1. Linkage disequilibrium (leftovers)

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- 2. PopG program
- 3. Probability of fixation
- 4. Time to fixation

- Q: What statistical test is appropriate for testing for disequilbrium?
  - You can use  $\chi^2$  to test observed versus expected numbers of each chromosome
  - A significant  $\chi^2$  rejects linkage equilibrium
  - There is no rule for "this value of D is significant" as it depends on sample size and allele frequencies
- Q: I don't understand the disequilibrium coefficient D

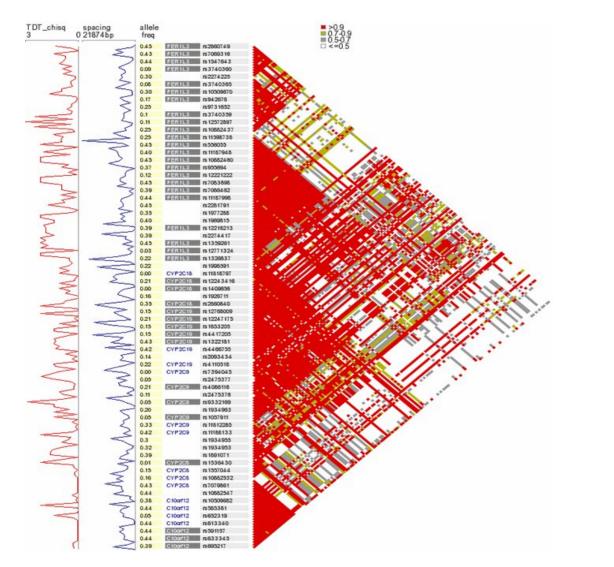
- If there is no disequilibrium:
  - expected  $pAB = pA \times pB$
- If there is disequilibrium, measure the discrepancy:

– observed pAB = expected pAB + D

- Example:
  - pA = 0.2, pB = 0.3
  - expected  $pAB = 0.2 \times 0.3 = 0.06$
  - observed pAB = 0.05
  - D = -0.01

# What forces can break down disequilibrium?

- Without recombination:
  - back mutation
  - repeated mutation
- With recombination and sexual reproduction:
  - chromosome segregation
  - recombination
  - back mutation
  - repeated mutation
- Disequilibrium areas are MUCH bigger and longer-lived without recombination



Blocks of LD around cytochrome P450 genes. Walton et al. Nature Genetics 2005.

- Graphs like this plot D' between sites
- In humans, conspicuous blocks of LD are usually seen
- Different ideas about their origin:
  - Recombination hotspots (at edges of blocks)
  - Population admixture followed by a few recombinations
  - Random chance
- I wish we could apply more statistics to this graph!

- The student on my left receives 1 red bean and 1 black bean
- They choose a bean at random and "reproduce" it
- They do this again
- They pass the new beans to the next student
- Repeat all the way around the table

What can we say about the color of the beans at the end?

- Average over many trials is 50% red and 50% black
- Any single trial generally either 100% red or 100% black
- This is genetic drift in a tiny population

This sort of experiment becomes cumbersome quickly with real beans. We will use computer simulation instead.

The program PopG is available from:

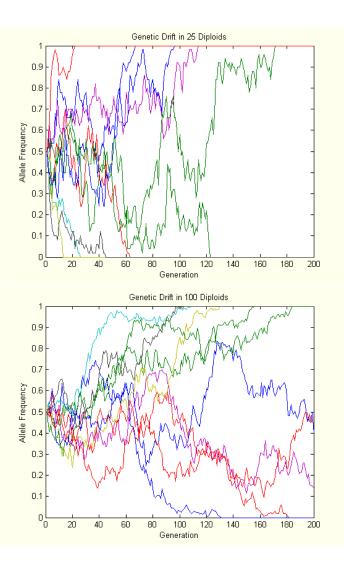
http://evolution.gs.washington.edu/popgen/popg.html

and will run on Windows, Mac and Linux/Unix machines. We will be using it for the next homework, so you will need a copy. Anyone who can't get one or doesn't have a computer to run it on can contact me for help.

PopG demonstration goes here!

Genetic drift: random change in allele frequencies due to finite population size

- Often called "sampling error" but it is not an experimental error
- The process of producing offspring IS a random sampling process
- The smaller the population, the more strongly it is affected by drift



What is the chance that drift will fix A as opposed to a?

- Drift is a random walk that continues until it touches 1 or 0.
- The closer it is to 1, the more likely it is to reach 1 before it reaches 0.
- Probability of fixing A: pA
- Probability of fixing a: pa
- With no other forces, eventually one or the other must fix.

Imagine extending this to a population in which everyone started with two unique alleles.

N individuals would have 2N alleles.

After a few generations, some alleles will be lost.

Eventually all but 1 will be lost.

The chance that any given allele is a "winner" must be:

 $\frac{1}{2N}$ 

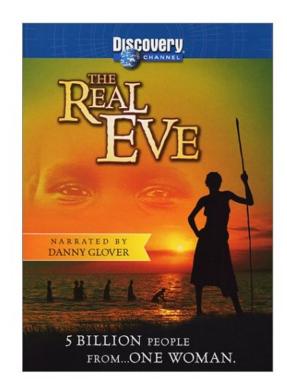
#### **Practice problem**

- Suppose you have a new, neutral mutant allele at your ADH locus.
- What is the chance that this allele will become the new "wild type"?
- Useful facts:
  - World population: 7 billion  $(7x10^9)$
  - Don't forget that you are diploid!
  - Assume the population size remains constant.

- World population contains  $14x10^9$  copies of this allele
- Your allele therefore has a chance of  $1/14x10^9$  to be the winner
- This is about  $7x10^{-11}$ . You'd have better luck playing the lottery.
- If we ask if ANY gene of yours will win, it's about 0.0000035

# **Mitochondrial Eve**

- Popular press was surprised that "mitochrondrial Eve" existed
- Statisticians were surprised by:
  - Where she lived
  - When she lived
- For a non-recombining locus there MUST be a single common ancester if you go back far enough



How long will the "winner" take to win on average? This requires some assumptions:

- Diploid organism (two gene copies per individual)
- Random mating
- Non-overlapping generations
- Each individual has an equal chance to contribute to the next generation
- Constant population size

(Humans don't quite fit here....)

With these assumptions:

- Average time to fixation is 2N generations.
- Variance is also 2N (uncertainty is high!)
- At any point, the chance that an allele will be the eventual winner is equal to its frequency
- mtDNA has an average fixation time of N/2 generations—why the difference?

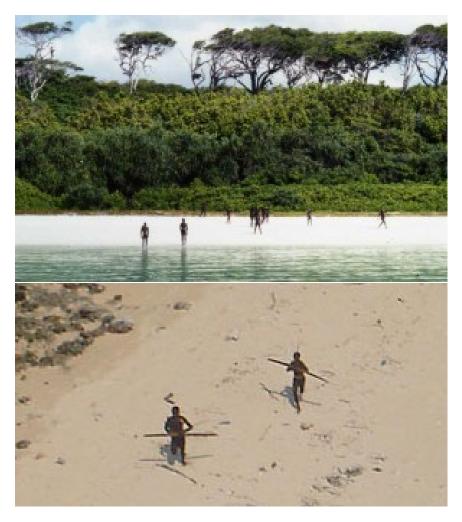
- If one of your alleles will become the new wild type, on average how long will it take (in years)?
- Assume 20 years per generation
- World population: 7 billion  $(7x10^9)$  assume it's constant

- If one of your alleles will become the new wild type, on average how long will it take (in years)?
- Assume 20 years per generation
- World population: 7 billion  $(7x10^9)$
- $14x10^9$  generations
- $2.8x10^{11}$  years
- Estimated future lifespan of Earth:  $5x10^9$  years

- At our current population size, humans can't fix rare alleles by drift in feasible time
- Things were different during most of our evolution, with an approximate population size around 5,000-50,000
- Humans **do** lose rare alleles by drift
- Fixation by drift is still significant for isolated human populations

### South Sentinel Island

- Island in the Bay of Bengal
- Current population estimated at 50-400 adults
- Isolated for 60,000 years
- Violent rejection of outside contact
- Rate of genetic drift is probably VERY high in this population



- How fast does a population fix new variants by drift?
- Each generation:
  - Mutation rate  $\mu$  per gene copy
  - -2N gene copies (diploid)
  - New mutations produced:  $2N\mu$
- $\bullet$  Exactly one copy out of 2N will be the long-term winner
- If the winner has a mutation, it will be fixed
- Rate of fixation of new mutants is  $2N\mu/2N=\mu$

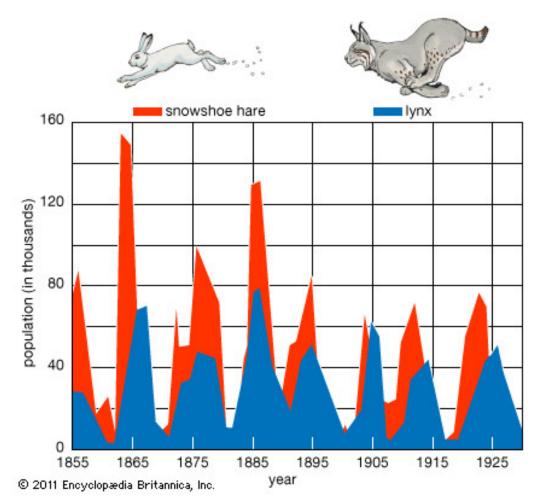
- Small and large populations will fix the same number of mutations per unit time
- Small population:
  - Few mutations arise
  - Those that arise fairly often fix
- Large population:
  - Many mutations arise
  - Each one has a very low chance to fix

### Rate of change in a species due to drift

- Standing variation (mutations waiting to fix or be lost) is larger in a large population
- Divergence rate (rate at which two populations become genetically different) does not depend on population size
- (This isn't quite true due to non-neutral mutations, but it's close)

# **Effective population size**

- Idealized population: constant size, random mating, no migration
- What about real populations?
- We can define an effective population size N<sub>e</sub> which compares a real population to the idealized population



If a population grows and shrinks,  $N_e$  is the harmonic mean of the various sizes. For two generations:

$$\frac{1}{N_e} = (\frac{1}{N_1} + \frac{1}{N_2})/2$$

The generalized solution for k generations:

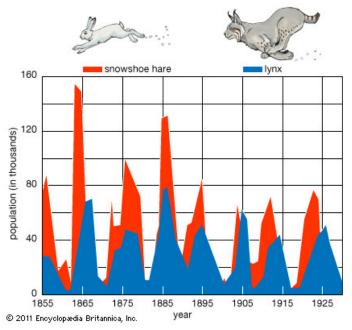
$$\frac{1}{N_e} = (\sum \frac{1}{N_i})/k$$

- Harmonic mean is close to minimum value
- Genetic diversity is lost when the population is small and is not rapidly regained when it is larger

**Effective population size** 

Consider a rabbit population which cycles between size 40,000 and size 100,000:

$$\frac{1}{N_e} = (1/40000 + 1/100000)/2$$
$$N_e = 57,143$$



- Long-term human effective population size: 5,000 to 50,000
- We do not yet have the genetic diversity of a species of size 7 billion
- $N_e$  is usually lower than N, often much lower
- Unusual mating systems can make  $N_e$  slightly higher than N but this is rare

### **Effective population size**

- Red drum are large fish of the Gulf of Mexico
- Effective size 1000 times lower than census size
- This species has the numbers of a big population but the genetic drift of a small one
- Likely explanation is very unequal reproductive success



- Red drum spawn in very specific estuary environments
- A few lucky clutches have thousands of survivors; most have none
- Allele frequencies change substantially from one generation to the next, reflecting the few lucky individuals

- Unequal males and females (domestic cattle)
- Overlapping generations (redwoods)
- Highly nonrandom mating success (racehorses)

All of these tend to reduce  $N_e$  compared to  ${\rm N}$ 

- Tear off a half-sheet of paper
- Write one line about the lecture:
  - Was anything unclear?
  - Did anything work particularly well?
  - What could be better?
- Leave at the back on your way out