ESS 524 Class #9

Highlights from last Wednesday – Erich
Today's highlights report on next Wednesday – Shashank

Today

- Time stepping – Linear Computational Instability
  - Ed's note on transfer functions for diffusion equation
- Linear instability analysis, transfer functions
- HW #3 due Wednesday. How's it going?

Next – including advection (when velocity is known)

- Patankar – Chapter 5
- Versteeg and Malalasekara – Chapter 5
- Ed's Notes on transfer functions with advection

Time stepping

Read *Ed's notes on time stepping* under the READING tab on the class web page.

Time-splitting parameter α

- α =0 Fully Explicit
- α =0.5 Crank Nicolson
- α =1 Fully Implicit

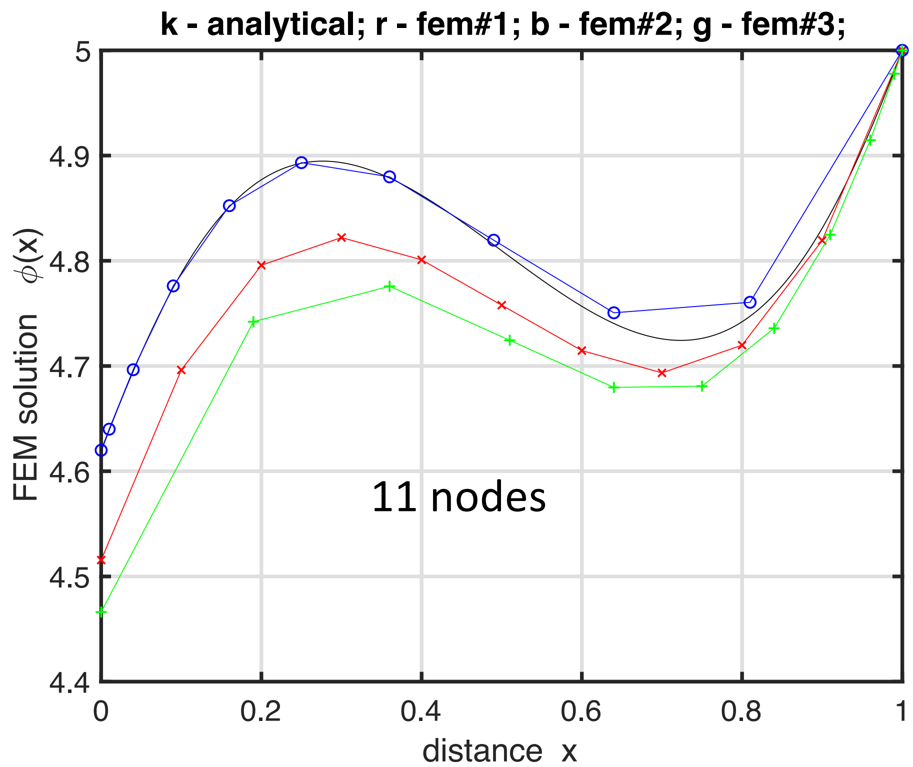
Patankar uses simplest possible model to illustrate each new concept

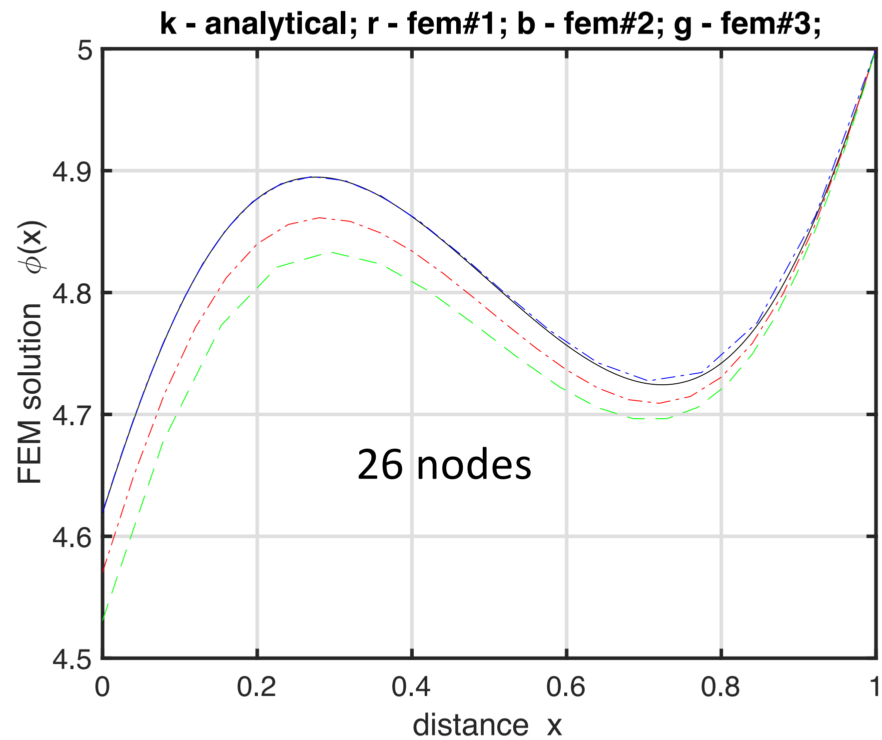
e.g. Behavior of a single node with fixed neighbors

- When space and time are continuous (analytical solution)
- When space is discretized and time is continuous
- When space and time are both discretized (as in a typical numerical code)

HW #3

Finite Element Solution
#1 Red – nodes uniformly spaced
#2 Blue – nodes concentrated near x=0
#3 Green – nodes concentrated near x=1

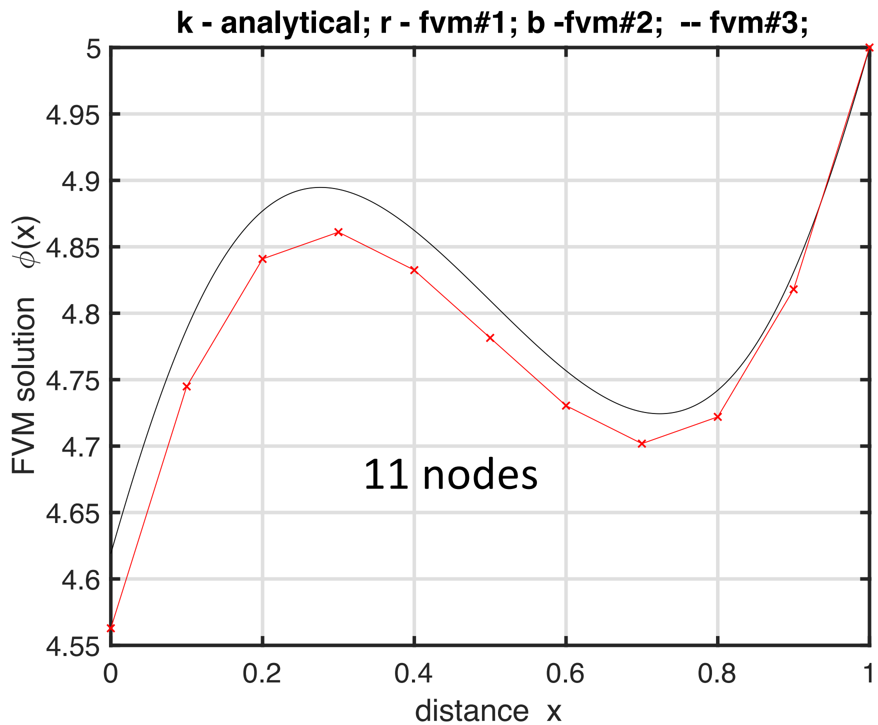


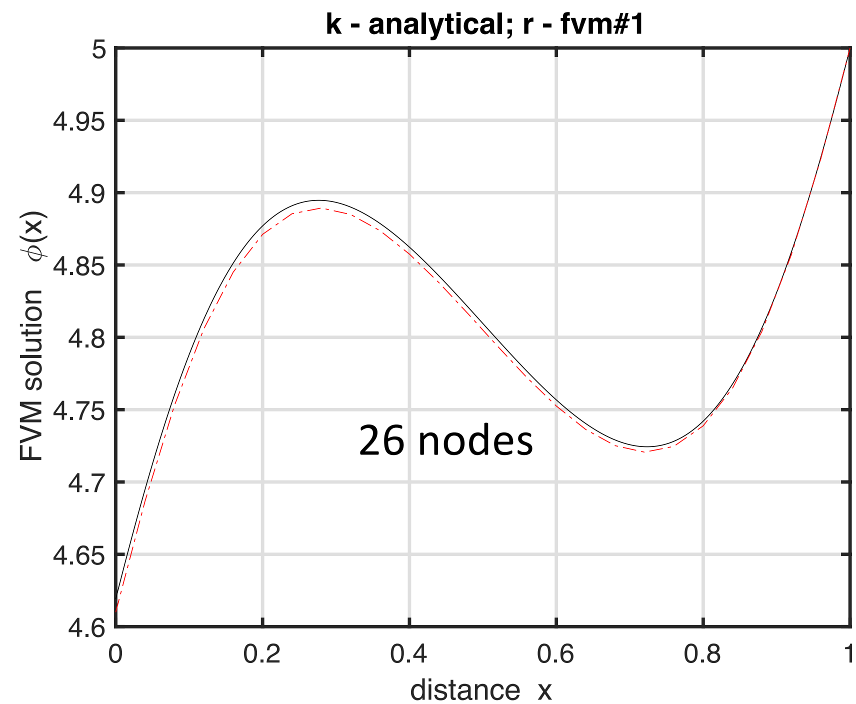


HW #3

Finite Volume Solution

- Nodes uniformly spaced

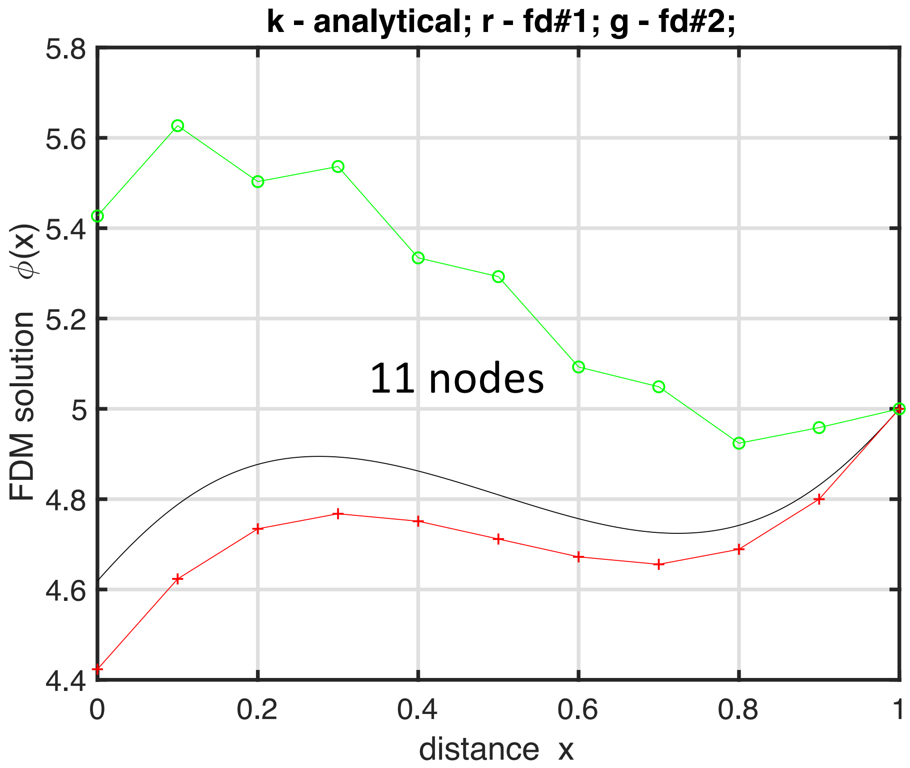


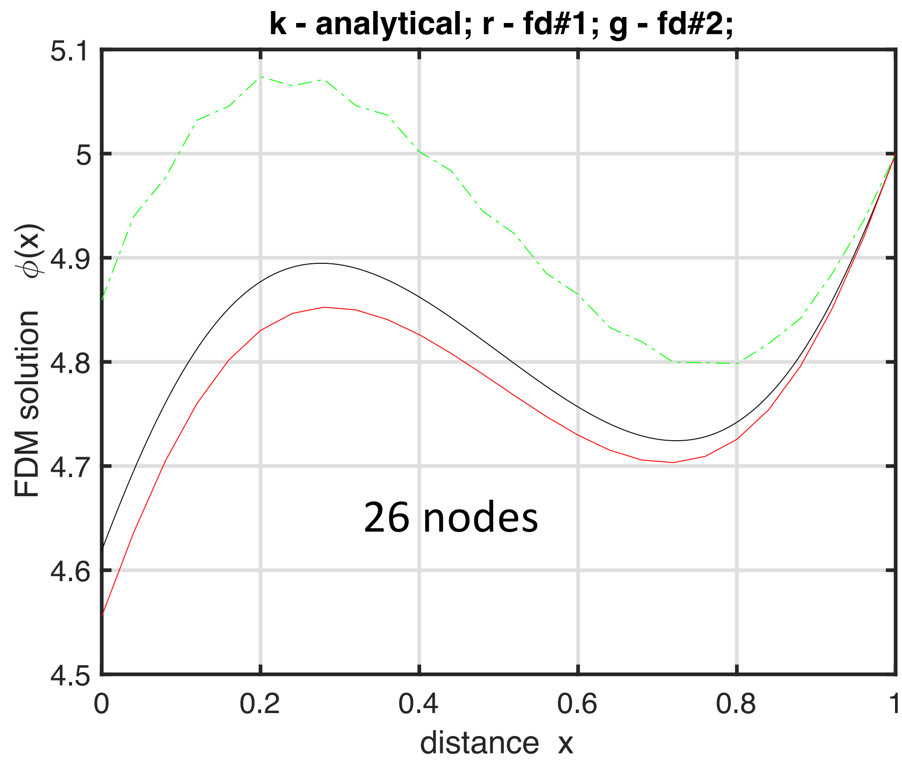


HW #3

Finite-difference Solution

- Nodes uniformly spaced
- 2 poor discretization schemes

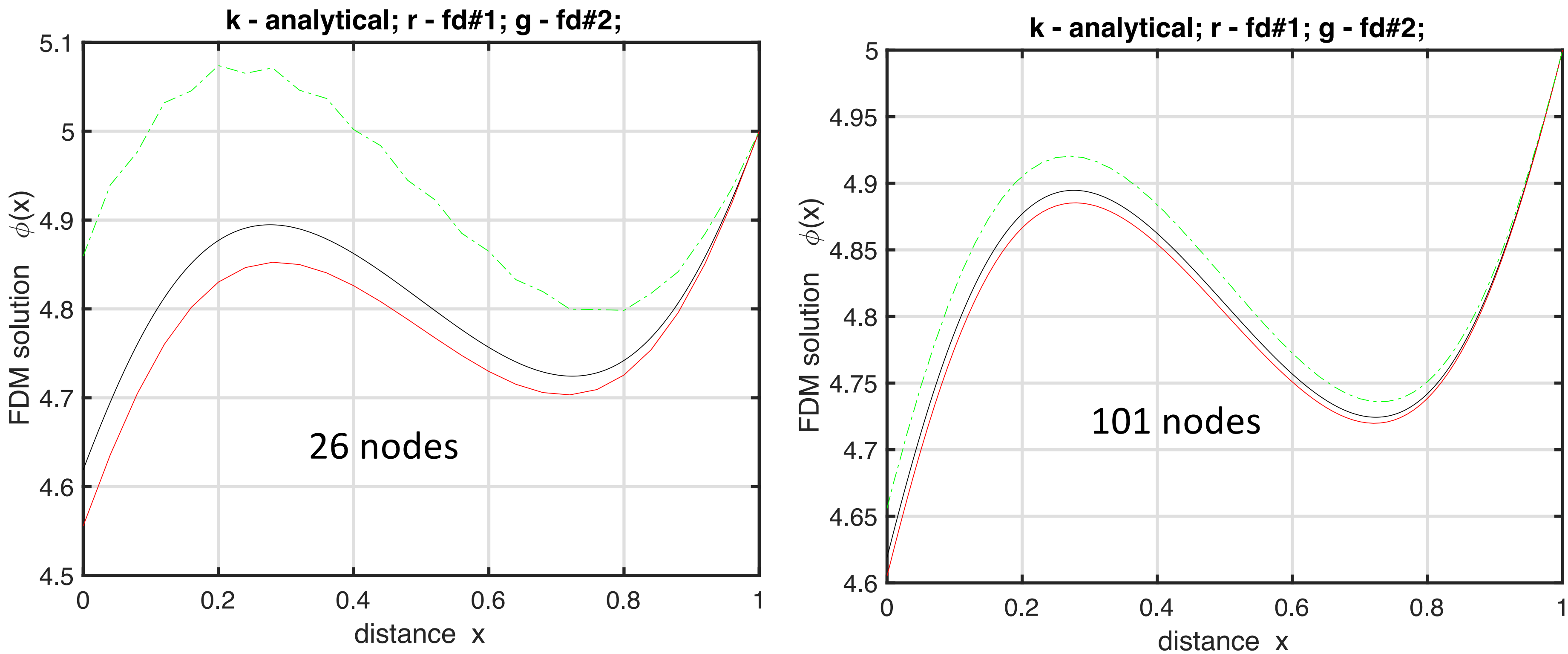




HW #3

Finite-difference Solution

- Nodes uniformly spaced
- 2 poor discretization schemes



Accuracy and time steps – value at central point

Black- Continuous space and time

Red – Discrete space, continuous time

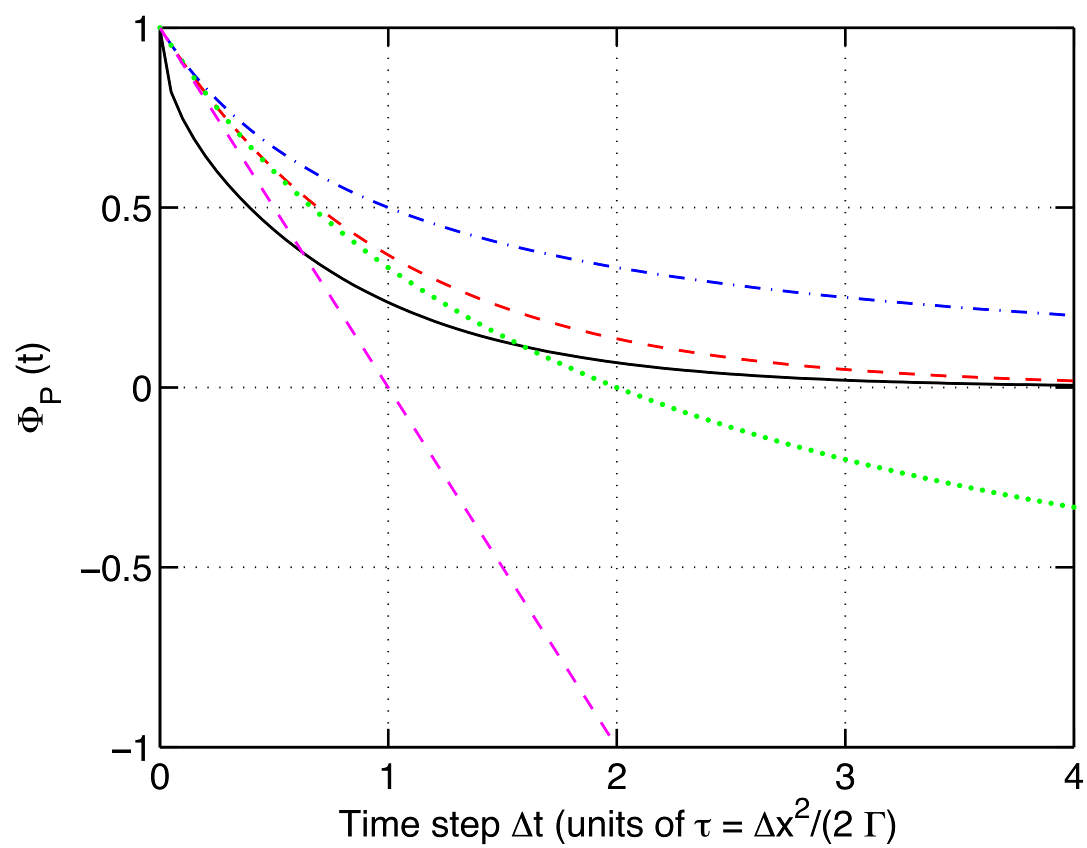
Blue –Discrete space and time, $\alpha = 1$ (Implicit scheme)

Green –Discrete space and time, $\alpha = 0.5$ (Crank-Nicolson scheme)

Magenta –Discrete space and time, $\alpha = 0$ (Explicit scheme)

$$\phi_P = \phi_P^0 \left[\frac{1-(1-\alpha)\,(\Delta t/\tau)}{1+\alpha\,(\Delta t/\tau)} \right]$$

$$\tau = \frac{\Delta x^2}{2\Gamma}$$



Today

Another look at time stepping

- with Transfer Functions