# Roadmap

- Midterm
- Gene flow versus selection
- Inbreeding
  - Effect on heterozygosity
  - Effect on  $N_e$
  - Interaction with selection
- F Statistics and migration

### Midterm

- Distribution of grades: 91, 86, 85, 84, 69, 68, 67
- Exam too long for given time
- All exams met acceptable graduate student standards and grades will be interpreted accordingly

#### Midterm

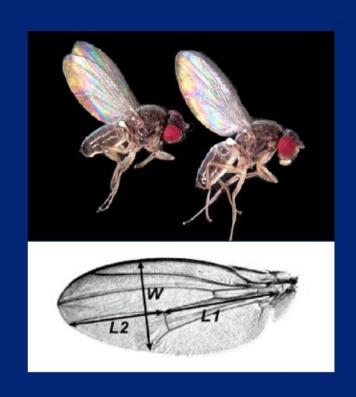
Issues to watch for on future homeworks and final:

- Don't give one-generation solution if problem states "for a long time"
- Check if the explanation you are proposing goes in the right direction:
  - Selection against something should not make it more common
  - Inheritance of very old haplotypes should not reduce diversity
  - ...etc.
- Key is available on web site

### Clines

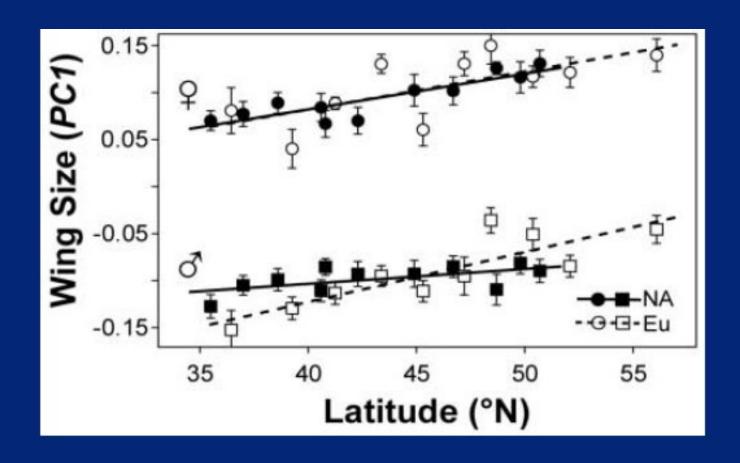
- Region between two populations where allele frequencies shift
- Could be due to drift:
  - Two populations are isolated and diverge by drift
  - When they re-contact, a cline forms
- Could also be due to divergent selection

# Drosophila subobscura





# Drosophila subobscura cline



### **Inbreeding**

- Ways to look at inbreeding:
  - Increased chance of mating with a related individual
  - Increased chance of two alleles with a recent common ancestor ending up in the same individual
- Population subdivision can be looked at as a form of inbreeding (more on this later)

### Identity by descent

- We agree to an artificial distinction:
  - Two alleles with a common ancestor within the time depth we're considering are identical by descent (IBD)
  - Alleles that do not have a common ancestor within this depth are not IBD, even if they are identical (identity by state, IBS)
- The time depth could be:
  - Within the known pedigree
  - An arbitrary cutoff time
  - Back to a non-inbred source population
  - Something nebulous and not discussed
- Note that two alleles IBD might not be IBS if there is a new mutation

### Inbreeding coefficient

- Single generation inbreeding of any intensity can be abstracted as:
  - Fraction f of offspring contain two gene copies IBD from the previous generation
  - Fraction 1 f are not IBD (though they may be IBS)
- This is tightly related to the coefficient of kinship
  - Coefficient of kinship between two individuals is the f of a hypothetical offspring

# **Pedigree examples**

- Self-fertilization
- Half siblings
- Full siblings

# H-W for a population with average inbreeding f

- In one generation of inbreeding, starting with no IBD
- (Using the convention that pA = p and pa = q)  $pAA = p^2(1-f) + pf$  pAa = 2pq(1-f)  $paa = q^2(1-f) + qf$
- ullet A fraction f of heterozygotes is being changed into homozygotes due to inbreeding
- Note that allele frequencies are not changing
- Another way to write this:

$$pAA = p^{2} + fpq$$

$$pAa = 2pq - 2fpq$$

$$paa = q^{2} + fpq$$

# Repeated inbreeding-discuss!

- If allele frequencies are not changing, why does heterozygosity continue to go down over time?
- Previous generation already has alleles IBD
- Repeated self-fertilization:
  - f = 0.5
  - In one generation, lose half of heterozygosity
  - Next generation, lose half of what's left; individuals can be IBD with respect to their parent, or failing that their grandparent
- ullet Does this predict that any non-zero f eventually makes everyone homozygous?

# Selection in the presence of inbreeding

Mean population fitness:

$$(p^{2}(1-f)+pf)wAA + 2pq(1-f)wAa + (q^{2}(1-f)+qf)waa$$

- Rearrange terms to get a difference from outbreeding
- Mean fitness is lower in inbred population if wAa higher than mean of wAA and wAa
  - Any degree of dominance of the better allele
  - Overdominance
- Mean fitness is higher in inbred if:
  - Any dominance of the worse allele
  - Underdominance

# Why is inbreeding even tolerable?

- In most cases, population mean fitness is lower with inbreeding for a given set of allele frequencies
- However, this causes selection to change the allele frequencies faster:
  - Bad recessives eliminated sooner
  - Good recessives fixed sooner
- ullet Populations like Arabidopsis which habitually inbreed do not see a fitness drop due to inbreeding, because bad recessives already purged
- Overdominance is still an issue

# Inbreeding and overdominance

• If f = 1 no overdominant equilibrium exists:

Ğenotype	Fitness	Frequency	before selection
AA	8.0	p	
Aa	1.0	0	
aa	0.7	$\overline{q}$	

- $\bullet$  For any non-zero starting frequency, A will fix (barring drift)
- ullet If f < 0 the equilibrium exists, but with high f is very vulnerable to drift as heterozygotes are so rare

# Self-fertilization vs. cloning

#### • Self-fertilization:

- Decrease in heterozygotes every generation
- Overdominance can't be maintained
- Recombination can happen, but becomes less useful as everything is homozygous

#### • Cloning:

- Random drift of genotype frequencies
- Overdominant heterozygote can fix
- Recombination probably not happening
- In the very long run, diploid state likely to be lost (rotifers)

# Population subdivision as a form of inbreeding

- An isolated subpopulation is inbred relative to the whole population, even if it has random mating internally
- Wright conceptualized this as the increase in IBD for two alleles from the same subpopulation relative to the total population
- Three terms:
  - $F_{IS}$  excess IBD in an Individual relative to its Subpopulation (measures inbreeding such as non-random mating)
  - $F_{ST}$  excess IBD in two alleles drawn from a Subpopulation relative to the Total population
  - $F_{IT}$  necessarily determined by the other two
- $\bullet$   $F_{ST}$  frequently used as a measure of population differentiation

# $F_{ST}$ in terms of variances

• 
$$F_{ST} = \frac{\sigma_S^2}{\sigma_T^2} = \frac{\sigma_S^2}{\hat{p}(1-\hat{p})}$$

 $\bullet$  Here  $\sigma_S^2$  is the variance in allele frequency across populations, and  $\hat{p}$  is a measure of variance within the total population

# $F_{ST}$ in terms of inbreeding coefficient

$$\bullet F_{ST} = \frac{f_0 - \hat{f}}{1 - \hat{f}}$$

 $\bullet$  Here f is the probability of IBD of two draws from a subpopulation, and  $\hat{f}$  is the probability for two draws from the total population

# $F_{ST}$ in terms of the coalescent

- $\bullet$   $F_{ST} \approx 1 \frac{t_S}{t_T}$
- (Note correction on this slide!)
- ullet  $t_S$  is the expected time back to the subpopulation common ancestor
- ullet  $t_T$  is the expected time back to the total population common ancestor

# $F_{ST}$ as it might actually be measured!

- $F_{ST} \approx \frac{\pi_B \pi_W}{\pi_B}$
- $\pi_B$  is mean pairwise difference between individuals from different subpopulations
- ullet  $\pi_W$  is mean pairwise difference between individuals from the same subpopulation
- This will react to:
  - Divergence time (longer separation is higher  $F_{ST}$ )
  - Gene flow (more gene flow is lower  $F_{ST}$ )

### The dark side of $F_{ST}$

• 
$$F_{ST} = \frac{\sigma_S^2}{\sigma_T^2} = \frac{\sigma_S^2}{\hat{p}(1-\hat{p})}$$

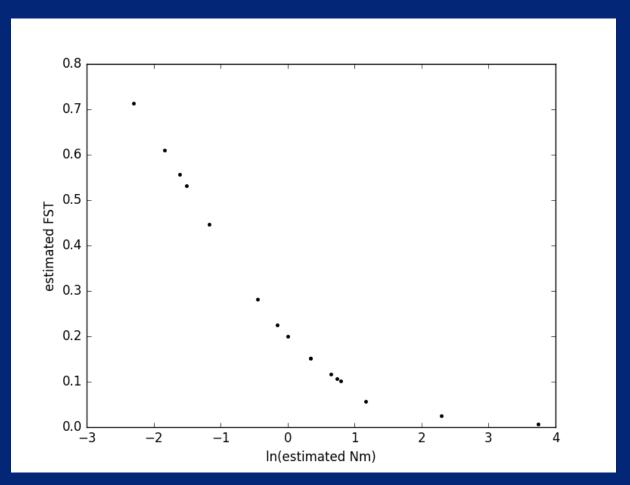
- If  $\hat{p}$  is very large or small (one allele is very frequent),  $F_{ST}$  can't get big
- None the less, people attempt to give it an absolute interpretation
- Need to average across multiple loci to get meaningful results
- Still vulnerable to unexpected allele frequency spectrum (e.g. in growing/shrinking populations)

# The dark side of $F_{ST}$

- $F_{ST} \approx \frac{\pi_B \pi_W}{\pi_B}$
- Good luck if your sampling yields  $\pi_B < \pi_W$ , and it can!
- You can do better with ANOVA or AMOVA, or with the coalescent methods to be described later

# $F_{ST}$ estimates from Slatkin 1985

Type of organism	Estimated Nm	Estimated $F_{ST}$
Annual plant	1.4	0.152
Mollusc	42.0	0.006
Insect	9.9	0.025
Insect	1.0	0.200
Fish	3.2	0.056
Frog	1.4	0.152
Salamander	2.1	0.106
Salamander	0.22	0.532
Salamander	0.10	0.714
Salamander	0.64	0.281
Salamander	0.20	0.556
Salamander	0.16	0.610
Lizard	1.9	0.116
Mouse	2.2	0.102
Mouse	0.31	0.446
Gopher	0.86	0.225



Data from Slatkin 1985

### Estimate Nm from $F_{ST}$

- As this graph shows, you can!
- Unfortunately:
  - Assumes same migration in all directions
  - Assumes similar population sizes
  - Quite biased if these assumptions not met
- There are better, but computationally complex, ways to do this

### Wednesday

- $\bullet$  Note that Monday 2/18 is a holiday! Homework will be assigned Wednesday, due the following Monday
- Quantitative traits
- Heritability
  - What it is
  - (Important) What it isn't

# **One-minute responses**

#### • Please:

- Tear off a slip of paper
- Give me one comment or question on something that worked, didn't work, needs elaboration, etc.