ESS 524 Class #2

Highlights from Monday – Shashank
Today's highlights next Monday
___Surabhi__________________

- Next Monday – 90-second initial ideas about projects.
- HW #1 (Matlab) due next Wednesday. I will set up a Canvas site where you can turn it in.
- Week 1 journals due Monday. I will set up a Canvas site.

Today
- Matlab Basics (code is available under MATLAB CODE tab)
- Derivation of Conservation Laws for Mass and Energy

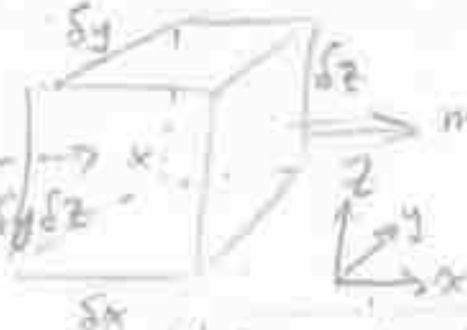
Conservation Equations

"stuff" into/out of boxes $\Rightarrow$ changes inside box

- revisit mass conservation

internal change: $\frac{\partial \rho}{\partial t}\delta x \delta y \delta z$

mass in: $\left(\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z$



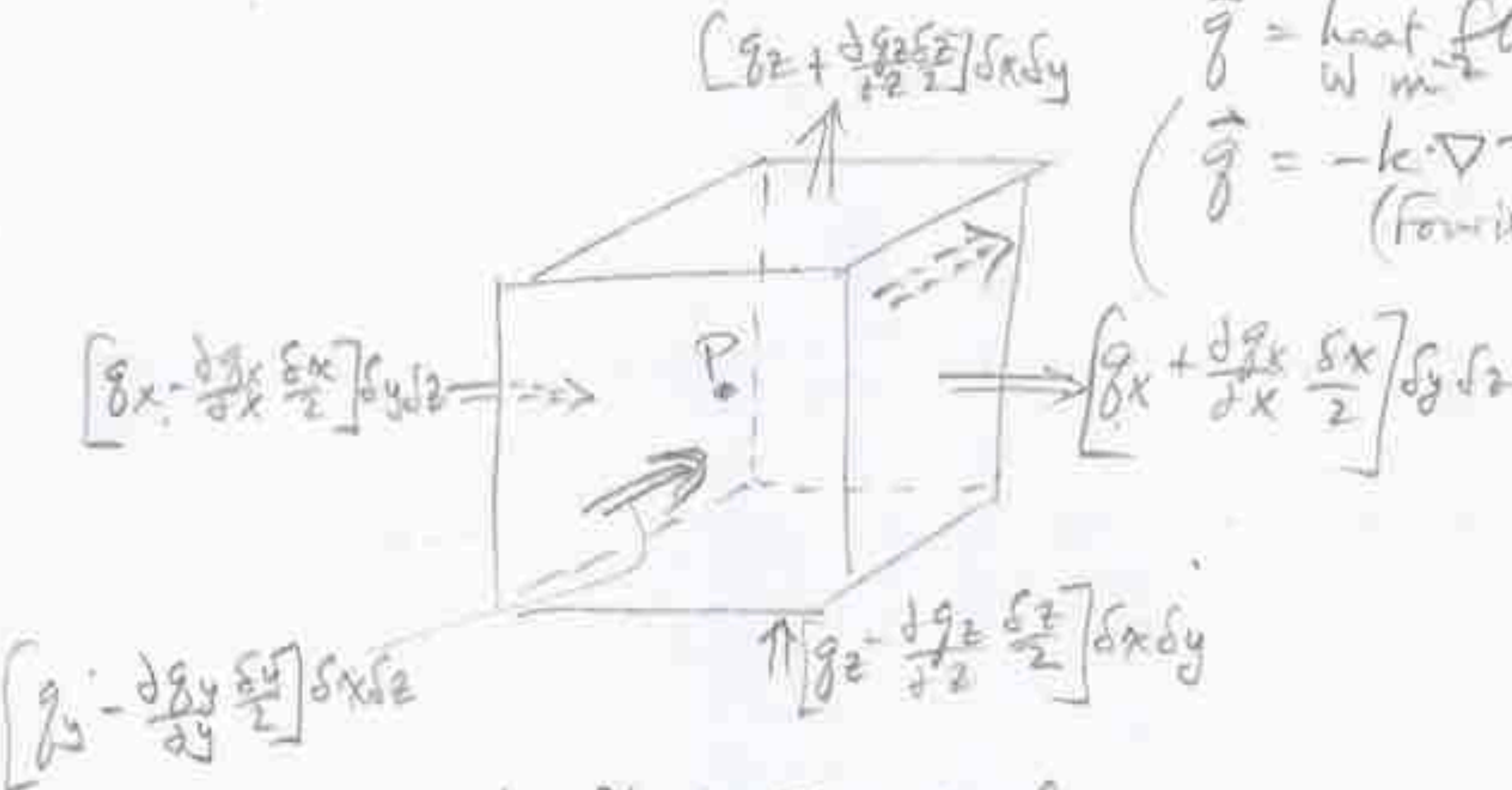
mass out $= \left[\rho u + \frac{\partial(\rho u)}{\partial x}\frac{\delta x}{2}\right]\delta y \delta z$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \vec{u}) = 0}$$

Energy Equation

$\vec{q}$ = heat flux W m^{-2}

$$\left(\vec{q} = -k \cdot \nabla T \quad \text{(Fourier)} \right)$$



Add all the fluxes through faces.

$$\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] \delta x \delta y \delta z = \nabla \cdot \vec{q}\, \delta x \delta y \delta z$$

Flux $\vec{q}$ can have diffusive & advective contributions

$$\vec{q}_{dif} = -k \nabla T \qquad \vec{q}_{adv} = \rho E \vec{u} \qquad E = \text{enthalpy}$$

$$= \int_0^T C(T')\, dT'$$

$$\nabla \cdot \vec{q}_{dif} = -\nabla \cdot (k \nabla T)$$

$C(T)$ = specific heat $J kg^{-1} K^{-1}$

Thermal physics independent of space

$$\nabla \cdot \vec{q}_{adv} = \nabla \cdot (\rho E \vec{u})$$

$$= E \nabla \cdot (\rho \vec{u}) + \rho \vec{u} \cdot \nabla E$$

$$= E \nabla \cdot (\rho \vec{u}) + \rho C(T) \vec{u} \cdot \nabla T$$

$$\left\{ \begin{aligned} \frac{\partial E}{\partial x} &= C(T) \frac{\partial T}{\partial x} \\ \frac{\partial E}{\partial y} &= C(T) \frac{\partial T}{\partial y} \\ \frac{\partial E}{\partial z} &= C(T) \frac{\partial T}{\partial z} \end{aligned} \right.$$

So -

Net flux through all sides

Change of Energy inside box

$$\frac{\partial}{\partial t}(\rho E)\,\delta x\,\delta y\,\delta z = \left(\rho\frac{dE}{dt} + E\frac{d\rho}{dt}\right)\delta x\,\delta y\,\delta z$$

$$= \left(\rho c\frac{dT}{dt} + E\frac{\partial\rho}{\partial t}\right)\delta x\,\delta y\,\delta z$$

Enthalpy

$$E = \int_0^T c(T')\,dT'$$

$$\frac{dE}{dt} = c(T)\frac{dT}{dt}$$

c doesn't depend on time (physics is invariant through time)

Sources $S(\vec{x},t)$ [energy/unit vol/unit time]

$$S\,\delta x\,\delta y\,\delta z$$

All together now

$$\delta x\delta y\delta z\left[\rho c\frac{\partial T}{\partial t} + E\frac{\partial\rho}{\partial t} + E\nabla\cdot(\rho\vec{u}) + \rho c\,\vec{u}\cdot\nabla T - \nabla\cdot(k\nabla T) - S\right] = 0$$

Cancel $\delta x\,\delta y\,\delta z$

$$\left[\rho c\frac{dT}{dt} + E\Big(\underbrace{\frac{d\rho}{dt} + \nabla\cdot(\rho\vec{u})}_{=0}\Big) + \rho c\,\vec{u}\cdot\nabla T - \nabla\cdot(k\nabla T) - S\right] = 0$$

Cancel $\delta x \delta y \delta z$

$$\left[\rho c \frac{dT}{dt} + E\left(\underbrace{\frac{d\rho}{dt} + \nabla\cdot(\rho\vec{u})}_{=0}\right) + \rho c\,\vec{u}\cdot\nabla T - \nabla\cdot(k\nabla T) - S\right] = 0$$

$$\underbrace{\rho c\frac{dT}{dt}}_{\text{transient}} + \underbrace{\rho c\,\vec{u}\cdot\nabla T}_{\text{advection}} - \underbrace{\nabla\cdot(k\nabla T)}_{\text{diffusion}} - \underbrace{S}_{\text{source}} = 0$$