ESS 524 Class #10

Highlights from last Monday – Shashank
Today's highlights report on next Monday – Erich

Today

- HW #3 is due today. How's it going?
- When to hold project presentations?
  - Scheduled time for (non)exam Thursday June 11, 2:30
- Time stepping – Linear Computational Instability
  - Wrap up time splitting parameter
- including advection (when velocity is known)
  - Patankar's use of simple examples to give insight

Reading

- Patankar – Chapter 5
- Versteeg and Malalasekara – Chapter 5
- Ed's notes Transfer functions for advective-diffusion equation

Time stepping

Read *Ed's notes on time stepping* under the READING tab on the class web page.

Time-splitting parameter α

- α =0 Fully Explicit
- α =0.5 Crank Nicolson
- α =1 Fully Implicit

Patankar uses simplest possible model to illustrate each new concept, e.g. Behavior of a single node with fixed neighbors

Ed's notes extended this idea -

- When space and time are continuous (analytical solution)
- When space is discretized and time is continuous
- When space and time are both discretized (as in a typical numerical code)

Transfer Function $F(k)$

We can also look at a general spatial signal $\phi(x,0)$

- $\phi(x,0)$ is a signal at time $t=0$
- k is wavenumber or spatial frequency
- $\tilde{\phi}(k,0)$ is a wavenumber component of $\phi(x,0)$ after taking the spatial Fourier transform of $\phi(x,0)$

$F(k)$ expresses how each sinusoidal component $\tilde{\phi}(k,0)$ with wavenumber k looks at a later time t.

For the diffusion equation,

$$\tilde{\phi}(k,t) = \tilde{\phi}(k,0)\exp(-\Gamma k^2 t)$$

- $F(k) = \exp(-\Gamma k^2 t)$ is the transfer function.

Transfer Function $F(k_N)$

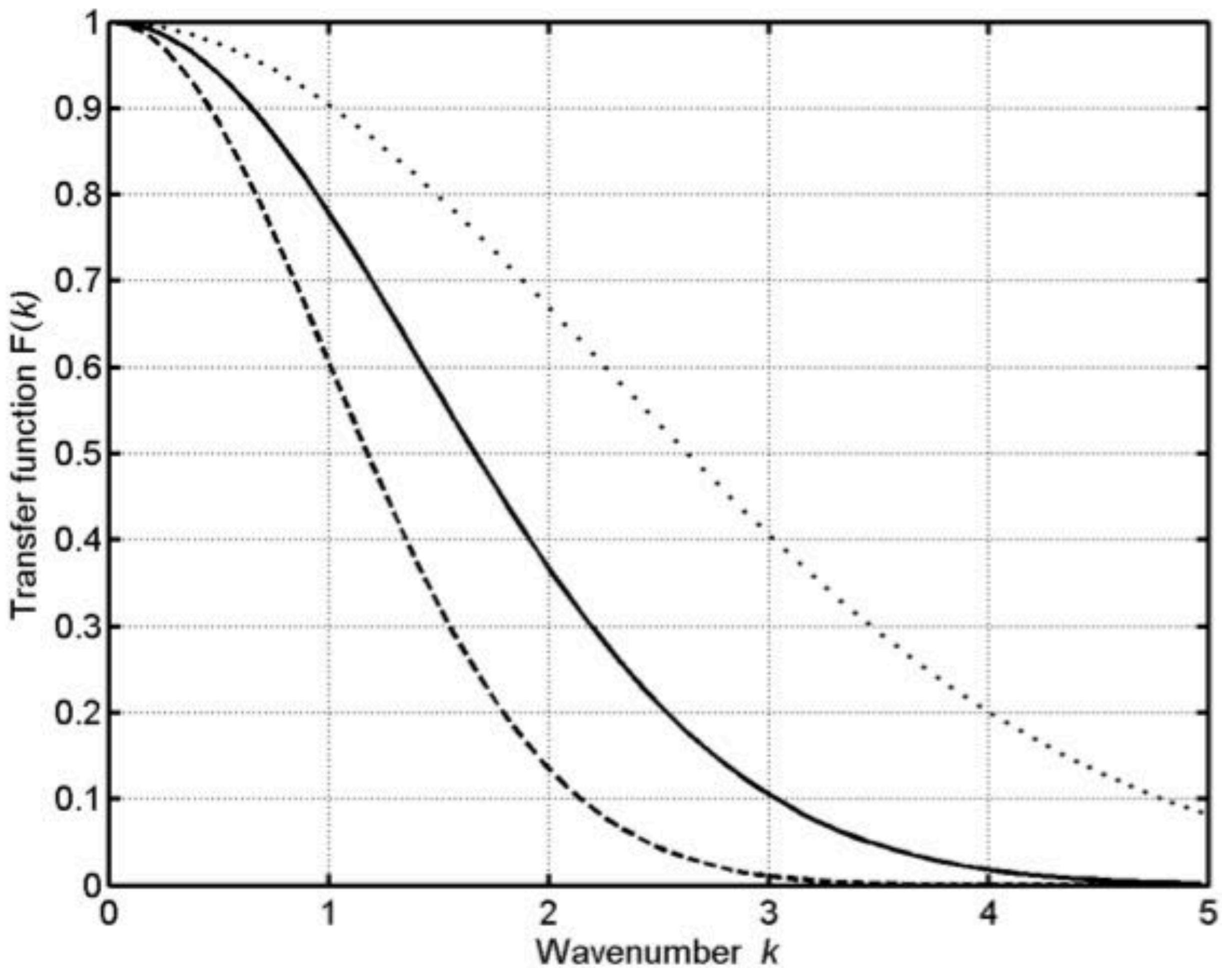

Figure 1. Transfer function F(k) for analytical diffusion equation, as a function of wavenumber k, for time steps Δt*=0.1 (dotted),* Δt*=0.25 (solid), and* Δt*=0.5 (dashed). Diffusion parameter is* Γ*=1.*

Transfer Function $F(k_N)$

$$F(k_N) = \left(\frac{1-2(1-\alpha)\Delta\tau}{1+2\alpha\Delta\tau} \right)$$

Time-splitting parameter α

k_N is Nyquist wavenumber

- 2 nodes per cycle ($\lambda = 2\Delta x$)
- Highest wave number that can be resolved on a grid at Δx

Characteristic time $\tau = \frac{\Delta x^2}{2\Gamma}$

Dimensionless time step $\Delta\tau = 2\Gamma\Delta t / \Delta x^2 = \Delta t/\tau$

α	$F(k_N)$	$\Delta\tau$	*Range of* $F(k_N)$	Result
0 (Explicit)	$(1-2\Delta\tau)$	$\Delta\tau<1$ $\Delta\tau>1$	$-1<F(k_N)<1$ $F(k_N)<-1$	OK Bad
0.5 (Crank-Nicolson)	$(1-\Delta\tau)/(1+\Delta\tau)$	Any $\Delta\tau>0$	$-1<F(k_N)<1$	OK
1.0 (Implicit)	$1/(1+2\Delta\tau)$	Any $\Delta\tau>0$	$0<F(k_N)\leq 1$	Good

Fun with Code

Check out the MATLAB codes on the web site

- One-point Transient Response
- Linear Computational Instability