ESS 524
Introduction to Heat and Mass Flow Modeling in Earth Sciences

Homework - Steady 2D Diffusion

From the class web site, follow the MATLAB CODE link to SS 2D diffusion Practice B. The m-files posted there solve the steady-state diffusion equation in 2D.
The code is designed to handle

- spatially variable volume dimensions.
- spatially variable diffusion coefficient K.
- spatially variable source term $S\_C + S\_P \times phi$

Currently, however, the code is set up to run a simple example.

- The volumes have uniform dimensions.
- One boundary accepts specified values for *phi*.
- The other 3 boundaries accept gradients in *phi*.
- The conductivity *K* is uniform, except for a central block of volumes which can have a different (but uniform) conductivity.
- The source terms are set to zero.

As you do the following exercises, you may want to save versions of the code after each step ☺. I don't need to see all your code for this assignment, just results (graphs) and some prose describing your procedures, modifications, and the results.

1. A really simple configuration
Set the conductivity of the central block equal to the conductivity of its surroundings.

- With the initially set boundary conditions, what should the solution be?
- Show that you get the answer that you expect.

2. Non-uniform node spacing
With the source terms all zero, and uniform conductivity, set the volume widths to a non-uniform pattern in both *x* and *z*. I suggest creating a variation of at least a factor of 3, and potentially concentrating nodes around the central block.

- Show that you still get the same answer as in **#1**.

3. Conductivity

- Run the default conductivity configuration in which the central block is a good conductivity (*K* is 100 times background).
- Make the central block a good insulator (*K* is 0.01 times background) and run the model again.
- Show results in both cases, and explain the two outcomes in prose.

4. Background source *S_C*

With uniform conductivity, and the initially set boundary conditions (zero flux on *x=0* and *x=10*) as in **#3**, and keeping *S_P=0*, set the source term *S_C* to a uniform but small positive value, for example *S_C=0.1*.

- Show the result and explain in prose why it differs from your result in **#1**. **(**For example, discuss values at *z=10*, and curvature.)

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5. *phi*-dependent source *S_P × phi*

Set *S_C=0* to turn off the background source term, and turn on the phi-dependent source term by setting *S_P=-0.0025*.

- Show the result and explain in prose why it differs from your results in **#2** and **#4.** **(**For example, discuss curvature and values at *z=10*.)

6. Positive coefficient *S_P*

Repeat **#5**, but set *S_P = +0.0025*.

- Explain the differences from **#5**. **(**For example, discuss curvature and values at *z=10*.)

7. Extreme Positive coefficient *S_P*

Repeat **#5**, but now set *S_P* = +0.025, and then *S_P* = +0.25.

- Describe what's going on now.

  (Points to consider: How does the source term act when *phi* becomes negative? Is this physically meaningful, if for example *phi* was absolute temperature? If *phi* was Celsius temperature?).

8. Boundary Conditions - I

Change the boundary condition on the *x=0* face to be a fixed *phi* condition, with the set *phi* varying linearly with *z*. At the *z=0* end, it should match the value of *phi* set on the adjoining interface. Retain the specified flux conditions on the other 2 boundaries (nonzero flux on *z=10*, zero flux on *x=10*).

- Find solutions using 3 different variations of your specified *phi* along the *x=0* boundary. Use

$$\left[\frac{\partial \phi}{\partial z}\right]_{x=0} = 0 \;,\; \left[\frac{\partial \phi}{\partial z}\right]_{x=0} = 1, \text{ and } \left[\frac{\partial \phi}{\partial z}\right]_{x=0} = -1.$$

- Explain why your solutions look reasonable in terms of your intuition about diffusion.

9. Boundary Conditions - II

Repeat **#8**, but set the flux BC on *x=10* to be equal in magnitude, but opposite in sign to the specified flux on *z=10*.

- Explain why your solutions look reasonable in terms of your intuition about diffusion.