ME 354, MECHANICS OF MATERIALS LABORATORY

STRAINS, DEFLECTIONS AND BEAM BENDING LABORATORY*

04 september 2000 / mgj

PURPOSE

The purposes of this exercise are a) to familiarize the user with strain gages and associated instrumentation, b) to measure deflections and strains and to compare these results to predicted values, and c) to verify certain aspects of stress-strain relations and simple beam theory.

EQUIPMENT

- Simply-supported 6061-T6 aluminum channel beam instrumented with uniaxial and rosette strain gages.
- Strain gage conditioning equipment and readout unit for an analog strain conditioning system
- A deflectometer (dial indicator) and a load cell ring.

PROCEDURE

- Read the reference document "NOTES on Strain Gages."
- Carefully examine attached Figs. 1 to 3. Note that a total of 10 Wheatstone bridge circuits are involved. Identify all strain gage circuits and strain gage channel numbers on the Figs 1 to 3 as well as on the aluminum beam itself.
- Record the location along the beam length of the turnbuckle loading device.
- Loosen the turnbuckle and prepare the strain gage conditioning equipment according to the manufacturer's instructions. (Note: Use the F_g appropriate for the strain gages and circuit).
- If not already done so, balance each strain gage circuit to zero or a reasonable minimum value. Record this offset strain (starting value with no force applied) on the data sheet for each channel.
- Record the starting reading of the dial indicator on the data sheet and note its position along the beam.
- Apply a modest concentrated force to the beam by tightening the turnbuckle to achieve a reading of ~400 µm/m for the load cell ring (channel 10).
- Record the reading for each channel on the data sheet.
- Record the reading of the dial indicator
- Repeat the force application and data recording for a total of 3 forces. (Note: use absolute (after correcting for any initial offset) values for the load cell ring of ~400, ~800, and ~1200 µm/m).
- Loosen the turnbuckle after completing all force cases.

BACKGROUND AND ANALYSIS OF RESULTS

Experimental mechanics is that branch of engineering mechanics involving the measurement of strains, displacements, stresses and forces acting on or within models, components and/or structures. One of the most useful and widespread measurement devices used in experimental mechanics is the resistance strain gage.

Properly configured multiple strain gages can be arranged in such a way that the complete strain state acting at a point can be determined. Then, by using the correct constitutive relations (e.g., Hooke's law), the complete stress state at that point can be calculated. The principal stresses and strains along with principal directions can be determined from the resulting information. Thus, without even knowing the magnitude or direction of applied loads (e.g., forces, temperature, pressure, etc), the complete stress and strain states along with their maximum values and their orientations can be determined.

For simple structures such as simple beams in bending, mechanics of materials solutions which describe the strain and stress state anywhere in the beam exist. Therefore, it is often not necessary to use experimental mechanics to determine the strain and stress state because the values are readily calculated from the applied force, the loading configuration and the beam dimensions.

However, it is still useful to apply strain gages to a simple beam in bending to verify analytical models or to confirm the response of a particular measurement system. In this laboratory exercise, the force (reaction force) at the load cell support is first calculated as:

$$P_{RLC}\ (\mathrm{N}) = C \times \varepsilon_{10}\ (\mu\mathrm{m/m}). \qquad (1)$$

where C is the calibration constant for the load cell ring in units of N / µm/m. Note that the readout for channel number 10 (ε_{10}) is in micro strain (µ m/m=10^{-6} m/m) for a full strain gage bridge (i.e., four strain gages) adhered to the load cell ring. Note also that P_{RLC} is the reaction force at the load cell ring and not the applied force, P, from tightening the turnbuckle.

After calculating the reaction force, the applied force can be calculated using simple statics relations. Shear (V) and moment (M) diagrams for the test setup and test specimen can then be determined. Using the beam dimensions, the moment of inertia and position of the neutral axis for the aluminum channel beam can be determined.

According to beam theory, a bending moment, M, causes a uniaxial normal stress, σ_X, given by Eq. 2. Since the stress is uniaxial,(although not uniform across the height), Eq. 3 can be used to predict the uniaxial normal strain, ε_X, using the stress calculated from the applied force, test setup dimensions, test specimen dimensions (y for distance from the neutral axis, and I for the moment of inertia) and the elastic modulus, E, of the material.

$$\sigma_x = \frac{My}{I} \qquad (2)$$

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad (3)$$

Strains predicted using Eqs 2 and 3 can then be compared to those measured with the strain gages at cross section A-A (See Figs. 1 and 3). Principal strains induced at the sites of the rectangular and delta strain gage rosettes can also be determined from the measurements and compared to strains calculated using Eqs. 2 and 3 at each force level. The orientation of the principal strain coordinate system with respect to the longitudinal axis of the beam, X (see Figs. 1 and 2) can also be determined.

It is well known that the strain and stress distributions across the height of a beam in bending is non uniform. For example, the stress distribution may be maximum at the bottom surface of the beam and a minimum at the top surface. The location in the beam where the stress is zero is called the neutral axis (N/A) coincides with the centroid of the beam's cross section in straight beams. The location of the centroid can be calculated from the dimensions of the beam's cross section. The measured strains (e.g., strains from strain gages 7, 8 and 9 as shown in Fig. 3) across this cross section can be plotted as a function of distance from a fixed reference such as the bottom of the beam. It is then possible to compare the distance at which the measured strain is zero to the theoretical calculation of the distance for the centroid (neutral axis).

According to beam theory, the vertical deflection of the beam at any longitudinal location can be related to the applied force, the moment of inertia, and the beam's dimensions. The relation for this deflection also depends on the type of reaction supports. Knowledge of mechanics of materials can be used to determine the deflection relation for this setup. The predicted deflection and measured deflection can be compared for each force.

Analytical and numerical models have been demonstrated and compared to experimental results. Although for simple loading geometries there may be little advantage for measuring the beam response, there may be advantages and disadvantages of using each method for predicting (or measuring) bending response.

* REFERENCES

ME354 NOTES on Strain Gages

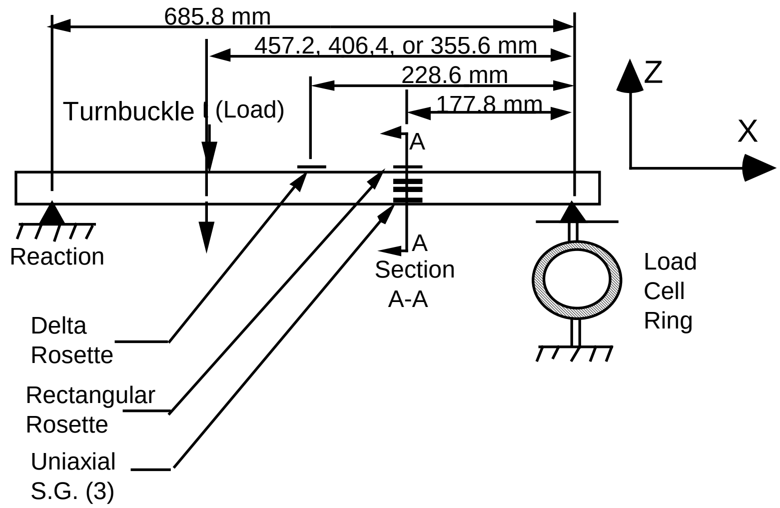


FIGURE 1 - Overall view of Test Specimen Geometry and Setup

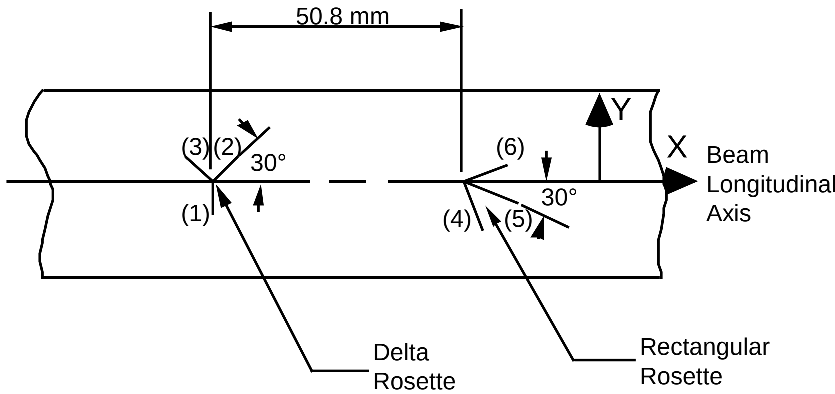
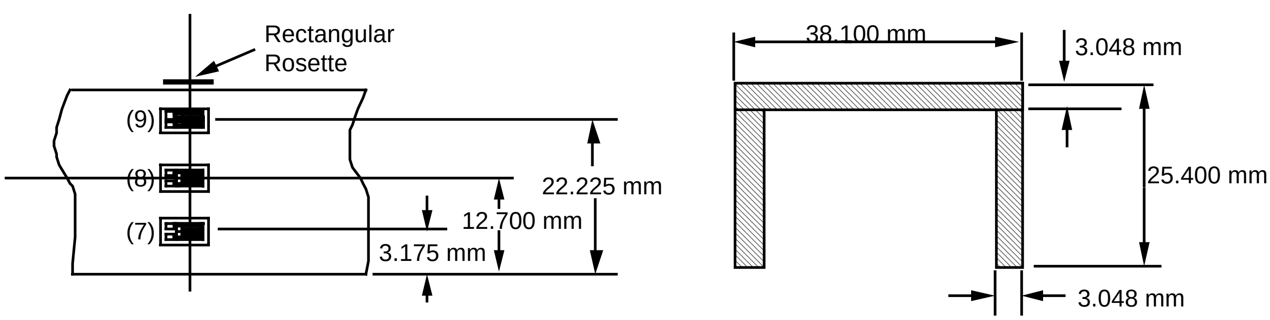


FIGURE 2 - Top View of Specimen Geometry Showing Orientations of 3-Element Strain Gage Rosettes. Note: Strain Gage Channel Numbers are Shown in Parentheses.



a) Side View of Uniaxial Strain Gage Locations.

b) Cross Section of Beam

FIGURE 3 - Strain Gage Locations and Cross Sectional Dimensions of the Beam. Note: Strain Gage Channel Numbers are Shown in Parentheses.

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DATA SHEET

NAME____**DATE**____________

LABORATORY PARTNER
NAMES____

____

EQUIPMENT IDENTIFICATION____

____

____

	STRAIN CONDITIONER CHANNEL NUMBER*										
Test No.	SG 1 (μm/m)	SG 2 (μm/m)	SG 3 (μm/m)	SG 4 (μm/m)	SG 5 (μm/m)	SG 6 (μm/m)	SG 7 (μm/m)	SG 8 (μm/m)	SG 9 (μm/m)	SG 10 Proving Ring (μm/m)	Deflection (mm)	Reaction Force (N)
Initial Offset												
1												
2												
3												

* The load cell is connected to channel 10. Strain gages are connected to the remaining nine channels as shown in Figs. 2 and 3.

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WORK SHEET

NAME____**DATE**______________

1) Some properties of 6061-T6 aluminum alloy are contained in Table 1.

Table 1 Some Properties of 6061-T6 Aluminum Alloy

Elastic Modulus, E (MPa)	69000
Yield Strength (0.2% offset), S_{YP} (MPa)	275
Poisson's ratio, ν	0.33
Ultimate Tensile Strength, S_{UTS} (MPa)	324
Percent Elongation, %el (50.8 mm gage length)	12

Note: Ranges represent ± 3 standard deviations about a mean

2) Analytical approach to the stress analysis of a beam in bending requires determination of the applied force (i.e. force at the turnbuckle) using the reaction force measured at one support.

For Force #1	For Force #2	For Force #3
Uncorrected strain from load cell ring, SG_{10}=_____ µm/m	Uncorrected strain from load cell ring, SG_{10}=_____ µm/m	Uncorrected strain from load cell ring, SG_{10}=_____ µm/m
Corrected strain from load cell ring, (SG_{10}-Offset) =ε_{10} =_____µm/m	Corrected strain from load cell ring, (SG_{10}-Offset) =ε_{10} =_____µm/m	Corrected strain from load cell ring, (SG_{10}-Offset) =ε_{10} =_____µm/m
Calibration Constant, C=_______ N/(µm/m)	Calibration Constant, C=_______ N/(µm/m)	Calibration Constant, C=_______ N/(µm/m)
Reaction force at load cell ring = P_{RLC} = C x ε_{10} = ________ N.	Reaction force at load cell ring = P_{RLC} = C x ε_{10} = ________ N.	Reaction force at load cell ring = P_{RLC} = C x ε_{10} = ________ N.

3) Applied force at turn buckle determined using statics.

For Force #1	For Force #2	For Force #3
Total length of beam = L=___________mm	Total length of beam = L=___________mm	Total length of beam = L=___________mm
Distance from Load Cell Ring to Turnbuckle = L_b=________mm	Distance from Load Cell Ring to Turnbuckle = L_b=________mm	Distance from Load Cell Ring to Turnbuckle = L_b=________mm
Distance from Opposite Support to Turnbuckle = L_a=L- L_b =________mm	Distance from Opposite Support to Turnbuckle = L_a=L- L_b =________mm	Distance from Opposite Support to Turnbuckle = L_a=L- L_b =________mm
Applied Force at turnbuckle = P =. $\frac{P_{RLC}L}{L_a}$ =_________N	Applied Force at turnbuckle = P =. $\frac{P_{RLC}L}{L_a}$ =_________N	Applied Force at turnbuckle = P =. $\frac{P_{RLC}L}{L_a}$ =_________N

4) A free body diagram (FBD) along with Shear and Moment diagrams are used to determined the shear and moments at the strain gage locations. Draw the generalized free body, shear and moment diagrams for the beam. Calculate the shear and moment values at each of the strain gage locations for each of the applied forces using the table.

FBD

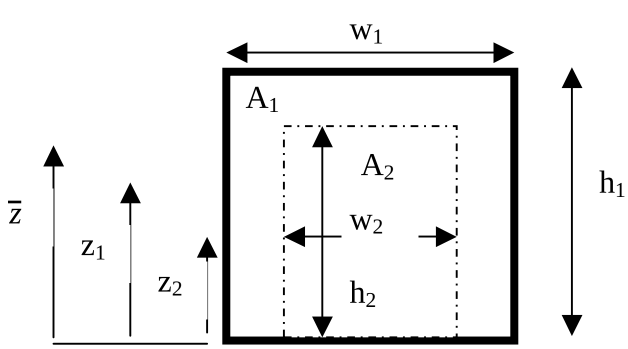
R_a R_b

Shear V

Moment M

For Force #1	For Force #2	For Force #3
Shear force at Delta Rosette $=V_D=P_{RLC}=$________N	Shear force at Delta Rosette $=V_D=P_{RLC}=$________N	Shear force at Delta Rosette $=V_D=P_{RLC}=$________N
Shear force at Rectangular Rosette $=V_R=P_{RLC}=$_______N	Shear force at Rectangular Rosette $=V_R=P_{RLC}=$_______N	Shear force at Rectangular Rosette $=V_R=P_{RLC}=$_______N
Distance from the Load Cell Ring to the Delta Rosette$=X_D=$__________mm	Distance from the Load Cell Ring to the Delta Rosette$=X_D=$__________mm	Distance from the Load Cell Ring to the Delta Rosette$=X_D=$__________mm
Distance from the Load Cell Ring to Rectangular Rosette$=X_R=$__________mm	Distance from the Load Cell Ring to Rectangular Rosette$=X_R=$__________mm	Distance from the Load Cell Ring to Rectangular Rosette$=X_R=$__________mm
Moment at Delta Rosette $=M_D=\frac{PL_aX_D}{L}=$________N-mm	Moment at Delta Rosette $=M_D=\frac{PL_aX_D}{L}=$________N-mm	Moment at Delta Rosette $=M_D=\frac{PL_aX_D}{L}=$________N-mm
Moment at Rectangular Rosette $=M_R=\frac{PL_aX_R}{L}=$________N-mm	Moment at Rectangular Rosette $=M_R=\frac{PL_aX_R}{L}=$________N-mm	Moment at Rectangular Rosette $=M_R=\frac{PL_aX_R}{L}=$________N-mm

5) The dimensions of the beam can be used to determine the moment of inertia and centroid of the beams cross section. The cross section of the beam can be divided into two areas, A_1 (total area) and A_2 (removed, inner area). The centroid can be determined relative to the bottom (i.e. open side) of the beam.



Area / Section	Height (mm)	Width (mm)	Area (mm^2)	Distance from beam bottom (mm)	Area times distance from beam bottom (mm^3)
i=1	h_1=	w_1=	A1=h_1*w_1=	z_1=h_1/2=	A_1*z_1=
i=2	h_2=	w_2=	A_2=h_2*w_2=	z_2=h_2/2=	A_2*z_2=

The vertical distance of the centroid from the bottom of the beam is calculated as $\bar{z} = \frac{\sum A_i z_i}{\sum A_i} = \frac{A_1 z_1 - A_2 z_2}{A_1 - A_2}$ =_____________mm. (Note that a negative sign is used with A_2 to take into account the "removed" area of A_2). Because of symmetry in the width direction the horizontal distance of the centroid from the front of the beam is the half the distance of the outer width of the beam=$\bar{y}$ =w_1/2=_________mm.

The moment of inertia can be calculated from the parallel axis theorem $I=\Sigma(I+A\ dz^2)$

Area / Section	Centroid of Area (mm)	Difference betweenc entroid of area and centroild of section (mm)	Area (mm^2)	Area times difference squared (mm^4)	Moment of inertia of Area (mm^4)
i=1	z_1=h_1/2=	dz_1=$\bar{z}$ -z_1=	A_1 =	A_1* $(dz_1)^2$=	I_1=(w_1*($h_1)^3$)/12=
i=2	z_2=h_2/2=	dz_2=$\bar{z}$ -z_2=	A_2=	A_2* $(dz_2)^2$=	I_2=(w_2*($h_2)^3$)/12=

The parallel axis theorem is now used to calculate the moment of inertia $I = \sum\left(I_i + A_i\ dz_i^2\right) = \left(I_1 + A_1\ dz_1^2\right) - \left(I_2 + A_2\ dz_2^2\right)$=_____________$mm^4$.

6) The uniaxial stresses and strains are calculated at each strain gage location

For Force #1	For Force #2	For Force #3
Distance from N/A to Delta Rosette=z_D-$\bar{z}$=_________mm	Distance from N/A to Delta Rosette=z_D-$\bar{z}$=_________mm	Distance from N/A to Delta Rosette=z_D-$\bar{z}$=_________mm
Distance from N/A to Rectangular Rosette= z_R -$\bar{z}$ =_________mm	Distance from N/A to Rectangular Rosette= z_R -$\bar{z}$ =_________mm	Distance from N/A to Rectangular Rosette= z_R -$\bar{z}$ =_________mm
Distance from N/A to Strain Gage 7 =z_7-$\bar{z}$=_________mm	Distance from N/A to Strain Gage 7 =z_7-$\bar{z}$=_________mm	Distance from N/A to Strain Gage 7 =z_7-$\bar{z}$=_________mm
Distance from N/A to Strain Gage 8 =z_8-$\bar{z}$=_________mm	Distance from N/A to Strain Gage 8 =z_8-$\bar{z}$=_________mm	Distance from N/A to Strain Gage 8 =z_8-$\bar{z}$=_________mm
Distance from N/A to Strain Gage 9 =z_9-$\bar{z}$=_________mm	Distance from N/A to Strain Gage 9 =z_9-$\bar{z}$=_________mm	Distance from N/A to Strain Gage 9 =z_9-$\bar{z}$=_________mm

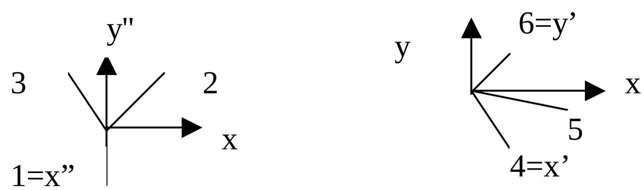
Use Table 1 in this worksheet for any mechanical properties

For Force #1	For Force #2	For Force #3
Uniaxial Normal X-Stress at Delta Rosette = $\sigma_{XD}=\frac{-M_D(z_D-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Delta Rosette = $\sigma_{XD}=\frac{-M_D(z_D-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Delta Rosette = $\sigma_{XD}=\frac{-M_D(z_D-\bar{z})}{I}$ =________MPa
Uniaxial Normal X-Stress at Rectangular Rosette = $\sigma_{XR}=\frac{-M_R(z_R-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Rectangular Rosette = $\sigma_{XR}=\frac{-M_R(z_R-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Rectangular Rosette = $\sigma_{XR}=\frac{-M_R(z_R-\bar{z})}{I}$ =________MPa
Uniaxial Normal X-Stress at Strain Gage 7 = $\sigma_{X7}=\frac{-M_R(z_7-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Strain Gage 7 = $\sigma_{X7}=\frac{-M_R(z_7-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Strain Gage 7 = $\sigma_{X7}=\frac{-M_R(z_7-\bar{z})}{I}$ =________MPa
Uniaxial Normal X-Stress at Strain Gage 8 = $\sigma_{X8}=\frac{-M_R(z_8-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Strain Gage 8 = $\sigma_{X8}=\frac{-M_R(z_8-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Strain Gage 8 = $\sigma_{X8}=\frac{-M_R(z_8-\bar{z})}{I}$ =________MPa
Uniaxial Normal X-Stress at Strain Gage 9 = $\sigma_{X9}=\frac{-M_R(z_9-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Strain Gage 9 = $\sigma_{X9}=\frac{-M_R(z_9-\bar{z})}{I}$ =________MPa	Uniaxial Normal X-Stress at Strain Gage 9 = $\sigma_{X9}=\frac{-M_R(z_9-\bar{z})}{I}$ =________MPa
Uniaxial Normal X-Strain at Delta Rosette = $\varepsilon_{XD}=\frac{\sigma_{XD}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Delta Rosette = $\varepsilon_{XD}=\frac{\sigma_{XD}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Delta Rosette = $\varepsilon_{XD}=\frac{\sigma_{XD}}{E}\times10^6$ =______µm/m
Uniaxial Normal X-Strain at Rectangulaur Rosette = $\varepsilon_{XR}=\frac{\sigma_{XR}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Rectangular Rosette = $\varepsilon_{XR}=\frac{\sigma_{XR}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Rectangular Rosette = $\varepsilon_{XR}=\frac{\sigma_{XR}}{E}\times10^6$ =______µm/m
Uniaxial Normal X-Strain at Strain Gage 7 = $\varepsilon_{X7}=\frac{\sigma_{X7}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Strain Gage 7 = $\varepsilon_{X7}=\frac{\sigma_{X7}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Strain Gage 7 = $\varepsilon_{X7}=\frac{\sigma_{X7}}{E}\times10^6$ =______µm/m
Uniaxial Normal X-Strain at Strain Gage 8 = $\varepsilon_{X8}=\frac{\sigma_{X8}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Strain Gage 8 = $\varepsilon_{X8}=\frac{\sigma_{X8}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Strain Gage 8 = $\varepsilon_{X8}=\frac{\sigma_{X8}}{E}\times10^6$ =______µm/m
Uniaxial Normal X-Strain at Strain Gage 9 = $\varepsilon_{X9}=\frac{\sigma_{X9}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Strain Gage 9 = $\varepsilon_{X9}=\frac{\sigma_{X9}}{E}\times10^6$ =______µm/m	Uniaxial Normal X-Strain at Strain Gage 9 = $\varepsilon_{X9}=\frac{\sigma_{X9}}{E}\times10^6$ =______µm/m

7) The deflection of the beam at the location of the rectangular rosette can be calculated from the applied force, the geometry of the setup and the beam dimensions

For Force #1	For Force #2	For Force #3
Deflection at the Rectangular Rosette= $\delta_R = -\left(\frac{PL_a X_R}{6EIL}\right)\left(L^2 - L_a^2 - X_R^2\right) =$ ________mm	Deflection at the Rectangular Rosette= $\delta_R = -\left(\frac{PL_a X_R}{6EIL}\right)\left(L^2 - L_a^2 - X_R^2\right) =$ ________mm	Deflection at the Rectangular Rosette= $\delta_R = -\left(\frac{PL_a X_R}{6EIL}\right)\left(L^2 - L_a^2 - X_R^2\right) =$ ________mm

8) The strain gage rosettes can be used to find the values and directions of the principal strains and stresses.



First the offset needs to be removed from each strain gage reading:

For Force #1	For Force #2	For Force #3
Corrected strain from SG1, (SG$_1$-Offset) =ε_1 =_____µm/m	Corrected strain from SG1, (SG$_1$-Offset) =ε_1 =_____µm/m	Corrected strain from SG1, (SG$_1$-Offset) =ε_1 =_____µm/m
Corrected strain from SG2, (SG$_2$-Offset) =ε_2 =_____µm/m	Corrected strain from SG2, (SG$_2$-Offset) =ε_2 =_____µm/m	Corrected strain from SG2, (SG$_2$-Offset) =ε_2 =_____µm/m
Corrected strain from SG3, (SG$_3$-Offset) =ε_3 =_____µm/m	Corrected strain from SG3, (SG$_3$-Offset) =ε_3 =_____µm/m	Corrected strain from SG3, (SG$_3$-Offset) =ε_3 =_____µm/m
Corrected strain from SG4, (SG$_4$-Offset) =ε_4 =_____µm/m	Corrected strain from SG4, (SG$_4$-Offset) =ε_4 =_____µm/m	Corrected strain from SG4, (SG$_4$-Offset) =ε_4 =_____µm/m
Corrected strain from SG5, (SG$_5$-Offset) =ε_5 =_____µm/m	Corrected strain from SG5, (SG$_5$-Offset) =ε_5 =_____µm/m	Corrected strain from SG5, (SG$_5$-Offset) =ε_5 =_____µm/m
Corrected strain from SG6, (SG$_6$-Offset) =ε_6 =_____µm/m	Corrected strain from SG6, (SG$_6$-Offset) =ε_6 =_____µm/m	Corrected strain from SG6, (SG$_6$-Offset) =ε_6 =_____µm/m

Next the coordinate strains are calculated

For Force #1	For Force #2	For Force #3
For the Delta Rosette, strain gage 1 is used as the reference for axis, x" such that $\varepsilon_{x''} = \varepsilon_1$=________μm/m	For the Delta Rosette, strain gage 1 is used as the reference for axis, x" such that $\varepsilon_{x''} = \varepsilon_1$=________μm/m	For the Delta Rosette, strain gage 1 is used as the reference for axis, x" such that $\varepsilon_{x''} = \varepsilon_1$=________μm/m
Now the strain in the y" direction is $\varepsilon_{y''} = \frac{1}{3}\left(2(\varepsilon_2 + \varepsilon_3) - \varepsilon_1\right)$= ________μm/m	Now the strain in the y" direction is $\varepsilon_{y''} = \frac{1}{3}\left(2(\varepsilon_2 + \varepsilon_3) - \varepsilon_1\right)$= ________μm/m	Now the strain in the y" direction is $\varepsilon_{y''} = \frac{1}{3}\left(2(\varepsilon_2 + \varepsilon_3) - \varepsilon_1\right)$= ________μm/m
Now the shear strain in the x"y" direction is $\gamma_{x''y''} = \frac{2\sqrt{3}}{3}(\varepsilon_2 - \varepsilon_3)$= ________μm/m	Now the shear strain in the x"y" direction is $\gamma_{x''y''} = \frac{2\sqrt{3}}{3}(\varepsilon_2 - \varepsilon_3)$= ________μm/m	Now the shear strain in the x"y" direction is $\gamma_{x''y''} = \frac{2\sqrt{3}}{3}(\varepsilon_2 - \varepsilon_3)$= ________μm/m
Principal normal strain is $\varepsilon_{1D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} + \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2}$ = ________μm/m	Principal normal strain is $\varepsilon_{1D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} + \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2}$ = ________μm/m	Principal normal strain is $\varepsilon_{1D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} + \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2}$ = ________μm/m
Principal normal strain is $\varepsilon_{3D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} - \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2}$ = ________μm/m	Principal normal strain is $\varepsilon_{3D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} - \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2}$ = ________μm/m	Principal normal strain is $\varepsilon_{3D} = \frac{\varepsilon_{x''} + \varepsilon_{y''}}{2} - \sqrt{\left(\frac{\varepsilon_{x''} - \varepsilon_{y''}}{2}\right)^2 + \left(\frac{\gamma_{x''y''}}{2}\right)^2}$ = ________μm/m
Principal normal strain is $\varepsilon_{2D} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1D} + \varepsilon_{3D})$= ________μm/m	Principal normal strain is $\varepsilon_{2D} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1D} + \varepsilon_{3D})$= ________μm/m	Principal normal strain is $\varepsilon_{2D} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1D} + \varepsilon_{3D})$= ________μm/m
Principal angle of ε_{1D} relative to the X-axis is $\theta_D = -90^\circ + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x''y''}}{(\varepsilon_{x''} - \varepsilon_{y''})}\right)$ =________°	Principal angle of ε_{1D} relative to the X-axis is $\theta_D = -90^\circ + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x''y''}}{(\varepsilon_{x''} - \varepsilon_{y''})}\right)$ =________°	Principal angle of ε_{1D} relative to the X-axis is $\theta_D = -90^\circ + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x''y''}}{(\varepsilon_{x''} - \varepsilon_{y''})}\right)$ =________°

For Force #1	For Force #2	For Force #3
For the Rectangular Rosette, strain gage 1 is used as the reference for axis, x' such that $\varepsilon_{x'} = \varepsilon_4$=________µm/m	For the Rectangular Rosette, strain gage 1 is used as the reference for axis, x' such that $\varepsilon_{x'} = \varepsilon_4$=________µm/m	For the Rectangular Rosette, strain gage 1 is used as the reference for axis, x' such that $\varepsilon_{x'} = \varepsilon_4$=________µm/m
Now the strain in the y' direction is $\varepsilon_{y'} = \varepsilon_6$=________µm/m	Now the strain in the y' direction is $\varepsilon_{y'} = \varepsilon_6$=________µm/m	Now the strain in the y' direction is $\varepsilon_{y'} = \varepsilon_6$=________µm/m
Now the shear strain in the x'y' direction is $\gamma_{x'y'} = 2(\varepsilon_5) - (\varepsilon_4 + \varepsilon_6)$= ________µm/m	Now the shear strain in the x'y' direction is $\gamma_{x'y'} = 2(\varepsilon_5) - (\varepsilon_4 + \varepsilon_6)$= ________µm/m	Now the shear strain in the x'y' direction is $\gamma_{x'y'} = 2(\varepsilon_5) - (\varepsilon_4 + \varepsilon_6)$= ________µm/m
Principal normal strain is $\varepsilon_{1R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} + \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$ = ________µm/m	Principal normal strain is $\varepsilon_{1R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} + \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$ = ________µm/m	Principal normal strain is $\varepsilon_{1R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} + \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$ = ________µm/m
Principal normal strain is $\varepsilon_{3R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} - \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$ = ________µm/m	Principal normal strain is $\varepsilon_{3R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} - \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$ = ________µm/m	Principal normal strain is $\varepsilon_{3R} = \frac{\varepsilon_{x'} + \varepsilon_{y'}}{2} - \sqrt{\left(\frac{\varepsilon_{x'} - \varepsilon_{y'}}{2}\right)^2 + \left(\frac{\gamma_{x'y'}}{2}\right)^2}$ = ________µm/m
Principal normal strain is $\varepsilon_{2R} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1R} + \varepsilon_{3R})$= ________µm/m	Principal normal strain is $\varepsilon_{2R} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1R} + \varepsilon_{3R})$= ________µm/m	Principal normal strain is $\varepsilon_{2R} = \left(\frac{-\nu}{1-\nu}\right)(\varepsilon_{1R} + \varepsilon_{3R})$= ________µm/m
Principal angle of ε_{1R} relative to the X-axis is $\theta_R = -75° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x'y'}}{(\varepsilon_{x'} - \varepsilon_{y'})}\right)$ =________°	Principal angle of ε_{1R} relative to the X-axis is $\theta_R = -75° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x'y'}}{(\varepsilon_{x'} - \varepsilon_{y'})}\right)$ =________°	Principal angle of ε_{1R} relative to the X-axis is $\theta_R = -75° + \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{x'y'}}{(\varepsilon_{x'} - \varepsilon_{y'})}\right)$ =________°

9) The analytical and experimental results can be tabulated and compared.

Rosette strains and deflections at each force

		Force #1	Force #2	Force #3
	Applied Force (N)			
Rectangular rosette	$\varepsilon_1 = \varepsilon_{1R}$ (μ m/m)			
	$\varepsilon_{p2} = \varepsilon_{2R}$ (μ m/m)			
	$\varepsilon_{p3} = \varepsilon_{3R}$ (μ m/m)			
Analytical	$\varepsilon_{XR} = \frac{\sigma_{XR}}{E}$			
Relative to X	$\theta_{p1} = \theta_R$ (°) (CW or CCW?)			
Relative to X	θ_{p2} (3?) (°) (CW or CCW?)			
Delta rosette	$\varepsilon_1 = \varepsilon_{1D}$ (μ m/m)			
	$\varepsilon_2 = \varepsilon_{2D}$ (μ m/m)			
	$\varepsilon_3 = \varepsilon_{3D}$ (μ m/m)			
Analytical	$\varepsilon_{XD} = \frac{\sigma_{XD}}{E}$			
Relative to X	$\theta_1 = \theta_D$ (°) (CW or CCW?)			
Relative to X	θ_2 (3?) (°) (CW or CCW?)			

Comment on the results. Are the results what you expected? How do the analytical results for the X-strain compare to the experimental results? Are the principal directions what you expected? Explain any difference and similarities…

10) The analytical and experimental results can be tabulated and compared.

The percent difference between calculated and measured values of strain and deflection can be computed as $\%diff = 100\dfrac{measured - calculated}{measured}$ because the measured is taken as the "correct" parameter.

First the offset needs to be removed from each strain gage reading:

For Force #1	For Force #2	For Force #3
Corrected strain from SG7, (SG $_7$-Offset) = ε_7 =_____µm/m	Corrected strain from SG7, (SG $_7$-Offset) = ε_7 =_____µm/m	Corrected strain from SG7, (SG $_7$-Offset) = ε_7 =_____µm/m
Corrected strain from SG8, (SG $_8$-Offset) = ε_8 =_____µm/m	Corrected strain from SG8, (SG $_8$-Offset) = ε_8 =_____µm/m	Corrected strain from SG8, (SG $_8$-Offset) = ε_8 =_____µm/m
Corrected strain from SG9, (SG $_9$-Offset) = ε_9 =_____µm/m	Corrected strain from SG9, (SG $_9$-Offset) = ε_9 =_____µm/m	Corrected strain from SG9, (SG $_9$-Offset) = ε_9 =_____µm/m

Next, the uniaxial strains and deflections at each force are compared

Force #1	Calculated value	Measured value	% Difference
Strain Gage #7, ε_7 (µ m/m)			
Strain Gage #8, ε_8 (µ m/m)			
Strain Gage #9, = ε_9 (µ m/m)			
Deflection, δ (mm)			

Force #2	Calculated value	Measured value	% Difference
Strain Gage #7, ε_7 (µ m/m)			
Strain Gage #8, ε_8 (µ m/m)			
Strain Gage #9, = ε_9 (µ m/m)			
Deflection, δ (mm)			

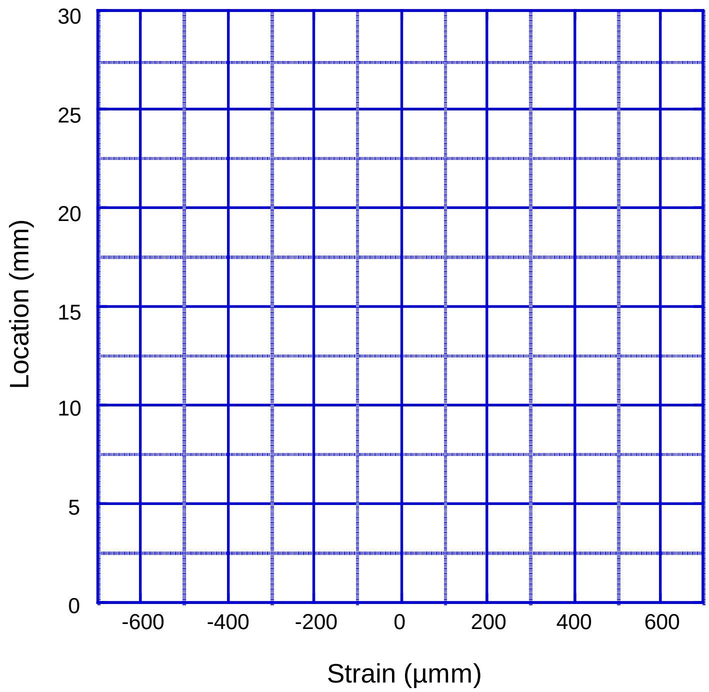
Force #3	Calculated value	Measured value	% Difference
Strain Gage #7, ε_7 (µ m/m)			
Strain Gage #8, ε_8 (µ m/m)			
Strain Gage #9, = ε_9 (µ m/m)			
Deflection, δ (mm)			

Comment on the results. Are the results what you expected? How do the analytical results for the X-strain compare to the experimental results? Are the principal directions what you expected? Explain any difference and similarities…

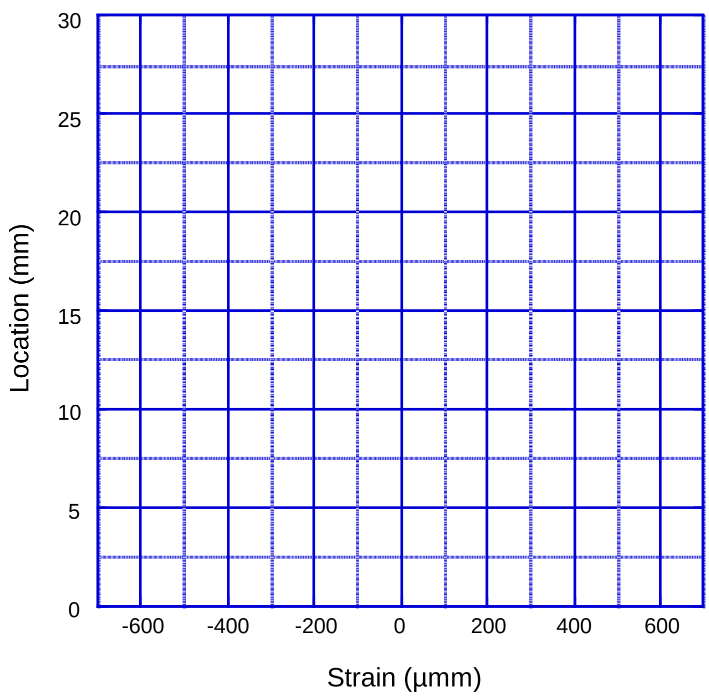
11) Plot the locations of strain gages 7, 8 and 9 as functions of the measured and calculated strains for each force case. Assuming a linear relation of location vs strain, the y-intercept will be the location at which the strain is zero (i.e., the neutral axis). Compare the location of the neutral axis determined using three methods: the measured strains, the calculated strains and the calculated centroid. Explain any differences and similarities.

	For Force #1		For Force #2		For Force #3	
Location	Measured Strain (µm/m)	Calculated Strain (µm/m)	Measured Strain (µm/m)	Calculated Strain (µm/m)	Measured Strain (µm/m)	Calculated Strain (µm/m)
SG7, z_7=____mm						
SG8, z_8=____mm						
SG9, z_9=____mm						
Rectangular Rosette, Principal strain in X direction, z_R=___mm						

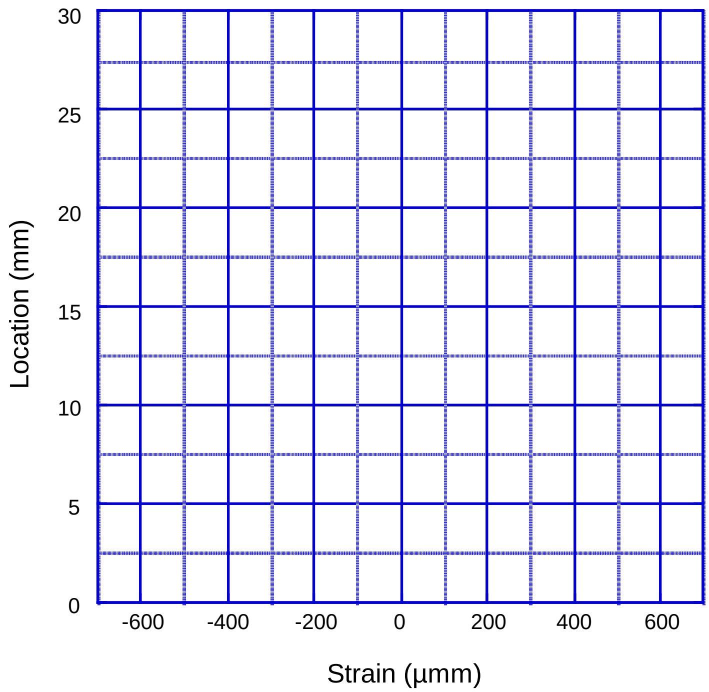
For Force #1



For Force #2



For Force #3



11 cont'd)

12) Compare stress and strain distributions and values from the experimental and analytical results. Explain and comment on any similarities and differences.