ESS 595B: Polar Firn: Thermal, Mechanical and Chemical Properties

Fall Glaciology Seminar

First Meeting: Herron, M.M. and C.C. Langway. Firn Densification-An Empirical Model. J. Glac. 1980.

Presenters: Ed and Jessica

…a little bookkeeping

Potential topics and papers:

- Densification modelling:
  - Steady state
    - Herron and Langway 1980
    - Spencer et al. 2001
  - Time dependent
    - Arthern and Wingham 1998
    - Cuffey 2001
    - Goujon et al. 2004
    - Li Jun and Jay Zwally 2004, Zwally and Li Jun 2002
    - Reeh 2008, In press
- Firn chemistry:
  - Severinghaus et al. 1998
  - Papers from physics of Ice Core Records: Schwander, Blunier, Arnaud

Herron and Langway: Major concepts

Developed empirical model for 2 densification zones, defined by 2 ‘critical densities’.

Steady-state model:

accumulation-rate and temperature don’t change, coefficients remain constant, tuned to current density profiles

Pros: model is simple to use, quick to compute

Cons: questionable outside of calibration

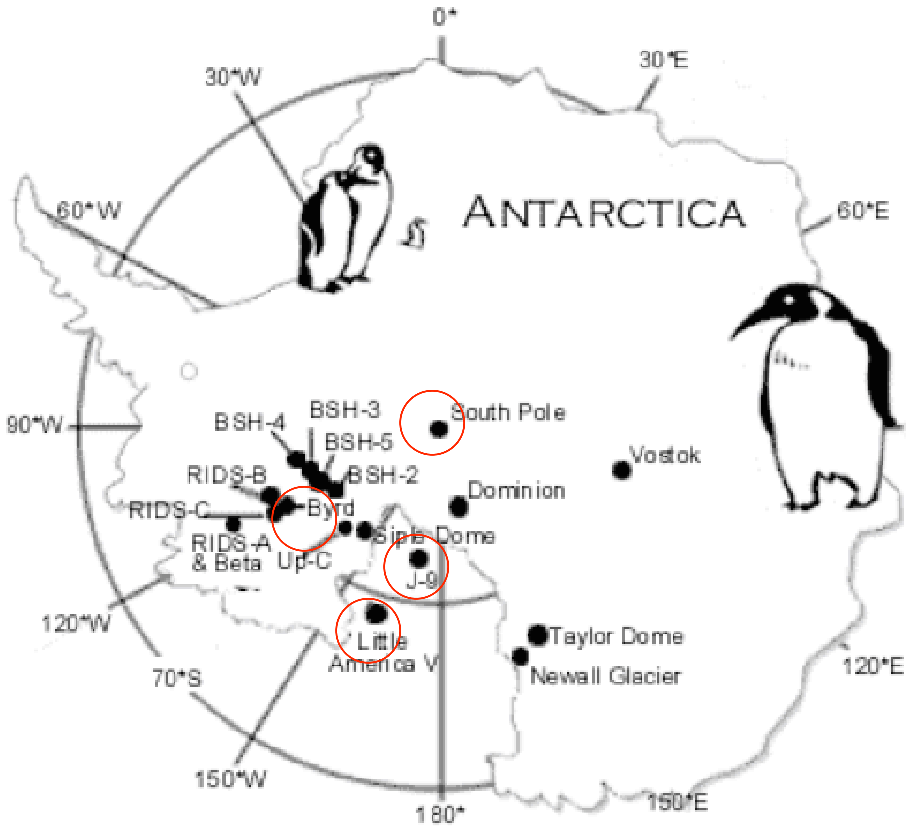
Zone	Density (kg/m^3)	Dominate mechanism(s) for densification
1	$\rho < 550$	Grain boundary sliding, packing
2	$550 < \rho < 800$	Unknown in HL Power-law creep (Artz, 1982, 1983) Pressure sintering
3	$\rho > 800$	Bubble compression

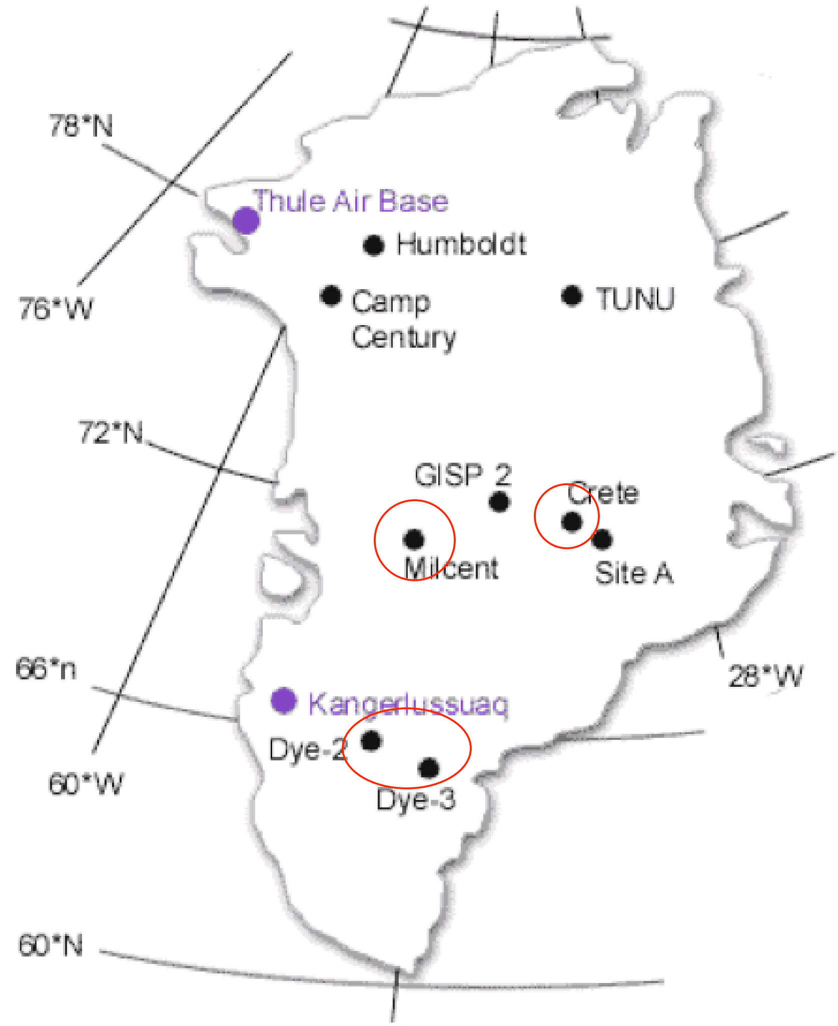
Selected sites from Greenland and Antarctica
Temp range: -57 to -15F
Accumulation rates A range: 0.022-0.5 m water year^-1

TABLE I. GLACIOLOGICAL DATA FOR GREENLAND AND ANTARCTIC ICE CORES

Site	*Location* *Latitude*	*Location* *Longitude*	*A* m water year^{-1}	*T* °C	*References for ρ, T, A data*
			Greenland		
North Central	74° 37′ N.	39° 36′ W.	0.11	−31.7	This work
Crête	71° 07′ N.	37° 19′ W.	0.265	−30	This work; personal communication from B. L. Hansen; Hammer and others (1978); Reeh and others (1978)
Site 2	76° 59′ N.	56° 04′ W.	0.4	−23.3	Langway (1970)
Milcent	70° 18′ N.	44° 19′ W.	0.50	−22	This work; personal communication from B. L. Hansen; Hammer and others (1978); Reeh and others (1978)
South Dome	63° 33′ N.	44° 36′ W.	0.50	−21.5	This work; personal communication from W. Dansgaard
Dye 3	65° 11′ N.	43° 50′ W.	0.50	−19.6	This work; personal communication from B. L. Hansen; Hammer and others (1978); Reeh and others (1978)
Dye 2	66° 29′ N.	46° 20′ W.	0.5	−16.7	This work

Map of 8/17 locations





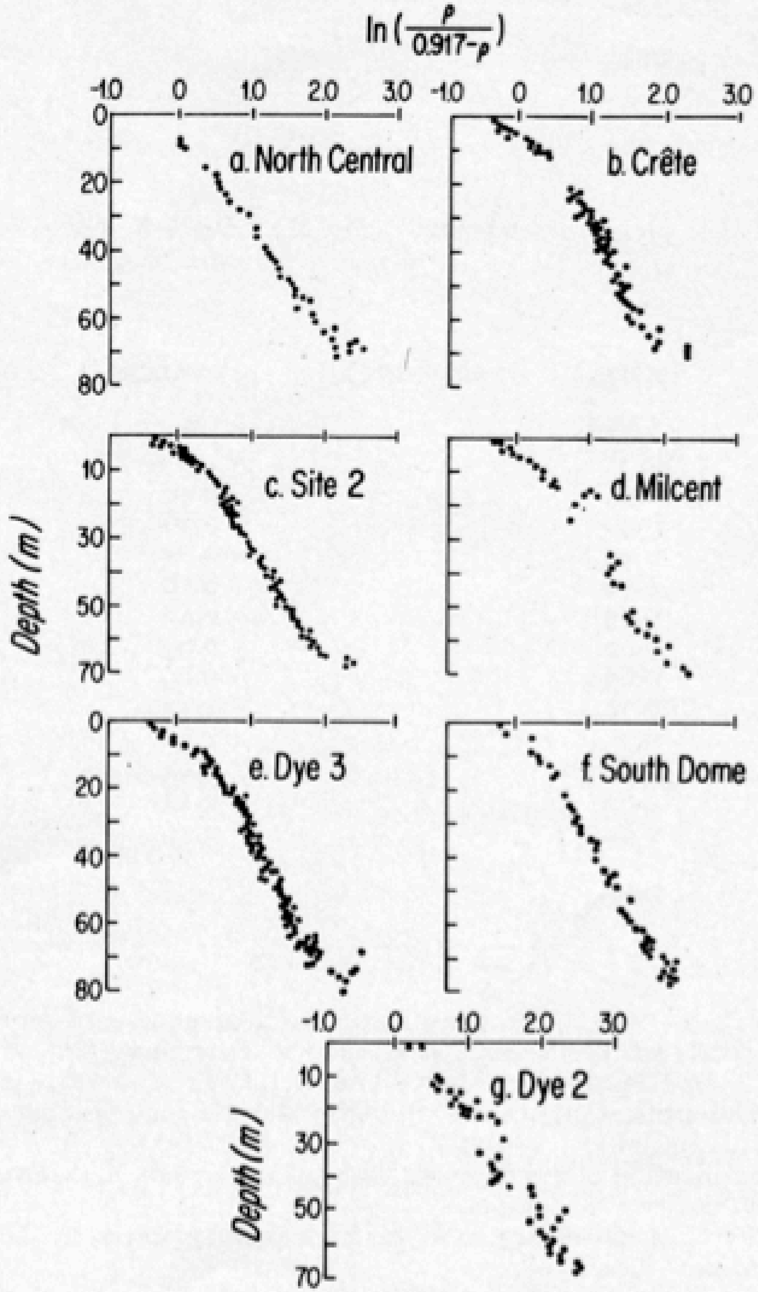


*Fig. A1. Depth–*ln $[\rho/(\rho_i - \rho)]$ *data for inland Greenland sites.*

Appendix
linear relationship between depth and

$$\ln\left(\frac{\rho}{\rho_i - \rho}\right)$$

for 17 Greenland and Antarctic sites

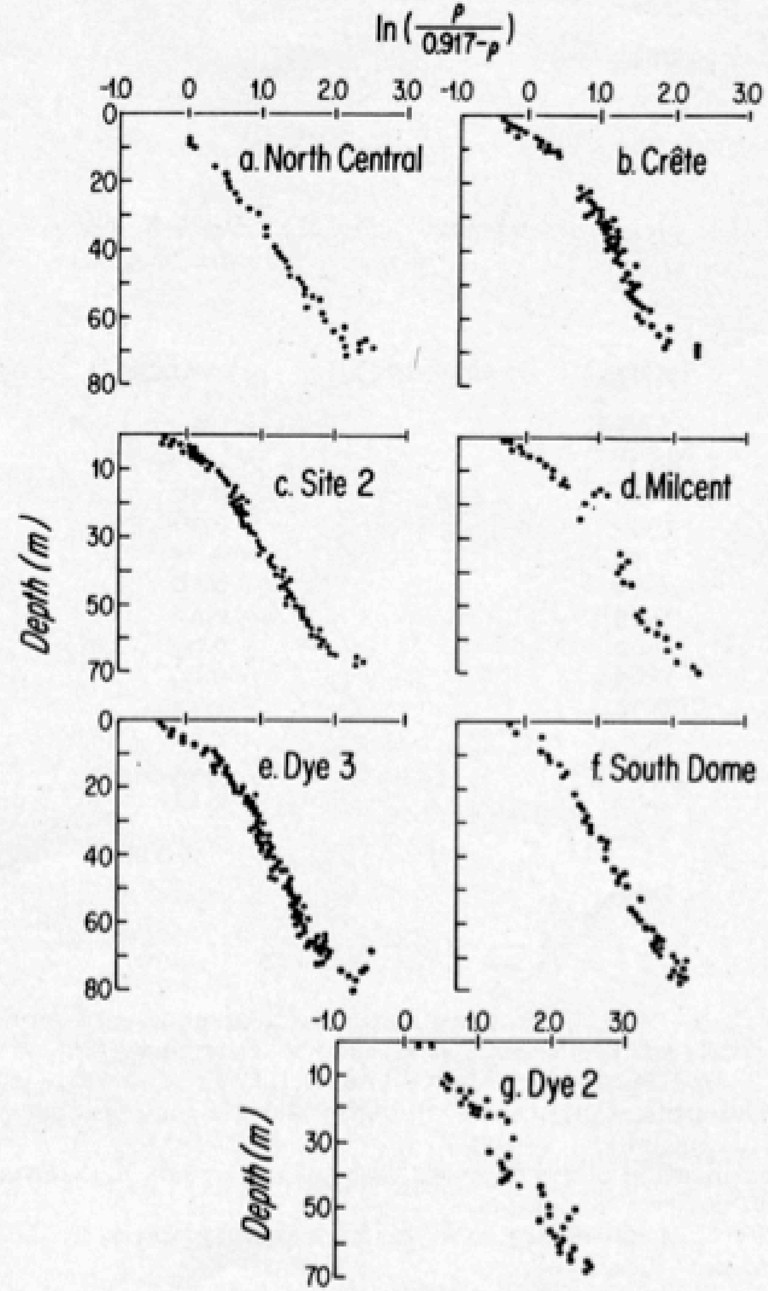


Fig. A1. Depth–ln $[\rho/(\rho_1-\rho)]$ *data for inland Greenland sites.*

Find coefficients C and C', derivative of lines, for 2 ranges of densities

$$C = \frac{\mathrm{d}\ln[\rho/(\rho_1-\rho)]}{\mathrm{d}h} \qquad \text{for } \rho < 0.55 \text{ Mg m}^{-3},$$

$$C' = \frac{\mathrm{d}\ln[\rho/(\rho_1-\rho)]}{\mathrm{d}h} \qquad \text{for } 0.55 \text{ Mg m}^{-3} < \rho < 0.8 \text{ Mg m}^{-3},$$

Fit linear depth-density curves

Now, solve for densification with time

$$\mathrm{d}\rho/\mathrm{d}t = \frac{CA}{\rho_i}(\rho_i - \rho), \qquad \rho < 0.55 \text{ Mg m}^{-3},$$

$$\mathrm{d}\rho/\mathrm{d}t = \frac{C'A}{\rho_i}(\rho_i - \rho), \qquad 0.55 \text{ Mg m}^{-3} < \rho < 0.8 \text{ Mg m}^{-3}.$$

By rearranging and substituting into Schytt equation 1

$$\frac{\mathrm{d}\rho}{\rho_i - \rho} = \text{constant } \rho \, \mathrm{d}h,$$

Can then solve for temp and accum-rate dependencies, through coefficients k_0, k_1, a and b

$$\mathrm{d}\rho/\mathrm{d}t = k_0 A^a (\rho_i - \rho), \qquad \rho < 0.55 \text{ Mg m}^{-3},$$

$$\mathrm{d}\rho/\mathrm{d}t = k_1 A^b (\rho_i - \rho), \qquad 0.55 \text{ Mg m}^{-3} < \rho < 0.8 \text{ Mg m}^{-3},$$

Fit linear depth-density curves

Can then solve for temp and accum-rate dependencies, through coefficients k_0, k_1, a and b

$$\mathrm{d}\rho/\mathrm{d}t = k_0 A^a(\rho_i - \rho), \qquad \rho < 0.55\ \mathrm{Mg\ m^{-3}},$$

$$\mathrm{d}\rho/\mathrm{d}t = k_1 A^b(\rho_i - \rho), \qquad 0.55\ \mathrm{Mg\ m^{-3}} < \rho < 0.8\ \mathrm{Mg\ m^{-3}},$$

k_0, k_1 are arrhenius-type rate constants dependent on temp (Fig 1)

Activation energies, E

$$k_0 = 11 \exp\left[-\frac{10\,160}{RT}\right],$$

$$k_1 = 575 \exp\left[-\frac{21\,400}{RT}\right],$$

R = gas constant, T = temp (K

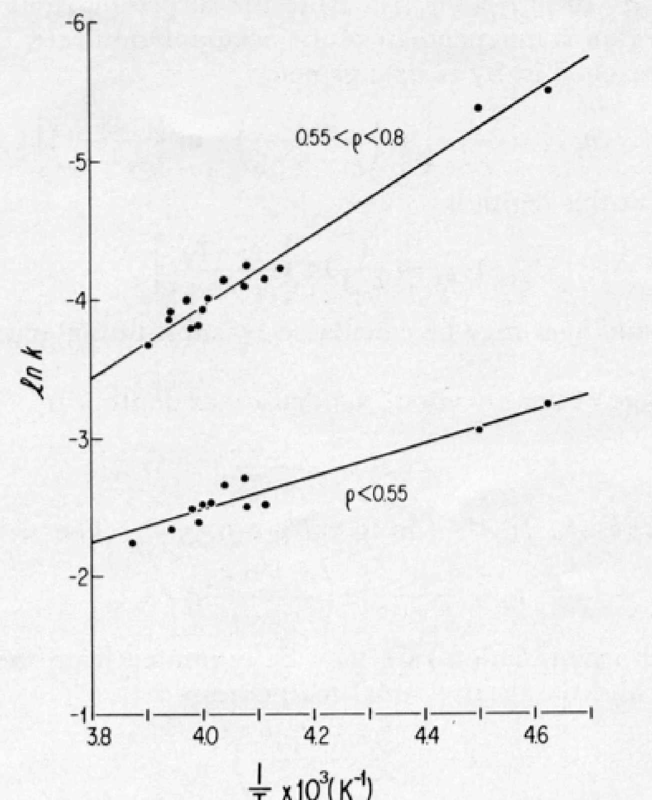


Fig. 1. Arrhenius plots for k_0 (lower line) and k_1 (upper line).

17 sites in Greenland and Antarctica plotted for 2 density regions to determine k coefficients, dependent on T

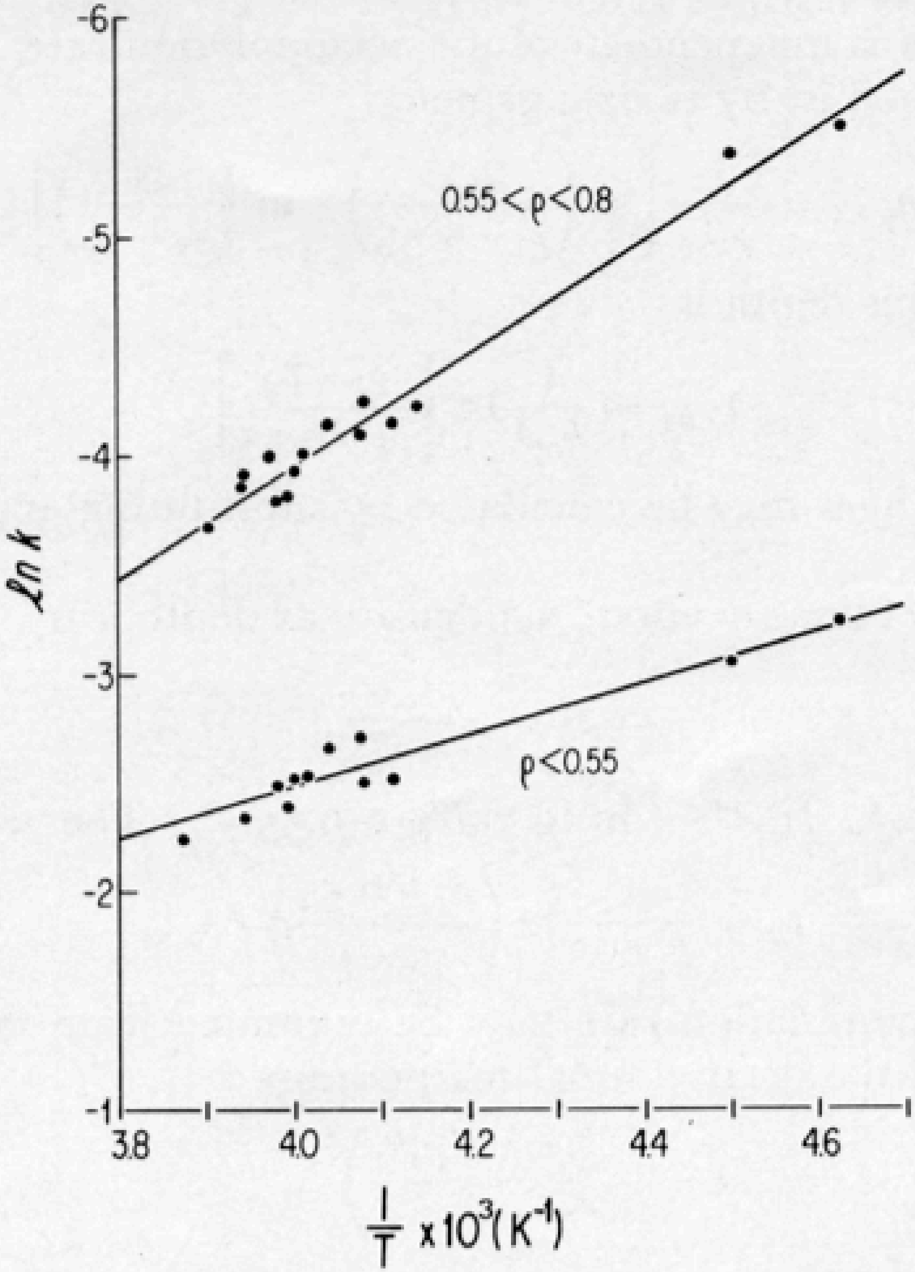


Fig. 1. Arrhenius plots for k_0 (lower line) and k_1 (upper line).

Fit linear depth-density curves

Can then solve for temp and accum-rate dependencies, through coefficients k_0, k_1, a and b

$$d\rho/dt = k_0 A^a(\rho_i - \rho), \qquad \rho < 0.55 \text{ Mg m}^{-3},$$

$$d\rho/dt = k_1 A^b(\rho_i - \rho), \qquad 0.55 \text{ Mg m}^{-3} < \rho < 0.8 \text{ Mg m}^{-3},$$

a and b are determined by comparing slopes for each stage of densification at sites of nearly equivalent temp and different accum. rates, shown in table II

$$a = \frac{\ln(C_1/C_2)}{\ln(A_1/A_2)} + 1,$$

b solved using same equation using C'

TABLE II. VALUES OF THE CONSTANTS a AND b DERIVED FROM ACCUMULATION RATES AND SLOPES DERIVED FROM GRAPHS OF DEPTH AGAINST $\ln[\rho/(\rho_i - \rho)]$ FOR PAIRS OF SITES

Site pair	*a*	*b*
Site 2–RID	0.8	0.3
Site 2–Milcent	1.2	ID
Site 2–LAS	1.0	0.6
Site 2–C-7-3	1.1	0.6
C-7-3–LAS	1.2	0.7
C-7-3–Old Byrd	1.4	0.4
Old Byrd–J-9	1.4	0.3
RID–Milcent	0.9	0.5
RID–South Dome	ID*	0.3
LAS–Milcent	1.1	ID
Wilkes S2–Dye 3	ID	0.5
Crête–North Central	ID	0.6
Average and standard deviation	1.1±0.2	0.5±0.2

* ID = Insufficient data.

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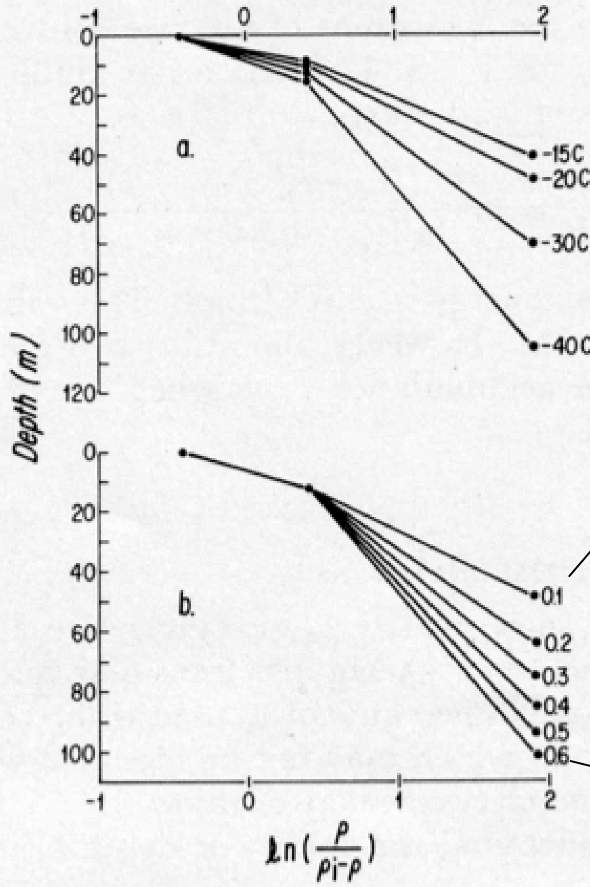
C-7-3–Old Byrd	1.4	0.4
Old Byrd–J-9	1.4	0.3
RID–Milcent	0.9	0.5
RID–South Dome	ID*	0.3
LAS–Milcent	1.1	ID
Wilkes S2–Dye 3	ID	0.5
Crête–North Central	ID	0.6
Average and standard deviation	1.1 ± 0.2	0.5 ± 0.2

* ID = Insufficient data.

Now we can calculate depth and time for given density in density zones 1 and 2 eqns (8-11).

Can estimate accumulation rate in second zone based on slope of C’, eqn (12)

Effects of varying T,
A is constant



Depth of zone 1 increases with colder temps

Deeper and longer to reach critical density 550 kg/m^3

Smaller A: 310 years, 49m depth

Effects of varying A,
T is constant

Densification rate of zone 1 invariant of accum. rate A

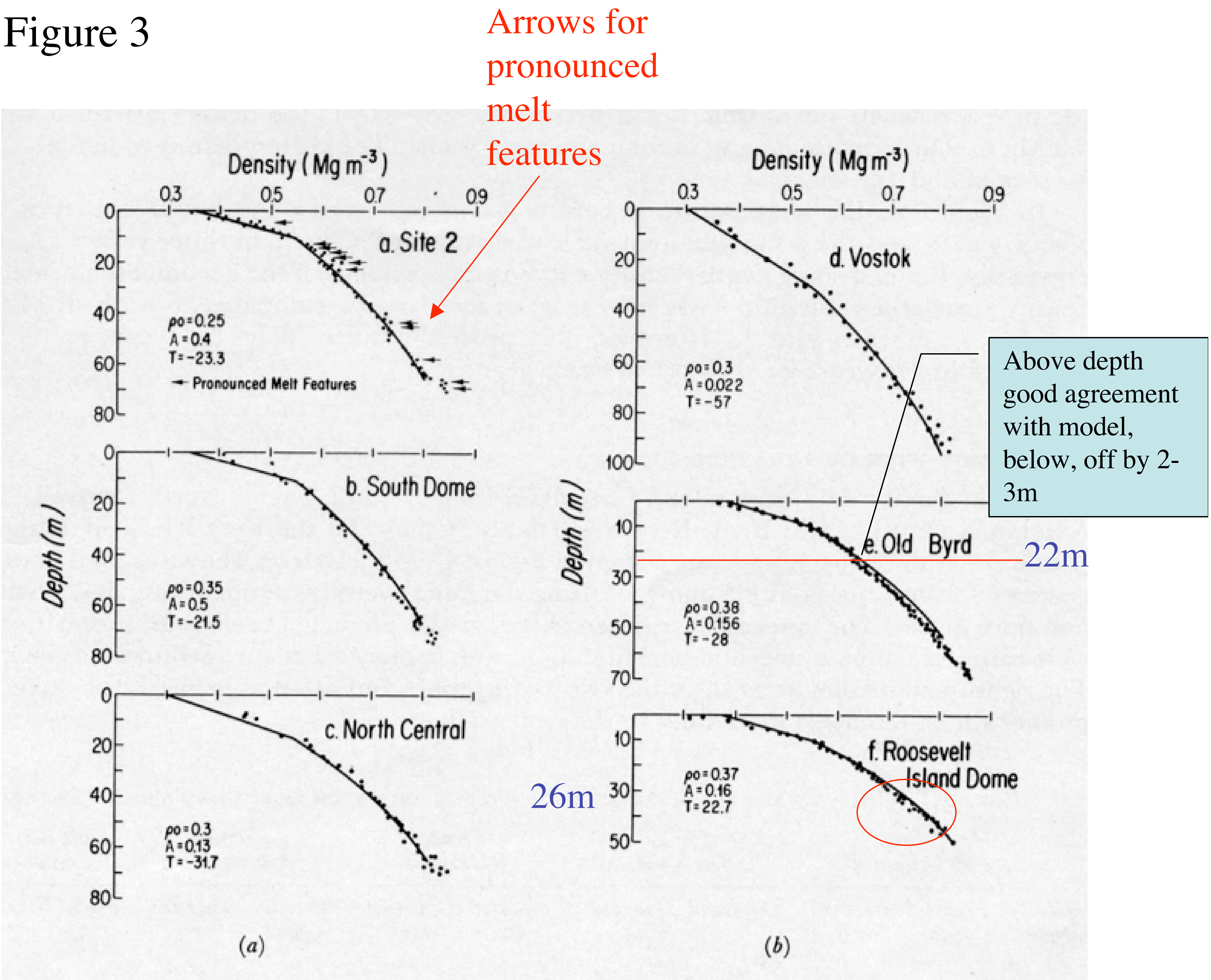
Greater A: 113 years, 102m

Fig. 2. Predicted depth–ln [$\rho/(\rho_1-\rho)$] curves for: (a) various temperatures at a constant accumulation rate of 0.30 m water year^{-1} and initial density of 0.36 Mg m^{-3}, and (b) various accumulation rates (in m water year^{-1}) at a constant temperature of −30°C and initial density 0.36 Mg m^{-3}.

Same plot as Appendix, here with experimental data

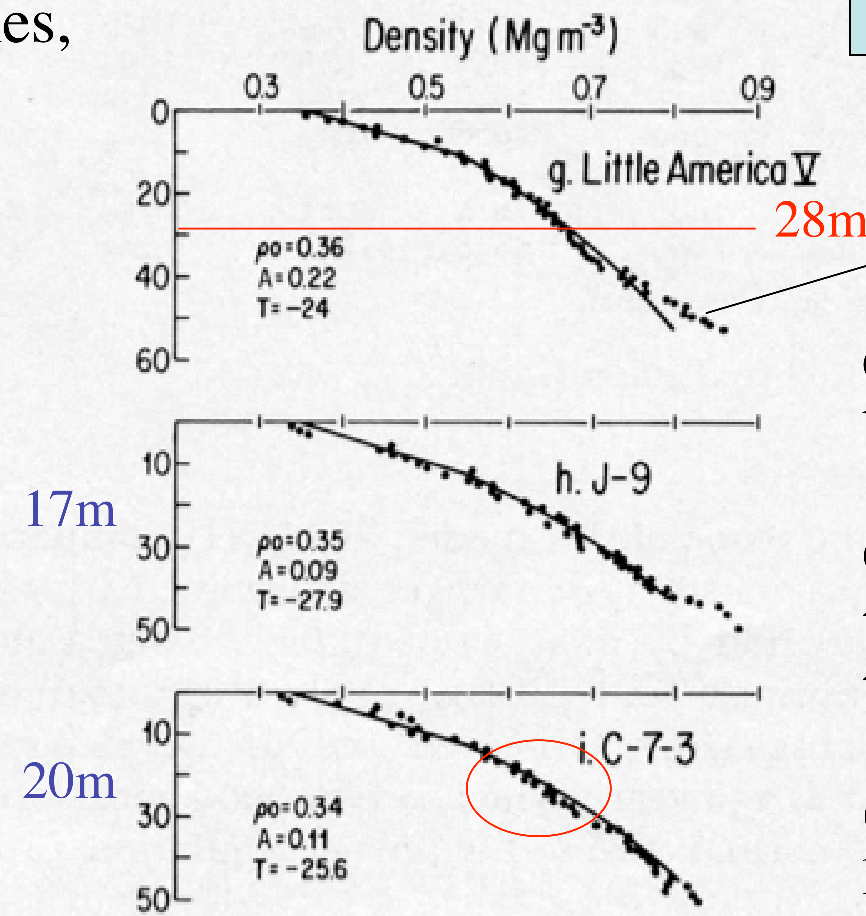
T and b usually coupled, increase T, increase A, affect is opposite on densification rate.

Figure 3


Arrows for pronounced melt features
Density (Mg m^{-3})
0.3
0.5
0.7
0.9
a. Site 2
ρo = 0.25
A = 0.4
T = -23.3
Pronounced Melt Features
b. South Dome
ρo = 0.35
A = 0.5
T = -21.5
c. North Central
ρo = 0.3
A = 0.13
T = -31.7
Depth (m)
(a)
d. Vostok
ρo = 0.3
A = 0.022
T = -57
e. Old Byrd
ρo = 0.38
A = 0.156
T = -28
22m
Above depth good agreement with model, below, off by 2-3m
f. Roosevelt Island Dome
ρo = 0.37
A = 0.16
T = 22.7
26m
(b)

Eqns 7-10 used to generate depth-density model lines, given initial density, accum. rate., and temp

High densification rate from local area of high stress, (Gow 1968)



(c)

Fig. 3. Depth–density curves for: (a) Greenland, (b) inland Antarctic, and (c) Ross Ice Shelf ice cores. Initial density, accumulation rate, and temperature values shown were used to generate model curves shown as solid lines. Arrows in the Site 2 profile refer to pronounced melt features (Langway, 1970).

GenerallyGood agreement between data and model

Of course the model is tuned to the data for all of these 9 sites.

(What do the other 8 sites look like?)

TABLE III. OBSERVED AND PREDICTED AGES FOR FIVE ICE CORES FROM GREENLAND AND ANTARCTICA

Depth m	*Crête, Greenland* Predicted age	Observed[1] age	*Site 2, Greenland* Predicted age	Observed[2] age	*Milcent, Greenland* Predicted age	Observed[3] age	*Byrd Station, Antarctica* Predicted age	Observed[4] age	*Little America V, Antarctica* Predicted age	Observed[4] age
0		1974·5		1957		1973·5		1959		1959
10	1957	1958	1945	c. 1945	1964	1964	1929	1927	1938	1937
20	1936	1938	1931	c. 1930	1952	1952	1892	1888	1912	1912
30	1912	1914	1915	c. 1915	1940	1939	1850	1845	1882	1884
40	1887	1888	1898	c. 1900	1926	1925	1804	1802	1853 (39 m)	1854 (39 m)
50	1860	1861	1880	c. 1880	1912	1911	1754	1754		
60	1832	1831	1860	c. 1860	1896	1894	1703	1703		

[1] Personal communication from W. Dansgaard.
[2] Langway (1970).
[3] Hammer and others (1978); Reeh and others (1978).
[4] Gow (1968).

Further tested model using 5 ice cores with:

- predicted dates: model equations 9 and 11
- observed dates: stratigraphy, stable O isotopes and ion concentrations

Agreement within 5 years between predicted and observed dates

Ages go back 256 years, (recent enough for steady-state assumption?)

(equation 12)

TABLE IV. OBSERVED AND PREDICTED ACCUMULATION RATES

Site	Accumulation rate m water year^{-1} Observed	Predicted	Deviation	% deviation
Greenland				
North Central	0.13	0.11	−0.02	15
Crête	0.26	0.22	−0.04	15
Milcent	0.50	0.40	−0.10	20
Site 2	0.40	0.39	−0.01	3
South Dome	0.50	0.61	+0.11	22
Dye 3	0.50	0.61	+0.11	22
Dye 2	0.5	0.45	−0.05	10
Antarctica				
Vostok	0.022	0.019	−0.003	14
South Pole	0.07	0.09	+0.02	29
Old Byrd Station	0.16	0.19	+0.03	19
Roosevelt Island Dome	0.2	0.16	−0.04	20
Wilkes S-2	0.133	0.15	+0.017	13
Antarctic ice shelves				
J-9	0.09	0.08	−0.01	11
C-7-3	0.11	0.13	+0.02	18
Little America V	0.22	0.22	0.00	0
Maudheim	0.37	0.37	0.00	0
Roi Baudouin	0.38	0.54	+0.16	42

Average deviation between predicted and modelled values is 0.04my^{-1}, with an average of 16%

Max deviation is 0.16my^{-1}, with 42% deviation, (noted to have abundant ice layers)

HL conclusions

- Empirical rate equations firn densification of 1st and 2nd stage have been developed for 17 sites in GL and Ant.
  - Require inputs of initial density, accumulation rate and temp at 10m depth
- Can predict depth-density curves and depth-age relationships
- Deviations between observed and predicted depth-density may indicate changing climate conditions
- High densities at site 2, GL are melt layers from warm summers
- Synchronous event with increased A occurring in 1880s may be seen in Old Byrd, Little America V, and potentially C-7-3, J-9, and Roosevelt Island Dome.