ESS 524 Class #7

Highlights from last Wednesday – Erich
Today's highlights report on this coming Wednesday – David

Today

- Project updates
- FVM Basics
    - 1-way and 2-way coordinates
    - Notation: Δx vs $(\delta x)_e$; a_P vs a_e; k_P vs k_e
    - Interpolation in a volume
    - Source term - $S(\phi)$ dependence
    - Formation of matrix equations
    - 4 Rules

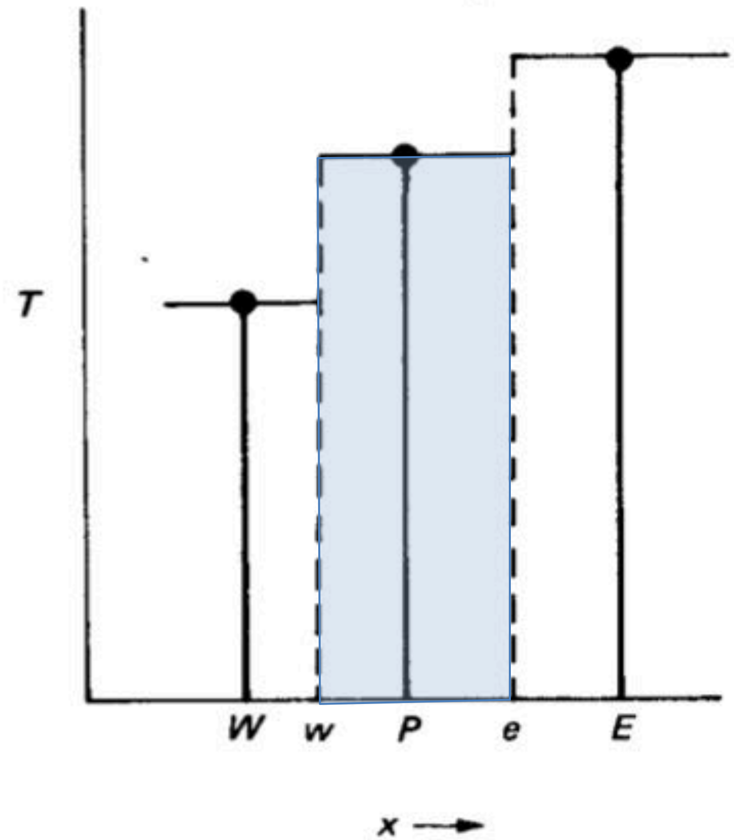
Next Week

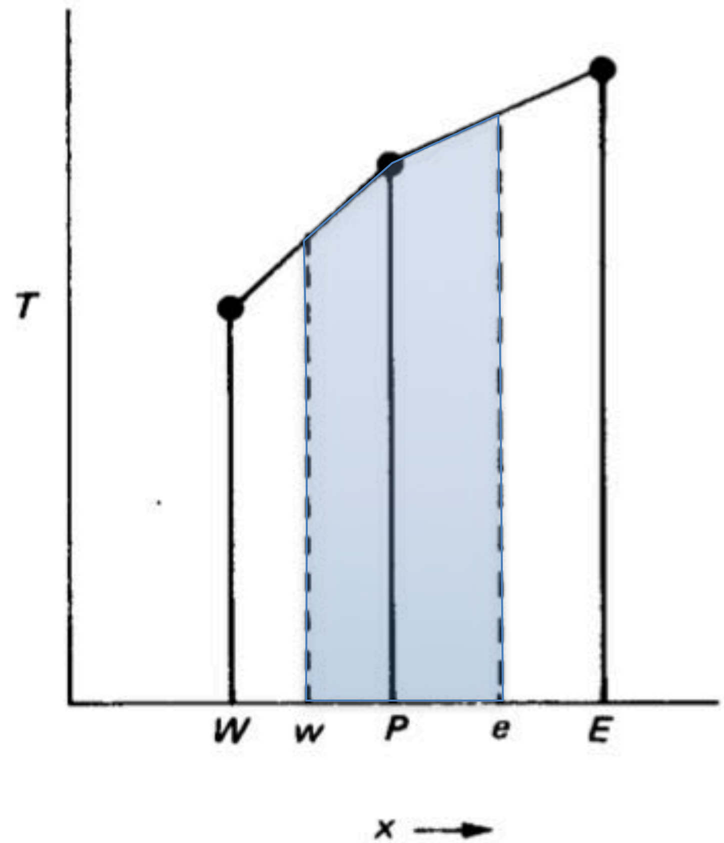
- Time stepping

Options for Interpolation Function

Notation for Finite Volume around node *P*

- Nodes are upper case - *W*est, *P*oint, *E*ast
- Volume edges are lower case - *w*est edge, *e*ast edge
- In 2-D add *N,S, n,s;* in 3-D, add *U,D,u,d*

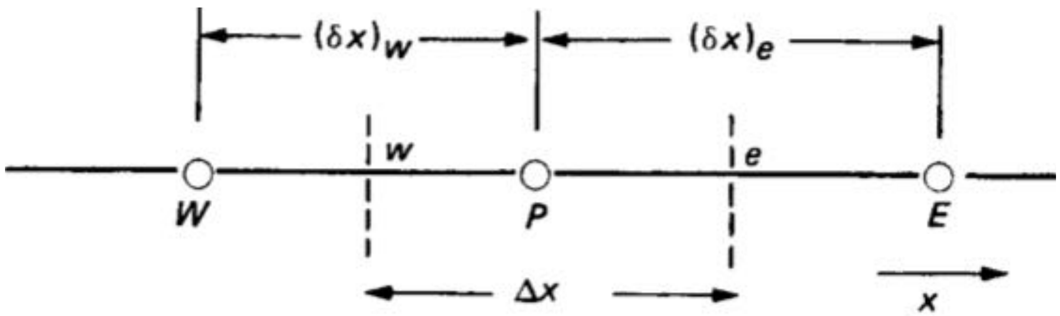




Differential Equation for steady 1-D heat flow

$$\frac{d}{dx}\left(k\,\frac{dT}{dx}\right) + S = 0\,, \qquad (3.10)$$

Mesh around node *P*



Integrate Diff Eq over the finite volume

$$\left(k\,\frac{dT}{dx}\right)_e - \left(k\,\frac{dT}{dx}\right)_w + \int_w^e S\,dx = 0\,. \qquad (3.11)$$

Difference in boundary fluxes

+ internal source production = 0

Setting up a matrix equation

$$\frac{k_e(T_E - T_P)}{(\delta x)_e} - \frac{k_w(T_P - T_W)}{(\delta x)_w} + \bar{S}\,\Delta x = 0\,, \qquad (3.12)$$

Collect terms multiplying each nodal value

$$a_P T_P = a_E T_E + a_W T_W + b\,, \qquad (3.13)$$

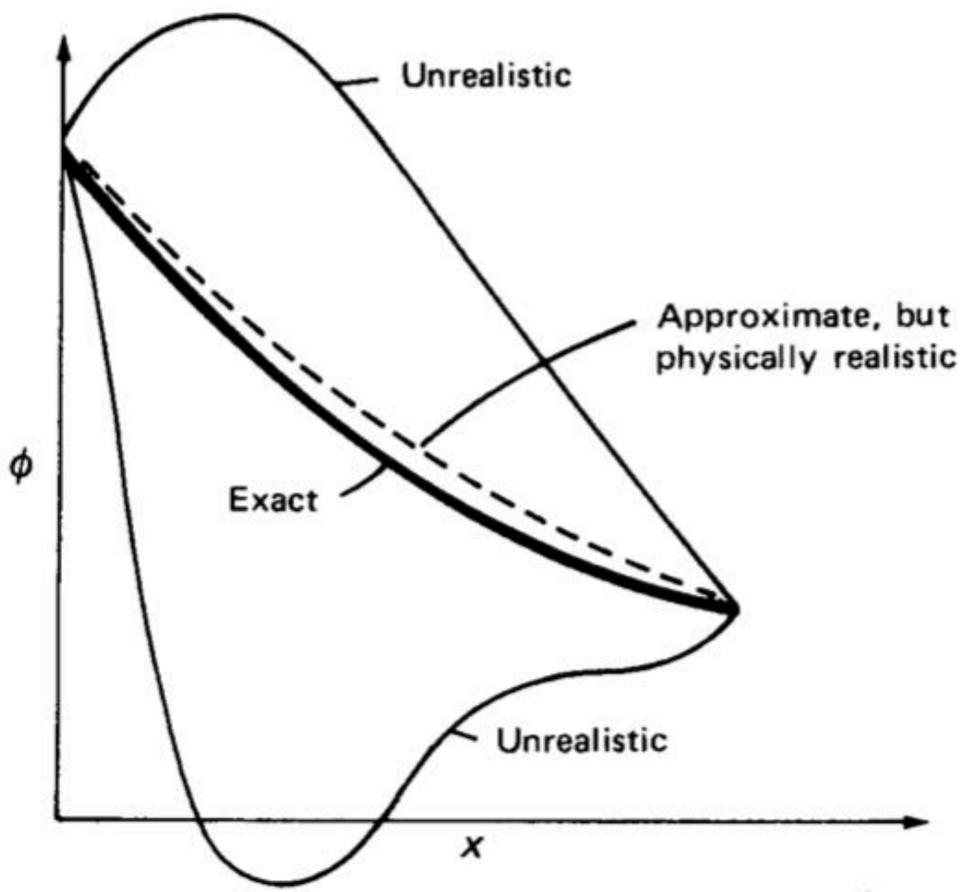
$$a_E = \frac{k_e}{(\delta x)_e}\,, \qquad (3.14a)$$

$$a_W = \frac{k_w}{(\delta x)_w}\,, \qquad (3.14b)$$

$$a_P = a_E + a_W\,, \qquad (3.14c)$$

$$b = \bar{S}\,\Delta x\,. \qquad (3.14d)$$

Common-sense Discretization Schemes

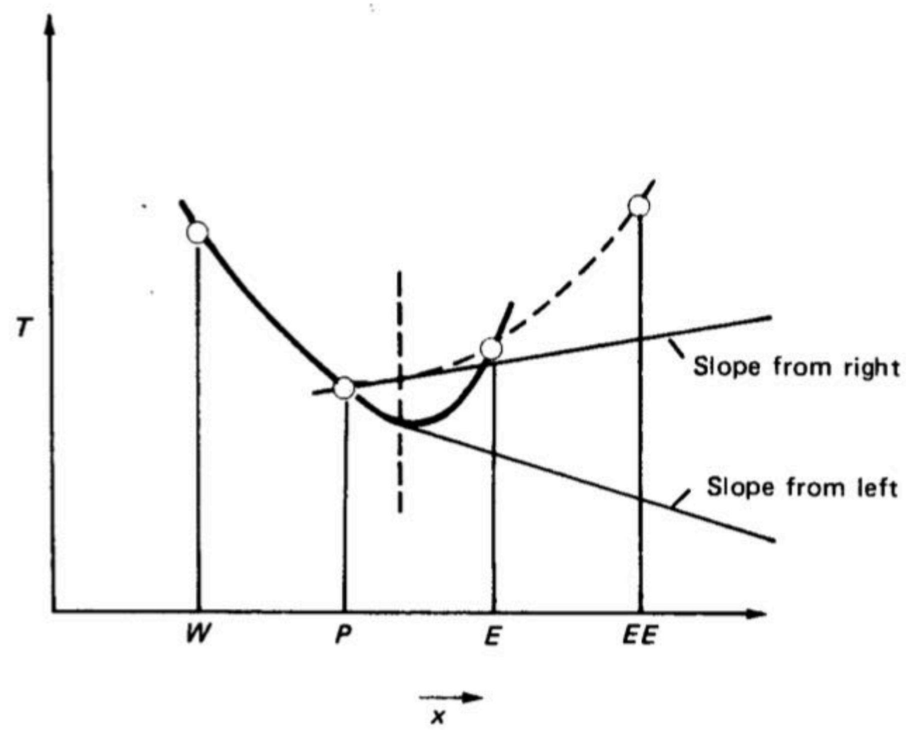


- A realistic approximation should have same trends as exact solution
- Conservation Laws should hold globally, even with a coarse mesh.
- Fluxes across a boundary between 2 volumes must be consistent.
- Source linearization should not create numerical instability. (Solving equations for a real physical instability is tricky already.)

The Four Rules

1. Fluxes across a boundary between 2 volumes must be consistent from the left and from the right.

- Surely a quadratic estimates of the slope of the solution must be better than linear estimate, right?
- Wrong! The estimate must be identical whether considering Volume *P* or Volume *E*.



The Four Rules

2. In absence of T-dependent sources, all coefficients a_P, a_E etc must have the same sign.
   - Otherwise, an increase in T at node E could cause a decrease in T at node P.
   - Diffusion doesn't work that way!

3. Negative-slope linearization of the source term. $\bar{S} = S_C + S_P T_P$
   - If an estimate of T is too big, then so is the source, and then T increases again at the next iteration

4. In absence of T-dependent sources, a_P should equal the sum of neighbor coefficients a_E etc. $a_P = \Sigma\, a_{nb}$
   - When only derivatives of T are present, $T=T+c$ should also be a solution.
   - Solution T_P should be a weighted average of the surrounding values.
   - When all neighbors T_{nb} are equal, the center point T_P must have the same value (steady state)

Equations with *T*-dependent Source Term

$$\frac{d}{dx}\left(k\,\frac{dT}{dx}\right) + S = 0\,. \qquad (4.1)$$

This leads to the discretization equation

$$a_P T_P = a_E T_E + a_W T_W + b\,, \qquad (4.2)$$

where

$$a_E = \frac{k_e}{(\delta x)_e}\,, \qquad (4.3a)$$

$$a_W = \frac{k_w}{(\delta x)_w}\,, \qquad (4.3b)$$

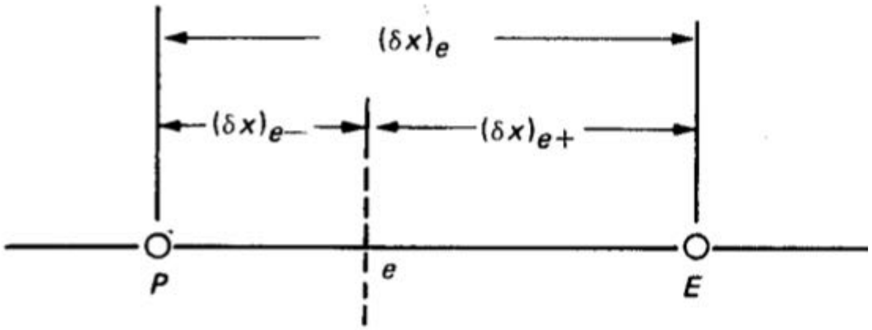
$$a_P = a_E + a_W - S_P\,\Delta x\,, \qquad (4.3c)$$

and

$$b = S_C\,\Delta x\,. \qquad (4.3d)$$

Interface conductivity k_e

$$q_e = \frac{k_e(T_P - T_E)}{(\delta x)_e}$$



Linear mix

$$k_e = f_e k_P + (1 - f_e) k_E,$$

$$f_e \equiv \frac{(\delta x)_{e+}}{(\delta x)_e}$$

$$q_e = \frac{T_P - T_E}{(\delta x)_{e-}/k_P + (\delta x)_{e+}/k_E}.$$

$$k_e^{-1} = 0.5(k_P^{-1} + k_E^{-1})$$

$$k_e = \left(\frac{1 - f_e}{k_P} + \frac{f_e}{k_E}\right)^{-1}$$

$$k_e = \frac{2k_P k_E}{k_P + k_E}.$$