ESS 524 Class #8

Highlights from last Monday – David
Today's highlights report on next Monday – Erich
Today

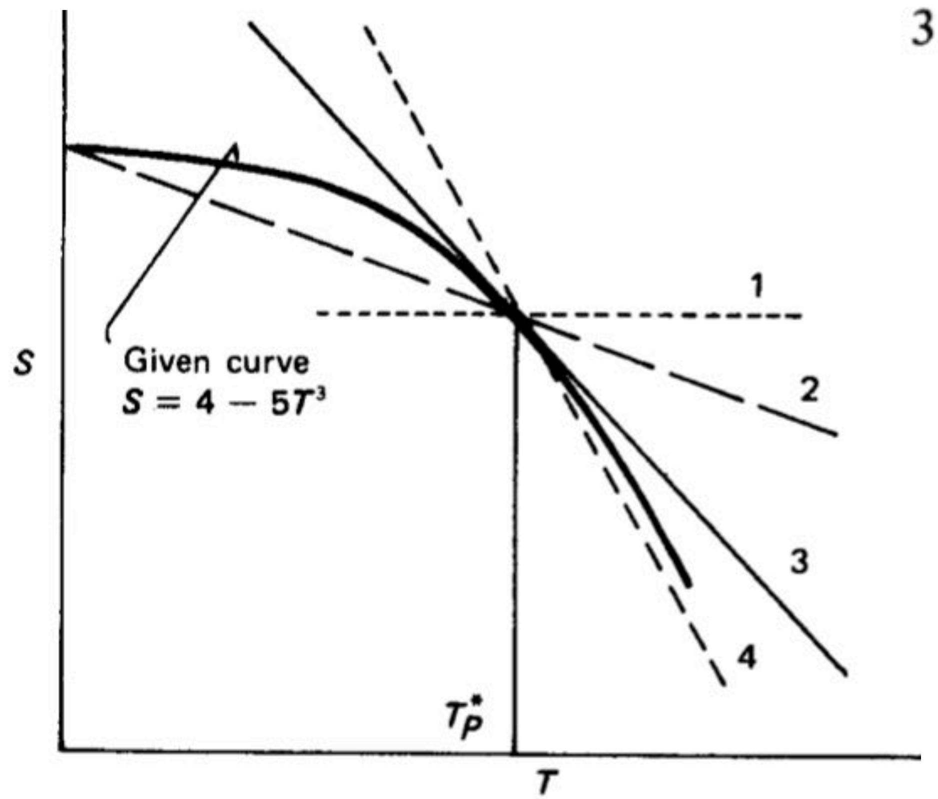
- More on source linearizations
- Treatment of boundaries
- Transient problems
  - Patankar Section 4.3
  - Ed's Notes on time stepping

Next Week

- Time stepping - instability
  - Ed's note on transfer functions for diffusion equation
- Linear instability analysis, transfer functions
- HW #3 due Wednesday. How's it going?

Source linearization if $S = 4 - 5\,T^3$

Iterations will be required.
S^* and T^* indicate values at current iteration.



Desired **linear** form:

$$S = S_c + S_P\,T$$

How to choose S_c and S_P?

1. $S_C = 4 - 5T_P^{*3}$, $S_P = 0$.

2. $S_C = 4$, $S_P = -5T_P^{*2}$

3. Recommended method:
   tangent to the $S \sim T$ curve at T_P^*.

$$S = S^* + \left(\frac{dS}{dT}\right)^* (T_P - T_P^*)$$

$$= 4 - 5T_P^{*3} - 15T_P^{*2}(T_P - T_P^*)$$

$$S_C = 4 + 10T_P^{*3}, \qquad S_P = -15T_P^{*2}.$$

4. $S_C = 4 + 20T_P^{*3}$, $S_P = -25T_P^{*2}$

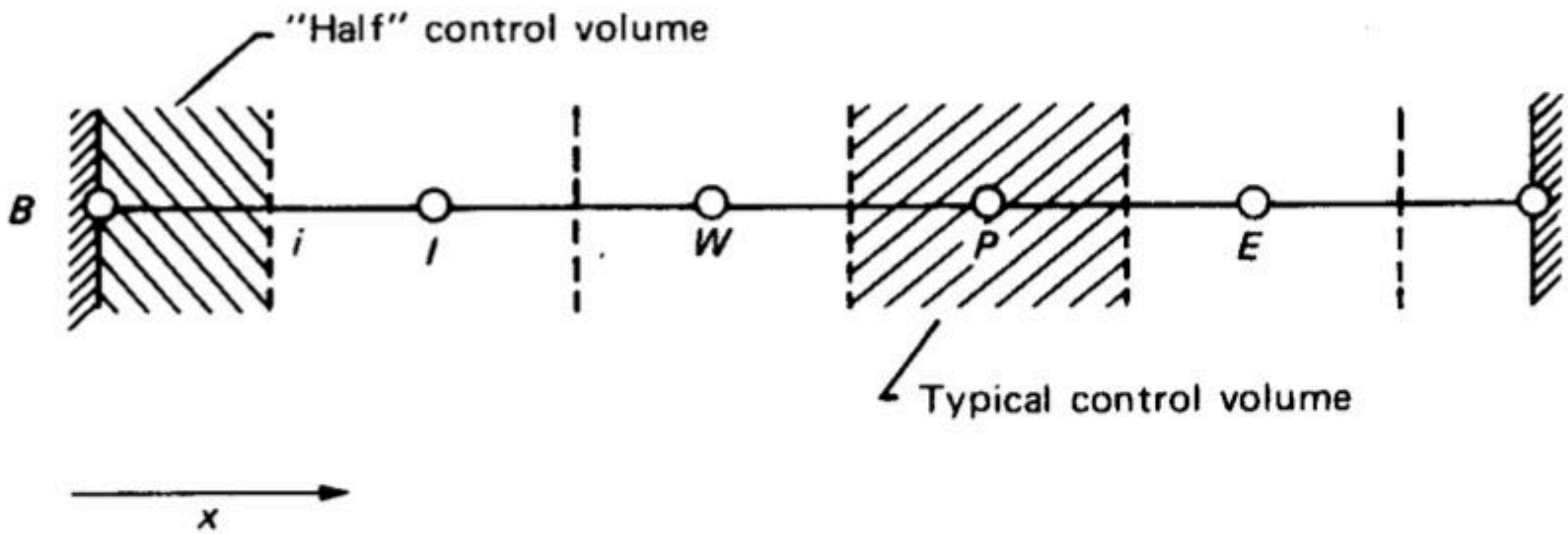
Nonlinear coefficients also arise, e.g.

- thermal conductivity $k(T)$
- Specific heat capacity $c_p(T)$

They can also be treated through iterations

The Challenge at Boundaries – half-volume solution

- Specified-value boundary conditions are applicable at nodes.
- Flux boundary conditions are applicable at volume edges .



- Specified-value at x_B: $T_B = T_0$
-
- Specified q_B flux at x_B: $q_B - q_i + (S_C + S_P T_B)\,\Delta x = 0$

Flux difference across half volume

+ total source in half volume = 0

Extension to 2-D

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + S$$

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b$$

$$a_E = \frac{k_e \, \Delta y}{(\delta x)_e},$$

$$a_W = \frac{k_w \, \Delta y}{(\delta x)_w},$$

$$a_N = \frac{k_n \, \Delta x}{(\delta y)_n},$$

$$a_S = \frac{k_s \, \Delta x}{(\delta y)_s},$$

$$a_P^0 = \frac{\rho c \, \Delta x \, \Delta y}{\Delta t},$$

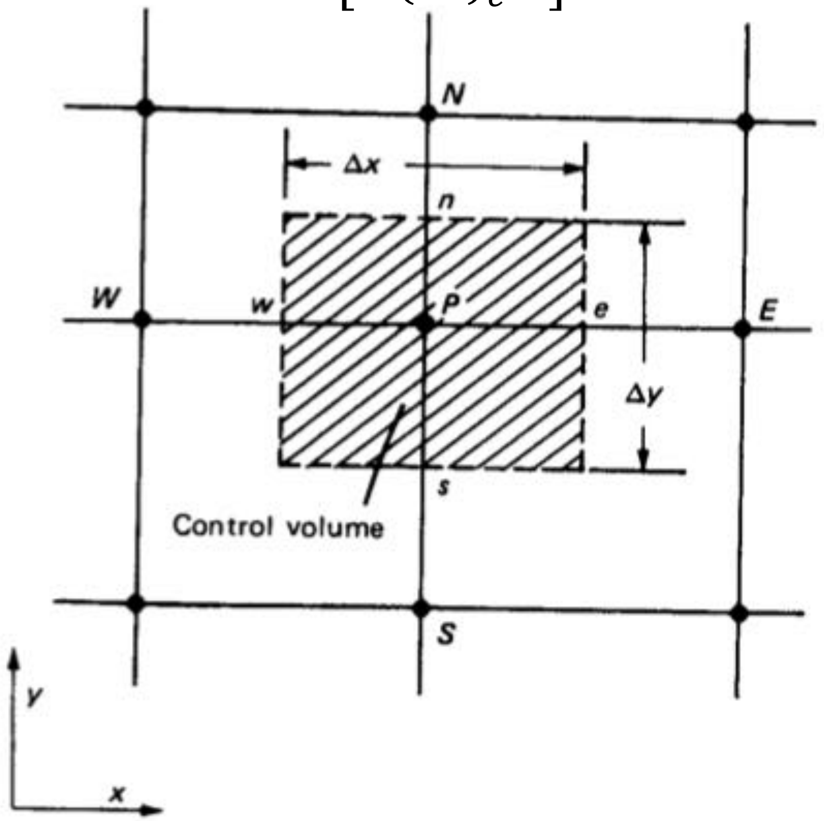
$$b = S_C \, \Delta x \, \Delta y + a_P^0 T_P^0,$$

$$a_P = a_E + a_W + a_N + a_S + a_P^0 - S_P \, \Delta x \, \Delta y .$$

Flux q_e across edge x_e is assumed to be

- uniform along the volume edge Δy
- calculated from nodes P and E

$$q_e = k_e \left[\frac{(T_E - T_P)}{(\delta x)_e}\right] \Delta y$$



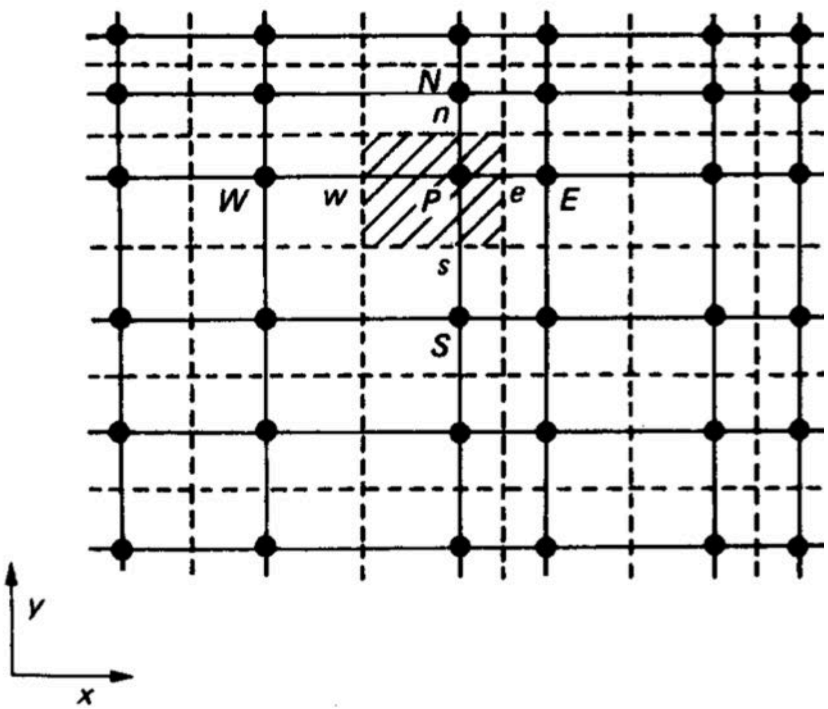
Additional considerations in 2-D

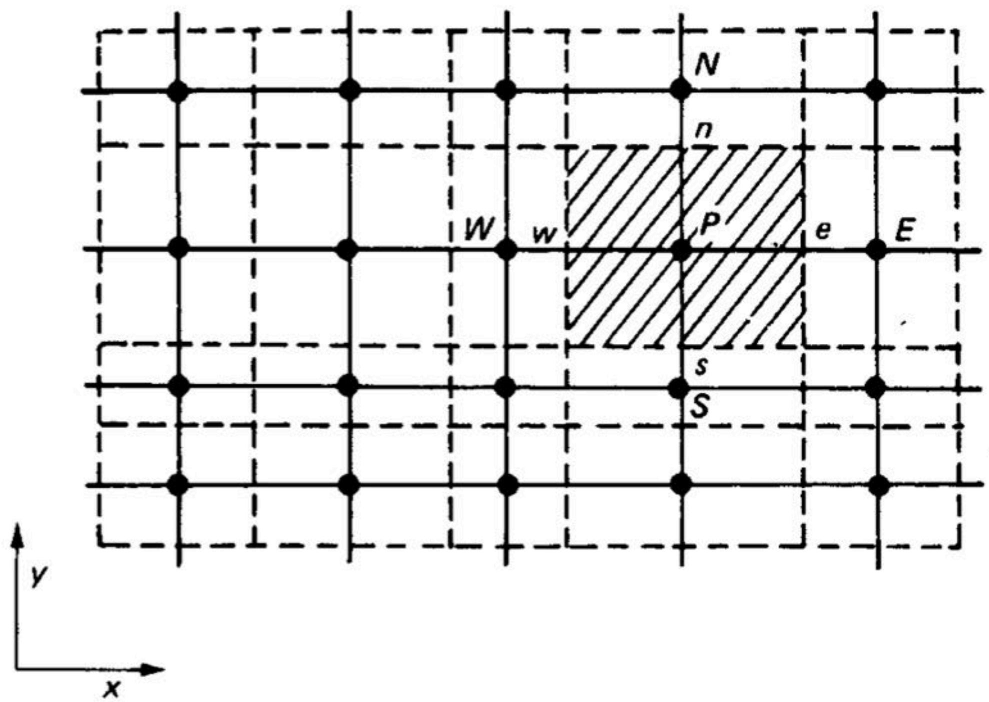
Practice A

- volume edges are halfway between nodes
- Fluxes are calculated halfway between nodes
- But not at center of faces
- P is not in center of volume

Practice B

- nodes are halfway between volume edges
- P is in center of volume
- Fluxes are calculated at face centers
- but not halfway between nodes
- Works well for composite materials





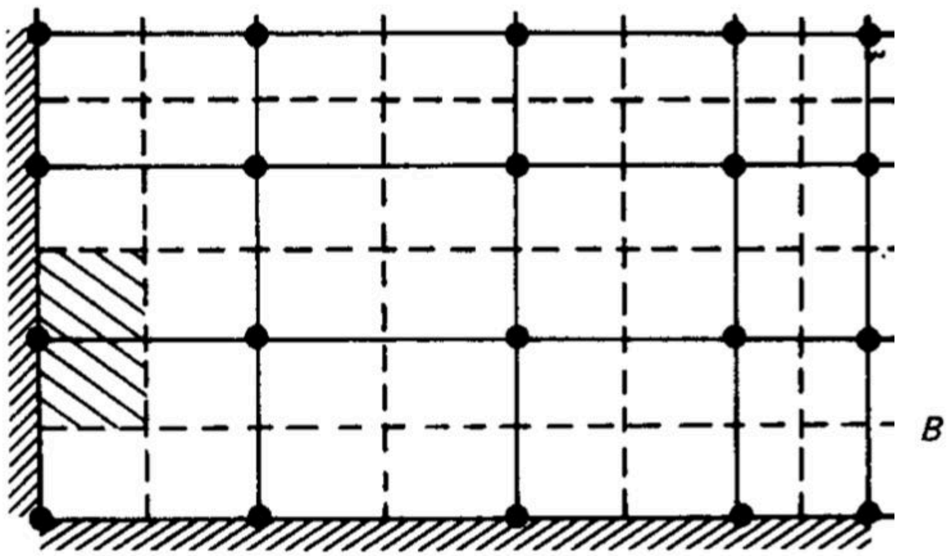
Boundary volumes in 2-D

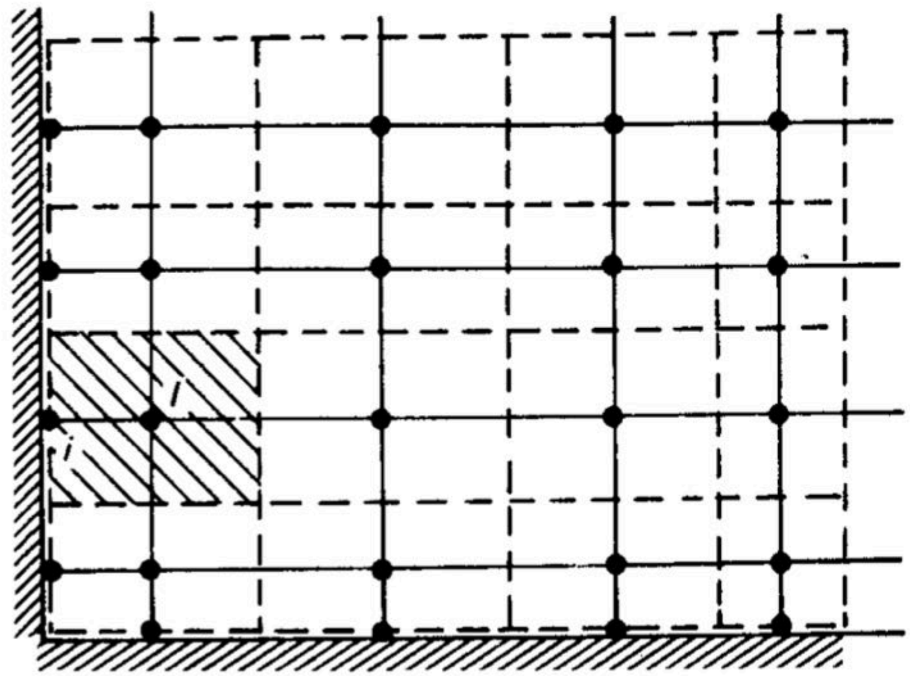
Practice A

- Boundary volumes are half-volumes.
- Treat same as in 1-D

Practice B

- Boundary volumes are infinitesimally thick
- Boundary node and first two edges coincide, so either type of BC can be applied easily.





Time stepping

Read *Ed's notes on time stepping* under the READING tab on the class web page.

Time-splitting parameter α

- Fully Explicit
- Crank Nicolson
- Fully Implicit

Patankar uses simplest possible model to illustrate each new concept

e.g. Behavior of a single node with fixed neighbors

- When space and time are continuous (analytical solution)
- When space is discretized and time is continuous
- When space and time are both discretized (as in a typical numerical code)