ESS 524
Introduction to Heat and Mass Flow Modeling in Earth Sciences

Transient Diffusion

In HW#3, you wrote a finite-volume solver for the steady-state diffusion equation (1),

$$\frac{d}{dx}\left(\Gamma(x)\frac{d}{dx}\phi(x)\right) + S(x,\phi) = 0 \tag{1}$$

on the interval $0 \le x \le 1$, with conductivity $\Gamma(x)$, source strength $S(x)$, and boundary conditions given by

$$\begin{aligned} &\Gamma(x) = \left(2x^2 - 2x + 1\right)^{-1} \\ &S(x) = 100\left(2x^3 - 3x^2 + x\right) \\ &\left[\frac{d\phi}{dx}\right]_0 = 2 \\ &\phi(1) = 5 \end{aligned} \tag{2}$$

1) Now, modify your code to solve the transient problem,

$$\frac{\partial\phi}{\partial t} = \frac{\partial}{\partial x}\left(\Gamma(x)\frac{\partial}{\partial x}\phi(x,t)\right) + S(x,\phi) \tag{3}$$

   with the same conductivity, source, and boundary conditions in (2), using fully implicit time steps Δt. You will need to modify your coefficients a_P *etc* to incorporate the time derivative, and then include a time-step loop.

2) For a transient problem, you will need an initial condition that also satisfies the boundary conditions in (2). Try

$$\phi(x,0) = -3x^2 + 2x + 6 \tag{4}$$

   and run your model until the profile reaches the steady-state solution that you expect from HW#2 and #3.

3) Now try different initial conditions and show that you can still reach the same steady state. The function

$$\psi(x) = a\cos\left(\left(\frac{2n+1}{2}\right)\pi x\right) \tag{5}$$

   has zero slope at x=0, and zero amplitude at x=1, for any a and for any integer n. That means that you can add any multiple of $\psi(x)$ in (5) to the initial condition (4) and still satisfy the boundary conditions in (2). Try, for example, a=0.5, and n=4.

4) Turn in your fully and explicitly documented code along with your resulting figures.

5) Describe in prose any insights or knowledge that you learned about transients in numerical methods from the exercise (suggested length approximately 1 page). For example, how did you select your space intervals Δx and time intervals Δt, and why?