ESS 524
Introduction to Heat and Mass Flow Modeling in Earth Sciences

Oh My God! Yet another Numerical Solution in Matlab!

1. Nonlinear source term

The steady heat equation (1) can have spatially variable sources that also depend on the solution $\phi(x)$.

$$\frac{d}{dx}\left(k(x)\frac{d}{dx}\phi(x)\right) + S(x,\phi) = 0 \qquad (1)$$

For example,

$$S(x) = A + B\phi^n(x) \qquad (2)$$

- When $k(x)=k_0$ is a constant, A=0, and c_0, c_1 and m are constants, show that
$$\phi(x) = c_0(x-c_1)^m \qquad (3)$$
is a solution to (1).
- With a boundary condition $\phi(x_0)=\phi_o$, find expressions for c_0, c_1 and m in terms of k_0, B, n, x_0, and ϕ_0.
- Incorporate the nonlinear source term (2) into your Finite Volume solver from Problems #3. Use linearization procedures
$$S(x,\phi) = S_c(\phi^*) + S_P(\phi^*)\phi \qquad (4)$$
as described in Patankar Section 4.2-5, with an iteration loop to update $S_P(\phi^*)$. Note that ϕ^* is the solution at the previous iteration. Document all steps in your code.
- With $n=2$, $A=0$, $B=-1$, $k_0=1$, and the boundary condition $\phi(0)=10$, find the gradient $d\phi/dx=\phi_1'$ at x=1 from your analytical solution.
- With the boundary conditions $\phi(0)=10$, and your value ϕ_1' for $d\phi/dx$ at x=1, solve numerically for $\phi(x)$.
- On the same graph, plot the analytical solution, and plot and label your successive solutions as they converge to the analytical solution.
- On a second graph, show the RMS mismatch between the exact solution and your numerical solution as a function of iteration number.
- Write a paragraph summarizing what you learned from this.

2. Extra challenges

- Repeat, using at least 2 different linearizations. For example, you could use estimates of S_P that were steeper or less steep than the tangent $dS/d\phi$.
- Incorporate the same source linearization and iterations into your Finite-*Difference* solver from Problems #2, and explore impacts of grid size and linearization.