ESS 524
Introduction to Heat and Mass Flow Modeling in Earth Sciences

My Second and Third Numerical Solutions in Matlab!

We are still using your analytical solution for the steady heat equation (1) on the domain [0,1] with spatially variable sources and conductivity

$$\frac{d}{dx}\left(k(x)\frac{d}{dx}\phi(x)\right)+S(x)=0 \qquad (1)$$

when *S(x)* is a polynomial in *x*, and 1/*k(x)* is a polynomial in *x*. As in Problem Set #2,

$$\frac{1}{k(x)}=2x^2-2x+1$$

$$S(x)=100\left(2x^3-3x^2+x\right)$$

$$\left[\frac{d\phi}{dx}\right]_0=2$$

$$\phi(1)=5$$

1. **Numerical Solution by Finite Volumes**
   - Write a Matlab m-file to solve the above equation using ***finite volumes***.
   - Determine when your grid interval *dx* is small enough by comparing your numerical answer to your analytical answer. Try using 5 nodes -> 100 nodes.
   - Thoroughly document your Matlab code, explaining what all the variables are, and explaining what every calculation is doing. Include graphs of all results.

2. **Numerical Solution by Finite Elements**
   - Write a Matlab m-file to solve the above equation using ***finite elements***.
   - Using uniform element widths *dx*, determine when your *dx* is small enough by comparing your numerical answer to your analytical answer. Try using 5 nodes -> 100 nodes.
   - Now try a model in which the elements are not uniform in width. If your uniformly spaced nodes are at positions x_j, show that you can use nodes at the nonuniform positions x_j^2, and still solve the equation correctly. Again explore the element size limitations for a good solution by comparing with your analytical solution.
   - Thoroughly document your Matlab code, explaining what all the variables are, and explaining what every calculation is doing. Include graphs of all results.

3. **Conclusions**
   - Describe in prose any insights or knowledge that you learned about the 3 different numerical methods (FDM, FEM, and FVM) numerical methods from the last 2 problem sets (suggested length approximately 1 page).