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Incorporating uncertainty into a supplier selection problem

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ABSTRACT

Supplier selection is an important strategic supply chain design decision. Incorporating uncertainty of demand and supplier capacity into the optimization model results in a robust selection of suppliers. A two-stage stochastic programming (SP) model and a chance-constrained programming (CCP) model are developed to determine a minimal set of suppliers and optimal order quantities with consideration of business volume discounts. Both models include several objectives and strive to balance a small number of suppliers with the risk of not being able to meet demand. The SP model is scenario-based and uses penalty coefficients whereas the CCP model assumes a probability distribution and constrains the probability of not meeting demand. Both formulations improve on a deterministic mixed integer linear program and give the decision maker a more complete picture of tradeoffs between cost, system reliability and other factors. We present Pareto-optimal solutions for a sample problem to demonstrate the benefits of the SP and CCP models. In order to describe the tradeoffs between costs and risks in an analytical form, we use multi-parametric programming techniques to more completely analyze the alternative Pareto-optimal supplier selection solutions in the CCP model. This analysis gives insights into the robustness of the solutions with respect to number of suppliers, costs and probability of not meeting demand.

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1. Introduction

Under the pressure of global competition, companies strive to achieve excellence in delivering high quality and low cost products to their customers on time and rely on the efficiency of their supply chain to gain competitive advantage. At the frontier of a supply chain, suppliers act as a key component for success because the right choice of suppliers reduces costs, increases profit margins, improves component quality and ensures timely delivery. Current supplier management trends show increasing interests in global sourcing, reducing the supplier base and establishing long-term relationships with the suppliers (Minner, 2003). Selecting suppliers is no longer an operational function but becomes a strategic level decision (Crama et al., 2004).

When consolidating and reducing the number of suppliers, companies run the risk of not having sufficient raw materials to meet their fluctuating demand. These risks may be caused by natural disasters or man-made actions. A recent example is the fire that happened at one of Phillips' microchip plants in 2000. Phillips lost about $40 million in sales. As a major customer of the chip plant, cell phone manufacturer Ericsson lost $2.34 billion in its mobile phone division (Bartholomew, 2006). The risks are further amplified by the current focus on supply chain efficiency and lean practices. A small disruption may ripple along the whole supply chain and cause significant business losses. As a result, there is a need to be able to evaluate the tradeoffs between the benefits of managing a few selected suppliers and the risks of not being able to meet the required demand. There can be substantial benefits if the companies plan flexibility into their supply chain to handle risks proactively.

Another source of risk is associated with global sourcing. With long lead times and transportation routes, the expanded supply chain is vulnerable to disruptions along the routes. Even though overseas suppliers offer competitive price schedules, they also increase the risk of late delivery of sufficient quantity. Instead of increasing inventory levels to ensure a sufficient supply of raw materials, another option is to strategically determine the number and location of suppliers. By establishing relationships with carefully selected local and overseas suppliers, companies can add flexibility to their supply chain and reduce the risks of disruption without stockpiling.

We develop stochastic mathematical programming models to capture the risk associated with uncertain customer demand and supplier capacity and to create a strategic purchasing plan. Moreover, we use multi-parametric programming techniques to analyze tradeoffs and determine a robust set of suppliers with balanced costs and risks.

Recognizing the importance of the supplier selection decision, an extensive literature exists to address this kind of decision. These existing decision making models are essentially trying to answer the following basic questions: how many suppliers are

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appropriate, which suppliers to choose, and what is the optimal ordering/replenishing policy. Many deterministic models have been developed to answer these questions with varying considerations of quantity discount, lot size, or inventory management decisions (e.g., Dahel, 2003; Dai and Qi, 2007; Ghodsypour and O'Brien, 2001; Narsimhan et al., 2006). The main disadvantage of deterministic models is their incapability of handling randomness embedded in the real system. Other researchers have been working on various probabilistic models and demonstrate the importance of incorporating randomness in the supplier selection problem. Typically they study the effect of random customer demand but do not incorporate uncertainty in the supply and the impact of potential disruption (Gutiérrez and Kouvelis, 1995; Kasilingam and Lee, 1996; Velarde and Laguna, 2004). Two studies make an all-or-none assumption for supplier availability (Berger and Zeng, 2006; Ruiz-Torres and Mahnoodi, 2007).

Another complication to the supplier selection decision is the multi-criteria aspect. However, most of the literature that addresses uncertainties focuses on a single objective (e.g., Basnet and Leung, 2005; Bollapragada et al., 2004; Dada et al., 2007; Yang et al., 2007). Dickson (1966) listed 23 selection criteria; however, quality, delivery and price have been identified as the prime criteria when purchasing industrial raw materials (Akarte et al., 2001; Cameron and Shipley, 1985). Price mentioned here has a broader meaning nowadays; it includes the costs associated with the whole purchasing process and over the purchased item's entire life in addition to the purchasing price. Among these costs, transportation and inventory costs constitute a significant bulk. Therefore, our models consider quality, delivery, and cost (including the transportation and inventory costs) as selection goals in addition to a probabilistic measure of risk.

We develop two optimization models to find a minimal set of suppliers to achieve quality and delivery goals while minimizing cost and the risk of having insufficient supply to meet demand. We incorporate uncertainties that may originate at the suppliers, or may be due to uncertain demand in our models. We also include business volume discounts to represent financial advantages in consolidating the supplier base. Globalization in the supplier base is reflected implicitly by the supplier capacity, the quoted price, the transportation cost, and the pipeline inventory cost in this study. Our models provide a means to explore the balance between the risk of not meeting the demand, the benefits of reduced number of suppliers, and the cost. The uncertainties in demand and supplier capacity are captured either by scenarios or with a probability distribution in the models. Not only the optimal supplier set but also ordering quantities are determined by the models. A multi-parametric analysis provides a means to explore tradeoffs between cost, risk, and number of suppliers in a closed form. A sample problem demonstrates the possibility to guard against supplier disruption by carefully weighing costs and risks in selecting a robust set of suppliers.

This paper is organized as follows: Section 2 gives the problem formulations of a stochastic programming model and a chance-constrained programming model. Section 3 presents numerical results obtained from a sample problem and provides some guidelines for the sourcing decision. Section 4 discusses the multi-parametric programming approach to analyze the robustness of solutions and illustrates it on the sample problem. Section 5 summarizes this paper, points out the importance of inclusion of uncertainties into modeling, and the advantages of using a chance-constrained programming model with multi-parametric analysis to determine the robustness of the supplier selection decisions.

2. Mathematical models

We formulate a stochastic programming (SP) model, and a chance-constrained programming (CCP) model for a multi-criteria supplier selection problem. We incorporate the uncertainty of demand and supplier capacity with either probabilistic scenarios or a closed-form probability distribution. Our SP model is scenario-based and includes a second-stage recourse problem for the order quantities. Our CCP model uses probability distributions for demand and supplier capacity, and assumes independence. Our models also include business volume discounts, transportation costs, as well as costs associated with pipeline inventory. Lead time differences among suppliers are captured in the transportation and inventory costs. Since our models involve high level decisions, the problem has no time dimension. Both models obtain a minimal set of suppliers that balance risks and costs.

We consider a set of plants which demand different sets of components. Coordination between plants is allowed to enable plants to use the business volume discounts offered by suppliers. However, there are certain costs associated with the coordination between plants. The initial set of potential suppliers includes their individual capacities, quality, and delivery performances. Each supplier offers its own business volume discount schedule on the total dollar amount of sales awarded with applicable discount rates. Since not only domestic suppliers but also overseas suppliers are considered in this problem, there are different transportation and pipeline inventory costs associated with the suppliers.

The multiple objectives include: (1) minimizing the total purchasing and shipping costs; (2) maximizing the probability of satisfying demand and staying within supplier capacity; (3) minimizing the total number of chosen suppliers; (4) maximizing the quality of received components; and (5) minimizing the late deliveries. Other models (e.g., Basnet and Leung, 2005; Narsimhan et al., 2006; Velarde and Laguna, 2004) assign a fixed cost per supplier to capture the effects of reducing the number of suppliers. However, we avoid the use of a fixed cost in this paper, because this fixed cost is very difficult to quantify and interpret in practice. Instead we present the Pareto-optimal solutions so that the decision maker can evaluate the tradeoffs and sensitivity associated with changing the number of suppliers. Since quality and on-time delivery are crucial factors when making the supplier selection decision, companies generally will not consider any supplier which may have a problem with quality and delivery. Therefore, we include (4) and (5) as hard constraints in the following two models. However, the right-hand-sides of the quality and delivery constraints can be modified to explore alternative solutions. In this case, it is equivalent to treating them as multiple objectives.

We use our optimization models to create a set of efficient solutions (also known as Pareto-optimal solutions) which achieve an optimum in one objective with compromises in other objectives. We apply the $\varepsilon-$constraint method (Deb, 2001) to approach this multi-objective problem because it is suitable for mixed integer programs, whereas manipulating a weighted objective function does not guarantee an efficient frontier in the presence of non-convexity (introduced by the binary variables). The $\varepsilon-$constraint method keeps one of the multiple objectives as a single objective in the model (cost in our models), and restricts the rest of the objectives in the constraints (e.g., number of suppliers and risk). A set of Pareto-optimal solutions can be found by changing the specified right-hand-sides (ε) for the objectives included in the constraints. In this paper, we apply the $\varepsilon-$constraint method to find the Pareto-optimal solutions and conduct a sensitivity analysis on the ε values to provide insights into this multi-objective supplier selection problem.

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2.1. Stochastic programming model

We first present our stochastic programming model for the supplier selection problem. The SP model incorporates uncertainties by the inclusion of the recourse problem, and probabilistic scenarios for demand and supplier capacity. The SP model is suitable when the decision maker does not have a clear definition about the distributions of the random variables but he/she may have some historical data to define scenarios or want to explore possible future scenarios. Scenarios may represent dependencies between demand and supplier capacity, which could be caused by global recession, political events, effects due to geographical location, etc.

Since it is not always possible to find a feasible solution under all scenarios for demand and supplier capacity, when supplier capacity is exceeded a penalty cost is incurred. An interpretation is that when a supplier's capacity is insufficient to meet demand, the supplier may take other measures, such as outsourcing, to provide the necessary components. The penalty cost may reflect a potential loss in quality, possible delayed deliveries, lost sales as in the Phillips–Ericsson example, and hence a loss in customer relations for unsatisfied demand. The parameters and decision variables used in the SP model are defined in Tables 1 and 2.

A multi-objective two-stage stochastic programming model with recourse follows:

$$\text{Min} \quad f_v = \sum_{i \in I} y_i \tag{1}$$

Table 1
Problem sets and parameters in the SP model.

Sets	Description
I	Set of suppliers
J	Set of plants
K	Set of items
M	Set of volume discount intervals
Ξ	Set of scenarios
Parameters	**Description**
$d_{jk}(\xi)$	Demand (in units) for item k at plant j given scenario ξ, $j \in J$, $k \in K$ and $\xi \in \Xi$
$cap_i(\xi)$	Aggregate capacity limit (in facility hours) of supplier i given scenario ξ, $i \in I$ and $\xi \in \Xi$
$ucap_{ik}$	Unit capacity (in facility hours) used by item k for supplier i, $i \in I$ and $k \in K$
p_{ijk}	Unit price (in \$/unit) of item k quoted by supplier i to plant j, $i \in I$, $j \in J$ and $k \in K$
inv_{ijk}	Unit pipeline inventory cost (in \$/unit) of item k shipped by supplier i to plant j, $i \in I$, $j \in J$ and $k \in K$
$ship_{ijk}$	Unit transportation cost (in \$/unit) of item k shipped by supplier i to plant j, $i \in I$, $j \in J$ and $k \in K$
$c_{jj'k}$	Unit shipping cost (in \$/unit) of item k from plant j to plant j', $j, j' \in J, j' \neq j$ and $k \in K$
q_{ik}	Fraction of poor quality items of type k from supplier i, $i \in I$ and $k \in K$
t_{ik}	Fraction of late items of type k from supplier i, $i \in I$ and $k \in K$
u_{im}	Upper cutoff point (in \$) of discount interval m from supplier i, $i \in I$ and $m \in M$
r_{im}	Discount rate (fraction) associated with discount interval m offered by supplier i, $i \in I$ and $m \in M$
e_i	Unit penalty cost (in \$/facility hour) of exceeding capacity of supplier i, $i \in I$
$\tau_q(\xi)$	Pre-set quality tolerance (in units) given scenario ξ, equal to $0.05\sum_{j,k} d_{jk}(\xi)$
$\tau_d(\xi)$	Pre-set delivery tolerance (in units) given scenario ξ, equal to $0.05\sum_{j,k} d_{jk}(\xi)$

Table 2
Decision variables in the SP model.

Decision variables	Description
y_i	Binary variable, has value of 1 if supplier i is chosen, 0 otherwise, $i \in I$
Y	Vector of y_i's
$v_{im}(\xi)$	Binary variable, has value of 1 if the volume of business awarded to supplier i falls in the m-th interval given scenario ξ, 0 otherwise, $i \in I, m \in M$ and $\xi \in \Xi$
$x_{ijk}(\xi)$	Amount of item k purchased from supplier i at plant j given scenario ξ, $i \in I, j \in J$, $k \in K$ and $\xi \in \Xi$
$z_{jj'k}(\xi)$	Amount of item k ordered from plant j for plant j' given scenario ξ, $j, j' \in J, j' \neq j$, $k \in K$ and $\xi \in \Xi$
$ocap_i(\xi)$	Amount of exceeded capacity of supplier i given scenario ξ, $i \in I$ and $\xi \in \Xi$
$b_{im}(\xi)$	Amount of business volume awarded to supplier i falling in the m-th discount interval given scenario ξ, $i \in I, m \in M$ and $\xi \in \Xi$

$$\text{Min} \quad f_c = E_\xi[C_f(Y,\xi)] \tag{2}$$

$$\text{s.t.} \quad y_i \in \{0,1\} \quad \forall i \in I \tag{3}$$

where for Y supplier selection values and any scenario $\xi \in \Xi$, we solve the following second-stage problem with decision variables $x_{ijk}(\xi)$, $z_{jj'k}(\xi)$, $b_{im}(\xi)$, $v_{im}(\xi)$ and $ocap_i(\xi)$,

$$C_f(Y,\xi) = \text{Min} \sum_{i \in I} \sum_{m \in M} (1-r_{im}) \cdot b_{im}(\xi) + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} (ship_{ijk} + inv_{ijk}) \cdot x_{ijk}(\xi)$$
$$+ \sum_{j \in J} \sum_{\substack{j' \in J \\ j' \neq j}} \sum_{k \in K} c_{jj'k} \cdot z_{jj'k}(\xi) + \sum_{i \in I} e_i \cdot ocap_i(\xi) \tag{4}$$

s.t.

$$\sum_{i \in I} \sum_{k \in K} q_{ik} \cdot \sum_{j \in J} x_{ijk}(\xi) \le \tau_q(\xi) \tag{5}$$

$$\sum_{i \in I} \sum_{k \in K} t_{ik} \cdot \sum_{j \in J} x_{ijk}(\xi) \le \tau_d(\xi) \tag{6}$$

$$\sum_{i \in I} x_{ijk}(\xi) + \sum_{j' \in J \setminus j} z_{j'jk}(\xi) - \sum_{j' \in J \setminus j} z_{jj'k}(\xi) \ge d_{jk}(\xi) \quad \forall j \in J, \ k \in K \tag{7}$$

$$\sum_{k \in K} ucap_{ik} \cdot \sum_{j \in J} x_{ijk}(\xi) - ocap_i(\xi) \le cap_i(\xi) \quad \forall i \in I \tag{8}$$

$$\sum_{j \in J} \sum_{k \in K} p_{ijk} \cdot x_{ijk}(\xi) = \sum_{m \in M} b_{im}(\xi) \quad \forall i \in I \tag{9}$$

$$b_{im}(\xi) \le u_{im} \cdot v_{im}(\xi) \quad \forall i \in I, \ m \in M \tag{10}$$

$$b_{i,m+1}(\xi) \ge u_{im} \cdot v_{i,m+1}(\xi) \quad \forall i \in I, \ m \in \{1,2,\ldots,|M|-1\} \tag{11}$$

$$\sum_{m \in M} v_{im}(\xi) = 1 \cdot y_i \quad \forall i \in I \tag{12}$$

$$v_{im}(\xi) \in \{0,1\} \quad \forall i \in I, \ m \in M \tag{13}$$

$$x_{ijk}(\xi), z_{jj'k}(\xi), b_{im}(\xi), ocap_i(\xi) \ge 0 \quad \forall i \in I, \ j \in J, \ j' \in J, \ k \in K, \ m \in M \tag{14}$$

Our stochastic programming model is a two-stage model with recourse. The first stage has two objectives: (1) to minimize the number of suppliers selected $\sum_{i \in I} y_i$ and (2) to minimize the *expected* total costs $E_\xi[C_f(Y,\xi)]$. The cost function $C_f(Y,\xi)$ is optimized at the second stage given the selected supplier set Y and the realized scenario ξ of the random demand and supplier capacity. It includes purchasing costs with business volume discounts

$\sum_{i\in I,m\in M}(1-r_{im})\cdot b_{im}(\xi)$, pipeline inventory costs and transportation costs $\sum_{i\in I,j\in J,k\in K}(inv_{ijk}+ship_{ijk})\cdot x_{ijk}(\xi)$, coordination costs $\sum_{j\in J,j'\in J,j'\neq j,k\in K}c_{jj'k}\cdot z_{jj'k}(\xi)$, and penalty costs for exceeding supplier's capacity $\sum_{i\in I}e_i\cdot ocap_i(\xi)$. The first stage decision is which suppliers to select y_i. The second stage decisions are how much to purchase from selected suppliers x_{ijk}, how to coordinate between plants $z_{jj'k}$, the volume awarded to selected suppliers in business discount b_{im}, and the amount of capacity exceeded $ocap_i$, given the selection decision y_i and the realized scenario ξ. The optimal value $C_f(Y,\xi)$ of the second stage problem (4)–(14) is a function of the first stage decision variables y_i and the realized scenario ξ. An interpretation of the two-stage SP with recourse is that while the suppliers are selected in the first stage (e.g., annually), the amount to order and shipment plan is decided in the second stage (e.g., weekly) when the random variables are realized through different scenarios.

The constraint sets (5) and (6) specify the requirements for high quality and on-time delivery of the received items. Since the requirements for high quality and on-time delivery of the received items are generally expressed as a percentage of the demand in practice and the demand varies at every scenario, hence different $\tau_q(\xi)$ and $\tau_d(\xi)$ are set under different scenarios. In our sample problem, we set $\tau_q(\xi)$ and $\tau_d(\xi)$ equal to $0.05\sum_{j,k}d_{jk}(\xi)$. The value of 0.05 can be modified to explore the impact of different quality and/or delivery requirements.

The constraint sets (7) and (8) model uncertainty of demand, $d_{jk}(\xi)$, and supplier capacity, $cap_i(\xi)$. Our model ensures that demand is met for each scenario ξ by allowing suppliers to exceed capacity, $ocap_i(\xi)$, and assuming that there are some measures to accommodate variable demand, such as outsourcing. There is a penalty cost, e_i, associated with this. The constraint set (7) describes the flow conservation at each plant and ensures demand at each plant is satisfied. This constraint set allows coordination between plants, $z_{jj'k(\xi)}$, with a coordination cost, $c_{jj'k}$. The constraint set (8) measures the amount that the total number of items purchased from a supplier is over the capacity limit of that supplier. This constraint accounts for a reduction in capacity due to a low probability event (such as fire or flood).

The constraint sets (9)–(12) link the total business volume with its corresponding discount rate. They also enforce a logic relationship: items can be purchased from selected suppliers only. The constraint sets (13) and (14) specify the binary and non-negative properties of the decision variables.

As mentioned before, we use the $\varepsilon-$constraint method to solve this multi-objective problem. The $\varepsilon-$constraint method is chosen because it is not difficult to implement and non-convexity of the mixed integer program does not present any problem in finding the Pareto-optimal solution set. When the $\varepsilon-$constraint method is implemented, objective function (1) is moved to be a constraint:

$$\sum_{i\in I}y_i\leq\varepsilon_s \tag{15}$$

where ε_s represents the limit on number of suppliers and is varied in the solution process. The reason we chose to move the objective function (1) instead of the objective function (2) to the constraint is that the right-hand-side ε_s for number of suppliers has naturally discrete values and bounds to explore the Pareto-optimal solutions.

2.2. Chance-constrained programming model

In addition to the two-stage stochastic programming model with recourse, we develop a chance-constrained programming model as an alternative way to incorporate the uncertainties. Unlike the stochastic programming model, which penalizes the average amount an obtained solution violates the capacity constraints over all scenarios, the chance-constrained programming model requires the demand and capacity constraints to be satisfied with some predetermined probability. In this model, it is assumed that random variables for demand and supplier capacity can be represented by closed-form probability distributions and are independent, whereas in the SP model, the probabilities for demand and supplier capacity may be dependent and are captured in the scenarios.

The parameters and decision variables used in the chance-constrained programming model are mostly the same as those used in the SP model, except the scenario notation is dropped. In addition, the CCP model uses the mean and standard deviation for demand and supplier capacity as well as two threshold levels, which are defined in Table 3.

A multi-objective chance-constrained programming model is formulated as follows:

$$\mathrm{Min}\sum_{i\in I}y_i \tag{16}$$

$$\mathrm{Min}\sum_{i\in I}\sum_{m\in M}(1-r_{im})\cdot b_{im}+\sum_{i\in I}\sum_{j\in J}\sum_{k\in K}(ship_{ijk}+inv_{ijk})\cdot x_{ijk}+\sum_{j\in J}\sum_{\substack{j'\in J\\ j'\neq j}}\sum_{k\in K}c_{jj'k}\cdot z_{jj'k} \tag{17}$$

s.t.

$$\sum_{i\in I}\sum_{k\in K}q_{ik}\cdot\sum_{j\in J}x_{ijk}\leq\tau_q \tag{18}$$

$$\sum_{i\in I}\sum_{k\in K}t_{ik}\cdot\sum_{j\in J}x_{ijk}\leq\tau_d \tag{19}$$

$$Pr\left(\sum_{i\in I}x_{ijk}+\sum_{j'\in J\backslash j}z_{j'jk}-\sum_{j'\in J\backslash j}z_{jj'k}\geq D_{jk}\right)\geq\varepsilon_d\quad\forall j\in J,\ k\in K \tag{20}$$

$$Pr\left(\sum_{k\in K}ucap_{ik}\cdot\sum_{j\in J}x_{ijk}\leq CAP_i\right)\geq\varepsilon_c\quad\forall i\in I \tag{21}$$

$$\sum_{j\in J}\sum_{k\in K}p_{ijk}\cdot x_{ijk}=\sum_{m\in M}b_{im}\quad\forall i\in I \tag{22}$$

$$b_{im}\leq u_{im}\cdot v_{im}\quad\forall i\in I,\ m\in M \tag{23}$$

$$b_{i,m+1}\geq u_{im}\cdot v_{i,m+1}\quad\forall i\in I,\ m\in\{1,2,\ldots,|M|-1\} \tag{24}$$

$$\sum_{m\in M}v_{im}=1\cdot y_i\quad\forall i\in I \tag{25}$$

$$y_i\in\{0,1\},v_{im}\in\{0,1\}\quad\forall i\in I,\ m\in M \tag{26}$$

$$x_{ijk},z_{jj'k},b_{im}\geq 0\quad\forall i\in I,\ j\in J,\ \ j'\in J,\ k\in K,\ m\in M \tag{27}$$

Table 3
Additional parameters in the CCP model.

Parameters	Description
D_{jk}	Demand (in units) for item k at plant j, $j\in J$ and $k\in K$, random variable with mean $\mu_{D_{jk}}$, standard deviation $\sigma_{D_{jk}}$, and cumulative probability distribution function $F_{D_{jk}}$
CAP_i	Aggregate capacity limit (in facility hours) of supplier i, $i\in I$, random variable with mean μ_{CAP_i}, standard deviation σ_{CAP_i}, and cumulative probability distribution function F_{CAP_i}
ε_d	Pre-determined satisfaction level of probabilistic demand constraint
ε_c	Pre-determined satisfaction level of probabilistic capacity constraint
τ_q	Pre-set quality tolerance (in units), equal to $0.05\sum_{j,k}\mu_{D_{jk}}$
τ_d	Pre-set delivery tolerance (in units), equal to $0.05\sum_{j,k}\mu_{D_{jk}}$

The objective of this model is to minimize the number of suppliers selected and the costs of purchasing, pipeline inventory, transportation and coordination. To find the Pareto-optimal solutions, the $\varepsilon-$constraint method will be applied to objective function (16), and is treated as a constraint, see (15). The constraint sets (18) and (19) are analogous to (5) and (6) for high quality and on-time delivery of items. The constraint sets (20) and (21) provide a lower limit on the probability that demand is met and are analogous to (7) and (8) in the SP model. There are two sources of uncertainty; either the demand could be more than what was expected and then the order is insufficient to meet demand, or the supplier's capacity is reduced by an unforeseen event and the order cannot be filled. Both random events result in demand not being met. We treat the two sources of uncertainty separately, where the constraint set (20) ensures that demand at each plant be satisfied with a probability of at least ε_d. The constraint set (21) requires that the probability that a supplier's capacity exceeds the total amount purchased is at least ε_c. Simply put, we use these two constraints to ensure a certain level of the system reliability.

For ease of computation, constraint sets (20) and (21), which include the probabilistic statements, can be replaced by the following linear functions, since the terms inside the probabilistic statements are linear, regardless of the underlying cumulative distribution of random demand and supplier capacity (Prékopa, 1995)

$$\sum_{i \in I} x_{ijk} + \sum_{j' \in J \setminus j} z_{j'jk} - \sum_{j' \in J \setminus j} z_{jj'k} \geq F^{-1}_{D_{jk}}(\varepsilon_d) \quad \forall j \in J, \ \ k \in K \tag{28}$$

$$\sum_{k \in K} ucap_{ik} \cdot \sum_{j \in J} x_{ijk} \leq F^{-1}_{CAP_i}(1-\varepsilon_c) \quad \forall i \in I \tag{29}$$

where $F^{-1}_{D_{jk}}$ and $F^{-1}_{CAP_i}$ are the inverse cumulative probability distribution functions for random demand and random supplier capacity.

3. Experimental results

The value of including uncertainties in the models is demonstrated by a sample problem. There are 10 potential suppliers to choose from and four plants with demand for 50 types of components. Suppliers provide three discount rates which vary from 2% to 6% with the monetary business volume awarded to them. The big suppliers with high capacity levels expect to be awarded with large sales, and therefore, they offer higher discount rates to large sales. The big suppliers with high capacity levels generally offer 4–12% lower prices than the mid and small sized suppliers. However, the big suppliers are possibly overseas, and their transportation costs and pipeline inventory costs are 50–70% higher than the mid and small sized suppliers. The transportation costs and pipeline inventory costs account for approximately 10% of the purchase price. The big suppliers also tend to have 40–70% higher poor-quality rate and late-delivery rate. The poor-quality rate and the late-delivery rate are about 2% and 4%, respectively, of the total shipment. Even though the big suppliers offer competitive prices on most of the components, every supplier gives the cheapest price on a certain component. In this study, suppliers 1, 2 and 3 are the biggest ones, suppliers 4, 5, 6, and 7 are mid sized having about half capacity of the big ones, and suppliers 8, 9 and 10 are the smallest ones having about half capacity of the mid-sized ones. The detailed data sets are available upon request in electronic format.

For computational purposes, we use either normal distributions or triangular distributions to represent demand and supplier capacity for both SP and CCP models. The reason to choose these two distributions in this experiment is because they are easy to understand and it is intuitive for practitioners to describe the uncertain demand and supplier capacity in these forms. The normal distribution is a common assumption and is easily described by two parameters, the mean and the variance. Like the well-known beta distribution used in PERT/CPM, the triangular distribution allows practitioners to describe the random demand and random supplier capacity in a limited region with three parameters; the minimum value, the most likely value, and the maximum value. Whereas the normal distribution is symmetric, the triangular distribution can be skewed. When triangular distributions are used, customer demands are negatively skewed and supplier capacities are positively skewed. Our study analyzes the effects of the probability distribution shape (symmetric versus skewed) on the supplier selection decision.

Although the scenarios for the SP model can be quite complicated with interdependencies, for comparison purposes, we construct 10 scenarios for the SP model by taking sample data of demand and supplier capacity from the same independent normal and triangular distributions as used in the CCP model. A scenario consists of one sample data drawn independently from each demand and supplier distribution. The SP model is transformed into its deterministic equivalent form in order to solve it (Birge and Louveaux, 1997). For a larger problem, new SP algorithms developed for solving mixed integer problems may be more efficient computationally (Sherali and Fraticelli, 2002; Sen and Higle, 2005; Sen and Sherali, 2006).

For the CCP model, we assume that random demands and random supplier capacities follow either normal or triangular distributions. The mean and standard deviation for D_{jk} and CAP_i are given by $\mu_{D_{jk}}, \sigma_{D_{jk}}$, μ_{CAP_i}, and σ_{CAP_i}. We can replace the probability constraint sets (20) and (21) with (28) and (29) and explicit right-hand-sides for both normal and triangular distributions.

With the normal distribution assumption, an explicit form of the inverse cumulative probability distribution function is used, and (28) and (29) become

$$\sum_{i \in I} x_{ijk} + \sum_{j' \in J \setminus j} z_{j'jk} - \sum_{j' \in J \setminus j} z_{jj'k} \geq \mu_{D_{jk}} + \sigma_{D_{jk}} \cdot Z_{\varepsilon_d} \quad \forall j \in J, \ \ k \in K \tag{30}$$

$$\sum_{k \in K} ucap_{ik} \cdot \sum_{j \in J} x_{ijk} \leq \mu_{CAP_i} + \sigma_{CAP_i} \cdot Z_{1-\varepsilon_c} \quad \forall i \in I \tag{31}$$

where Z_ε is a standard normal random variate with cumulative probability of ε and can be obtained from the standard normal table when the ε levels are set.

We can similarly provide an explicit form of the inverse cumulative probability distribution functions in (28) and (29) when D_{jk} and CAP_i follow triangular distributions. Let a, b, and c be the three parameters of the distribution: the lower limit a, the mode b, and the upper limit c. We subscript parameters a, b, and c with the random variables D_{jk} and CAP_i. The probability constraint (28) for a triangular distribution can be derived as

$$\sum_{i \in I} x_{ijk} + \sum_{j' \in J \setminus j} z_{j'jk} - \sum_{j' \in J \setminus j} z_{jj'k} \geq \begin{cases} a_{D_{jk}} + \sqrt{\varepsilon_d (b_{D_{jk}} - a_{D_{jk}})(c_{D_{jk}} - a_{D_{jk}})} & \text{if } \varepsilon_d \leq \dfrac{b_{D_{jk}} - a_{D_{jk}}}{c_{D_{jk}} - a_{D_{jk}}} \\ c_{D_{jk}} - \sqrt{(1-\varepsilon_d)(c_{D_{jk}} - a_{D_{jk}})(c_{D_{jk}} - b_{D_{jk}})} & \text{if } \varepsilon_d \geq \dfrac{b_{D_{jk}} - a_{D_{jk}}}{c_{D_{jk}} - a_{D_{jk}}} \end{cases} \tag{32}$$

for all $j \in J$ and $k \in K$. Similarly, the probability constraint (29) for a triangular distribution can be derived as

$$\sum_{k \in K} ucap_{ik} \cdot \sum_{j \in J} x_{ijk} \leq \begin{cases} c_{CAP_i} - \sqrt{\varepsilon_c (c_{CAP_i} - a_{CAP_i})(c_{CAP_i} - b_{CAP_i})} & \text{if } 1-\varepsilon_c \geq \dfrac{b_{CAP_i} - a_{CAP_i}}{c_{CAP_i} - a_{CAP_i}} \\ a_{CAP_i} + \sqrt{(1-\varepsilon_c)(c_{CAP_i} - a_{CAP_i})(b_{CAP_i} - a_{CAP_i})} & \text{if } 1-\varepsilon_c \leq \dfrac{b_{CAP_i} - a_{CAP_i}}{c_{CAP_i} - a_{CAP_i}} \end{cases} \tag{33}$$

for all $i \in I$.

For the sample problem, we choose values for a, b, and c to keep the mean and standard deviation the same as for the normal distributions. Our triangular distributions are skewed; to the right for demand to capture unanticipated high demand, and to the left for supplier capacity to represent sudden reduction in capacity. This provides a contrast to the symmetric normal distribution.

The constraints for the chance-constrained programming model are given in (18)–(27) where (20) is replaced by (30) for the normal distribution and (32) for the triangular distribution, and (21) is replaced by (31) for the normal distribution and (33) for the triangular distribution, depending on the parameter values of ε_d and ε_c.

Solutions of the deterministic mixed integer programming (MIP) model are obtained for comparison purposes, using the expected values of the random demand and random supplier capacities. The MIP model is based on the CCP model rather than the SP model and is formulated as follows:

Min (17)

s.t. (15), (18), (19), (22)−(27)

$$\sum_{i\in I} x_{ijk} + \sum_{j'\in J\setminus j} z_{j'jk} - \sum_{j'\in J\setminus j} z_{jj'k} \geq \mu_{D_{jk}} \quad \forall j\in J,\ k\in K$$

$$\sum_{k\in K} ucap_{ik}\cdot \sum_{j\in J} x_{ijk} \leq \mu_{CAP_i} \quad \forall i\in I$$

where $\mu_{D_{jk}}$ is the expected demand for item k at plant j and μ_{CAP_i} is the expected capacity limit of supplier i.

By varying the number of suppliers ε_s from 1 to 10 in constraint (15) with the objective to minimize the total costs (17), a set of Pareto-optimal solutions is found by using the MIP model. For the SP model, we vary the number of suppliers ε_s from 1 to 10 in constraint (15) and vary the penalty cost e_i in the objective function (4) from 20 to 100 with an increment of 10 assuming equal penalty cost e_i applies to each supplier to obtain the Pareto-optimal solutions. The reason we start with 20 on the penalty cost is because we treat a unit of capacity as worth at least 10 monetary units for each supplier, and when suppliers overflow their capacity, we set the penalty cost to be at least twice the value of their capacity, i.e., 20. For the CCP model, we change the number of suppliers ε_s from 1 to 10 in constraint (15) and the satisfaction level of the probabilistic demand ε_d (20) and capacity ε_c (21) from 0.1 to 0.9 with an increment of 0.1 to find a set of Pareto-optimal solutions. We let $\varepsilon_d=\varepsilon_c$ during the solution process. It is possible to explore the solutions with different ε_d and ε_c values. However, the system reliability level described by different ε_d and ε_c levels are less meaningful for practitioners to interpret. It is more convenient for practitioners to achieve the same satisfaction levels for meeting probabilistic demands and respecting probabilistic supplier capacities.

The Pareto-optimal solutions obtained by these three models (SP, CCP and MIP) are illustrated in Figs. 1 and 2, for normal and triangular distributions, respectively. The detailed solutions are presented in Table 4 in the appendix. For Pareto-optimality, four performance measurements are considered: the number of suppliers, the system reliability level ($\varepsilon_d=\varepsilon_c$), the total costs as evaluated by the CCP model, and the *expected* total costs as evaluated by the SP model.

To have an appropriate comparison between models, we evaluate the performance measurements by taking every solution y_i^* found, and evaluating it using both the CCP model and the SP model under both normal and triangular distributions. The reason to evaluate solutions for the probability level ε_d and ε_c using the CCP model instead of the SP model is because the SP model cannot capture the probability of meeting demand explicitly. This is partly due to the limited number of scenarios considered in the SP model. Also, the SP model ensures that demand is met while penalizing exceeding supplier capacities under each scenario. While the CCP model calculates the total costs based on a single ordering plan, the SP model computes the *expected* total costs where the ordering plans are based on a different set of 10 scenarios drawn from the normal and triangular distributions. The appendix includes a detailed description of the evaluation process.

Figs. 1 and 2 list the Pareto-optimal sets of suppliers along the horizontal axis, summarizing the evaluation results under the

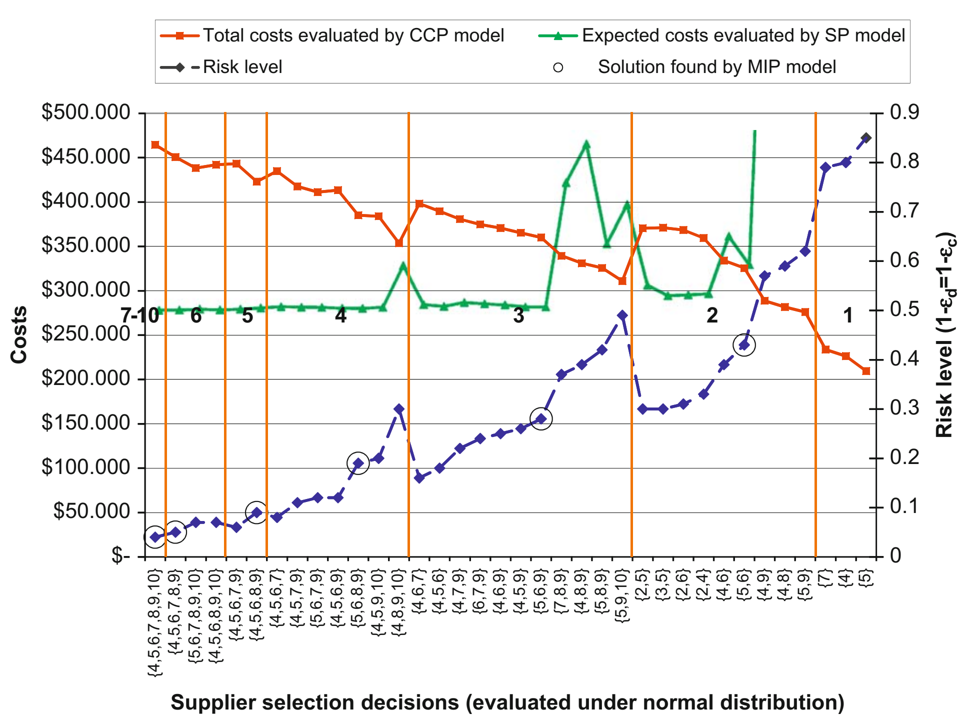

Fig. 1. Evaluation results under normal distribution.

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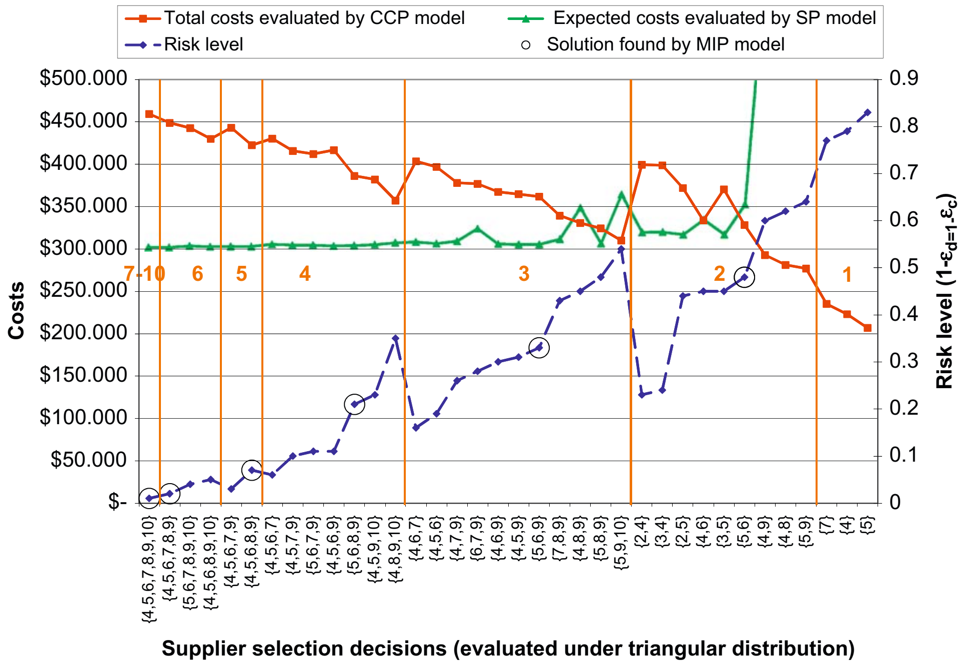

Fig. 2. Evaluation results under triangular distribution.

normal and triangular distributions. The gray vertical lines and gray numbers are used to group the solutions according to the number of suppliers allowed. For this sample problem, seven suppliers is the largest number of suppliers needed, so the left-most group includes 7–10 suppliers with a single Pareto-optimal solution. The solutions found by the MIP model are marked by black circles. The line with squares represents the total cost evaluated by the CCP model, the line with triangles represents the expected total cost evaluated by the SP model and the dashed line with diamonds represents the risk associated with *NOT* having sufficient supplier capacity or meeting demand $(1-\varepsilon_d = 1-\varepsilon_c)$. Their values can be found at the left vertical axis representing the costs and the right vertical axis giving the probability. Since we have extremely high *expected* costs for solutions with low system reliability when evaluated under the SP model, we cut off the scale of the left vertical axis at $500K to avoid having the high costs mask the trends in the costs. Note that the lines connecting the markers are used to show the trends in the data only.

As we look at the costs versus the number of suppliers, the total costs tend to drop with fewer suppliers using the CCP model evaluation whereas the expected costs increase when using the SP model evaluation. This observation is anticipated because the CCP model allows fulfilling partial demand as specified by the probabilistic constraints and thereby the lower the system reliability, the fewer items need be purchased from the suppliers, which results in lower costs. The SP model requires all demands be met by penalizing the overflowing capacities and hence having more suppliers avoid incurring the high penalty costs. If we fix the number of suppliers to be used, we can observe the total costs as evaluated by the CCP model are decreasing while the risks $(1-\varepsilon_d = 1-\varepsilon_c)$ are increasing when solutions are evaluated under both normal and triangular distributions. This is predictable since a solution with high total costs and high risk will be dominated by another solution with lower costs and lower risk. In other words, we should expect to see the total costs going down while the risks are going up within the same number of suppliers in Figs. 1 and 2. When comparing the figures, there is no clear indication that distribution skewness has an evident impact on the overall trends.

The expected cost as evaluated under the SP model has a different trend from the total cost evaluated by the CCP model. When the number of suppliers are enough to provide low risk, the expected cost is fairly flat. This indicates that the recourse decision can be used to modify the ordering and shipping plan with the same set of suppliers without incurring a large penalty cost of exceeding capacity. However, as the risk increases, the penalty cost increases and hence the expected cost increases. As we can observe from the figures, the penalty cost kicks in when the risk is above 0.3 or so. Since the penalty costs are difficult to estimate, one can generally use the CCP model to incorporate uncertainties into the supplier selection decision. However, if one is particularly interested in recourse decisions, then the SP model gives more insightful information.

From Figs. 1 and 2, and Table 4 in the appendix, we observe that *single* sourcing incurs high expected costs and a high risk of not having sufficient capacities to meet demand. On the other hand, *multiple* sourcing reduces the expected costs and ensures a high system reliability. For this sample problem, choosing four suppliers seems appropriate in terms of balancing cost, the number of suppliers, and high system reliability. With four suppliers, the system reliability level is at least 65% for both distributions. If a practitioner is more concerned with high system reliability than the total costs, then the supplier sets {4, 5, 6, 7} and {4, 5, 7, 9} should be examined first. These are the mid-sized and small suppliers. The supplier sets {5, 6, 8, 9} and {4, 5, 9, 10} may be considered if the costs are more of a concern. The marginal changes in cost can be contrasted with a marginal change in system reliability to achieve an ultimate decision. Taking two of these three solutions as an example, we can see that there is a 14% decrease in the system reliability, a 0.5% decrease in the expected costs and a 11% decrease in the total costs if we switch from {4, 5, 6, 7} to {5, 6, 8, 9}. There is a 16% decrease in the system reliability with an 0.2% decrease in the expected costs and 11% decrease in the total costs if we switch from {4, 5, 6, 7} to {4, 5, 9, 10}. In this case, the decision maker may choose {4, 5, 6, 7} as the best if he/she is more concerned with the risk since there is no obvious benefit in the expected cost reduction with sacrifice of the system reliability. When making the selection decisions,

Table 4
Performance of supplier selection decisions evaluated under normal (N) and triangular (T) distributions.

Number and set of selected suppliers	N/T	System reliability level $\varepsilon_d = \varepsilon_c$	Total costs CCP evaluation	Expected costs SP evaluation	Solution model
7-10 Suppliers	N	0.96	$ 464,548	$ 297,606	MIP, SP(N), SP(T),
{4, 5, 6, 7, 8, 9, 10}	T	0.99	$ 459,344	$ 302,354	CCP(N), CCP(T)
6 Suppliers	N	0.95	$ 450,694	$ 297,775	MIP, SP(N), SP(T),
{4, 5, 6, 7, 8, 9}	T	0.98	$ 448,770	$ 302,400	CCP(N)
6 Suppliers	N	0.93	$ 438,353	$ 298,853	SP(T)
{5, 6, 7, 8, 9, 10}	T	0.96	$ 442,640	$ 303,793	
6 Suppliers	N	0.93	$ 442,070	$ 298,091	SP(N), SP(T),
{4, 5, 6, 8, 9, 10}	T	0.95	$ 429,909	$ 303,115	CCP(T)
5 Suppliers	N	0.94	$ 443,256	$ 298,846	SP(N), SP(T)
{4, 5, 6, 7, 9}	T	0.97	$ 442,987	$ 303,567	
5 Suppliers	N	0.91	$ 422,974	$ 300,211	MIP, SP(N), SP(T),
{4, 5, 6, 8, 9}	T	0.93	$ 422,495	$ 303,386	CCP(N), CCP(T)
4 Suppliers	N	0.92	$ 434,893	$ 301,561	CCP(N), CCP(T)
{4, 5, 6, 7}	T	0.94	$ 430,189	$ 305,985	
4 Suppliers	N	0.89	$ 417,647	$ 301,174	SP(N)
{4, 5, 7, 9}	T	0.90	$ 415,661	$ 305,071	
4 Suppliers	N	0.88	$ 411,163	$ 300,757	SP(N)
{5, 6, 7, 9}	T	0.89	$ 412,013	$ 305,059	
4 Suppliers	N	0.88	$ 413,414	$ 299,914	CCP(N), CCP(T),
{4, 5, 6, 9}	T	0.89	$ 416,588	$ 303,911	SP(N), SP(T)
4 Suppliers	N	0.81	$ 385,091	$ 299,769	MIP, CCP(N)
{5, 6, 8, 9}	T	0.79	$ 386,216	$ 304,700	
4 Suppliers	N	0.80	$ 383,965	$ 300,927	CCP(T)
{4, 5, 9, 10}	T	0.77	$ 382,028	$ 305,413	
4 Suppliers	N	0.70	$ 353,609	$ 348,047	CCP(T)
{4, 8, 9, 10}	T	0.65	$ 357,127	$ 307,800	
3 Suppliers	N	0.84	$ 398,231	$ 303,980	SP(N)
{4, 6, 7}	T	0.84	$ 403,500	$ 308,723	
3 Suppliers	N	0.82	$ 389,616	$ 302,068	CCP(N), CCP(T),
{4, 5, 6}	T	0.81	$ 396,799	$ 306,731	SP(N)
3 Suppliers	N	0.78	$ 380,756	$ 306,402	CCP(T), SP(T)
{4, 7, 9}	T	0.74	$ 377,932	$ 309,610	
3 Suppliers	N	0.76	$ 374,742	$ 304,863	SP(T)
{6, 7, 9}	T	0.72	$ 376,787	$ 324,383	
3 Suppliers	N	0.75	$ 370,705	$ 303,681	CCP(N), CCP(T)
{4, 6, 9}	T	0.70	$ 367,338	$ 306,202	
3 Suppliers	N	0.74	$ 365,338	$ 301,477	SP(N), SP(T),
{4, 5, 9}	T	0.69	$ 364,706	$ 305,846	CCP(N)
3 Suppliers	N	0.72	$ 360,073	$ 301,346	MIP, SP(T),
{5, 6, 9}	T	0.67	$ 361,771	$ 305,859	CCP(N)
3 Suppliers	N	0.63	$ 339,304	$ 440,909	SP(N)
{7, 8, 9}	T	0.57	$ 339,302	$ 311,943	
3 Suppliers	N	0.61	$ 330,882	$ 484,461	CCP(T)
{4, 8, 9}	T	0.55	$ 330,608	$ 349,036	
3 Suppliers	N	0.58	$ 325,643	$ 372,343	CCP(N)
{5, 8, 9}	T	0.52	$ 324,307	$ 306,882	
3 Suppliers	N	0.51	$ 310,900	$ 416,154	CCP(T)
{5, 9, 10}	T	0.46	$ 309,939	$ 364,711	
2 Suppliers	N	0.70	$ 370,430	$ 325,787	CCP(N), SP(T)
{2, 5}	T	0.56	$ 371,898	$ 317,307	
2 Suppliers	N	0.70	$ 371,118	$ 314,119	SP(T)
{3, 5}	T	0.55	$ 370,383	$ 317,357	
2 Suppliers	N	0.69	$ 368,402	$ 314,958	SP(N)
{2, 6}	T	0.34	$ 374,768	$ 342,157	
2 Suppliers	N	0.67	$ 359,397	$ 316,325	SP(T)

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Table 4 (*continued*)

Number and set of selected suppliers	N/T	System reliability level $\varepsilon_d = \varepsilon_c$	Total costs CCP evaluation	Expected costs SP evaluation	Solution model
{2, 4}	T	0.77	$ 399,404	$ 320,027	
2 Suppliers	N	0.67	$ 360,989	$ 375,304	SP(T)
{3, 4}	T	0.76	$ 398,715	$ 320,376	
2 Suppliers	N	0.61	$ 333,959	$ 381,001	CCP(N), CCP(T)
{4, 6}	T	0.55	$ 333,681	$ 334,838	
2 Suppliers	N	0.57	$ 325,460	$ 348,901	MIP, SP(N),
{5, 6}	T	0.52	$ 328,264	$ 352,858	CCP(N), CCP(T)
2 Suppliers	N	0.43	$ 288,703	$ 882,115	CCP(N)
{4, 9}	T	0.40	$ 292,818	$ 629,728	
2 Suppliers	N	0.41	$ 281,657	$ 830,438	CCP(N), CCP(T)
{4, 8}	T	0.38	$ 281,121	$ 664,540	
2 Suppliers	N	0.38	$ 276,017	$ 667,325	CCP(N)
{5, 9}	T	0.36	$ 276,924	$ 509,151	
1 Supplier	N	0.34	$ 287,210	Infeasible	CCP(N), CCP(T)
{2}	T	0.33	$ 288,039	Infeasible	
1 Supplier	N	0.21	$ 233,673	$1,478,587	SP(N), SP(T)
{7}	T	0.23	$ 235,122	$1,336,047	
1 Supplier	N	0.20	$ 226,235	$1,498,125	CCP(N), CCP(T),
{4}	T	0.21	$ 222,961	$1,338,709	SP(N), SP(T)
1 Supplier	N	0.15	$ 209,380	$1,265,621	CCP(N), CCP(T)
{5}	T	0.17	$ 206,779	$1,141,054	

Dominated values are shown in gray.

the practitioner needs to make a tradeoff between costs, risks, number of suppliers and most likely use other criteria, such as his/her previous experience with certain suppliers, the financial stability of the suppliers, the technical support of the suppliers, etc., to favor one supplier set against another.

While selection decisions are mostly determined by the supplier limit, cost and system reliability, the amount of components to be ordered also plays a role in determining which suppliers to choose. Even though ordering from a supplier with high capacity can take advantage of business volume discount, the savings in the costs may be outweighed by their high transportation cost. In the sample problem, supplier 1 has the highest capacities and the highest transportation costs, and this is why it is not chosen by any of the models. This demonstrates the capability for the models to provide tradeoffs between business volume discounts and transportation costs, because the models include an interaction between the supplier selection decision and the ordering and shipping plan.

It is clear from this sample problem that the CCP and SP models provide more information about how supplier selection decisions are affected by the tradeoffs between different factors as compared to the MIP model. Out of 38 Pareto-optimal solutions, the CCP model found 26 of them, the SP model found 24 while MIP found 6. Both the CCP model and the SP model provide a similar number of non-dominated solutions. When the number of suppliers is set to seven or more, the difference between the three approaches (SP, CCP and MIP) is not evident because they all select the same set of suppliers. However, when the number of suppliers is set to four, for example, we can see a big difference between the approaches. The SP model provides {4, 5, 7, 9}, {5, 6, 7, 9}, and {4, 5, 6, 9} as the alternative sets of suppliers whereas the MIP model provides {5, 6, 8, 9} as the optimal solution and the CCP model provides {4, 5, 6, 7}, {4, 5, 6, 9}, {5, 6, 8, 9}, {4, 5, 9, 10} and {4, 8, 9, 10}. The SP model includes {5, 9}, a mid-sized and a small supplier in every solution whereas the CCP model uses different combinations of {4, 5, 9} in the solutions. This gives the decision maker a hint that these are good candidates to include in the selection decision. From this example, we can see that the SP and CCP models provide more robust solutions and more information than the MIP model to allow decision makers to quantify the tradeoffs between risks and costs.

In this sample problem, the SP model gives a similar number of solutions as the CCP model but with considerably more computational effort. The computational times to solve the CCP and MIP models were not significantly different for a single run, while the time to solve the SP model with 10 scenarios was substantially more. The models were run using GAMS/CPLEX on a PC Intel Pentium processor and typically took less than five minutes for the CCP and MIP models, but took several hours (ranging between 1 h and one day) for each SP run. Since we use the deterministic equivalent form of the SP model to solve the problem, the computational time comparison between MIP, CCP, and SP models may not be fair. We expect to see a reduction in the computing time when more efficient SP algorithms (see Sherali and Fraticelli, 2002; Sen and Higle, 2005; Sen and Sherali, 2006) are implemented. Nonetheless, another advantage brought by the CCP model with the ε–constraint approach is that it is clear how to explore the Pareto-frontier using the CCP model by varying the values of the number of suppliers ε_s, the probability of meeting demand ε_d and the probability of not exceeding supplier capacity ε_c whereas for the SP model, the relationship between Pareto-optimal solutions and the values of the number of suppliers ε_s and unit penalty cost of exceeding supplier capacity e_i is less obvious. Due to the computational considerations and ease of exploring solutions, we recommend the CCP model as an excellent way to incorporate uncertainties into the supplier selection decision. However, if scenarios are more meaningful than an assumed probability distribution, one may turn to the SP model in the case

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of correlated uncertainties and for more accurate approximation of the real costs.

A rough estimation of tradeoffs between costs and system reliability can be visualized in Figs. 1 and 2. However, it would be more informative to the decision maker if we were able to identify the relationship between the costs and the system reliability in a closed form and determine the valid interval for which a selection decision remains Pareto-optimal. In this sample problem, different efficient solutions are identified under different combinations of cost levels, risk levels and supplier limit levels. Instead of choosing probability levels at an increment of 0.1 in the CCP model or weights in the SP model (e_i chosen by trial-and-error), we need a more systematic approach to explore how the changes in the cost levels, risk levels and supplier limit levels affect the supplier selection decision and total costs, which leads us to the use of multi-parametric programming techniques.

4. Multi-parametric analysis

Sensitivity analysis, postoptimality analysis, and parametric programming have been used interchangeably in the literature to describe the characterization of the changes of the optimal solution and the objective function value with respect to the changes presented in the problem parameter values. This information is of great value in practice in several ways. First, it helps answer what-if questions. Second, it helps identify scarce resources and the bottlenecks of the system. Finally, it helps determine how reliable the obtained optimal solution is (Acevedo and Pistikopoulos, 1999; Jansen et al., 1997).

Using parametric programming approaches to analyze the uncertainty in the problem parameters has received a lot of attention due to the recognition of the lack of exact and reliable data in practice. Therefore, researchers intend to find robust solutions to the real applications from the developed models by the use of the parametric programming techniques. However, as declared in the stochastic and probabilistic programming literature, parametric programs do not deliver good solutions for the case when there *are* uncertainties in the problem, since it only provides *posterior* analysis of the solution using a *deterministic* tool (Wallace, 2000). In order to obtain a robust solution, it is crucial to plan for uncertainty proactively beforehand, that is, to incorporate the uncertainty into the modeling.

Under the framework of our multi-objective CCP model with probabilistic constraints with the implementation of the ε−constraint method, we avoid the disadvantages of the sensitivity analysis—posterior analysis of the solution using a deterministic tool, by inclusion of the probabilistic constraints to plan for uncertainty. Furthermore, we utilize the advantages of the parametric optimization (ability to handle the multiple objectives and characterize the effects of changing problem parameters on the objective function value) through the use of the ε constraints on the supplier limits, the demand requirements, and the capacity limits. Therefore, our multi-parametric program is able to provide the decision maker with a complete map of robust solutions. In Section 3, we used predetermined ε levels on the number of the suppliers, the probabilistic demand level, and the probabilistic capacity level and stepped through the ε levels with an increment of 0.1 to obtain the Pareto-optimal solutions. In the multi-parametric analysis, we have undetermined ε variables and attempt to investigate valid ranges of ε on which the selection decision remains efficient.

Dua and Pistikopoulos (2000) presented new techniques for multi-parametric programming with binary variables. We adapt these techniques to perform multi-parametric programming to solve our mixed binary, multi-objective chance-constrained programming (CCP) model. Since we are interested in getting business insights from the parametric analysis results, we will not discuss the implementation process of the multi-parametric programming techniques in this paper. Interested readers may refer to Li (2007) for a detailed discussion on how to implement the multi-parametric techniques to our supplier selection problem.

For demonstration purposes, we apply the multi-parametric programming techniques to the CCP model under the normal distribution, where the probability constraint sets (20) and (21) are replaced by (30) and (31). Now in our parametric analysis setting, Z_{ε_d} and $Z_{1-\varepsilon_c}$ become variables instead of parameters. Since the valid range for a normal distribution is $(-\infty, +\infty)$ which

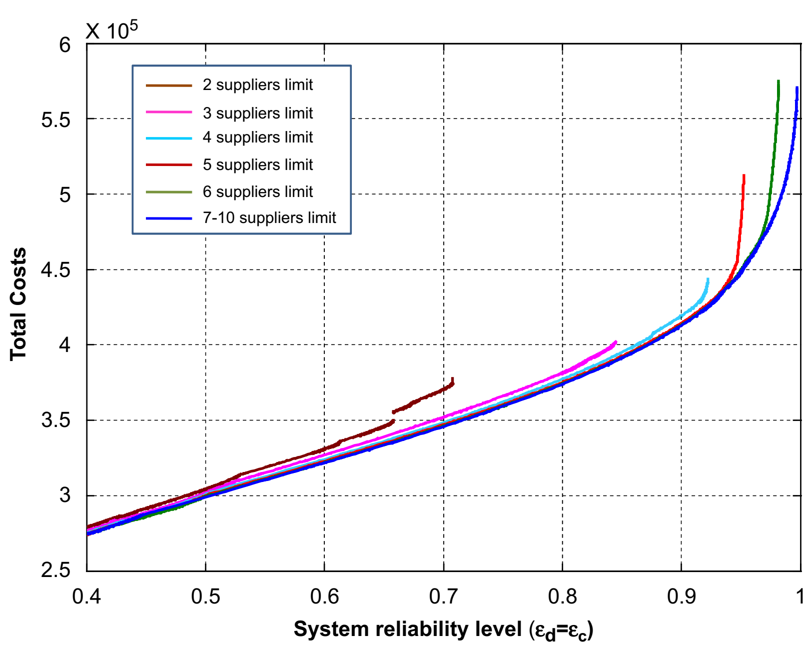

Fig. 3. Relationship between total costs, the number of suppliers, and system reliability under the normal distribution assumption.

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does not make sense in practice, we explore Z_{ε_d} and $Z_{1-\varepsilon_c}$ in the range of $[-0.25, 3]$, which represents varying ε_d and ε_c from 40% to 99.87%. The minimum 40% system reliability is chosen since it is unusual for the decision maker to accept less than 40% chance of having sufficient supplier capacity to meet demand. We choose an upper limit of 99.87% (Z value of 3) to explore the cost associated with an extremely reliable supply chain. The probability levels ε_d and ε_c are explicit in the constraint sets (30) and (31). However, the combined probability of meeting demand requires both constraints to be satisfied, that is, there is sufficient capacity for the amount ordered at the suppliers, and the demand can be satisfied by the amount ordered. To have a more detailed analysis and to provide insight into how changes in the system reliability level affect the changes in the selection decision and the total costs, we set $\varepsilon_d = \varepsilon_c = \varepsilon$ to represent the system reliability level.

The relationship between total costs and the system reliability level is shown in Fig. 3, where the different lines are associated with different numbers of suppliers. The horizontal axis represents the system reliability level ε. The vertical axis

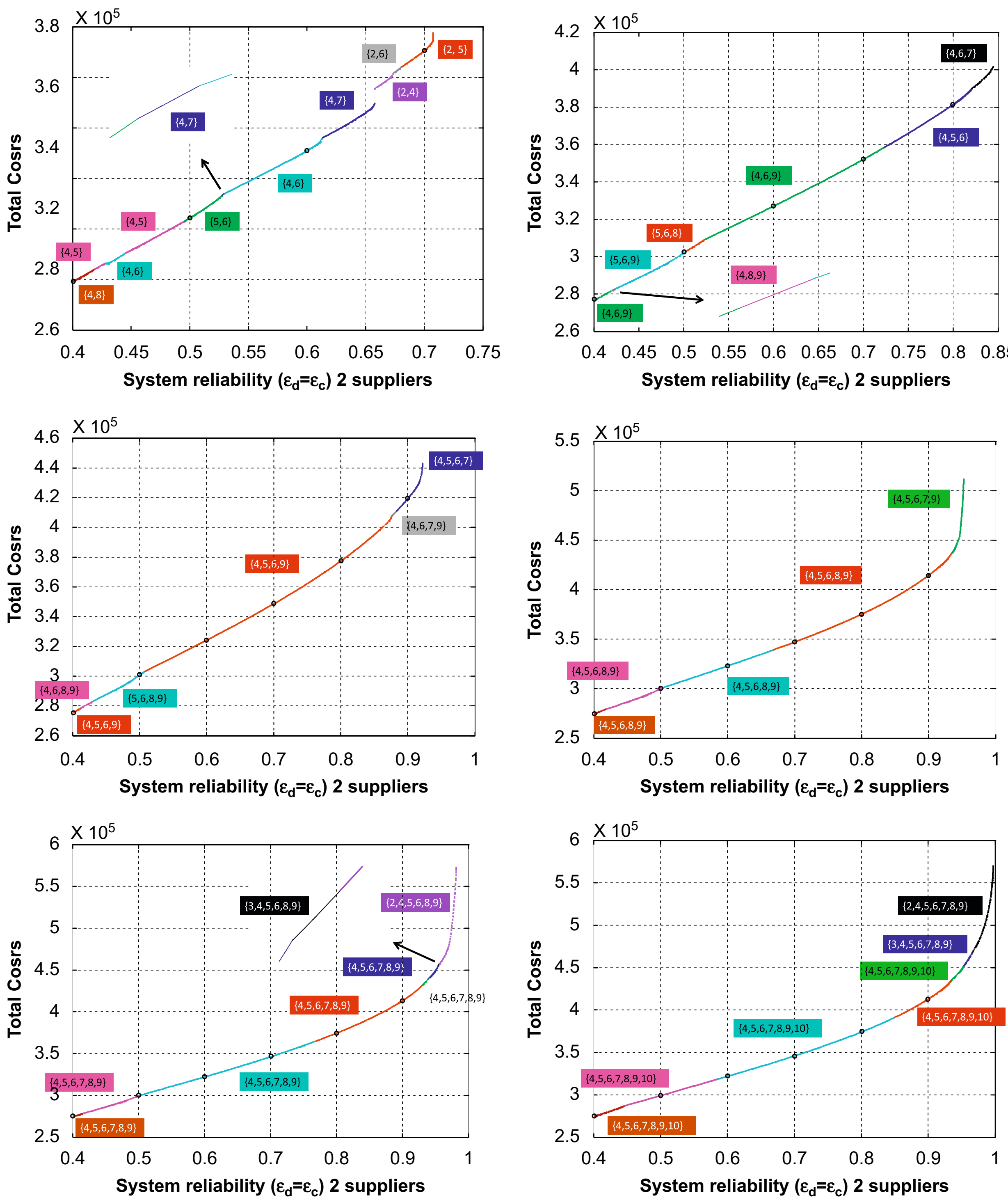

Fig. 4. Supplier selection decisions under normal distribution assumption.

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represents the total costs evaluated under CCP with the normal distribution. As we can see from the figure, with limited suppliers, there are limits on the highest system reliability we can achieve. The steep curve in this figure shows the decision maker where the marginal change in cost is extremely high as compared to the marginal change in the system reliability. The functions are linear in Z_ε, but appear curved in Fig. 3 because the x-axis is ε. With the same system reliability level, we can reduce total costs but with a drawback of managing more suppliers. This figure gives *exact* quantitative cost reduction values associated with different supplier limits. For example, it is more costly to go with fewer suppliers when we want 90% system reliability than when we want 80% system reliability. Similarly, with the same costs, the decision maker can achieve a higher system reliability level if he/she is willing to manage more suppliers. Following our previous argument on this, we can see if the decision maker is willing to spend about \$375K for the total costs, with two suppliers the system reliability level is about 70%, and with three suppliers is about 78%, and with four or more suppliers is about 80% which provides more accurate and complete information than we had in the previous section.

The detailed parametric solutions are shown separately in Fig. 4. From these graphs, we can see the benefit of using parametric analyses. It identifies new selection decisions, the break-even probability levels between different selection decisions, and the highest probability level where there exists no feasible decision. Note that there are cases when the same set of suppliers appears more than once on the same figure, which indicates those suppliers are robust and costs can be moderated by different ordering quantities. The dots on the lines represent the optimal solutions found by the CCP model when we enumerated the probability level at every 0.1 level. It is obvious that the parametric solutions provide more information than the solutions found earlier.

The parametric analyses also provide insight to the robustness of a set of suppliers within an acceptable range of system reliability. For example, comparing the supplier selection decisions for two suppliers with those for four suppliers, we see that, when there are only two suppliers, the supplier set is very sensitive to the level of system reliability (ε level). However, when we allow the number of suppliers to be four, the SP and CCP models in Section 3 identify six non-dominated solutions (under the normal assumption). With the parametric analysis for four suppliers in Fig. 4, it is clear that {4, 5, 6, 9} is very robust in the sense that the same set of suppliers can cover a wide range of costs and reliability by varying the ordering decisions. For three suppliers, the decision maker can choose between {4, 6, 9} and {4, 5, 6} to achieve low costs and high system reliability level. For five suppliers, {4, 5, 6, 8, 9} is a robust supplier set which provides low costs and high system reliability level. For six suppliers, {4, 5, 6, 7, 8, 9} is a robust supplier set and {4, 5, 6, 7, 8, 9, 10} is robust for more than seven suppliers. The tendency is that the larger the number of suppliers, the more robust the selection decisions are. When we want to achieve high system reliability, even with a large set of suppliers, the selection decisions are sensitive, as in the total costs. From those figures, we can determine whether there exists a robust set of suppliers within an acceptable range of system reliability.

GAMS/CPLEX is used to solve the problem and perform sensitivity analysis on ε. The computational time varies from hours to a day. It is possible to allow ε_d and ε_c to vary independently using the free multi-parametric Matlab toolbox (MPT) (see Kvasnica et al., 2004; Löfberg, 2004) for small to medium scale multi-parametric problems. However, the results are difficult to interpret.

5. Conclusion

Selecting a robust set of suppliers requires balancing cost and risk, somewhat akin to a portfolio investment problem. A diverse set of suppliers, with respect to large versus small, local versus far away, high costs versus low costs, can be selected to reduce risk while managing costs. A multi-objective stochastic supplier selection problem with business volume discounts is studied in this paper. The problem is formulated as a SP model and a CCP model. Demand and supplier capacity uncertainties are considered explicitly in these models. The best set of suppliers and order quantities are optimized in these two models. The $\varepsilon-$constrained method is used to generate Pareto-optimal solutions. These Pareto-optimal solutions give decision makers a clear picture about the tradeoffs between number of suppliers, cost, and system reliability. Moreover, the developed models provide more robust solutions as compared to a deterministic MIP model. If the uncertainty is represented by scenarios, the SP model is preferable to a deterministic MIP model. If distributions are available, the CCP model can provide the Pareto-frontier in a straightforward manner, and in less computational time than the SP model. However, the relationship between the number of selected suppliers, the risks and the total costs is discretized. With the multi-parametric programming techniques, we are able to describe the relationship more completely using explicit linear functions. A benefit of the chance-constrained model becomes manifest here: the ability of applying multi-parametric programming techniques to address multi-criteria and uncertainty.

Appendix

Table 4 provides the detailed numerical results for the sample problem as illustrated in Figs. 1 and 2. The first column provides the list of Pareto-optimal solutions. The second column indicates the distribution used, normal or triangular. The next three columns provide the system reliability level as evaluated by the CCP model, the total costs as evaluated by the CCP model, and the expected costs as evaluated by the SP model. The last column indicates which models detected the solution during the solution process. Since the results for both the normal and triangular distributions are summarized in one table, it may occur that one solution is dominated (not Pareto-optimal) under one distribution, but is Pareto-optimal under the other distribution. This occurred in two cases, supplier sets {2, 6} and {3, 4}; the values for the dominated solution are colored gray in Table 4. No feasible x values can be found for solution {2} when evaluated under the SP model due to the delivery constraints. Therefore, *infeasible* appears in that column.

To evaluate the system reliability, we execute the CCP model in (16)–(27) with a fixed y_i^* and solve for the other decision variables (x_{ijk}, $z_{jj'k}$, b_{im} and v_{im}). When we evaluate the solutions y_i^* under the normal and triangular distributions, we prioritize the probability of meeting demand over the total costs. That is, we repeatedly execute the CCP model with decreasing ε_d and ε_c values (dropping from 0.99 at every 0.01 level) until the fixed y_i^* yields feasible and optimal x_{ijk}, $z_{jj'k}$, b_{im} and v_{im}. We record the associated ε_d and ε_c values as the system reliability and the associated total costs. We evaluate the expected costs for each solution y_i^* under the SP model by solving the recourse problem for $x_{ijk}(\xi)$, $z_{jj'k}(\xi)$, $b_{im}(\xi)$ and $v_{im}(\xi)$ with a different set of 10 scenarios drawn from the normal and triangular distributions than used originally. The expected cost for the SP evaluation differs from the total costs under the CCP evaluation due to the recourse opportunity to modify the ordering plan under each scenario, the limited number of scenarios, and the penalty ($e_i = 60$) for exceeding supplier capacity while ensuring that demand is met.

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