Secondary or second arriving wave (Shear wave: particle motion is perpendicular to the direction of the ray path. A shear wave has two polarizations, SH is horizontal motion and SV is perpendicular to the ray and SH)
Measure arrival times, amplitudes

Surface Waves travel horizontally, are trapped at the free surface, amplitude decays with depth
Love Waves: particle motion is horizontal and perpendicular to propagation direction
Rayleigh Waves: elliptical retrograde particle motion in vertical plane containing the propagation direction
Measure dispersion (each frequency travels at a different speed because it has a different sensitivity to structure as a function of depth).

Normal Modes are excited by big earthquakes. These are standing (not traveling) waves that oscillate at fixed frequencies called eigenfrequencies, similar to the fundamental mode and overtones of a guitar string. The longest period mode is about 1000 s.
Measure eigenfrequencies.

Tomography
In a nutshell, the tomography problem is one of determining the wavespeeds (P and or S) as a function of three dimensions in the interior of Earth based on seismograms that record ground motion at the surface of Earth. We call the unknown wave speeds the *Model* and the seismograms the *Data*. Various physical principles of elasticity and wave propagation allow us to calculate *Data* we would observe from a given *Model*. This is known as the *Forward Problem*. *Inverse theory* allows us to invert the *Forward problem* to determine a model that is consistent with the data within its errors and to characterize the range of models that are consistent with the data.

The three key steps to any tomography problem are:
1) **Linearize** the forward problem - typically the data depend on the model in a weakly nonlinear fashion, so we make an approximation to the forward problem that is nearly linear in the vicinity of a starting model.
2) **Discretize** the model - The model is a continuous function of 3-D space making it an infinite dimensional space. It is convenient to parameterize the model, typically either by dividing earth up into a finite number of blocks, expanding the velocity model in global basis function such as spherical harmonics, or in local basis functions such as splines.
3) Regularize the inversion - Typically there are not enough data to fully determine the value of each of the parameters of the model so assumptions need to be made. This is a key part of inverse theory. Common solutions are to "find the smallest model that fits the data to with their uncertainties" or "find the smoothest model that fits the data to with their uncertainties" or "find the model that is closest to some preferred model and fits the data to with their uncertainties". The answers to these three questions should be similar where the data coverage is good, but will be very different where the model is not well constrained by the data.

Data used to locate earthquakes and determine earth structure
Arrival times of P and S waves
Phase and group velocities of surface waves
Eigenfrequencies of Normal modes

Model represents the unknown features of the earth that we want to know.

$Vs(r, \theta, \phi) = \mathrm{sqrt}(\mu/\rho)$ = shear wave velocity
$Vp(r, \theta, \phi) = \mathrm{sqrt}((\kappa + 4/3\mu)/\rho)$ = compressional wave velocity

μ = shear modulus
ρ = density
κ = incompressibility

Forward Problem is to calculate Data from a Model, for example what travel time would I get for a given 3D model of the earth. This represents the Physics of wave propagation. Travel time tomography typically relies on "ray theory" which is an asymptotic theory valid at high frequencies (but not valid at low frequencies). A hot topic now is the development of a higher-order theory known as banana-donut theory, but that is beyond the scope of this course.

Ray theory is based on the premise that the travel time (time for a wave to go from a source fixed in space to a fixed seismometer) is the integral of 1/v along a ray path. v is the P or S wave speed depending of the wave of interest. A ray path is the path along which the travel time is a minimum (or maximum). The hardest part of doing tomography, aside from collecting the data, is calculating the ray paths.

The first equation below defines the travel time through a velocity (v), the second gives the *travel-time anomaly* defined as the travel time through some perturbed model v+δv minus the travel time through a reference model v. The final equation is a *Linearized* version. The first step invokes Fermat's Principle, which is key to the whole process. Fermat's Principle states that for two points A and B on a ray, the ray itself is a path along which, in the velocity field v(**x**), the travel time from A to B is stationary. This means that the derivative of travel time with respect to any small perturbation to the path is zero, which is important because it means that we get the correct answer (to first order) even if we have small errors in the ray path.

$$T = \int_{ray} \frac{1}{v} ds$$

$$\delta T = \int_{ray(v+\delta v)} \frac{1}{v+\delta v} ds - \int_{ray(v)} \frac{1}{v} ds$$ exact expression for travel-time anomaly

$$\delta T = \int_{ray(v)} \frac{1}{v+\delta v} ds - \int_{ray(v)} \frac{1}{v} ds$$ note that first integral is along ray for the reference model v

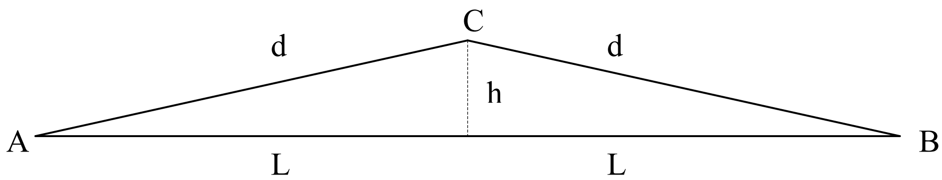
$$\delta T \cong \int_{ray(v)} \frac{-dv}{v^2} ds$$ Linearized Forward Problem (δT depends linearly on δv)

$$\delta T \cong G\delta v .$$ δT is vector of travel times, δv is a vector of parameters describing velocity, model G is a large matrix that approximates the integral expression above.

The bottom line is that we now have an equation that allows us to calculate the travel-time anomaly that you would get if the velocity was perturbed by some δv.

The inverse problem is to determine δv from the known data δT:

$\delta v = G^{-1}\delta T$ This is great if the matrix G is invertible, but it seldom is. The problem of regularization is to find an approximation to the inverse of G as described in the Regularization discussion above.



Simple example of Fermat's principle. In a homogeneous medium with wave speed v, the time from A to B through C is $T = 2\ d\ /\ v\ = 2/v\ (L^2+h^2)^{1/2}$
The value of h that corresponds to a ray path is where $dT/dh = 2/v\ (L^2+h^2)^{-1/2}\ \ h = 0$; This occurs at h=0, so the straight line (h=0) is a ray path.