

Earley algorithm

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Today's lecture

Earley algorithm

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Earley algorithm

Earley: introduction
Example of Earley
algorithm

- 1 Earley algorithm
 - Earley: introduction
 - Example of Earley algorithm

Top-down parsing

In naive search, top-down parsing is inefficient because structures are created over and over again.

- Need a way to record that a particular structure has been predicted
- Need a way to record *where* the structure was predicted wrt the input

Pros/cons of top-down strategy

- ✓ Never explores trees that aren't potential solutions, ones with the wrong kind of root node.

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- ✗ But explores trees that do not match the input sentence (predicts input before inspecting input).

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- ✗ But explores trees that do not match the input sentence (predicts input before inspecting input).
- ✗ Naive top-down parsers never terminate if G contains recursive rules like $X \rightarrow X Y$ (left recursive rules).

Pros/cons of top-down strategy

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- ✗ But explores trees that do not match the input sentence (predicts input before inspecting input).
- ✗ Naive top-down parsers never terminate if G contains recursive rules like $X \rightarrow X Y$ (left recursive rules).
- ✗ Backtracking may discard valid constituents that have to be re-discovered later (duplication of effort).

Pros/cons of top-down strategy

- ✓ Never explores trees that aren't potential solutions, ones with the wrong kind of root node.
- ✗ But explores trees that do not match the input sentence (predicts input before inspecting input).
- ✗ Naive top-down parsers never terminate if G contains recursive rules like $X \rightarrow X Y$ (left recursive rules).
- ✗ Backtracking may discard valid constituents that have to be re-discovered later (duplication of effort).

Use a top-down strategy when you know what kind of constituent you want to end up with (e.g. NP extraction, named entity extraction). Avoid this strategy if you're stuck with a highly recursive grammar.

Earley algorithm

The **Earley Parsing Algorithm**: an efficient top-down parsing algorithm that avoids some of the inefficiency associated with purely naive search with the same top-down strategy (cf. recursive descent parser).

- Intermediate solutions are created only once and stored in a chart (dynamic programming).
- Left-recursion problem is solved by examining the input.
- Earley is not picky about what type of grammar it accepts, i.e., it accepts arbitrary CFGs (cf. CKY).

Earley Parsing Algorithm (J&M, p. 444)

function EARLEY-PARSE(*words, grammar*) **returns** *chart*

```
ENQUEUE( $(\gamma \rightarrow \bullet S, [0,0])$ , chart[0])
for i  $\leftarrow$  from 0 to LENGTH(words) do
  for each state in chart[i] do
    if INCOMPLETE?(state) and NEXT-CAT(state) is not POS then
      PREDICTOR(state)
    elseif INCOMPLETE?(state) and NEXT-CAT(state) is POS then
      SCANNER(state)
    else
      COMPLETER(state)
  end
end
return(chart)
```

Setting up the Earley algorithm: the chart

The rationale is to fill in a chart with the solutions to the subproblems encountered in the top-down parsing process.

- Based on an input string of length n , build a 1D array (called a **chart**) of length $n + 1$ to record the solutions to subproblems
- Chart entries are lists of **states**, or info about partial solutions.
- States represent attempts to discover constituents.

Empty Earley chart

Chart[0]: ● astronomers saw stars with ears
partial solutions ...

Chart[1]: astronomers ● saw stars with ears
partial solutions ...

Chart[2]: astronomers saw ● stars with ears
partial solutions ...

Chart[3]: astronomers saw stars ● with ears
partial solutions ...

Chart[4]: astronomers saw stars with ● ears
partial solutions ...

Chart[5]: astronomers saw stars with ears ●

Assumed indexing scheme:

●₀ *astronomers* ●₁ *saw* ●₂ *stars* ●₃ *with* ●₄ *ears* ●₅

Setting up the Earley algorithm: the states

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A *state* consists of:

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Setting up the Earley algorithm: the states

A *state* consists of:

- 1 a subtree corresponding to a grammar rule
 $S \rightarrow NP VP$

Setting up the Earley algorithm: the states

A *state* consists of:

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 $S \rightarrow NP \bullet VP$

Setting up the Earley algorithm: the states

A *state* consists of:

- 1 a subtree corresponding to a grammar rule
 $S \rightarrow NP VP$
- 2 info about progress made towards completing this subtree
 $S \rightarrow NP \bullet VP$
- 3 the position of the subtree wrt input
 $S \rightarrow NP \bullet VP, [0, 3]$

Setting up the Earley algorithm: the states

A *state* consists of:

- 1 a subtree corresponding to a grammar rule
 $S \rightarrow NP VP$
- 2 info about progress made towards completing this subtree
 $S \rightarrow NP \bullet VP$
- 3 the position of the subtree wrt input
 $S \rightarrow NP \bullet VP, [0, 3]$
- 4 pointers to all contributing states in the case of a parser
(cf. recognizer)

Setting up the Earley algorithm: dotted rules

Definition

A **dotted rule** is a data structure used in top-down parsing to record partial solutions towards discovering a constituent.

- $S \rightarrow \bullet VP, [0, 0]$
Predict an S will be found which consists of a VP ; the S will begin at 0.
- $NP \rightarrow Det \bullet Nominal, [1, 2]$
Predict an NP starting at 1; an Det has been found; $Nominal$ is expected next.
- $VP \rightarrow V NP \bullet, [0, 3]$
A VP has been found starting at 0 and spanning to 3; the constituents of VP are V and NP .

Dotted rules and corresponding graph

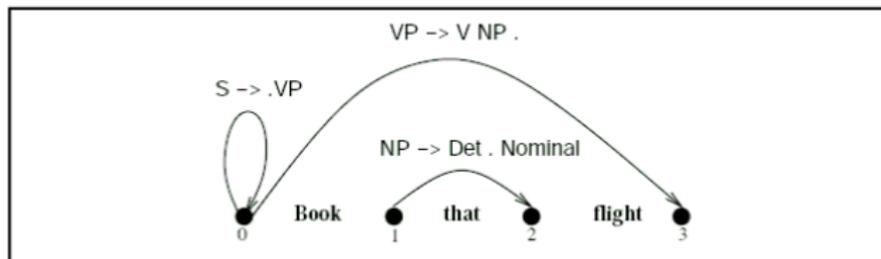
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Earley: fundamental operations

- **Predict** sub-structure (based on grammar)
- **Scan** partial solutions for a match
- **Complete** a sub-structure (i.e., build constituents)

Sample grammar from J&M

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$S \rightarrow NP VP$	$NP \rightarrow NP PP$
$PP \rightarrow P NP$	$NP \rightarrow N$
$VP \rightarrow V NP$	$N \rightarrow \textit{astronomers}$
$VP \rightarrow VP PP$	$N \rightarrow \textit{ears}$
$P \rightarrow \textit{with}$	$N \rightarrow \textit{stars}$
$V \rightarrow \textit{saw}$	$N \rightarrow \textit{telescopes}$

ambiguous, PP attachment

astronomers saw stars with ears

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An extra rule

How to represent progress towards finding an S node?

Add a dummy rule to grammar:

$$\gamma \rightarrow S$$

This seeds the chart as the base case for recursion.

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How to represent progress towards finding an S node?

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$$\gamma \rightarrow \bullet S$$

This seeds the chart as the base case for recursion.

Chart[0]: “● astronomers saw stars with ears”

state ID

dotted rule

position

back pointer

operation

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Chart[0]: “● astronomers saw stars with ears”

state ID dotted rule position back pointer operation

Enqueue dummy start state

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed

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Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed

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For each state in chart[i]... processing S0

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed

if state incomplete and NEXTCAT is not POS, then PREDICTOR

```
procedure PREDICTOR( $(A \rightarrow \alpha \bullet B \beta, [i,j])$ )  
  for each  $(B \rightarrow \gamma)$  in GRAMMAR-RULES-FOR( $B, grammar$ ) do  
    ENQUEUE( $(B \rightarrow \bullet \gamma, [j,j]), chart[j]$ )  
end
```

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed

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  for each  $(B \rightarrow \gamma)$  in GRAMMAR-RULES-FOR( $B, grammar$ ) do  
    ENQUEUE( $(B \rightarrow \bullet \gamma, [j,j])$ ,  $chart[j]$ )  
end
```

$S \rightarrow NP VP$	$NP \rightarrow NP PP$
$PP \rightarrow P NP$	$NP \rightarrow N$
$VP \rightarrow V NP$	$N \rightarrow astronomers$
$VP \rightarrow VP PP$	$N \rightarrow ears$
$P \rightarrow with$	$N \rightarrow stars$
$V \rightarrow saw$	$N \rightarrow telescopes$

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor

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Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
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processing S1

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor

if state incomplete and NEXTCAT is not POS, then PREDICTOR

```

procedure PREDICTOR( $(A \rightarrow \alpha \bullet B \beta, [i,j])$ )
  for each  $(B \rightarrow \gamma)$  in GRAMMAR-RULES-FOR( $B, grammar$ ) do
    ENQUEUE( $(B \rightarrow \bullet \gamma, [j,j], chart[j])$ )
  end

```

$S \rightarrow NP VP$	$NP \rightarrow NP PP$
$PP \rightarrow P NP$	$NP \rightarrow N$
$VP \rightarrow V NP$	$N \rightarrow astronomers$
$VP \rightarrow VP PP$	$N \rightarrow ears$
$P \rightarrow with$	$N \rightarrow stars$
$V \rightarrow saw$	$N \rightarrow telescopes$

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor

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Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor
S3	$NP \rightarrow \bullet N$	[0,0]	[]	Predictor

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Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor
S3	$NP \rightarrow \bullet N$	[0,0]	[]	Predictor

processing S2... but NP at position [0,0] has already been considered

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor
S3	$NP \rightarrow \bullet N$	[0,0]	[]	Predictor

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor
S3	$NP \rightarrow \bullet N$	[0,0]	[]	Predictor

processing S3 ...

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor
S3	$NP \rightarrow \bullet N$	[0,0]	[]	Predictor

if state incomplete and NextCat is a POS, then Scanner

```
procedure SCANNER( $(A \rightarrow \alpha \bullet B \beta, [i,j])$ )  
  if  $B \in \text{POS}(\text{word}[j])$  then  
    ENQUEUE( $(B \rightarrow \text{word}[j], [j,j+1]), \text{chart}[j+1]$ )
```

Chart[0]: “● astronomers saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S0	$TOP \rightarrow \bullet S$	[0,0]	[]	Seed
S1	$S \rightarrow \bullet NP VP$	[0,0]	[]	Predictor
S2	$NP \rightarrow \bullet NP PP$	[0,0]	[]	Predictor
S3	$NP \rightarrow \bullet N$	[0,0]	[]	Predictor

Scanner: Add $N \rightarrow \text{astronomers} \bullet$ [0,1] at Chart[0+1]

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner

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Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner

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Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner

processing S4...**else** COMPLETE

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner

```
procedure COMPLETER( $(B \rightarrow \gamma \bullet, [j,k])$ )  
  for each  $(A \rightarrow \alpha \bullet B \beta, [i,j])$  in chart[j] do  
    ENQUEUE( $(A \rightarrow \alpha B \bullet \beta, [i,k])$ , chart[k])  
end
```

Which states from Chart[0] require the current const. to be complete?

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer

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Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer

processing S5...**else** COMPLETER

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \text{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer

```

procedure COMPLETER( $(B \rightarrow \gamma \bullet, [j,k])$ )
  for each  $(A \rightarrow \alpha \bullet B \beta, [i,j])$  in chart[j] do
    ENQUEUE( $(A \rightarrow \alpha B \bullet \beta, [i,k])$ , chart[k])
  end

```

Which states from Chart[0] require the current const. to be complete?

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer

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Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer

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Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer

processing S6...PREDICTOR

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor

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Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor

processing S7...PREDICTOR

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor
S10	$PP \rightarrow \bullet P NP$	[1,1]	[]	Predictor

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor
S10	$PP \rightarrow \bullet P NP$	[1,1]	[]	Predictor

Processing S8...SCANNER: Add $V \rightarrow \textit{saw} \bullet$ [1,2] at Chart[1+1]

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor
S10	$PP \rightarrow \bullet P NP$	[1,1]	[]	Predictor

Processing S9...PREDICTOR, but VP already expanded in Chart[1]

Chart[1]: “astronomers • saw stars with ears”

state ID	dotted rule	position	back pointer	operation
S4	$N \rightarrow \textit{astronomers} \bullet$	[0,1]	[]	Scanner
S5	$NP \rightarrow N \bullet$	[0,1]	[S4]	Completer
S6	$S \rightarrow NP \bullet VP$	[0,1]	[S5]	Completer
S7	$NP \rightarrow NP \bullet PP$	[0,1]	[S5]	Completer
S8	$VP \rightarrow \bullet V NP$	[1,1]	[]	Predictor
S9	$VP \rightarrow \bullet VP PP$	[1,1]	[]	Predictor
S10	$PP \rightarrow \bullet P NP$	[1,1]	[]	Predictor

Chart[2]: “astronomers saw • stars with ears”

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state ID	dotted rule	position	back pointer	operation
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Chart[2]: “astronomers saw • stars with ears”

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Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S11	$V \rightarrow saw\bullet$	[1,2]	[]	Scanner

Chart[2]: “astronomers saw • stars with ears”

Earley algorithm

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S11	$V \rightarrow saw \bullet$	[1,2]	[]	Scanner
S12	$VP \rightarrow V \bullet NP$	[1,2]	[S11]	Completer

Chart[2]: “astronomers saw • stars with ears”

Earley algorithm

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S11	$V \rightarrow \text{saw} \bullet$	[1,2]	[]	Scanner
S12	$VP \rightarrow V \bullet NP$	[1,2]	[S11]	Completer
S13	$NP \rightarrow \bullet NP PP$	[2,2]	[]	Predictor

Chart[2]: “astronomers saw • stars with ears”

Earley algorithm

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Earley algorithm

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Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S11	$V \rightarrow saw \bullet$	[1,2]	[]	Scanner
S12	$VP \rightarrow V \bullet NP$	[1,2]	[S11]	Completer
S13	$NP \rightarrow \bullet NP PP$	[2,2]	[]	Predictor
S14	$NP \rightarrow \bullet N$	[2,2]	[]	Predictor

Chart[3]: “astronomers saw stars • with ears”

Earley algorithm

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Earley algorithm

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Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner

Chart[3]: “astronomers saw stars • with ears”

Earley algorithm

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner
S16	$NP \rightarrow N\bullet$	[2,3]	[S15]	Completer

Chart[3]: “astronomers saw stars • with ears”

Earley algorithm

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner
S16	$NP \rightarrow N\bullet$	[2,3]	[S15]	Completer
S17	$VP \rightarrow V NP\bullet$	[1,3]	[S11,S16]	Completer

Chart[3]: “astronomers saw stars • with ears”

Earley algorithm

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner
S16	$NP \rightarrow N\bullet$	[2,3]	[S15]	Completer
S17	$VP \rightarrow V NP\bullet$	[1,3]	[S11,S16]	Completer
S18	$NP \rightarrow NP\bullet PP$	[2,3]	[S16]	Completer

Chart[3]: “astronomers saw stars • with ears”

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Earley algorithm

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Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner
S16	$NP \rightarrow N\bullet$	[2,3]	[S15]	Completer
S17	$VP \rightarrow V NP\bullet$	[1,3]	[S11,S16]	Completer
S18	$NP \rightarrow NP\bullet PP$	[2,3]	[S16]	Completer
S19	$S \rightarrow NP VP\bullet$	[0,3]	[S5,S17]	Completer

Chart[3]: “astronomers saw stars • with ears”

Earley algorithm

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Earley algorithm

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Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner
S16	$NP \rightarrow N\bullet$	[2,3]	[S15]	Completer
S17	$VP \rightarrow V NP\bullet$	[1,3]	[S11,S16]	Completer
S18	$NP \rightarrow NP\bullet PP$	[2,3]	[S16]	Completer
S19	$S \rightarrow NP VP\bullet$	[0,3]	[S5,S17]	Completer
S20	$VP \rightarrow VP\bullet PP$	[1,3]	[S17]	Completer

Chart[3]: “astronomers saw stars • with ears”

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S15	$N \rightarrow stars\bullet$	[2,3]	[]	Scanner
S16	$NP \rightarrow N\bullet$	[2,3]	[S15]	Completer
S17	$VP \rightarrow V NP\bullet$	[1,3]	[S11,S16]	Completer
S18	$NP \rightarrow NP\bullet PP$	[2,3]	[S16]	Completer
S19	$S \rightarrow NP VP\bullet$	[0,3]	[S5,S17]	Completer
S20	$VP \rightarrow VP\bullet PP$	[1,3]	[S17]	Completer
S21	$PP \rightarrow \bullet P NP$	[3,3]	[]	Predictor

Chart[4]: “astronomers saw stars with • ears”

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S22	$P \rightarrow with\bullet$	[3,4]	[]	Scanner

Chart[4]: “astronomers saw stars with • ears”

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S22	$P \rightarrow with\bullet$	[3,4]	[]	Scanner
S23	$PP \rightarrow P\bullet NP$	[3,4]	[S19]	Completer

Chart[4]: “astronomers saw stars with • ears”

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S22	$P \rightarrow with\bullet$	[3,4]	[]	Scanner
S23	$PP \rightarrow P\bullet NP$	[3,4]	[S19]	Completer
S24	$NP \rightarrow \bullet NP PP$	[4,4]	[]	Predictor

Chart[4]: “astronomers saw stars with • ears”

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Earley algorithm

Earley: introduction

Example of Earley
algorithm

state ID	dotted rule	position	back pointer	operation
S22	$P \rightarrow with\bullet$	[3,4]	[]	Scanner
S23	$PP \rightarrow P\bullet NP$	[3,4]	[S19]	Completer
S24	$NP \rightarrow \bullet NP PP$	[4,4]	[]	Predictor
S25	$NP \rightarrow \bullet N$	[4,4]	[]	Predictor

Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
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Earley algorithm

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Example of Earley
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Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner

Earley algorithm

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Example of Earley
algorithm

Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer

Earley algorithm

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Example of Earley
algorithm

Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow PNP\bullet$	[3,5]	[S19,S27]	Completer

Earley algorithm

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Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer

Earley algorithm

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Chart[5]: “astronomers saw stars with ears ●”

state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer

Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer

Earley algorithm

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Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer
S32	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer

Earley algorithm

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Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer
S32	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S33	$VP \rightarrow V NP\bullet$	[2,5]	[S11,S29]	Completer

Earley algorithm

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Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer
S32	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S33	$VP \rightarrow V NP\bullet$	[2,5]	[S11,S29]	Completer
S34	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer

Earley algorithm

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Example of Earley
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Chart[5]: “astronomers saw stars with ears ●”

state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer
S32	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S33	$VP \rightarrow V NP\bullet$	[2,5]	[S11,S29]	Completer
S34	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S35	$S \rightarrow NP VP\bullet$	[0,5]	[S5,S30]	Completer

Chart[5]: “astronomers saw stars with ears ●”

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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer
S32	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S33	$VP \rightarrow V NP\bullet$	[2,5]	[S11,S29]	Completer
S34	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S35	$S \rightarrow NP VP\bullet$	[0,5]	[S5,S30]	Completer
S36	$S \rightarrow NP VP\bullet$	[0,5]	[S5,S31]	Completer

Earley algorithm

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Chart[5]: “astronomers saw stars with ears ●”

Earley algorithm

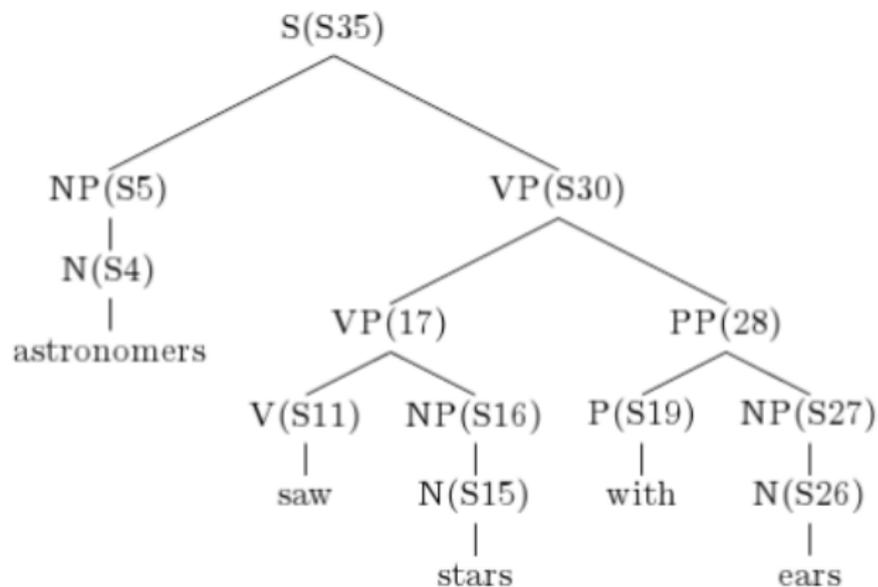
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state ID	dotted rule	position	back pointer	operation
S26	$N \rightarrow ears\bullet$	[4,5]	[]	Scanner
S27	$NP \rightarrow N\bullet$	[4,5]	[S26]	Completer
S28	$PP \rightarrow P NP\bullet$	[3,5]	[S19,S27]	Completer
S29	$NP \rightarrow NP PP\bullet$	[2,5]	[S16,S28]	Completer
S30	$VP \rightarrow VP PP\bullet$	[1,5]	[S17,S28]	Completer
S31	$VP \rightarrow V NP\bullet$	[1,5]	[S11,S29]	Completer
S32	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S33	$VP \rightarrow V NP\bullet$	[2,5]	[S11,S29]	Completer
S34	$NP \rightarrow NP \bullet PP$	[2,5]	[S29]	Completer
S35	$S \rightarrow NP VP\bullet$	[0,5]	[S5,S30]	Completer
S36	$S \rightarrow NP VP\bullet$	[0,5]	[S5,S31]	Completer
S37	$PP \rightarrow \bullet P NP$	[5,5]	[]	Predictor

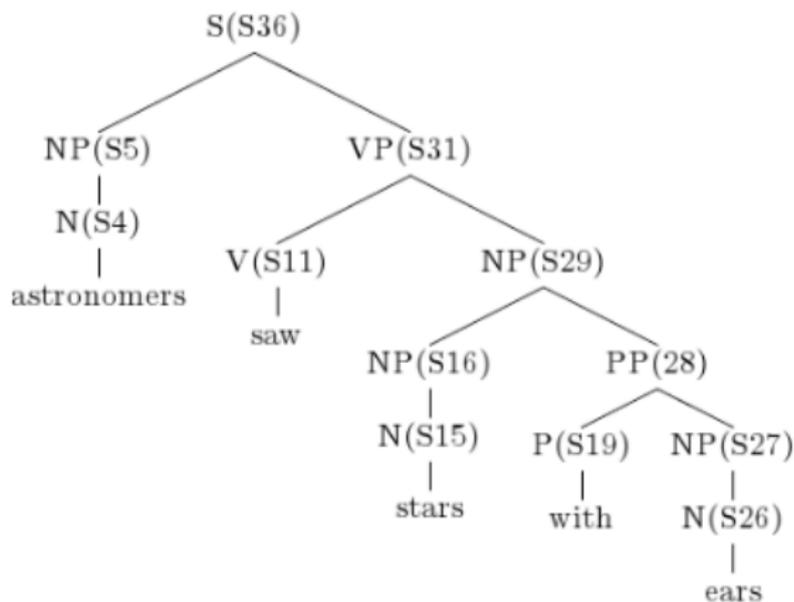
Earley algorithm

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Parse 1



Parse 2



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Earley algorithm

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