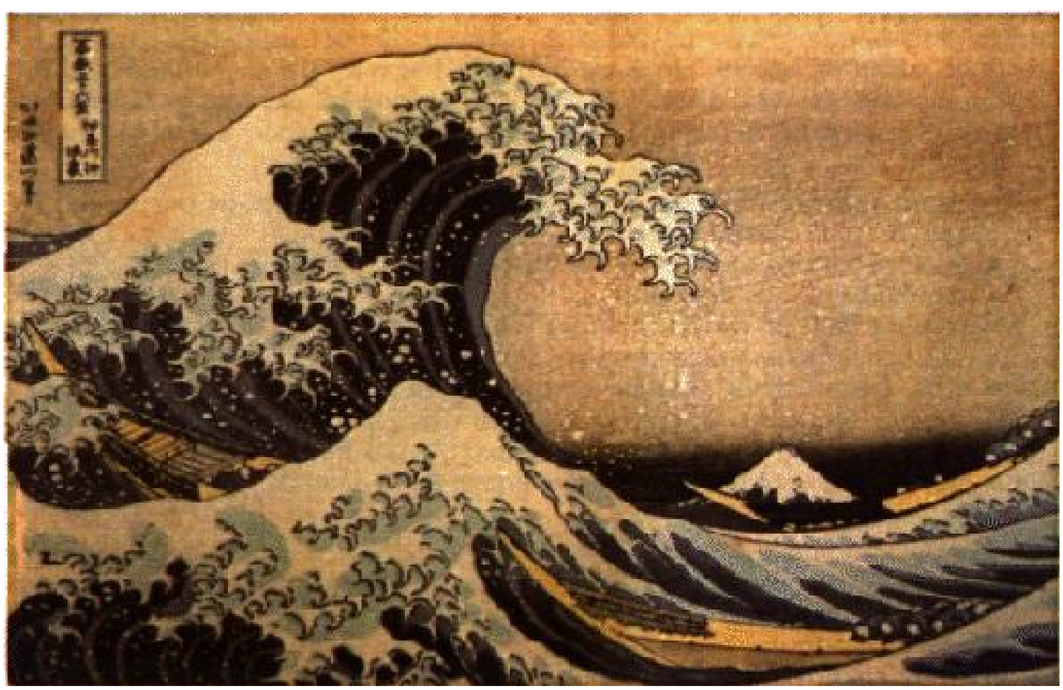
A Physical Introduction to Fluid Mechanics

Study Guide and Practice Problems

Spring 2014



A Physical Introduction to Fluid Mechanics

Study Guide and Practice Problems

Spring 2014

by

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Chapter 1

Introduction

1.1 Study Guide

- A fluid is defined as a material that deforms continuously and permanently under the application of a shearing stress.
- The pressure at a point in a fluid is independent of the orientation of the surface passing through the point; the pressure is isotropic.
- The force due to a pressure p acting on one side of a small element of surface dA defined by a unit normal vector $\mathbf{n}$ is given by $-p\mathbf{n}dA$.
- Pressure is transmitted through a fluid at the speed of sound.
- The *units* we use depend on whatever system we have chosen, and they include quantities like feet, seconds, newtons and pascals. In contrast, a *dimension* is a more abstract notion, and it is the term used to describe concepts such as mass, length and time.
- The specific gravity (SG) of a solid or liquid is the ratio of its density to that of water at the same temperature.
- A Newtonian fluid is one where the viscous stress is proportional to the rate of strain (velocity gradient). The constant of proportionality is the viscosity, μ, which is a property of the fluid, and depends on temperature.
- At the boundary between a solid and a fluid, the fluid and solid velocities are equal; this is called the "no-slip condition." As a consequence, for large Reynolds numbers ($>> 1$), boundary layers form close to the solid boundary. In the boundary layer, large velocity gradients are found, and so viscous effects are important.
- At the interface between two fluids, surface tension may become important. Surface tension leads to the formation of a meniscus, drops and bubbles, and the capillary rise observed in small tubes, because surface tension can resist pressure differences across the interface.

1.2 Worked Examples

Example 1.1: Units, and converting between units

Consider a rectangular block with dimensions $300\,mm \times 100\,mm \times 25\,mm$, of mass $10\,kg$, resting on a surface (Figure 1.1). The pressure acting over the area of contact can be found as follows.

Solution: Since pressure is a stress, it has dimensions of force per unit area. When in position (a), the force exerted on the table is equal to the weight of the block (= mass × gravitational acceleration = $98\,N$), and the average pressure over the surface in contact with the table is given by

$$\frac{98}{100 \times 25 \times 10^{-6}}\,N/m^2 \;=\; 39,200\,Pa$$

In position (b), the force exerted on the table is still equal to $98\,N$, but the average pressure over the surface in contact with the table is reduced to $98/\big(300 \times 25 \times 10^{-6}\big)\,N/m^2$, that is, $13,067\,Pa$.

We can repeat this example using engineering units. Let's take a rectangular block, made of a different material with dimensions $12\,in. \times 4\,in. \times 1\,in.$, of mass $20\,lb_m$ (this is similar to the case shown in Figure 1.1). Find the pressure acting over the area in the BG system.

Solution: The unit lb_m is not part of the engineering system of units (see Table 1.1), so we first convert it to *slugs*, where

$$\text{mass in } slugs \;=\; \frac{\text{mass in } lb_m}{32.2} \;=\; \frac{20}{32.2}\,slug \;=\; 0.622\,slug$$

So $20\,lb_m = 0.622\,slug$.

When in position (a), the force exerted on the table is equal to the weight of the block (= mass × gravitational acceleration = $20\,lb_f$), and the average pressure over the surface in contact with the table is $20/(4 \times 1)\,lb_f/in.^2$, that is, $5\,psi$. In position (b), the force exerted on the table is still equal to $20\,lb_f$, but now the average pressure over the surface in contact with the table is $20/(12 \times 1)\,lb_f/in.^2$, that is, $1.67\,psi$.

Example 1.2: Hydraulic presses and hoists

A hydraulic press uses the transmissibility of fluid pressure to produce large forces. A simple press consists of two connected cylinders of significantly different size, each fitted with a piston and filled with either oil or water (see Figure 1.2). The pressures produced by hydraulic devices are typically hundreds or thousands of *psi*, so that the weight of the fluid can usually be neglected. If there is an unobstructed passage connecting the two cylinders, then $p_1 \approx p_2$. Since pressure = (magnitude of force)/area,

$$\frac{F}{A} = \frac{f}{a}, \qquad \text{so that} \qquad F = \frac{A}{a}\,f$$

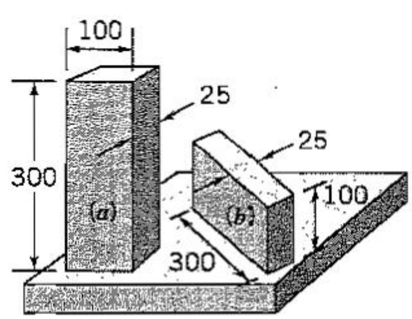


Figure 1.1: Pressure exerted by a weight resting on a surface. All dimensions in millimeters.

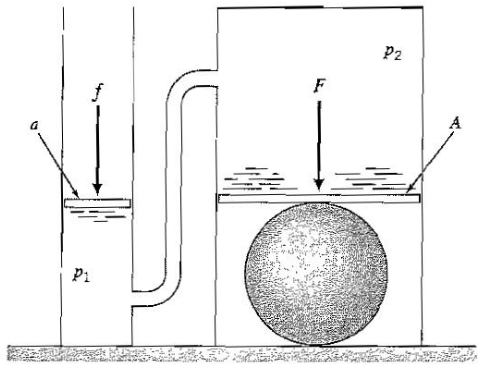


Figure 1.2: Hydraulic press.

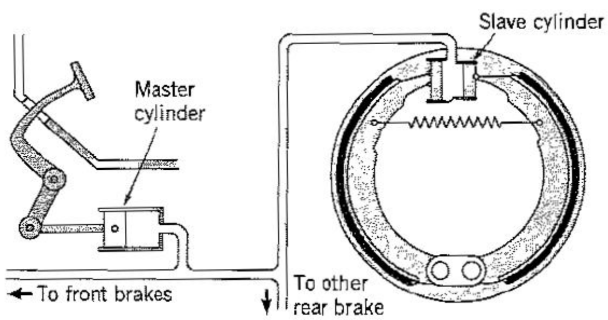


Figure 1.3: Hydraulic drum brake system.

The pressure transmission amplifies the applied force; a hydraulic press is simply a hydraulic lever.

A hydraulic hoist is basically a hydraulic press that is turned around. In a typical garage hoist, compressed air is used (instead of an actuating piston) to force oil through the connecting pipe into the cylinder under a large piston, which supports the car. A lock-valve is usually placed in the connecting pipe, and when the hoist is at the right height the valve is closed, holding the pressure under the cylinder and maintaining the hoist at a constant height.

A similar application of the transmissibility of pressure is used in hydraulic brake systems. Here, the force is applied by a foot pedal, increasing the pressure in a “master” cylinder, which in turn transmits the pressure to each brake or “slave” cylinder (Figure 1.3). A brake cylinder has two opposing pistons, so that when the pressure inside the cylinder rises the two pistons move in opposite directions. In a drum brake system, each brake shoe is pivoted at one end, and attached to one of the pistons of the slave cylinder at the other end. As the piston moves out, it forces the brake shoe into contact with the brake drum. Similarly, in a disk brake system, there are two brake pads, one on each side of the disk, and the brake cylinder pushes the two brake pads into contact with the disk.

Example 1.3: Finding the pressure in a fluid

Consider the piston and cylinder illustrated in Figure 1.4. If the piston had a mass $m = 1\,kg$, and an area $A = 0.01\,m^2$, what is the pressure p of the gas in the cylinder? Atmospheric pressure p_a acts on the outside of the container.

Solution: The piston is not moving so that it is in equilibrium under the force due to its own weight, the force due to the gas pressure inside of the piston (acting up), and the force due to air pressure on the outside of the piston (acting down). The weight of the piston $= mg$, where g is the acceleration due to gravity ($9.8\, m/s^2$, or $32.1\, ft/s^2$). The force due to pressure = pressure × area of piston. Therefore

$$pA - p_a A \;=\; mg \;=\; 1\, kg \times 9.8\, m^2/s = 9.8\, N$$

That is,

$$p - p_a \;=\; \frac{9.8}{0.01}\,\frac{N}{m^2} \;=\; 980\, Pa$$

where $Pa = pascal = N/m^2$.

What is this excess pressure in psi (pounds per square inch)? A standard atmosphere has a pressure of $14.7\, lb_f/in.^2$, or $101,325\, Pa$ (see Table 2.1). Therefore, $980\, Pa$ is equal to $14.7 \times 980/101325\, psi = 0.142\, psi$.

Example 1.4: Force due to pressure

In the center of a hurricane, the pressure can be very low. Find the force acting on the wall of a house, measuring $10\, ft$ by $20\, ft$, when the pressure inside the house is $30.0\, in$ of mercury, and the pressure outside is $26.3\, in.$ of mercury. Express the answer in lb_f and N.

Solution: A mercury barometer measures the local atmospheric pressure. A standard atmosphere has a pressure of $14.7\, lb_f/in.^2$, or $101,325\, Pa$ or N/m^2 Table 2.1). When a mercury barometer measures a standard atmosphere, it shows a reading of $760\, mm$ or $29.92\, in.$ To find the resultant force, we need to find the difference in pressure on the two sides of the wall and multiply it by the area of the wall, A. That is,

$$\begin{aligned}
\text{Force on wall, acting out} \;&=\; (p_{inside} - p_{outside})\, A \\
&= \left(\frac{30.0 - 26.3}{29.92} \times 14.7\right) \frac{lb_f}{in.^2} \times 10\, ft \times 20\, ft \times 12\, \frac{in.}{ft} \times 12\, \frac{in}{ft} \\
&= 52,354\ lb_f \\
&= 52,354 \times 4.448\, \frac{N}{lb_f} lb_f \\
&= 232,871\ N
\end{aligned}$$

We see that the force acting on the wall is very large, and without adequate strengthening, the wall can explode outward.

Example 1.5: Stresses in a pipe wall

Consider a section through a pipe of outside radius R and inside radius r, as shown in Figure 1.4. If the yield stress of the material is τ_y, what is the maximum gauge pressure p_{max} that can be contained in the pipe?

Solution: For a uniform pressure inside the pipe, the force dF acting radially outward on a segment of length dz with included angle θ is

$$\begin{aligned}
dF \;&=\; \text{pressure} \times \text{area} \\
&=\; pr\theta\, dz
\end{aligned}$$

Since the pipe is in static equilibrium (it is not tending to move), this force is exactly counterbalanced by the forces set up within the pipe material. If the stress is assumed to

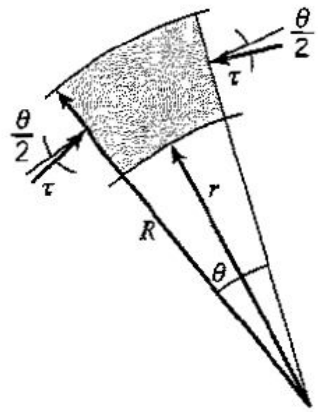


Figure 1.4: Stresses in a pipe wall due to pressure.

be uniform across the pipe wall thickness, then

$$\begin{aligned} dF &= \text{wall stress} \times \text{area} \\ &= 2\tau\,(R-r)\sin\tfrac{1}{2}\theta\,dz \end{aligned}$$

Therefore,

$$p\,r\theta\,dz = 2\tau\,(R-r)\sin\tfrac{1}{2}\theta\,dz$$

For small angles, $\sin(\frac{1}{2}\theta) \approx \frac{1}{2}\theta$, so that

$$p = \frac{(R-r)}{r}\,\tau$$

and

$$p_{max} = \frac{t}{r}\,\tau_y$$

where t is the wall thickness. In practice, the maximum allowable stress is considerably lower because of prescribed factors of safety, allowances for the way the pipe was manufactured and heat treated, possible corrosion effects, and the addition of fittings and joints, which all tend to weaken the pipe.

Example 1.6: Bulk modulus and compressibility

(a) Calculate the fractional change in volume of a fixed mass of seawater as it moves from the surface of the ocean to a depth of 5000 ft.
(b) Calculate the fractional change in density of a fixed mass of air as it moves isothermally from the bottom to the top of the Empire State Building (a height of 350 m, equivalent to a change in pressure of about 4100 Pa).

Solution: For part (a), from equation 1.1,

$$dp = -K\,\frac{dV}{V}$$

so that the fractional change in volume is given by

$$\frac{dV}{V} = -\frac{dp}{K}$$

For seawater, the bulk modulus $K = 2.28\times10^9\, N/m^2$ (see Table Appendix-C9). The change in pressure due to the change in depth may be found as follows. One standard atmosphere is equal to $101,325\, N/m^2$, but it can also be expressed in terms of an equivalent height of water, equal to $33.90\, ft$ (see Table 2.1). Therefore

$$\frac{dV}{V} = -\frac{dp}{K} = \frac{\frac{5000}{33.90}\times 101,325\, N/m^2}{2.28\times10^9\, N/m^2} = 0.0066 = 0.66\%$$

We see that seawater is highly incompressible.

For part (b), we use the ideal gas law (equation 1.3), where

$$p = \rho R T$$

with $R = 287.03\, m^2/s^2K$ for air. When the temperature is constant, we obtain

$$dp = RT d\rho$$

so that

$$\frac{dp}{p} = \frac{d\rho}{\rho}$$

Therefore,

$$\frac{d\rho}{\rho} = \frac{4100}{101325} = 0.0405 = 4.05\%$$

We see that air is much more compressible than seawater.

Example 1.7: Bulk modulus and compressibility

In Section 1.5, we considered pushing down on the handle of a bicycle pump to decrease its air volume by 50% (Figure 1.7). If the air was initially at atmospheric pressure and $20°C$, and its temperature remains constant, estimate the bulk modulus of air, and the force required to be applied on the handle if if the pump has an internal diameter of $1.25\, in$.

Solution: Halving the air volume at constant temperature increases its density and pressure by a factor of two. Since the air was initially at atmospheric pressure, its pressure increases by one atmosphere ($14.696\, psi$, or $101,325\, Pa$), and its density increases by an amount equal to $1.204\, kg/m^3$ (see Table Appendix-C.1). Using equation 1.2,

$$K = \rho\frac{dp}{d\rho} = 1.204\frac{101325}{1.204}\, Pa = 101,325\, Pa$$

Also, with a pump diameter of $1.25\, in.$, the required force is

$$14.7\, psi \times \tfrac{1}{4}\pi\,(1.25\, in)^2\ lb_f = 18.1\, lb_f$$

Example 1.8: Compressibility and Mach number

(a) Calculate the change in pressure Δp of water as it increases its speed from 0 to $30\, mph$ at a constant height.
(b) What air speed V corresponds to $M = 0.6$ when the air temperature is $270K$?

Solution: For part (a), using equation 1.4,

$$\Delta p \;=\; -\tfrac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

In this case,

$$\Delta p \;=\; -\tfrac{1}{2}\rho V_2^2$$

where $1000\,kg/m^3$ and $V_2 = 30\,mph = (30/2.28)\,m/s$ (Appendix B). Hence,

$$\Delta p \;=\; -\tfrac{1}{2} \times 1000 \times \left(\frac{30}{2.28}\right)^2 Pa = 86,565\,Pa$$

That is, the change in pressure is a little less than one atmosphere.

For part (b), from equation 1.6

$$a = \sqrt{\gamma RT}$$

For air, $R = 287.03\,m^2/s^2K$, and $\gamma = 1.4$. Since $M = V/a$,

$$V = aM = 0.6 \times \sqrt{1.4 \times 287.03 \times 270}\,m/s = 197.6\,m/s$$

Example 1.9: Dynamic and kinematic viscosity

What are the dynamic and kinematic viscosities of air at $20°C$? $60°C$? What are the values for water at these temperatures?

Solution: From Table Appendix-C.1, we find that for air at $20°C$, $\mu = 18.2 \times 10^{-6}\,N\,s/m^2$ and $\nu = 15.1 \times 10^{-6}\,m^2/s$. At $60°C$, $\mu = 19.7 \times 10^{-6}\,N\,s/m^2$ and $\nu = 18.6 \times 10^{-6}\,m^2/s$.

From Table Appendix-C.3, we find that for water at $20°C$, $\mu = 1.002 \times 10^{-3}\,N\,s/m^2$ and $\nu = 1.004 \times 10^{-6}\,m^2/s$. At $60°C$, $\mu = 0.467 \times 10^{-3}\,N\,s/m^2$ and $\nu = 0.475 \times 10^{-6}\,m^2/s$.

Note that: (a) the viscosity of air increases with temperature, while the viscosity of water decreases with temperature; (b) the variation with temperature is more severe for water than for air; and (c) the dynamic viscosity of air is much smaller than that of water, but its kinematic viscosity is much larger.

Example 1.10: Forces due to viscous stresses

Consider plane Couette flow, as shown in Figure 1.14. Note that if there is no force acting on the top plate, it will slow down and eventually stop because of the viscous stress exerted by the fluid on the plate. That is, if the plate is to keep moving at a constant speed, the viscous force set up by the shearing of the fluid must be exactly balanced by the applied force. Find the force required to keep the top plate moving at a constant speed U in terms of the fluid viscosity μ, U, its area A, and the gap distance h. Also find the power required.

Solution: The stress τ_w required to keep the plate in constant motion is equal to the viscous stress exerted by the fluid on the top plate. For a Newtonian fluid, we have

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{wall}$$

so that

$$F = \tau_w A = \mu \frac{U}{h} A$$

We see that as the velocity and the viscosity increase, the force increases, as expected, and as the gap size increases, the force decreases, again as expected. Also,

$$\mu = \frac{h\tau_w A}{U} = \frac{hF}{U}$$

We can use this result to determine the viscosity of a fluid by measuring F at a known speed and gap width.

The power required to drive the top plate is given by the applied force times the plate velocity. Hence,

$$\text{power} = FU = \tau_w AU \;= \mu A \frac{U^2}{h}$$

which shows that the power is proportional to the velocity squared.

Example 1.11: Viscous stress in a boundary layer

A particular laminar boundary layer velocity profile is given by

$$\frac{u}{U_e} = 2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2$$

for $y \leq \delta$ so that at $y = \delta$, $u = U_e$, where δ is the boundary layer thickness, and U_e is the freestream velocity, that is, the velocity outside the boundary layer region. Find the shear stress τ as a function of the distance from the wall, y.

Solution: We have

$$\tau = \mu \frac{du}{dy}$$

so that

$$\begin{aligned} \tau &= \mu \frac{d}{dy}\left[U_e\left(2\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^2\right)\right] \\ &= \mu U_e\left(\frac{2}{\delta} - \frac{2y}{\delta^2}\right) = \frac{2\mu U_e}{\delta}\left(1 - \frac{y}{\delta}\right) \end{aligned}$$

At the wall, where $y = 0$, $\tau = \tau_w$, and so

$$\tau_w = \frac{2\mu U_e}{\delta}$$

Example 1.12: Reynolds number

(a) Find the Reynolds number of a water flow at an average speed $\overline{V} = 20\,ft/s$ at $60°F$ in a pipe with a diameter $D = 6\,in$.
(b) If the flow is laminar at Reynolds numbers below 2300, what is the maximum speed at which we can expect to see laminar flow?

Solution: For part (a), we have the Reynolds number for pipe flow

$$Re = \frac{\rho \overline{V} D}{\nu} = \frac{\overline{V} D}{\nu}$$

where ν is the kinematic viscosity ($= \mu/\rho$). Using Table Appendix-C.4, we find that for water at $60°F$, $\nu = 1.21 \times 10^{-5}\,ft^2/s$, so that

$$Re = \frac{\overline{V} D}{\nu} = \frac{20\,ft/s \times 0.5\,ft}{1.21 \times 10^{-5}\,ft^2/s} = 8.26 \times 10^5$$

For part (b), we have

$$\overline{V}_{max} = \frac{Re_{max} \times \nu}{D} = \frac{2300 \times 1.21 \times 10^{-5}\,ft^2/s}{0.5\,ft} = 0.056\,ft/s$$

so that the flow is laminar for velocities less than $0.056\,ft/s$ ($0.67\,in/s$ or $15\,mm/s$). This is a very slow flow. In most practical applications on this scale, such as in domestic water supply systems, we would therefore expect to see turbulent flow.

Problems

1.1 A body requires a force of $400\,N$ to accelerate at a rate of $1.0\ m/s^2$. Find the mass of the body in kilograms, slugs, and lb_m.

1.2 What volume of fresh water at $20°C$ will have the same weight as a cubic foot of lead? Oak? Seawater? Air at atmospheric pressure and $100°C$? Helium at atmospheric pressure and $20°C$?

1.3 What is the weight of one cubic foot of gold? Pine? Water at $40°F$? Air at atmospheric pressure and $60°F$? Hydrogen at 7 atmospheres of pressure and $20°C$?

1.4 What is the weight of $5000\,liters$ of hydrogen gas at $20°C$ and $100\,atm$ on earth? On the moon?

1.5 What is the specific gravity of gold? Aluminum? Seawater? Air at atmospheric pressure and $-20°C$? Argon at atmospheric pressure and $20°C$?

1.6 When the temperature changes from $20°C$ to $30°C$ at atmospheric pressure:
(a) By what percentage does the viscosity μ of air change?
(b) Of water?
(c) By what percentage does the kinematic viscosity ν of air change?
(d) Of water?

1.7 (a) Find the altitude (in meters) where the density of air has fallen to half its value at sea level.
(b) Find the altitude (in meters) where the pressure has fallen to half its value at sea level.
(c) Why is the answer to part (a) different from the answer to part (b)?

1.8 How much will it cost to run a $10\,hp$ motor for $8\,hrs$ if $1\,kW \cdot hr$ costs 10 cents?

1.9 A scuba diving tank initially holds $0.25\,ft^3$ of air at $3000\,psi$. If the diver uses the air at a rate of $0.05\,kg/min$, approximately how long will the tank last? Assume that the temperature of the gas remains at $20°C$.

1.10 A hollow cylinder of $30\,cm$ diameter and wall thickness $20\,mm$ has a pressure of $100\,atm$ acting on the inside and $1\,atm$ acting on the outside. Find the stress in the cylinder material.

1.11 What is the change in volume of 1 kilogram of fresh water kept at $5°C$ as it moves from a depth of $1\,m$ to a depth of $100\,m$. Take the isothermal bulk modulus of water to be $2 \times 10^4\,atm$.

1.12 (a) Find the dynamic viscosity μ of water at 20C.
(b) What is the specific gravity of aluminum? Ethyl alcohol? Ice?
(c) Calculate the fractional change in volume of 1 kg of seawater at the ocean surface compared to its volume at a depth of 1000 m.

1.13 A square submarine hatch $60\,cm$ by $60\,cm$ has $3.2\,atm$ pressure acting on the outside and $2.6\,atm$ pressure acting on the inside. Find the resultant force acting on the hatch.

1.14 A Boeing 747 airplane has a total wing area of about $500\,m^2$. If its weight in level cruise is $500,000\,lb_f$, find the average pressure difference between the top and bottom surfaces of the wing, in psi and in Pa.

1.15 (a) Estimate the average pressure difference between the bottom and the top of a fully loaded (maximum take-off weight) Boeing 747 wing.
(b) Does this value change with altitude?
(c) What are the implications of your answer to 2(b)?

1.16 Find the force acting on the wall of a house in the eye of a hurricane, when the pressure inside the house is $1,000\,mbar$, and the pressure outside is $910\,mbar$. The wall measures $3.5\,m$ by $8\,m$. Express the answer in N and lb_f.

1.17 The pressure at any point in the atmosphere is equal to the total weight of the air above that point, per unit area. Given that the atmospheric pressure at sea level is about $10^5\,Pa$, and the radius of the earth is $6370\,km$, estimate the mass of all the air contained in the atmosphere.

1.18 Estimate the change in pressure that occurs when still air at $100°F$ and atmospheric pressure is accelerated to a speed of $100\,mph$ at constant temperature. Neglect compressibility effects.

1.19 For the previous problem, estimate the error in the calculation of the pressure change due to compressibility. Assume an isothermal acceleration.

1.20 Compute the Mach number of a flow of air at a temperature of $-50°C$ moving at a speed of $500\,mph$.

1.21 Compute the Mach number of bullet fired into still air at a temperature of $60°F$ at a speed of $1500\,ft/s$.

1.22 An airplane of length 100 ft travels at 300 mph at an altitude of 20,000 ft.
(a) Find the Mach number.
(b) Find the Reynolds based on length.

1.23 A circular hockey puck of diameter D slides at a constant speed V at a height h over a smooth surface. If the velocity distribution in the gap is linear, and the viscosity of air is μ, find the force acting on the puck required to overcome viscous effects.

1.24 A boogy board of area $1\,m^2$ slides at a speed of $3\,m/s$ over the beach supported on a film of water $3\,mm$ thick. If the velocity distribution in the film is linear, and the temperature of the water is $30°C$, find the force applied to the board.

1.25 An experiment is performed where a plate of area $A = 1\,m^2$ slides at a constant velocity $V = 1\,m/s$ over a stationary surface. The gap between the plates is $h = 1\,mm$, and it is filled with a fluid of viscosity μ. If the force required overcome the viscous drag opposing the motion of the plate is $1\,N$, and the velocity gradient is observed to be linear, what is the viscosity of the fluid?

1.26 A cylinder of radius r rotates concentrically inside a larger cylinder of radius R with an angular velocity ω. The gap between the two cylinders is filled with a fluid of viscosity μ. Find the force required to rotate the inner cylinder if the cylinders are of length L and the velocity varies linearly across the gap.

1.27 A circular shaft of radius R_1 and length L rotates concentrically within a stationary cylindrical bearing of radius R_2. The gap between the shaft and bearing is filled with an oil of viscosity μ. If the velocity profile is linear, find the angular speed of the shaft in terms of the torque T applied to the shaft and R_1, R_2, μ and L (torque is force times lever arm).

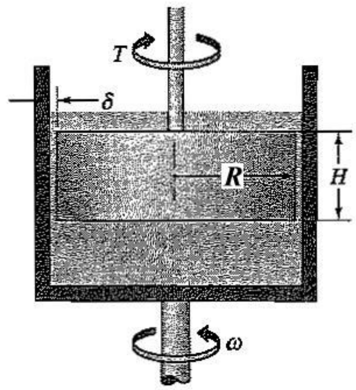


Figure P1.28

1.28 The viscosity of a Newtonian liquid can be estimated using the instrument shown in Figure P1.28. The outer cylinder rotates at an angular speed ω. If the torque (dimensions of force times length) required to keep the inner cylinder stationary is T, and the velocity distribution in the gap is linear, find an expression for the viscosity μ in terms of T, ω, H, δ, and R.

1.29 A long cylinder of diameter d_1 slides at a velocity U inside a long stationary tube of diameter d_2, so that the cylinder and tube are concentric. If the gap is filled with a fluid of viscosity μ, and if the fluid velocity varies linearly across the (small) gap, find the force per unit length required to keep the inner cylinder moving at a constant velocity.

1.30 A cylinder of length L and diameter D rotates concentrically in another cylinder of the same length with diameter $D + 2\epsilon$, where $\epsilon << D$. A fluid of viscosity μ fills the gap. The inner cylinder is driven by a motor so that it rotates at an angular speed ω, while the outer cylinder remains fixed. If the velocity profile in the gap is linear, find (in terms of L, D, ϵ, μ and ω):
(a) the torque (force times length) exerted on the outer cylinder, and
(b) the power expended by the motor.

1.31 A Taylor-Couette apparatus consists of an inner cylinder and an outer cylinder that can rotate independently. The gap between the cylinders is filled with a Newtonian fluid. For a particular experiment at $20°C$, we find that when the inner cylinder is held stationary and a force of $0.985N$ is applied to the outer cylinder, the outer cylinder rotates at a speed of $1\,Hz$. Find the viscosity of the fluid, assuming that the velocity profile in the gap is linear. The inner and outer cylinders have radii of $200\,mm$ and $150\,mm$, respectively, and the cylinders are $300\,mm$ long. Can you suggest which fluid is being tested?

1.32 A $10\,cm$ cube of mass $2\,kg$, lubricated with SAE 10 oil at $20°C$ (viscosity of $0.104N \cdot s/m^2$), slides down a $10°$ inclined plane at a constant velocity. Estimate the speed of the body if the oil film has a thickness of $1\,mm$, and the velocity distribution in the film is linear.

1.33 When water at $40°F$ flows through a channel of height h, width W and length L at low Reynolds number, the flow is laminar and the velocity distribution is parabolic, as shown in Figure P1.33. When $h = 2\,in.$, $W = 30\,in.$, $L = 10\,ft$, and the maximum velocity is $1\,ft/s$:
(a) Calculate the Reynolds number based on the channel height and the maximum velocity.

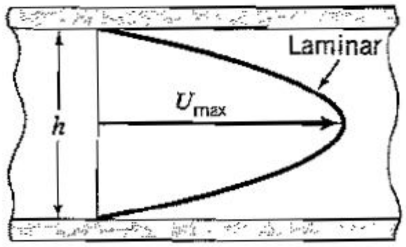


Figure P1.33

(b) Find the viscous stress at the wall τ_w, and the total viscous force acting on the channel, assuming τ_w is constant.

1.34 Given that the mean free path of a given gas is inversely proportional to its density, and that the mean free path of air at sea level is $0.089\,\mu m$, find the mean free path at $30\,km$ altitude, and at $50\,km$ altitude. Estimate the altitude where the mean free path is $5\,mm$. Can we apply the continuum approximation under these conditions?

1.35 By making reasonable estimates of characteristic speeds and lengths, estimate the Reynolds number for the following:
(a) a professionally pitched baseball
(b) a shark swimming at full speed
(c) a passenger jet at cruise speed
(d) the wind flow around the Empire State Building
(e) methane (CH_4) flowing in a $2\,m$ diameter pipe at $40\,kg/s$
(f) a mosquito flying in air.

1.36 Find the characteristic Reynolds number of a submarine of length $120\,m$ moving at $20\,m/s$ in water. If a 12:1 model is to be tested at the same Reynolds number, describe the test conditions if you were to use
(a) water
(b) air at standard temperature and pressure
(c) air at 200 atmospheres and $20°C$ (assume that the viscosity is not a function of pressure).

1.37 The largest artery in the body is the aorta. If the maximum diameter of the aorta is $2\,cm$, and the maximum average velocity of the blood is $20\,cm/s$, determine if the flow in the aorta is laminar or turbulent (assume that blood has the same density as water but three times its viscosity — blood, after all, is thicker than water).

1.38 Find the height to which water at $300°K$ will rise due to capillary action in a glass tube $3\,mm$ in diameter.

1.39 A liquid at $10°C$ rises to a height of $20\,mm$ in a $0.4\,mm$ glass tube. The angle of contact is $45°$. Determine the surface tension of the liquid if its density is $1,200\,kg/m^3$.

1.40 What is the pressure inside a droplet of water $0.1\,mm$ diameter if the ambient pressure is atmospheric?

1.41 A beer bubble has an effective surface tension coefficient of $0.073\,N/m$. What is the overpressure inside the bubble if it has a diameter of $1.0\,mm$?

1.42 A glass ring has a circular cross-section of diameter d and it has a mass of $5\,g$. What is the maximum value for d for the ring to continue to float on a water surface at $20°C$?

Chapter 2

Fluid Statics

2.1 Study Guide

- The hydrostatic equation, $dp/dz = -\rho g$, expresses the pressure variation with depth, where z is measured vertically up.
- For a constant density fluid, the pressure increases linearly with depth.
- Gauge pressure is the absolute pressure relative to the local atmospheric pressure, that is, $p_g = p - p_{atm}$.
- For fluids in rigid body motion, use Newton's second law, where the pressure gradient balances the inertial force acting on a fluid particle due to the total acceleration (including gravity).

To find the resultant force acting on a submerged surface due to hydrostatic pressure differences:

Step 1. Choose a coordinate system. It is best to choose a set which makes the task of expressing the shape of the surface as straightforward as possible. Clearly show the origin and direction of your coordinate system.

Step 2. Mark an element of area dA on the surface.

Step 3. Find the depth of dA, that is, its distance below the surface, measured vertically down (that is, in the direction of the gravitational vector).

Step 4. Determine if it is possible to use gauge pressure instead of absolute pressure. If the gauge pressure acting on dA is p_g, then $p_g = p - p_a = \rho g \times \textit{depth}$. Then the force acting on dA is $dF = p_g \times dA$ (if gauge pressure can be used).

Step 5. Integrate to find F. For a double integral, do the integral at a constant depth first. The shape of the surface sets the limits of integration.

To find the point of action of the resultant force acting on a submerged surface due to hydrostatic pressure differences, we take moments:

Step 1. Look for symmetry since this will always lead to simplifications. For instance, in the example given above, F will act on y-axis so that $\bar{x} = 0$.

Step 2. Choose the axis about which to take moments (the x-axis in the example given above). Then $dM = y \times dF$, where y is the moment arm of dF about the x-axis, and $F \times \bar{y} = \int dM = \int y \times dF = \int y p_g \, dA$, (if gauge pressure can be used).

Step 3. Integrate to find M. For a double integral, do the integral at a constant depth first. The shape of the surface sets the limits of integration.

2.2 Worked Examples

Example 2.1: Density and specific gravity

(a) Find the density of a rectangular block with dimensions $300\,mm \times 100\,mm \times 25\,mm$, of mass $10\,kg$.
(b) Find the density of a rectangular block with dimensions $12\,in. \times 4\,in. \times 1\,in.$, of mass $20\,lb_m$. (c) Find the specific gravity of the density of the material in part (a).

Solution: For part (a)

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{10}{300 \times 100 \times 25 \times 10^{-9}}\ kg/m^3 = 13,333\ kg/m^3$$

For part (b), the unit lb_m is not part of the engineering system of units, so we first convert it to $slugs$, where

$$\text{mass in } slugs = \frac{\text{mass in } lb_m}{32.1739}$$

Therefore,

$$\rho = \frac{20 \times 12 \times 12 \times 12}{32.2 \times 12 \times 4 \times 1}\ slug/ft^3 = 22.36\ slug/ft^3$$

For part (c), we see from Table Appendix-C.7 that this material has a density somewhere between lead and gold. Its specific gravity is equal to its density divided by the density of water at $20°C$. For the material in part (a), therefore, the specific gravity $= 13,333/998.2 = 13.36$.

Example 2.2: Manometers

Consider the manometer shown in Figure 2.5. Let z_1 be the height of point 1 above a horizontal reference level, z_2 be the height of point 2, and so on. If $\rho_1/\rho_3 = 2$, and $z_1 = 10\,in.$, $z_2 = 8\,in.$, $z_3 = 6\,in.$ and $z_6 = 14\,in.$, find the ratio ρ_2/ρ_3.

Solution: We know that $p_1 = p_6 = p_a$ and $p_3 = p_4$. Therefore, if we equate pressures at height z_3, we get

$$\begin{aligned} \rho_1 g\,(z_1 - z_3) &= \rho_3 g\,(z_6 - z_5) + \rho_2 g\,(z_5 - z_4) \\ &= \rho_3 g\,(z_6 - z_2) + \rho_2 g\,(z_2 - z_3) \end{aligned}$$

That is,

$$\rho_2\,(z_2 - z_3) = \rho_1\,(z_1 - z_3) - \tfrac{1}{2}\rho_1\,(z_6 - z_2)$$

and

$$\frac{\rho_2}{\rho_1} = \frac{(z_1 - z_3) - \frac{1}{2}\,(z_6 - z_2)}{(z_2 - z_3)} = \frac{1}{2} = \frac{\rho_2}{\rho_3}\frac{\rho_3}{\rho_1}$$

and since $\rho_1/\rho_3 = 2$, we see that $\rho_2/\rho_3 = 1$.

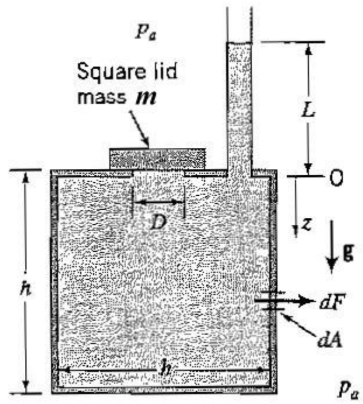


Figure 2.1: Tank, with stand-pipe.

Example 2.3: Forces and moments on vertical walls

A cubic tank of dimension h contains water of density ρ. It has a pipe open to atmosphere located in its top surface. This pipe contains water to a height L, as shown in Figure 2.1. A square vent of size D in the top surface is just held closed by a lid of mass m.
(a) Find the height L in terms of m, D and ρ.
(b) Find the force F due to the water acting on a vertical face of the tank in terms of ρ, g, h and L.
(c) Find where this force acts in terms of h and L.

Solution: For part (a), the weight of the lid mg is just balanced by the water pressure acting on the area of the lid. Using gauge pressure (since the atmospheric pressure acts everywhere equally), this pressure is given by $\rho g(depth) = \rho g L$. Hence,

$$\rho g L D^2 = mg$$

and so

$$L = \frac{m}{\rho D^2}$$

For part (b), we identify an element of area dA on a vertical wall of the tank. Atmospheric pressure acts everywhere, so dF, the force due to water pressure acting on dA is given by

$$dF = p_g\, dA = \rho g\, (depth)\; dA = \rho g\, (L + z)\; dA$$

Note that the depth is measured from the free surface, not the top of the tank Since the wall is of constant width ($= h$), $dA = h\, dz$, and

$$dF = \rho g\, (L + z)\, h\, dz$$

To find F we integrate

$$F = \int_0^h \rho g\, (L + z) h\, dz = \rho g h \left[Lz + \frac{z^2}{2} \right]_0^h$$

so that

$$F = \rho g h^2 \left(L + \frac{h}{2} \right)$$

For part (c), we take moments about the top of the tank side wall (where $z = 0$). If dM is the moment due to dF,

$$dM = z\,dF = \rho g\,(L + z)\,zh\,dA$$

Integrating, we obtain:

$$\begin{aligned} M \;=\; F \times \bar{z} \;&=\; \int_0^h \rho g\,(L+z)\,zh\,dz \\ &=\; \rho g h \left[L\frac{z^2}{2} + \frac{z^3}{3} \right]_0^h \\ &=\; \rho g h^2 \left(L\frac{h}{2} + \frac{h^2}{3} \right) \end{aligned}$$

Therefore, the resultant force acts at a distance $\bar{z}$ below the top of the tank, where

$$\bar{z} = \frac{M}{F} = \frac{L\frac{h}{2} + \frac{h^2}{3}}{L + \frac{h}{2}} = \frac{h(L + \frac{2h}{3})}{2L + h}$$

Example 2.4: Equilibrium of a hinged gate

Sometimes we have a problem where the equilibrium of a solid body depends on the sum of the moments. For instance, if a hinged gate is placed in a vertical wall, the water pressure will try to open the gate unless a moment of sufficient strength is exerted on the gate to keep it shut. Consider a simple case where the whole wall serves as a gate. The gate is vertical and rectangular in shape, of width W and height h [Figure 2.2(a)]. The top of the gate is level with the surface of the water, where it is supported by a frictionless hinge H. Halfway down the gate, an arm sticks out horizontally and a weight mg hangs at a distance a from the gate. The bottom of the gate rests against a stop. Atmospheric pressure acts everywhere. Neglect the weight of the gate and the arm. What is the minimum value of m necessary to keep the gate shut?

Solution: Consider the free body diagram for the gate [Figure 2.2(b)]. This diagram shows the forces and moments acting on the gate when it is at the point of opening, so that the reaction force exerted on the gate by the stop at the foot of the gate is zero. The force F exerted by the water pressure on the gate acts from the left in the horizontal direction, and it tends to rotate the gate in a counterclockwise direction. We know from our previous work that it has a magnitude $F = \frac{1}{2}\rho g W h^2$ and that it acts at a distance $\frac{2}{3}h$ from the top of the gate. The weight mg acts at a distance a horizontally out from the gate, and it tends to rotate the gate in a clockwise direction. There is also a force F_H exerted by the hinge on the gate, but the hinge exerts no moment since it is frictionless (a hinge without friction cannot exert a moment). Since the gate is not moving, it is in static equilibrium under the action of these forces and moments.

How do we find the critical value of m, where the gate is just on the point of opening? We know that $\Sigma\mathbf{F} = 0$ and $\Sigma\mathbf{M} = 0$. If we use $\Sigma\mathbf{F} = 0$, we see from the free body diagram that there is a force exerted at the hinge F_H that needs to be found separately before the force balance can be solved for m. We may be able to find F_H using the moment equation $\Sigma\mathbf{M} = 0$, but instead we can use the moment equation to find m directly. If we choose the moment axis to coincide with the top of the gate, then the hinge force F_H exerts no moment about this axis and we need not consider it any further. We simply balance the moments about the hinge exerted by the weight mg and the force due to water pressure. That is,

$$mg \times a - \frac{1}{2}\rho g W h^2 \times \frac{2h}{3} \;=\; 0$$

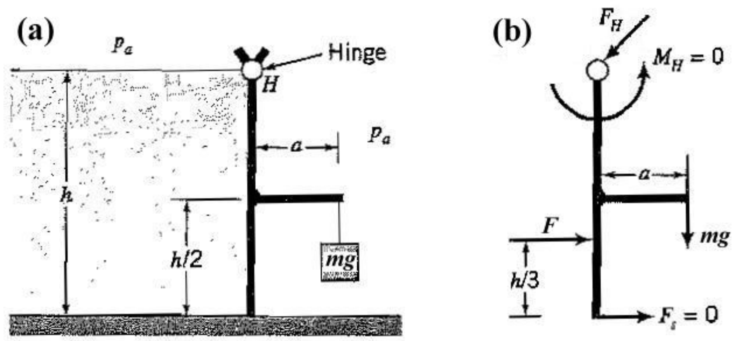


Figure 2.2: (a) Simple gate, hinged at the level of the free surface. (b) Free body diagram of gate.

and so we find

$$m = \frac{\rho W h^3}{3a}$$

Example 2.5: Another hinged gate

In Figure 2.3, a gate is shown, hinged at the top using a frictionless hinge H, located at the same level as the water surface. There is a rectangular overhang in the gate that sticks out a distance a horizontally from the gate. There is a stop at the bottom of the gate to resist the force of the water pressure on the inside of the gate, and to prevent it opening in a counterclockwise direction. However, as a is increased, there will come a point where the weight of the water in the overhang will be large enough to cause the gate to move away from the stop with a clockwise rotation. What is this critical value of a?

Solution: If we were to draw the free body diagram of this gate, we would see that it is best to take moments about the hinge line since the unknown force exerted by the hinge on the gate has no moment about this axis.

We could solve the problem by considering each vertical part of the wall separately, and find the moments about the hinge exerted by the forces acting on each surface (these moments are all counterclockwise), and then find the moment exerted by the forces acting on the horizontal parts of the overhang (these moments are clockwise). For equilibrium, the sum of the moments must be zero, and so we could find a.

However, there is a simpler way. The total horizontal force is equal to the sum of the forces acting on all the vertical parts of the gate, and therefore it is equal to the force acting on a vertical wall of the same height, given by $\frac{1}{2}\rho g W h^2$, where W is the width of the gate. It acts at a distance $\frac{2}{3}h$ from the top of the gate, and so its moment can be found directly.

As for the overhang, there are two approaches. First, we work in terms of the pressures acting on the top and bottom surfaces of the overhang. We can find the pressure acting on the bottom surface, multiply by the area of the bottom surface to find the force (since the bottom surface is at a constant depth, the pressure is constant over the area), and then multiply by its moment arm to find its (clockwise) moment. The moment arm is equal to $\frac{1}{2}a$, since the loading on the bottom surface is uniformly distributed. If we say that clockwise moments are positive, then, for the bottom surface of the overhang,

$$\begin{aligned}
\text{pressure on bottom surface} &= \tfrac{2}{3}\rho g h \\
\text{force on bottom surface} &= \tfrac{2}{3}\rho g h W a \\
\text{CW moment due to force on bottom surface} &= \tfrac{2}{3}\rho g h W a \times \tfrac{1}{2}a
\end{aligned}$$

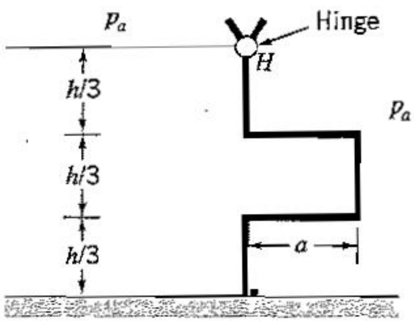


Figure 2.3: Gate with overhang, hinged at the level of the free surface.

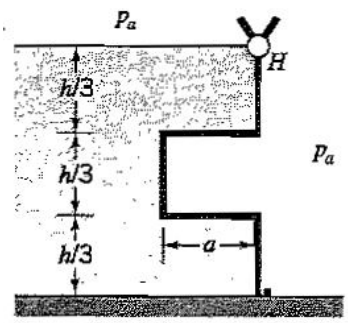


Figure 2.4: Gate with cut-out.

Similarly, for the top surface of the overhang, we find for the clockwise moment:

$$\begin{aligned}
\text{pressure on top surface} &= \tfrac{1}{3}\rho g h \\
\text{force on top surface} &= \tfrac{1}{3}\rho g h W a \\
\text{CW moment due to force on top surface} &= -\tfrac{1}{3}\rho g h W a \times \tfrac{1}{2} a
\end{aligned}$$

Therefore the resultant clockwise moment exerted by the overhang is $\frac{1}{3}\rho g h W a \times \frac{1}{2}a$.

In the second, alternative approach, we note that the moment produced by the overhang is due to the weight of water contained in it, and this weight is the volume multiplied by the density, that is, $\frac{1}{3}\rho g h W a$. The moment arm of this weight is given by the distance to the centroid of the volume, located at a point $\frac{1}{2}a$ out from the hinge, so that the total clockwise moment exerted by the overhang is $\frac{1}{3}\rho g h W a \times \frac{1}{2}a$, as before.

The sum of all the moments is given by the moment due to the weight of water contained by the overhang, plus the moment due to the water acting on the vertical portions of the gate. That is, a can be found from:

$$\tfrac{1}{3}\rho g h W a \times \tfrac{1}{2}a - \tfrac{1}{2}\rho g W h^2 \times \tfrac{2}{3}h \;=\; 0$$

That is,

$$a = \sqrt{2}h$$

Example 2.6: A final hinged gate

What happens if the overhang in Example 2.4 was negative, as shown in Figure 2.4? That is, if instead of an overhang there was a cut-out of the same dimensions?

Solution: The moment due to the water acting on the vertical parts of the gate is the same as in Example 2.4: about the hinge, it is counterclockwise, of magnitude $\frac{1}{2}\rho g W h^2 \times \frac{2}{3}h$. For the cut-out, the moment due to pressure acting on the bottom surface is $\frac{2}{3}\rho g h W a \times \frac{1}{2}a$ in the clockwise direction, and for the top surface it is $\frac{1}{3}\rho g h W a \times \frac{1}{2}a$ in the counterclockwise direction. The resultant moment produced by the horizontal surfaces of the cut-out is therefore $\frac{1}{3}\rho g h W a \times \frac{1}{2}a$ in the clockwise direction, exactly the same as that found in Example 2.4. So the moment equilibrium of the gate shown in Figure 2.4 is the same as for the gate shown in Figure 2.3, and the critical value of a is also the same.

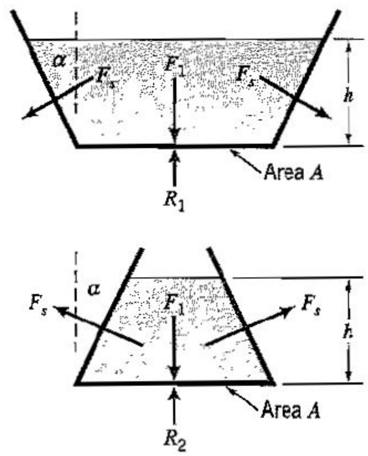


Figure 2.5: Pressure exerted on the bottom of different shaped containers.

Figure 2.6: Pascal's rain barrel. From Martin and Conner, *Basic Physics*, 8th ed., published by Whitcombe & Tombs Pty. Ltd., Melbourne, Australia, 1962, with permission.

Example 2.7: Weight and forces due to pressure

Consider the two containers shown in Figure 2.5. They have a width w, and they are filled with water to the same height, h. Assume the weight of each container is negligible. From the hydrostatic equation, we know that the pressure at depth h will be the same for both vessels. Therefore, if the bottom area A is the same, the force exerted on the bottom of the container will be equal, that is, $F_1 = F_2$, in spite of the obvious difference in the total weight of liquid contained. Is there a paradox?

Solution: We must be careful to consider all the forces, including the forces acting on the side walls, the bottom of the container, and the reaction from the surface it is resting on. Note that the forces on the side walls (F_s) act at right angles to the wall, and that the horizontal components of the side-wall forces cancel out since they act in opposing directions.

Consider the forces acting on the container shown in the upper part of Figure 2.5. The vertical component of of F_s acts downward, and for static equilibrium

$$R_1 = \rho g h A + 2F_s \sin\alpha$$

The force F_s is given by equation 2.14, where $\alpha = \frac{1}{2}\pi - \theta$. Hence,

$$R_1 = \rho g h A + \rho g W h^2 \tan\alpha$$

which is exactly equal to the weight of the water in the container, as it should be.

Consider now the forces acting on the container shown in the lower part of Figure 2.5. The vertical component of F_s acts upward, and for static equilibrium

$$R_2 = \rho g h A - 2F_s \sin\alpha$$

Hence,

$$R_2 = \rho g h A - \rho g W h^2 \tan\alpha$$

which is exactly equal to the weight of the water in this particular container. So the external reaction forces on the base in each case (R_1 and R_2) are equal to the weight of the liquid in the container, so that $R_1 > R_2$, but the force acting on the base from the inside of the container is the same in each case, since it is due to hydrostatic pressure which only depends on depth.

In 1646, the French scientist Blaise Pascal gave an interesting illustration of this principle. He placed a long vertical pipe in the top of a barrel filled with water and found that by pouring water into the pipe he could burst the barrel even though the weight of water added in the pipe was only a small fraction of the force required to break the barrel (Figure 2.6). This is a case where the force due to hydrostatic pressure is overwhelmingly greater than the external reaction force acting on the base of the barrel.

Example 2.8: Choosing moment axes

Consider a rectangular tank filled with water, with a triangular gate located in its side-wall (Figure 2.7). The top edge of the gate is level with the surface of the water. Atmospheric pressure acts everywhere outside the tank. The gate is held on by three bolts. Find the force in each bolt.

Solution: Since the gate is in static equilibrium, the sum of the forces in any given direction must be zero. That is, in the horizontal direction,

$$\Sigma F_x = F_1 + F_2 + F_3 - F = 0$$

where F is the force exerted by the water pressure on the gate. It is clear that we need additional information to solve for F_1, F_2 and F_3. This information will come from the moment equation; since the gate is in static equilibrium, the sum of the moments must also be zero. This is true for any axis we choose, but some axes are better than others. For example, if we choose the z-axis, F_1 and F_2 need not be considered since they have no moment about the z-axis (their moment arm about the z-axis is zero). Therefore the moment exerted by F_3 about the z-axis must balance the moment exerted by F about the z-axis, and F_3 can be found directly. Similarly, F_2 can be found by taking moments about the y-axis, and so, together with $\Sigma F_x = 0$, we have three equations for three unknowns.

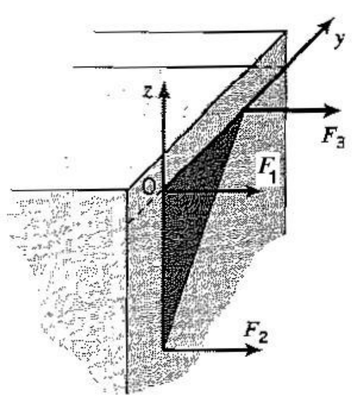


Figure 2.7: Wall with triangular gate.

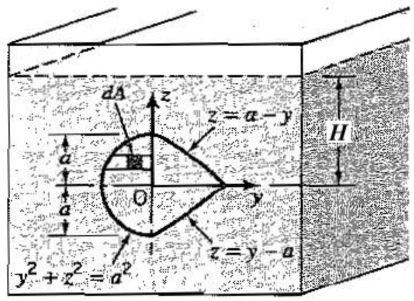


Figure 2.8: Gate with circular section on one side, and triangular section on the other.

Example 2.9: Complex two-dimensional surfaces

What is the resultant force on the gate shown in Figure 2.8, given that atmospheric pressure acts everywhere?

Solution: The shape of the gate is rather complex, and it is best to treat it in two parts, where the forces acting on the left hand side and the right hand side of the gate are found separately, and the resultant force is found by simple addition. We will not give the full solution here, only an outline of how the problem may be solved. Here is the basic result for the left hand side

$$F_1 = \iint p_g \, dy \, dz = \int_{-a}^{a} p_g(z) \left(\int_{-\sqrt{a^2-z^2}}^{0} dy \right) dz = \int_{-a}^{a} p_g(z) \sqrt{a^2 - z^2} \, dz$$

where $p_g(z) = \rho g(H - z)$. The integration can be completed using a table of standard integrals.

To find the force acting on the right hand side, we need to subdivide the area further: the top half (A_{2t}) is described by the equation $z = a - y$, and the bottom half (A_{2b}) is described by the equation $z = y - a$. Here is the basic result for the top half

$$F_{2t} = \int p_g dA_{2t} = \int_{0}^{a} p_g(z) \left(\int_{0}^{a-z} dy \right) dz = \int_{0}^{a} p_g(z) \, (a - z) \, dz$$

For the bottom half

$$F_{2b} = \int p_g dA_{2b} = \int_{-a}^{0} p_g(z) \left(\int_{0}^{a+z} dy \right) dz = \int_{-a}^{0} p_g(z) \, (a + z) \, dz$$

Finally

$$F = F_1 + F_{2t} + F_{2b}$$

Example 2.10: The tip of an iceberg

Consider an iceberg floating in seawater. Find the fraction of the volume of the iceberg that shows above the sea surface.

Solution: If the volume of the iceberg is υ, and the fraction showing above the surface is $\Delta\upsilon$, the buoyancy force acting up on the iceberg is given by $\rho_{sw} g(\upsilon - \Delta\upsilon)$, where ρ_{sw} is the density of sea water. For static equilibrium, this must equal the weight of the iceberg, which is given by $\rho_{ice} g\upsilon$, where ρ_{ice} is the density of ice. That is

$$\rho_{sw} g \, (\upsilon - \Delta\upsilon) = \rho_{ice} g\upsilon$$

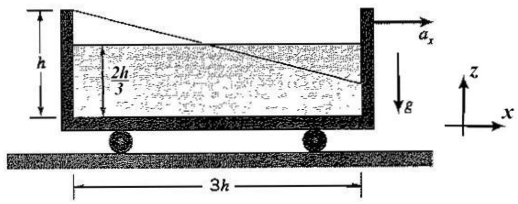


Figure 2.9: Fluid in rigid body motion under the action of gravity and a constant horizontal acceleration.

so that

$$\frac{\Delta v}{v} = 1 - \frac{\rho_{ice}}{\rho_{sw}}$$

Ice has a density of $920\,kg/m^3$ (it is made up of fresh water), and seawater has a density of $1025\,kg/m^3$ (Table Appendix-C.7). Hence

$$\frac{\Delta v}{v} = 0.102$$

so that only about 10% of the bulk of an iceberg is visible above the surface of the sea (see Figure 2.3).

Example 2.11: Rigid body motion

For the case shown in Figure 2.9, find the horizontal acceleration that would make the water spill out of the container.

Solution: From equation 2.30, we know that the slope of the water surface under a constant horizontal acceleration is given by

$$\frac{dz}{dx} = -\frac{a_x}{g + a_z} = -\frac{a_x}{g}$$

since $a_z = 0$. From the shape of the container, we see that the water will spill out of the container when

$$\frac{dz}{dx} = -\frac{\frac{1}{3}h}{\frac{3}{2}h} = -\frac{2}{9}$$

which requires a horizontal acceleration

$$a_x = \tfrac{2}{9}g$$

Problems

2.1 Express the following pressures in psi:
(a) $2.5 \times 10^5\,Pa$
(b) $4.3\,bar$
(c) $31\,in.$ Hg
(d) $20\,ft\,H_2O$

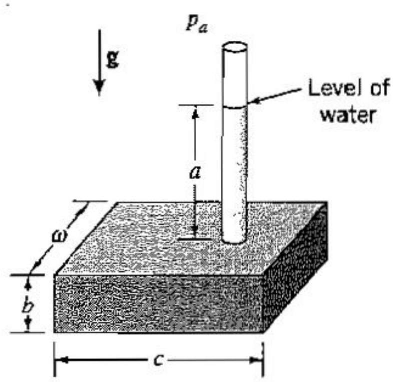


Figure P2.6

2.2 Express the following pressures in Pa:
(a) $3\,psia$
(b) $4.3\,bar$
(c) $31\,in.$ Hg
(d) $8\,m\,H_2O$

2.3 Express the following absolute pressures as gauge pressures in SI and BG units:
(a) $3\,psia$
(b) $2.5 \times 10^5\,Pa$
(c) $31\,in.$ Hg
(d) $4.3\,bar$
(e) $20\,ft\,H_2O$

2.4 In a hydraulic press, a force of $200\,N$ is exerted on the small piston (area $10\,cm^2$). Determine the force exerted by the large piston (area $100\,cm^2$), if the two pistons were at the same height.

2.5 For the hydraulic press described in the previous problem, what is the the force produced by the large piston if it were located $2\,m$ above the small piston? The density of the hydraulic fluid is $920\,kg/m^3$.

2.6 A device consisting of a circular pipe attached to a rectangular tank is filled with water as shown in Figure P2.6. Neglecting the weight of the tank and the pipe, determine the total force on the bottom of the tank. Compare the total weight of the water with this result and explain the difference.

2.7 Two cylinders of cross-sectional areas A_1 and A_2 are joined by a connecting passage, as shown in Figure P2.7. A force F_1 acts on one piston, and a force F_2 acts on the other. Find F_2 in terms of F_1, A_1, A_2, ρ, g and H, where ρ is the fluid density. Ignore the weights of the pistons.

2.8 If the average person can generate about 1 psi (7000 Pa) of negative gauge pressure (that is, suction) in their mouth, what's the longest straw they can possibly drink out of? The density of air is $1000\,kg/m^3$.

2.9 The gauge pressure at the liquid surface in the closed tank shown in Figure P2.9 is $4.0\,psi$. Find h if the liquid in the tank is
(a) water

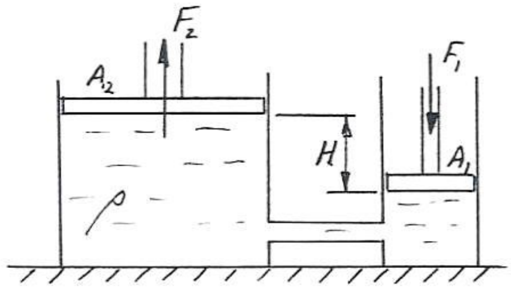


Figure P2.7

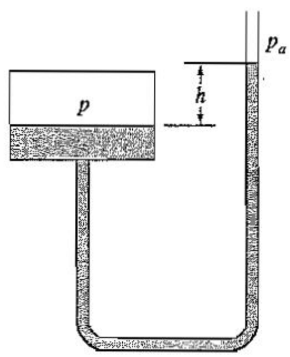


Figure P2.9

(b) kerosene
(c) mercury

2.10 Find the maximum possible diameter of the circular hole so that the tank shown in Figure P2.10 remains closed. The lid has a mass $M = 50\,kg$.

2.11 A hollow cylinder of diameter $1\,m$ has a closed bottom, which is pushed into a swimming pool to a depth of $3\,m$. The cylinder is open to atmosphere at the top, and the swimming pool is at sea level.
(a) Find the force acting on the bottom of the cylinder when the pool contains fresh water, and the air pressure is taken to be constant everywhere.
(b) How does the answer to part (a) change when the pool contains sea water?
(c) How does the force found in part (a) change when the variation in air pressure inside

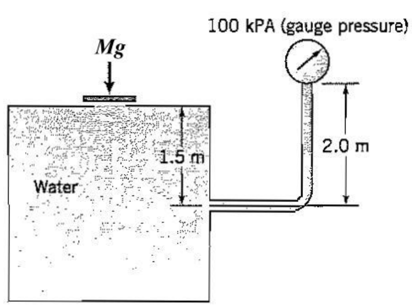


Figure P2.10

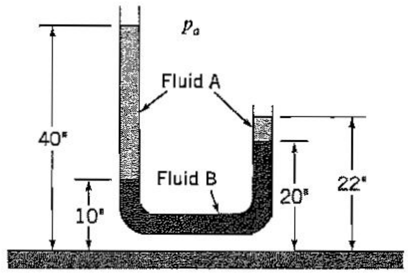


Figure P2.12

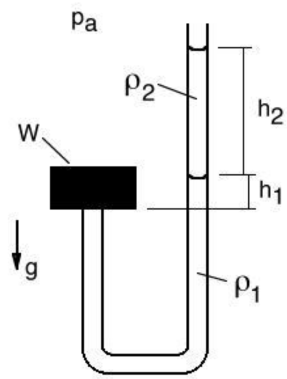


Figure P2.14

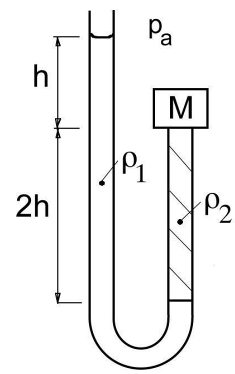


Figure P2.15

the cylinder is taken into account?
(d) Do the answers to parts (a), (b) and (c) change if the swimming pool is moved to the top of a 5000 m mountain?

2.12 For the manometer shown in Figure P2.12, both legs are open to the atmosphere. It is filled with liquids A and B as indicated. Find the ratio of the liquid densities.

2.13 A U-tube manometer consists of a glass tube bent into a U-shape, and held vertically. Water is poured in one side and allowed to settle so that the free surface in both legs is at the same height. Alcohol (specific gravity 0.8) is added to the right hand leg without mixing so that after the liquids have settled the depth of alcohol in the right hand leg is 100 mm. Find the difference in height between the left and the right leg of the manometer.

2.14 A manometer of constant cross-sectional area A contains two fluids of density ρ_1 and ρ_2, as shown in Figure P2.14. One end of the manometer is closed by a weight W, and the other end is open to atmospheric pressure p_a. Find W in terms of ρ_1, ρ_2, h_1, h_2, g, and A when h_2 is large enough to be on the point of lifting up the weight.

2.15 A manometer tube is filled with a two fluids with densities $\rho_1 = 1000$ kg/m^3, and $\rho_2 = 800$ kg/m^3, as shown in Figure P2.15. The tube has a diameter of 10 mm. One end is open to atmospheric pressure, and the other end is blocked by a block of steel of mass $M = 0.1$ kg. Find the height h where the steel block is about to be dislodged.

2.16 Find the pressure at an elevation of 3000 m if the temperature of the atmosphere decreases at a rate of $0.006°K/m$. The ground-level temperature is $15°C$, and the barometer reading is 29.8 in. Hg. (The gas constant for air is $= 287.03\ m^2 s^2 K$.)

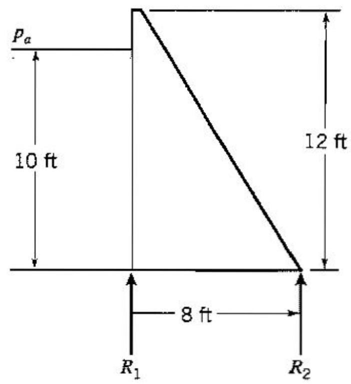


Figure P2.22

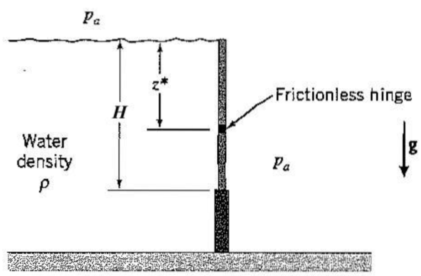


Figure P2.23

2.17 Find the reduction in pressure in N/m^2 at an altitude of $1000\,m$ if the density of air ρ in kg/m^3 decreases with altitude z (in meters) according to $\rho = 1 - 2 \times 10^{-4} z$.

2.18 At a particular point in the Pacific Ocean, the density of sea water increases with depth according to $\rho = \rho_0 + mz^2$, where ρ_0 is the density at the surface, z is the depth below the surface and m is a constant. Develop an algebraic equation for the pressure as a function of depth.

2.19 An underwater cave contains trapped air. If the water level in the cave is 60 m below the surface of the ocean, what is the air pressure in the cave? Express the answer in terms of gauge pressure in atmospheres.

2.20 The vertical wall of a dam can withstand a total force of $500,000\,N$. It has a width of $10\,m$. At what depth of (fresh) water will it fail?

2.21 Repeat the previous question, where this time the wall is inclined at $60°$ to the horizontal.

2.22 Given that the specific gravity of concrete is 2.4, find the vertical reactions R_1 and R_2 per unit width of the concrete dam shown in Figure P2.22.

2.23 The gate shown in Figure P2.23 has a width W and a height h and it is pivoted on a frictionless hinge at a point z^* below the surface of the water. The top of the gate is level with the surface of the water. The water is of density ρ, and outside the tank the pressure is uniform everywhere and equal to the atmospheric pressure.
(a) Find the magnitude of the resultant force F on the gate, in terms of ρ, g, W and h.

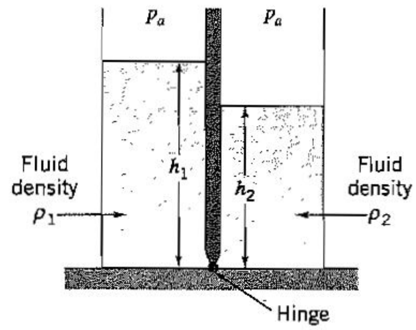


Figure P2.25

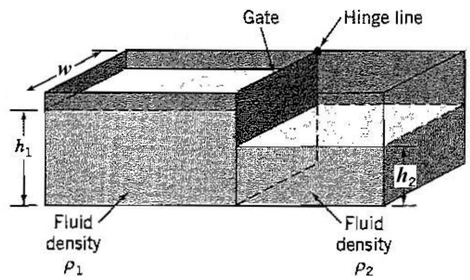


Figure P2.26

(b) Find the value of z^* so that there is no resultant moment about the hinge tending to open the gate.

2.24 A gate $3\,ft$ square in a vertical dam has air at atmospheric pressure on one side and water on the other. The resultant force acts $2\,in.$ below the center of the gate. How far is the top of the gate below the water surface?

2.25 A gate of width W stands vertically in a tank, as shown in Figure P2.25, and it is connected to the bottom of the tank by a frictionless hinge. On one side the tank is filled to a depth h_1 by a fluid density ρ_1; on the other side it is filled to a depth h_2 by a fluid density ρ_2. Find h_2/h_1 in terms of ρ_2/ρ_1 if the gate is in static equilibrium.

2.26 The tank shown in Figure P2.26 has a gate which pivots on a vertical, frictionless hinge locate at the top of the gate. Find the ratio of the depths of water h_1/h_2 in terms of the densities ρ_1 and ρ_2 when the gate is in static equilibrium.

2.27 A rectangular door of width w and height H is located in a vertical wall. There is water of density ρ on one side of the door, so that the top of the door is a distance d below the level of the water. The door is held in the wall by a frictionless hinge at the top of the door.
(a) Find the force acting on the door due to the water.
(b) Find the minimum moment about the hinge required to keep the door shut.

2.28 A rectangular door of width W and height $2b$ is placed in side wall of a tank containing water of density ρ and depth $3b$, as shown in Figure P2.28. The tank is open to the atmosphere. Find the resultant force due to the water acting on the door, and where it acts, in terms of ρ, W, g, and b. Show your coordinate system, and all working.

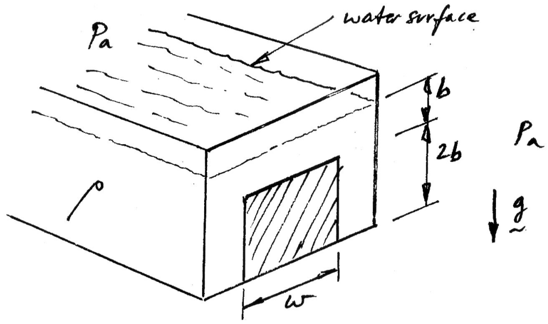


Figure P2.28

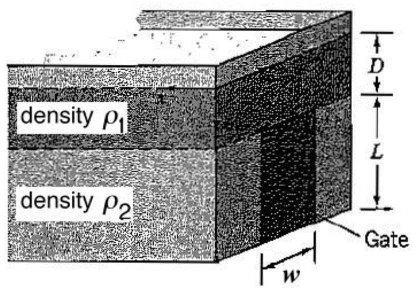


Figure P2.29

2.29 A rectangular gate is located in a dam wall as shown in Figure P2.29. The reservoir is filled with a heavy fluid of density ρ_2 to a height equal to the height of the gate, and topped with a lighter fluid of density ρ_1.
(a) Write down the variation of pressure for $z \leq D$, and for $D \leq z \leq D + L$.
(b) Find the resultant force acting on the gate due to the presence of the two fluids.
(c) Find the point where this resultant force acts.

2.30 A primitive safety valve for a pressure vessel containing water is shown in Figure P2.30. The water has a density ρ, and it has a constant depth H. The gauge pressure exerted on the surface of the water is p_w, and the pressure outside the vessel is atmospheric. The gate is rectangular, of height B and width w, and there is a spring at the hinge which exerts a constant clockwise moment M_h. Find p_w for which the gate is just on the point of opening.

2.31 A square gate of dimension b separates two fluids of density ρ_1 and ρ_2, as shown in Figure P2.31. The gate is mounted on a frictionless hinge. As the depth of the fluid on the right increases, the gate will open. Find the ratio ρ_2/ρ_1 for which the gate is just about to open in terms of H_1, H_2 and b.

2.32 The symmetric trough shown in Figure P2.32 is used to hold water. Along the length of the trough, steel wires are attached to support the sides at distances w apart. Find the magnitude of the resultant force exerted by the water on each side, and the magnitude of the tension in the steel wire, neglecting the weight of the trough.

2.33 A rigid uniform thin gate of weight Mg and constant width W is pivoted on a frictionless hinge as shown in Figure P2.33. The water depth on the left hand side of the gate is H and remains constant. On the right hand side of the gate the level of water is

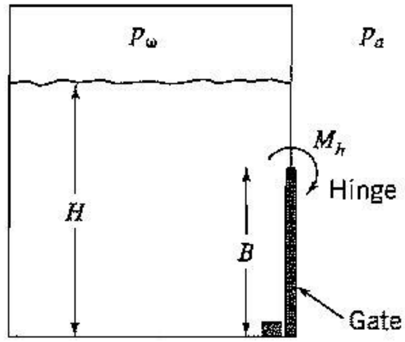


Figure P2.30

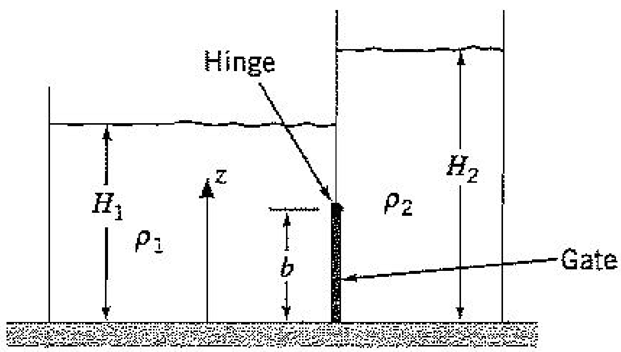


Figure P2.31

slowly decreased until the gate is just about to open. Find the depth D at which this occurs.

2.34 Repeat Problem 2.27, where this time the wall and the door are inclined at an angle θ to the horizontal.

2.35 The rectangular gate shown in Figure P2.35 (of width W and length L) is made of a homogeneous material and it has a mass m. The gate is hinged without friction at point B. Determine the mass required to hold the gate shut when the water depth at the point B is H.

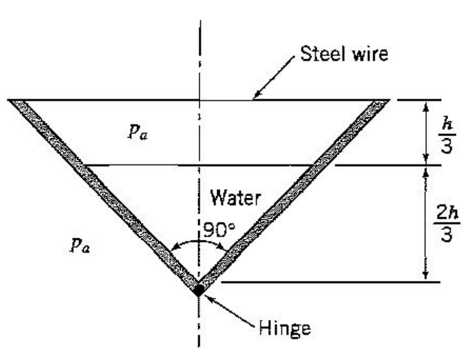


Figure P2.32

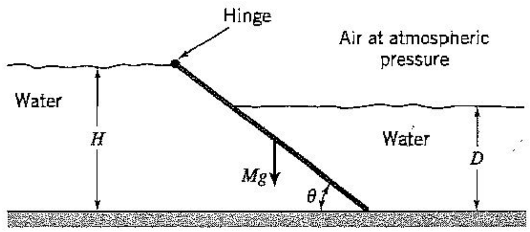


Figure P2.33

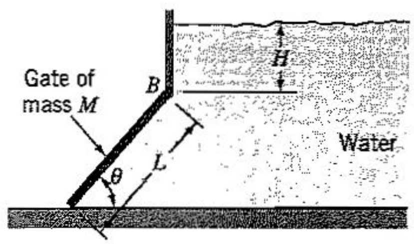


Figure P2.35

2.36 A gate of constant width W is hinged at a frictionless hinge located at point O and rests on the bottom of the dam at the point A, as shown in Figure P2.36. Find the magnitude and direction of the force exerted at the point A due to the water pressure acting on the gate.

2.37 A rectangular gate $1\,m$ by $2\,m$, is located in a wall inclined at 45°, as shown in Figure P2.37. The gate separates water from air at atmospheric pressure. It is held shut by a force F. If the gate has a mass of $100\,kg$, find F.

2.38 Figure P2.38 shows a very delicate balancing act. On one side of a weightless wedge there is fluid of density ρ_1, and on the other side there is fluid of density ρ_2. If the wedge is just balanced, find the ratio ρ_2/ρ_1 in terms of H_1, H_2 and θ.

2.39 A rectangular window of width W is set into the sloping wall of a swimming pool, as shown in Figure P2.39. Find the point of action of the resultant force acting on the window.

2.40 A triangular trough contains two fluids, with densities ρ_1 and ρ_2, and depths H_1 and H_2, respectively, as shown in Figure P2.40. Find the resultant hydrostatic force acting on the divider, and where it acts.

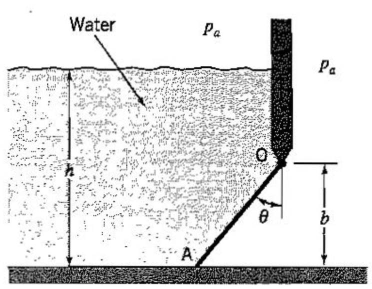


Figure P2.36

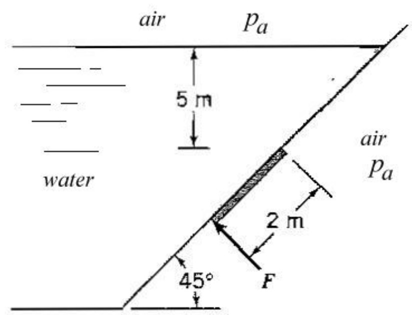


Figure P2.37:

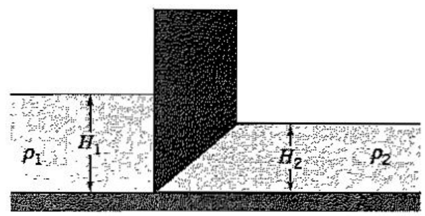


Figure P2.38

2.41 A rigid, weightless, two-dimensional gate of width W separates two liquids of density ρ_1 and ρ_2, respectively, as shown in Figure P2.41. The gate pivots on a frictionless hinge and it is in static equilibrium. Find the ratio ρ_2/ρ_1 when $h = b$.

2.42 A gate of uniform composition, and of length L, width w, and mass m, is held by a frictionless hinge at H and a string at A, as shown in Figure P2.42. Find the water depth h at which the tension in the string is zero, in terms of m, L, w, θ and ρ (the density of the water). Indicate all your working clearly, and state all your assumptions.

2.43 A symmetrical, triangular prism of uniform composition and width w is balanced vertically on its apex as shown in Figure P2.43. A fluid of depth H_1 and density ρ_1 acts on the left hand side, while a fluid of depth H_2 and density ρ_2 acts on the right hand side. Find the ratio of the densities in terms of the ratio of the depths. Show all your working, and state all your assumptions.

2.44 A gate of length L and width w separates two liquids of different density, as shown in Figure P2.44. The gate is held by a frictionless hinge at H. Find the ratio of densities ρ_2/ρ_1 in terms of L, θ, h_1 and h_2 when the gate is about to open. Indicate all your working clearly, and state all your assumptions.

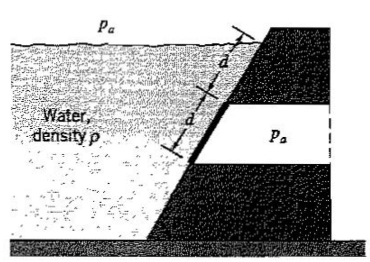


Figure P2.39

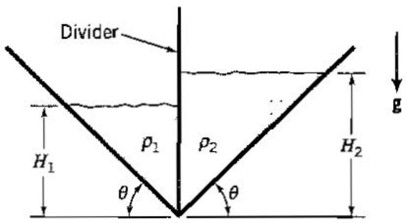


Figure P2.40

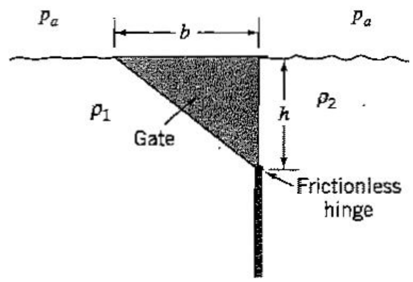


Figure P2.41

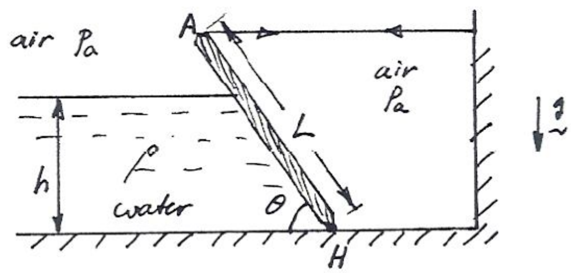


Figure P2.42

2.45 A rectangular door of width w and length b is placed in the sloping side wall of a tank containing water of density ρ and depth H. The top of the door is hinged without friction at point T, as shown in Figure P2.45. On the other side of the door is a pressurized chamber containing air at gauge pressure p_c. Find the value of p_c at the point where the door is just about to open, in terms of ρ, w, g, b, H and θ. The top of the tank is open to the atmosphere, and the door has no mass. Show your coordinate system(s).

2.46 An underwater cave has a gate of width w, uniform thickness, and weight W mounted on a frictionless hinge at a point 0, as shown in Figure P2.46. Find the internal gauge pressure p_c when the gate is on the point of opening. Indicate all your working clearly, and state all your assumptions. (Hint: use gauge pressure throughout).

2.47 A horizontal lever arm of length $3a$ is pivoted without friction at a point $2a$ along its length. A cubic mass of density ρ, dimension b and mass M_1 hangs from the long part of the lever arm and a mass M_2 hangs from the short part of the lever arm. The lever arm is in balance when mass M_1 is immersed in water to a depth c. When $M_1 = M_2$, express the specific gravity of the M_1 material in terms of b and c.

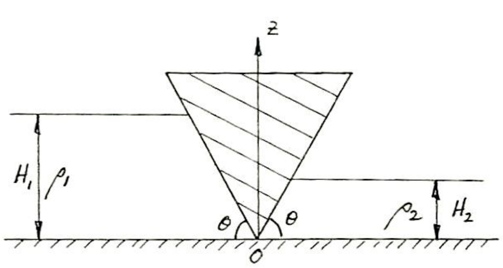


Figure P2.43

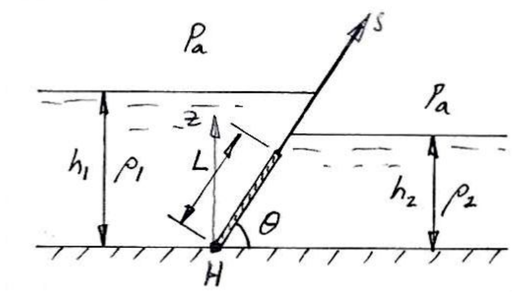


Figure P2.44

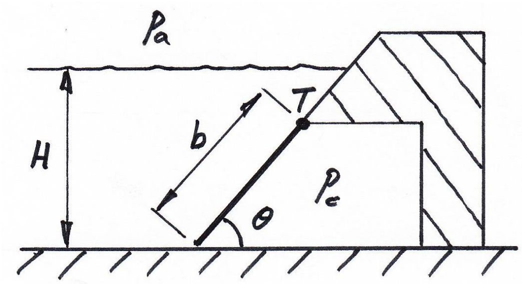


Figure P2.45

2.48 A gate of length L, width w, uniform thickness t, and density ρ_s is held by a frictionless hinge at O and rests against a vertical wall at P, as shown in Figure P2.48. On the one side the gate is exposed to atmospheric pressure, and on the other side there is water of density ρ. When the water depth is h, the gate is about to open. Under these conditions, find the thickness of the plate in terms of L, ρ_s ρ, θ, and h. Indicate all your working clearly, and state all your assumptions, and indicate your coordinate system.

2.49 A gate of width w, length L, mass M, and uniform thickness, separates two fluids of equal density ρ, with depths b and $2b$, as shown in Figure P2.49. The gate is pivoted without friction at the hinge H, and it is in equilibrium when it is inclined at an angle θ to the horizontal. Find the mass M in terms of ρ, b, w, θ and L. Show your coordinate system, and all your working.

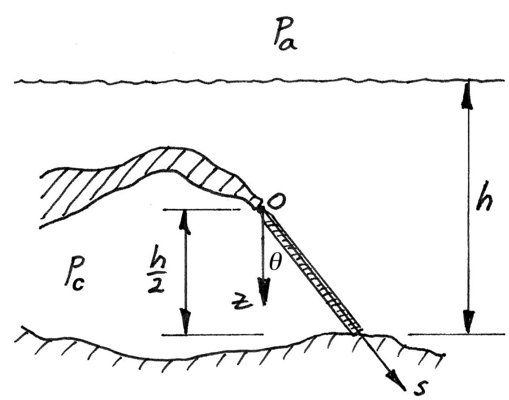


Figure P2.46

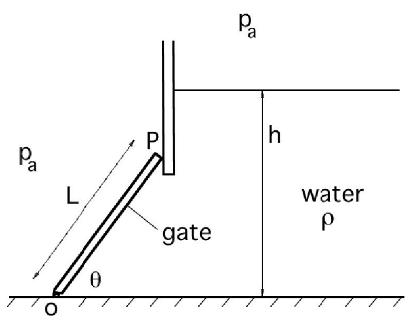


Figure P2.48

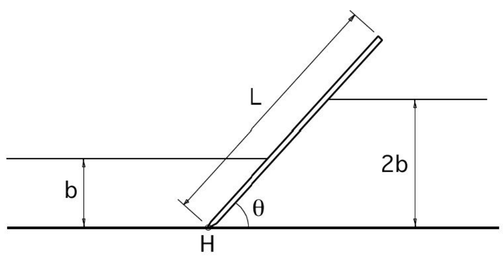


Figure P2.49

2.50 A sightseeing boat has a window in its side of length L and width w, as shown in Figure P2.50. The bottom of the window is a depth h below the surface. The water has a density ρ. Find the magnitude of the resultant force acting on the window due to the water pressure, and its location, in terms of L, ρ, θ, g and h. Indicate all your working clearly, and state all your assumptions, and indicate your coordinate system.

2.51 A weightless gate of width w separates two reservoirs of water of depths h_1 and h_2, as shown in Figure P2.51. The two sides of the gate are inclined to the horizontal at angles θ_1 and θ_2. The gate is pivoted without friction at the hinge H. Find the ratio h_2/h_1 when the gate is in static equilibrium in terms of θ_1 and θ_2. Show your coordinate system, and all your work.

2.52 A gate of length L and width w is hinged without friction at the point O, as shown in Figure P2.52. The gate is of uniform thickness and it has a weight of W. There is another

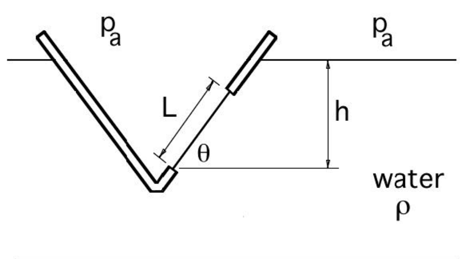


Figure P2.50

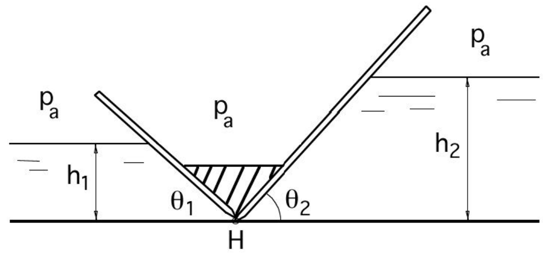


Figure P2.51

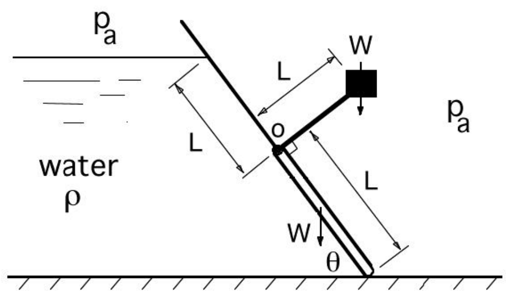


Figure P2.52

weight W suspended a distance L from the hinge by a link that is rigidly connected to the gate, and at right angles to it. The water has a density ρ. Find the magnitude of W, in terms of L, ρ, θ, and g so that the gate is on the point of opening. Show all your working and state all your assumptions. Indicate your coordinate system clearly.

2.53 A symmetrical trapdoor with doors of width w and length L is located on the bottom of a tank that is filled with water of density ρ to a depth h, as shown in Figure P2.53. The doors are hinged without friction at the point O, and they have a negligible mass. Find the value of p_g at which the doors are about to open, where p_g is the gauge pressure on the air side of the trapdoors.

2.54 The closed tank shown in Figure P2.54 is filled with water of density ρ to a depth h. The absolute pressure in the air trapped above the water is $p_1 = 2p_a$, where p_a is the atmospheric pressure. The tank has a width w (into the page)
(a) Find the height H of the open water column in terms of p_a, ρ, g and h.

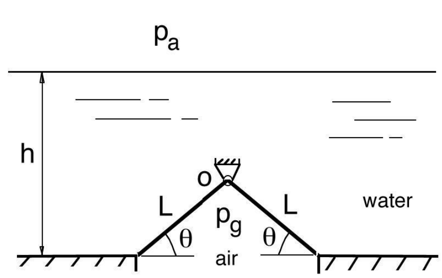


Figure P2.53

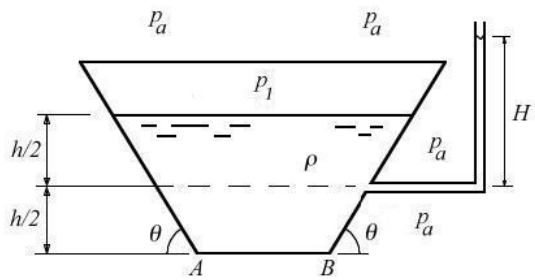


Figure P2.54

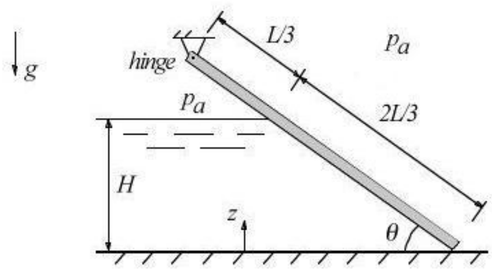


Figure P2.55

(b) Find the magnitude of the resultant force due to the water pressure acting on one of the sloping sides of the tank. Remember that $p_1 = 2p_a$.
(c) Find the moment (magnitude and direction) due to the water pressure acting on one of the sloping sides of the tank about the point B. Remember that $p_1 = 2p_a$.

2.55 A steel gate of density ρ_s, width w, thickness t, and length L is pivoted at a frictionless hinge, as shown in Figure P2.55. The dam is filled with water of density ρ until the gate is about to open. The atmospheric pressure is p_a.
(a) Find the resultant force (magnitude and direction) due to the water pressure acting on the gate in terms of ρ, g, θ, L, w and H.
(b) Find the density of steel in terms of ρ, g, θ, t, L and H.
(c) If the depth of the water was halved, what moment must be applied at the hinge if the gate is still about to open?

2.56 [1] Your first foray into ice fishing takes a bad turn when a sudden overnight thaw sends your shack to the bottom of the lake with you sleeping inside, as shown in Figure P2.56. Luckily your shack is completely watertight and maintains atmospheric pressure inside. It settles on the bottom at an angle θ. On one wall of the shack (the one pointing down in the figure), there is a square window measuring $H/3$ on a side that is hinged on the top and swings outward. The top of the window is $3H$ below the surface.
(a) What is the minimum force required to open the window, in terms of ρ, g, H and θ?
(b) Where would you need to apply this force, in terms of H and θ?

2.57 A rigid gate of width w is pivoted at a frictionless hinge as shown in Figure P2.57. A force F acts at the top of the gate so that it is at right angle to the gate. The dam is filled with water of density ρ of depth $2H$. The atmospheric pressure p_a acts everywhere outside the water.

[1] With thanks to Lester Su

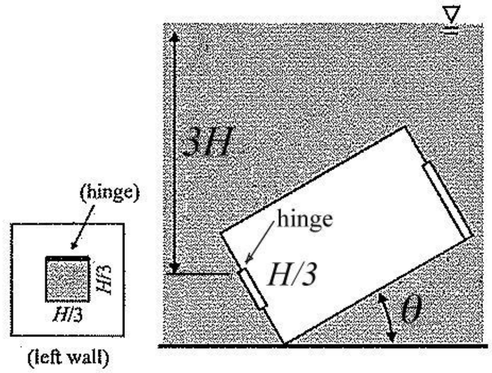


Figure P2.56

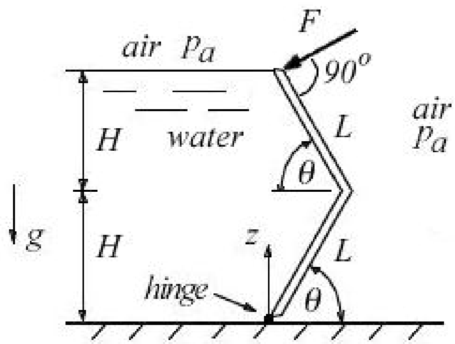


Figure P2.57

(a) Find the resultant force vector due to the water pressure acting on the gate in terms of ρ, g, θ, L, w and H.
(b) If the water depth was reduced to H, find the minimum force F required to keep the gate from opening in terms of ρ, g, w, θ, and H.
(c) If the water depth was increased again to $2H$, find the new value of the minimum force F required to keep the gate from opening in terms of ρ, g, w, θ, and H.

2.58 An aquarium of depth h has a passage under the main tank, as shown in Figure P2.58 (the passage goes into the paper).
(a) Find the magnitude and direction of F_p, where F_p is the resultant force due to the water pressure acting on one half of the passage.
(b) Find the line of action of F_p.
(c) Find the magnitude and direction of F_t, where F_t is the resultant force due to the water pressure acting on both halves of the passage.

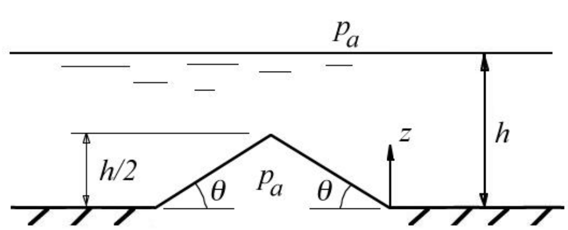


Figure P2.58

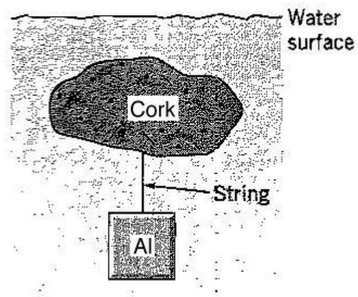


Figure P2.65

2.59 A rectangular block of wood of density ρ_w is floating in water of density ρ. If the block is immersed to 80% of its height, find the specific gravity of the wood.

2.60 A submarine of volume V and weight W is lying on the ocean bottom. What is the minimum force required to lift the submarine to the surface?

2.61 A beach ball of weight Mg and diameter D is thrown into a swimming pool. If the ball just floats, what is the diameter of the ball?

2.62 A square tray measuring $h \times h$, supporting a metal cube of dimension $h/4$, floats on water immersed to a depth $h/10$. Find the specific gravity of the metal cube, neglecting the weight of the tray.

2.63 A rectangular steel barge measuring $15\,m \times 4\,m$ in planform with a depth of $1.5\,m$ floats in water (density $\rho = 1000\,kg/m^3$) to a depth of $1\,m$. Find the weight of the barge. The barge develops a slow leak of $1\,liter/s$. Calculate how long it will take before the barge sinks.

2.64 Determine what fractions of the volume of an ice cube are visible above:
(a) the surface of a glass of fresh water, and (b) the surface of a glass of ethanol. Do these answers change if we were on the surface of the moon?

2.65 A $1\,m^3$ of aluminum, of specific gravity 2.7, is tied to a piece of cork, of specific gravity 0.24, as shown in Figure P2.65. What volume of cork is required to keep the aluminum block from sinking in water if both masses are completely submerged?

2.66 A concrete block rests on the bottom of a lake. The block is a cube, $1\,ft$ on a side. Calculate the force required to hold the block at a fixed depth. The density of concrete is $2400\,kg/m^3$.

2.67 A cube of an unknown material measuring h per side has a weight Mg in air, and an apparent weight of $Mg/3$ when fully submerged in water. Find the specific gravity of the material.

2.68 A triangular wooden block is resting on the bottom of a tank, as shown in Figure P2.68. Water is slowly added to the tank, and when the depth of the water is $h = a/2$, the block is on the point of lifting off the bottom. Find the specific gravity of the wood.

2.69 A two-dimensional symmetrical prism floats in water as shown in Figure P2.69. The base is parallel to the surface. If the specific gravity of the prism material is 0.25, find the ratio a/d.

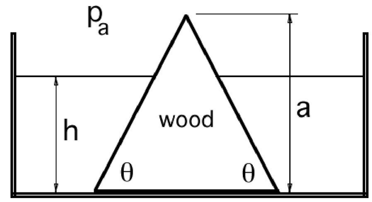


Figure P2.68

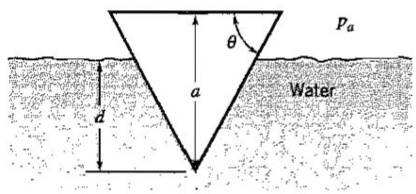


Figure P2.69

2.70 A cubic container of dimension b that is initially empty floats so that the water surface is $b/4$ from the bottom of the container. Water is then added slowly to the container. What is the depth of water inside the container (with respect to b) when it sinks?

2.71 A rectangular barge of length L floats in water (density ρ_w) and when it is empty it is immersed to a depth D, as shown in Figure P2.71. Oil of density ρ_o is slowly poured into the barge until it is about to sink. Find the depth of oil in terms of H, D, ρ_w and ρ_o.

2.72 A cylinder of diameter D and length L has one closed end and one open end. It is filled with air. The open end is lowered vertically down into a pool of water of density ρ, and then the air inside the cylinder is pressurized so that no water is allowed to enter the cylinder. The cylinder floats so that 75% of its length is below the surface of the water.
(a) Find the gauge pressure of the air trapped in the cylinder in terms of ρ, L, and the acceleration due to gravity g.
(b) Find the weight of the cylinder in terms of ρ, L, D and g.

2.73 For the example shown in Figure 2.16,what will happen if the steel cylinders are replaced by logs of wood?

2.74 A cubic tray measuring D along any of its edges floats on water. Gold coins of density ρ_g and volume V are slowly placed in the tray until it is about to sink. At this point there are n coins in the tray. Neglecting the mass of the tray, find the specific gravity of gold in terms of D, n, and V.

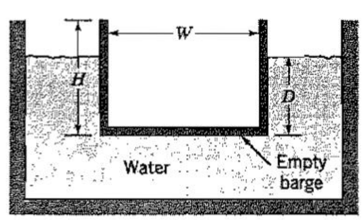


Figure P2.71

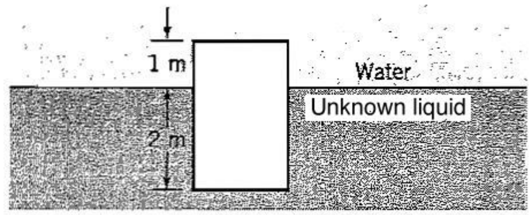


Figure P2.78

2.75 A cork float with dimensions $a \times a \times a$ and specific gravity 0.24 is thrown into a swimming pool with a water surface area $2a \times 2a$, and an initial depth of $2a$. Derive an expression for the pressure at the bottom of the pool.

2.76 A circular cylinder of length 3 ft and diameter 6 $in.$ floats vertically in water so that only 6 $in.$ of its length protrudes above the water level. If it was turned to float horixontally, how far would its longitudinal axis be below the water surface?

2.77 A rigid, helium-filled balloon has a total mass M and volume υ, and it is floating in static equilibrium at a given altitude in the atmosphere. Using Archimedes' Principle, describe what happens when ballast of mass m is dropped overboard. How does this answer change if the balloon is no longer rigid but it is allowed to stretch?

2.78 The circular cylinder shown in Figure P2.78 has a specific gravity of 0.9. (a) If the system is in static equilibrium, find the specific gravity of the unknown liquid. (b) Do you think the system is stable?

2.79 A sunken ship of mass M is to be raised using a spherical balloon. If the density of the gas in the balloon is ρ_b, find the minimum diameter of the balloon necessary to lift the ship when the ballon is fully under water. Neglect the weight of the balloon material, and the volume of the submarine.

2.80 A wooden cube with specific gravity of 1/3 floats on water. The cube measures $b \times b \times b$, and it is immersed to a depth of $b/6$. A spherical balloon of radius R filled with helium is attached to the wooden cube (its volume is $4\pi R^3/3$). The specific gravity of air is 1/800, and the specific gravity of helium is 1/6000. Neglect the weight of the balloon material. Find R.

2.81 A simple buoy can be made from a weighted piece of wood, as shown in Figure P2.81. If the wood has a specific gravity of 0.75, and a cross-section $w \times w$ and a length L, find the weight of steel W attached to the bottom of the buoy such that $7L/8$ is submerged in terms of ρ (density of water), L, w and g. Neglect the volume of the steel.

2.82 A square cylinder measuring $a \times a \times b$ floats on water so that the b-dimension is vertical. The cylinder is made of a uniform material of specific gravity equal to 0.8. When $b << a$, the cylinder is stable under a small angular deflection. When $b >> a$, it is unstable. Find b/a when the cylinder is neutrally stable.

2.83 A rectangular body of length a and width b has a specific gravity of 0.8. It floats at the interface between water and another liquid with a specific gravity of 0.7, as shown in Figure P2.83. If $a > b$, find the position of the body relative to the interface between the two fluids.

2.84 A float and lever system is used to open a drain valve, as shown in Figure P2.84.

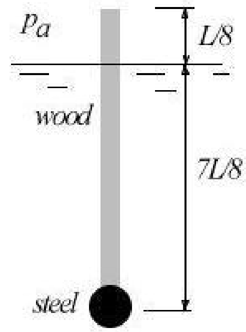


Figure P2.81

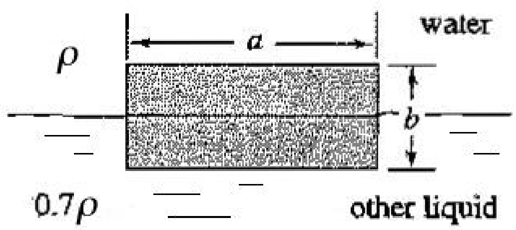


Figure P2.83

The float has a volume V and a density ρ_f. The density of the water is ρ_w. Find the maximum force available to open the drain valve, given that the hinge is frictionless. Hint: first consider the forces acting on the cork float, then consider the free-body diagram of the gate.

2.85 A rectangular cork float is attached rigidly to a vertical gate as shown in Figure P2.85. The gate can swing about a frictionless hinge. Find an expression for the depth of water D where the gate will just open, in terms of a, b, L, and h. The cork float has the same width W as the gate, and the specific gravity of cork is 0.24.

2.86 The two-dimensional gate shown in Figure P2.86 is arranged so that it is on the point of opening when the water level reaches a depth H. If the gate is made of a uniform material that has a weight per unit area of mg, find an expression for H.

2.87 A rectangular gate of width W and height h is placed in the vertical side wall of a tank containing water. The top of the gate is located at the surface of the water, and

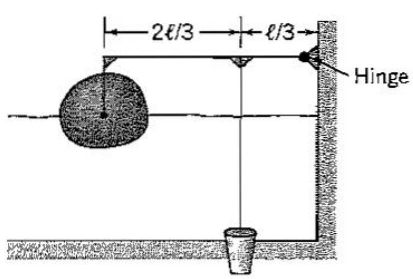


Figure P2.84

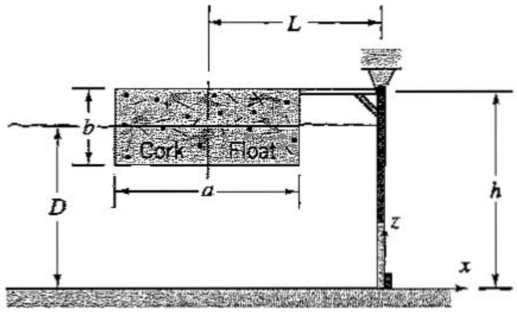


Figure P2.85

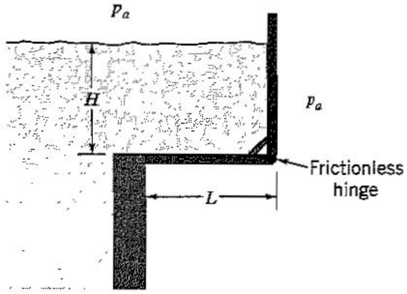


Figure P2.86

a rectangular container of width W and breadth b is attached to the gate, as shown in Figure P2.87. Find d, the depth of the water required to be put into the container so that the gate is just about to open, in terms of h and b. The top and sides of the tank and container are open to the atmosphere. Neglect the weight of the container.

2.88 A rigid gate of width W is hinged without friction at a point H above the water surface, as shown in Figure P2.88. Find the ratio b/H at which the gate is about to open. Neglect the weight of the gate.

2.89 If the weightless gate shown in Figure P2.89 was just on the point of opening, find an expression for B in terms of h.

2.90 A certain volume of water is contained in a square vessel shown in Figure P2.90. Where the sealing edges of the inclined plate come into contact with the vessel walls, the reactive forces are zero.
(a) Find the magnitude and direction of the force F required to hold the plate in position.

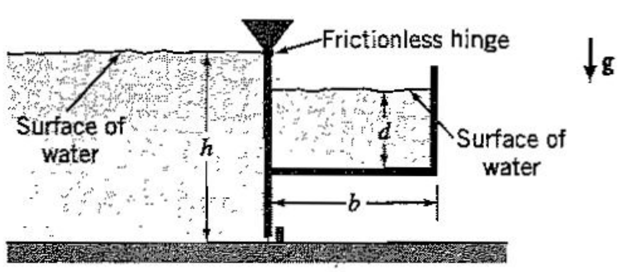


Figure P2.87

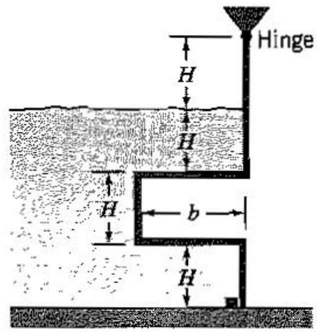


Figure P2.88

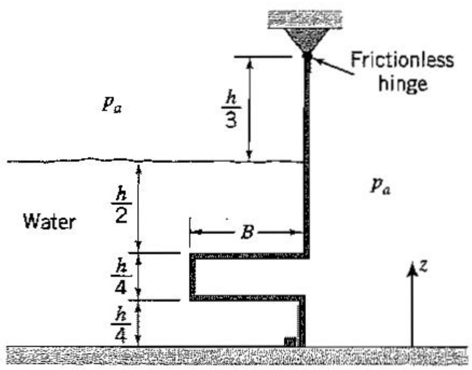


Figure P2.89.

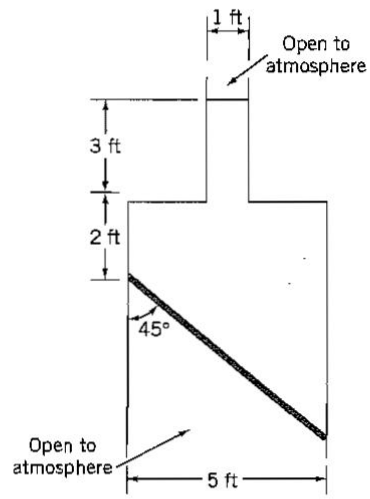


Figure P2.90

The weight of the plate may be neglected.
(b) Where does this force F act?
(c) If the inclined plate is replaced by a horizontal one, find the relative position of the new plate if the force used to maintain position has the same magnitude as before. The volume of fluid remains the same as before.

2.91 Figure P2.91 shows a vessel of constant width W that contains water of density ρ. Air is introduced above the surface of the water at pressure p_1. There is a rectangular blow-off safety lid of width W and length ℓ hinged without friction at point O, which is located at the same height as the water surface. At what pressure will the lid open? Express the answer in terms of W, d, ℓ, θ, ρ, and m, the mass keeping the lid closed.

2.92 A vessel of constant width W is filled with water, and the surface is open to atmospheric pressure. On one side, a relief valve is located, as shown in Figure P2.92. The relief valve has the same width as the vessel, and it has a length D. It is hinged without friction at point A. At point B, a mass m is connected to the gate to keep it shut. Find m in terms of H, D, θ and ρ, where ρ is the density of water.

2.93 The rectangular submarine escape hatch shown in Figure P2.93 (of width W and length L) will open when the constant pressure inside the chamber, p_i, exceeds a critical value p_{ic}. The hatch has negligible mass and it is hinged without friction at point A which

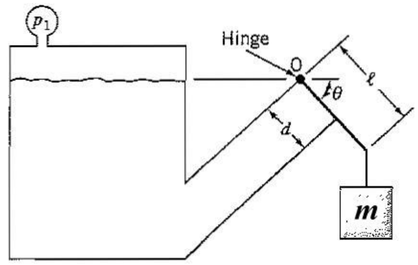


Figure P2.91

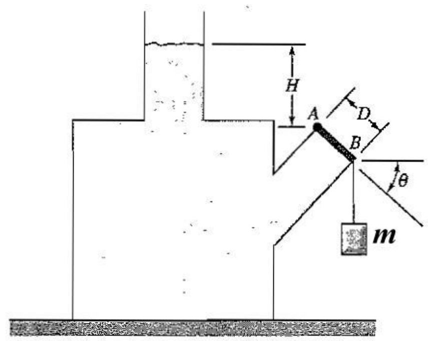


Figure P2.92

is located at a depth D below the surface. Find p_{ic} in terms of W, L, D, ρ, g and θ.

2.94 The gate AB shown in Figure P2.94 is rectangular with a length L and a width W. The gate has a frictionless hinge at point A and it is held against a stop at point B by a weight mg. Neglecting the weight of the gate, derive an expression for the water height h at which the gate will start to move away from the stop.

2.95 The gate shown in Figure P2.95 has a constant width W. The gate has a frictionless hinge at point A and it is held against a stop at point B. Neglecting the weight of the gate, find the magnitude of the resultant force exerted on the gate by the water, and the dimension b for which there is no force on the gate at point B.

2.96 A tank of water of density ρ has a symmetric triangular gate of height H and maximum width $2a$, as shown in Figure P2.96. Calculate the force F exerted by the water

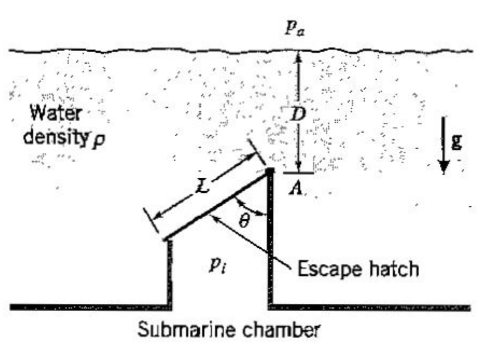


Figure P2.93

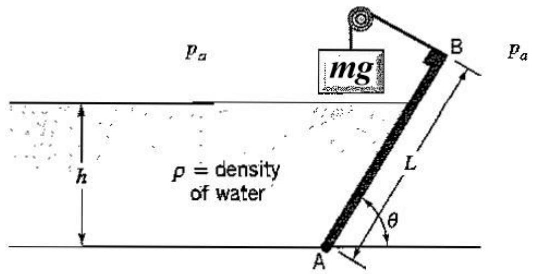


Figure P2.94

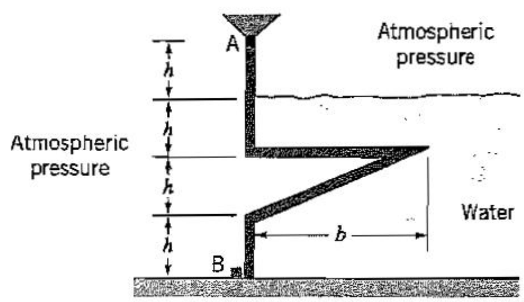


Figure P2.95.

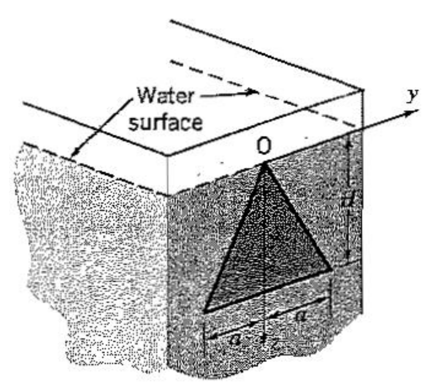


Figure P2.96

on the gate, and where it acts, in terms of ρ, g, h, H and a. The air pressure outside the tank is uniform everywhere.

2.97 A rigid, weightless, two-dimensional gate of width W separates two liquids of density ρ_1 and ρ_2 respectively. The gate has a parabolic face, as shown in Figure P2.97, and it is in static equilibrium. Find the ratio ρ_2/ρ_1 when $h = \ell$.

2.98 A rigid, submerged, triangular gate is hinged as shown in Figure P2.98. (a) Find the magnitude and direction of the total force exerted on the gate by the water. (b) A weight Mg on a rigid lever arm of length L is meant to keep this gate shut. What is the depth of water D when this gate is just about to open? Neglect the weight of the gate, and the weight of the lever arm.

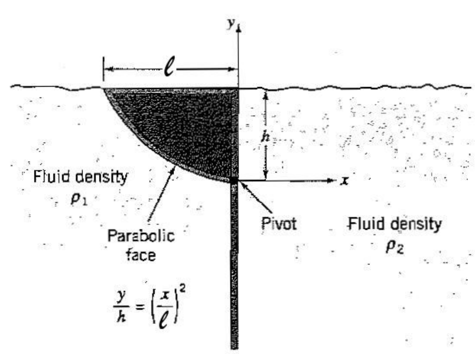


Figure P2.97

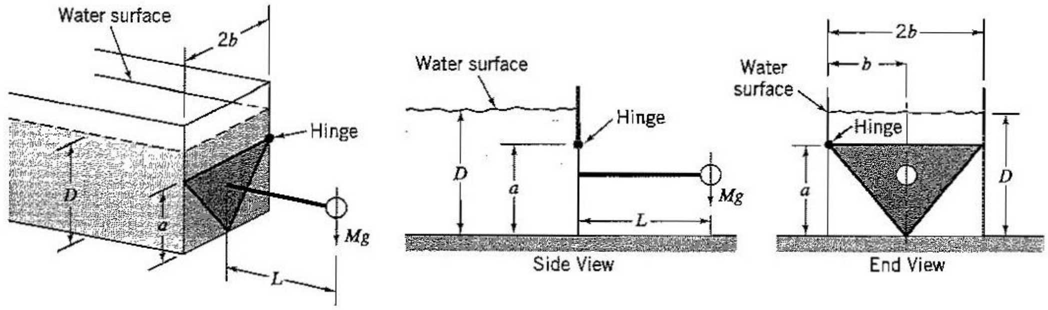


Figure P2.98

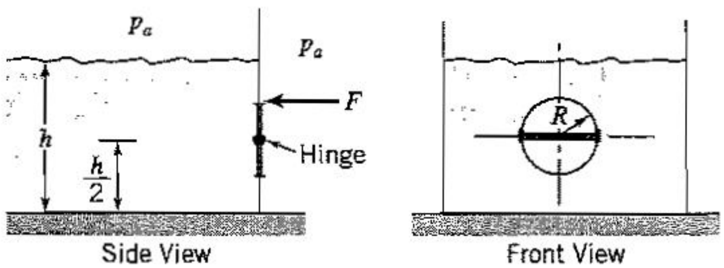


Figure P2.99

2.99 A circular gate of radius R is mounted halfway up a vertical dam face as shown in Figure P2.99. The dam is filled to a depth h with water, and the gate pivots without friction about a horizontal diameter.
(a) Determine the magnitude of the force due to the water pressure acting on the gate.
(b) Determine the magnitude and sign of the force F required to prevent the gate from opening.

2.100 An elevator accelerates vertically down with an acceleration of $\frac{1}{4}g$. What is the weight of a $120\,lb_f$ person, as measured during the acceleration?

2.101 A rocket accelerating vertically up with an acceleration of a carries fuel with a density ρ in tanks of height H. The top of the tank is vented to atmosphere. What is the pressure at the bottom of the tank?

2.102 A car accelerates at a constant rate from $0\,mph$ to $60\,mph$ in $10\,s$. A U-tube manometer with vertical legs $2\,ft$ apart is partly filled with water and used as an accelerometer.
(a) What is the difference in height of the water level in the two legs?
(b) Starting from rest, how fast would the car be going at the end of $10\,s$ if the difference in level were $1.0\,in.$ larger?

2.103 The cart shown in Figure 2.9 is now moving upward on a 5° incline at a constant acceleration a_i. Find the value of a_i that would make the water spill out of the container.

2.104 A $10\,cm$ diameter cylinder initially contains $10\,cm$ of water. It is then spun about its axis at an angular speed of ω. Find the value of ω where the bottom of the container just becomes exposed to air.

Chapter 3

Equations of Motion in Integral Form

3.1 Study Guide

- Write down the principle of conservation of mass in words.
- What are the dimensions of mass flow rate? What are its typical units?
- What are the dimensions of momentum flow rate? What are its typical units?
- At approximately what included angle do you expect the flow in a diffuser to separate?
- Describe the differences in the flow through a sudden contraction and the flow through a sudden expansion.
- Describe the differences between turbulent flow and separated flow.
- For unsteady flow, write down the integral form of the continuity equation. Explain each term in words.
- For unsteady inviscid flow, write down the integral form of the momentum equation. Explain each term in words.

3.2 Worked Examples

Example 3.1: Computing fluxes

A uniform flow of air with a velocity of $10\,m/s$ and density $1.2\,kg/m^3$ passes at an angle of 30° in the $[x,y]$-plane through an area of $0.1\,m^2$, as shown in Figure 3.1. Find the volume flux, the mass flux, the x-component of the momentum flux, and the kinetic energy flux passing through the area.

Solution: We first evaluate the volume flux, as per equation 3.3. We will need to express the velocity in vector notation, and so we choose a Cartesian coordinate system where

$$\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$$

and u, v and w are the velocity components in the x, y and z directions, respectively. In this particular case, $u = -10\cos 30^\circ = 8.66\,m/s$, $v = -10\sin 30^\circ = 5\,m/s$, and $w = 0$, so that

$$\mathbf{V} = -8.66\,\mathbf{i} + 5\,\mathbf{j}\,(m/s)$$

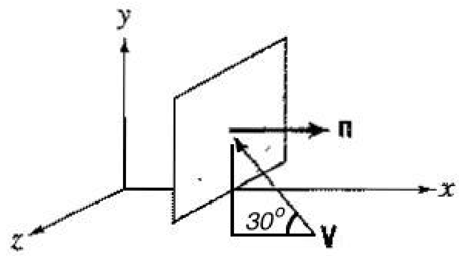


Figure 3.1: Notation for Example 3.2.

Also, the unit vector normal to the area lies in the x-direction, so that $\mathbf{n} = \mathbf{i}$. Therefore, for a uniform flow,

$$\begin{aligned} \text{volume flux} &= \int \mathbf{n} \cdot \mathbf{V}\, dA \;=\; (\mathbf{n} \cdot \mathbf{V})\; A \\ &= \mathbf{i} \cdot (-8.66\mathbf{i} + 5\mathbf{j})\; A \\ &= -8.66\, m/s \times 0.1\, m^2 \\ &= -0.866\, m^3/s \end{aligned}$$

We can find the mass flux by using equation 3.4, so that for a uniform flow,

$$\begin{aligned} \text{mass flux} &= \int \mathbf{n} \cdot \rho\mathbf{V}\, dA \;=\; (\mathbf{n} \cdot \rho\mathbf{V})\; A \;=\; (\mathbf{n} \cdot \mathbf{V}A)\, \rho \\ &= -0.866\, m^3/s \times 1.2\, kg/m^3 \\ &= -1.04\, kg/s \end{aligned}$$

For the momentum flux we use equation 3.5. We can obtain the x-momentum flux by taking the dot product of the momentum vector with the unit vector in the x-direction. That is, for a uniform flow,

$$\begin{aligned} x\text{-momentum flux} &= \mathbf{i} \cdot \int (\mathbf{n} \cdot \rho\mathbf{V})\mathbf{V}\, dA \;=\; \int (\mathbf{n} \cdot \rho\mathbf{V})\, \mathbf{i} \cdot \mathbf{V}\, dA \\ &= (\mathbf{n} \cdot \rho\mathbf{V})\, u\, A \;=\; (\mathbf{n} \cdot \rho\mathbf{V}A)\, u \\ &= -1.04\, kg/s \times (-8.66)\, m/s \\ &= 9.0\, kg\, m/s^2 \;=\; 9.0\, N \end{aligned}$$

Finally, we can find the kinetic energy flux by using equation 3.6, so that for a uniform flow,

$$\begin{aligned} \text{kinetic energy flux} &= \int (\mathbf{n} \cdot \rho\mathbf{V}) \tfrac{1}{2} V^2\, dA \;=\; (\mathbf{n} \cdot \rho\mathbf{V})\, \tfrac{1}{2} V^2\, A \;=\; (\mathbf{n} \cdot \rho\mathbf{V}A)\, \tfrac{1}{2} V^2 \\ &= -1.04\, kg/s \times \left(\tfrac{1}{2} \times 10^2\right)\; m^2/s^2 \\ &= -52\, kg \cdot m^2/s^3 \;=\; -52\, watt \end{aligned}$$

Example 3.2: Flux of mass and momentum

Water flows through a duct of height $2h$ and width W, as shown in Figure 3.2. The velocity varies across the duct according to

$$\frac{U}{U_m} = 1 - \left(\frac{y}{h}\right)^2$$

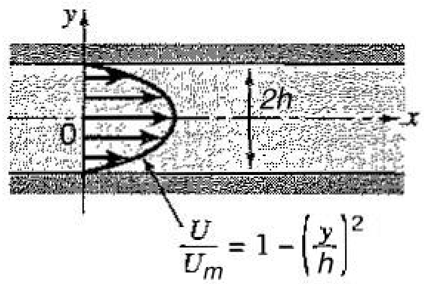


Figure 3.2: Fully-developed flow through a duct of constant width.

Find the volume, mass, and momentum fluxes over the cross-sectional area of the duct.

Solution: We note that the flow is always in the x-direction, so that $\mathbf{V} = U\mathbf{i}$, and the area of interest is the cross-sectional area of the duct, which has a unit normal vector $\mathbf{n} = \mathbf{i}$. Hence, $\mathbf{n} \cdot \mathbf{V} = \mathbf{i} \cdot U\mathbf{i} = U$. Also, $dA = W\,dy$, and so we have from equation 3.3

$$
\begin{aligned}
\text{volume flux} &= \int (\mathbf{n} \cdot \mathbf{V})\, dA \;=\; \int U\, W\, dy \;=\; \int_{-h}^{h} U_m \left(1 - \left(\frac{y}{h}\right)^2\right) W\, dy \\
&= 2U_m W \int_0^h \left(1 - \left(\frac{y}{h}\right)^2\right) dy \;=\; 2U_m W \left[y - \frac{y^3}{3h^2}\right]_0^h \\
&= \tfrac{4}{3} U_m W h
\end{aligned}
$$

To find the mass flux, we use equation 3.4. For a constant density flow,

$$
\begin{aligned}
\text{mass flux} &= \int (\mathbf{n} \cdot \rho\mathbf{V})\, dA \;=\; \rho \int (\mathbf{n} \cdot \mathbf{V})\, dA \\
&= \tfrac{4}{3} \rho U_m W h
\end{aligned}
$$

To find the momentum flux, we use equation 3.5. For a constant density flow,

$$
\begin{aligned}
\text{momentum flux} &= \int (\mathbf{n} \cdot \rho\mathbf{V})\, \mathbf{V}\, dA \;=\; \rho \int (U)\, U\mathbf{i}\, W\, dy \;=\; \rho W\, \mathbf{i} \int U^2\, dy \\
&= \rho W\, \mathbf{i} \int_{-h}^{h} U_m^2 \left(1 - \left(\frac{y}{h}\right)^2\right)^2 dy \\
&= 2\rho U_m^2 W\, \mathbf{i} \int_0^h \left(1 - 2\left(\frac{y}{h}\right)^2 + \left(\frac{y}{h}\right)^4\right) dy \\
&= 2\rho U_m^2 W\, \mathbf{i} \left[y - \frac{2y^3}{3h^2} + \frac{y^5}{5h^4}\right]_0^h \\
&= \tfrac{16}{15} \rho U_m^2 W h\, \mathbf{i}
\end{aligned}
$$

We see that the momentum flux is a vector, pointing in the positive x-direction.

Example 3.3: Mass conservation in steady, one-dimensional flow

Flow in a diverging duct is illustrated in Figure 3.3. Apply the principle of mass conservation for steady flow through the control volume.

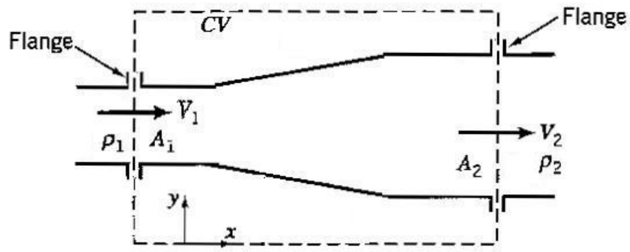


Figure 3.3: Flow through a diverging duct.

Solution: The continuity equation for steady flow (equation 3.11) gives

$$\int \mathbf{n} \cdot \rho \mathbf{V}\, dA \;=\; 0$$

The integral is over the entire surface of the control volume. Areas A_1 and A_2 are the only places where mass is entering or leaving the control volume, and so

$$\int \mathbf{n} \cdot \rho \mathbf{V}\, dA \;=\; \int \mathbf{n_1} \cdot \rho_1 \mathbf{V_1}\, dA_1 + \int \mathbf{n_2} \cdot \rho_2 \mathbf{V_2}\, dA_2 \;=\; 0$$

From Figure 3.3, we have $\mathbf{V_1} = V_1\mathbf{i}$, $\mathbf{V_2} = V_2\mathbf{i}$, $\mathbf{n_1} = -\mathbf{i}$, and $\mathbf{n_2} = \mathbf{i}$. Therefore,

$$-\int_{A_1} \rho_1 V_1 dA \;+\; \int_{A_2} \rho_2 V_2 dA \;=\; 0 \tag{3.1}$$

If the densities and velocities are uniform over their respective areas, we obtain

$$-\;\rho_1 V_1 A_1 \;+\; \rho_2 V_2 A_2 \;=\; 0$$

and we recover the result for a one-dimensional steady flow first given in Section 3.2.

Example 3.4: Conservation of mass

A cylinder of diameter d and length ℓ is mounted on a support in a wind tunnel, as shown in Figure 3.4. The wind tunnel is of rectangular cross section and height $4d$. The incoming flow is steady and uniform, with constant density and a velocity V_1. Downstream of the cylinder the flow becomes parallel again, and the shape of the velocity profile is as shown. Find the magnitude of V_2 in terms of V_1.

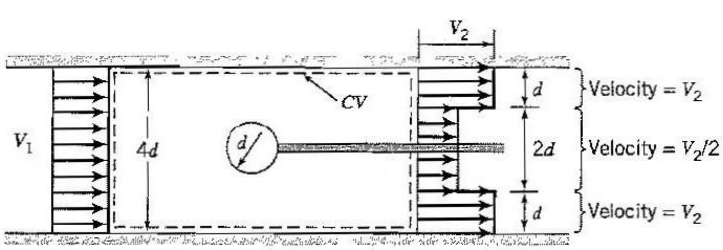


Figure 3.4: A cylinder in a wind tunnel.

Solution: We draw a large control volume that encloses the cylinder but does not cut the walls of the duct (CV in the Figure). As in any control volume problem, we examine each face of the control volume in turn. We see that there is flow into the control volume over the left hand face and flow out over the right hand face. The downstream flow is not actually one-dimensional since the velocity is varying across the cross-section of the wind tunnel. Nevertheless, it can be treated as one-dimensional by considering separately the two regions where the velocity is different, and then adding the result. By applying the continuity equation in this way we obtain

$$\rho V_1\,(4\ell d) - \rho V_2\,(2\ell d) - \rho \tfrac{1}{2} V_2\,(2\ell d) = 0$$

so that

$$V_2 = \tfrac{4}{3} V_1$$

Example 3.5: Mass conservation in steady, two-dimensional flow

In many cases the velocity is not uniform over the inlet and outlet areas and the one-dimensional assumption cannot be made. However, the streamlines of the flow entering and leaving the control volume are often parallel, and then it may be possible to assume that the pressure is uniform over the inlet and outlet areas (see Section 4.2.2.)

Consider the steady duct flow shown in Figure 3.5. The duct has a constant width W, and the inflow and outlet velocities vary along the flow in the x-direction, and across the flow in the y-direction. The flow is two-dimensional since the velocity distributions depend on two space variables (x and y). For this particular problem, the velocity distribution over the inlet is parabolic, but as the area expands the profile becomes less "full" at the exit.
(a) Find the average velocities over the inlet and outlet areas, where the average velocity $\overline{V}$ is defined by

$$\overline{V} = \frac{1}{A}\int u\,dA \tag{3.2}$$

so that $\overline{V}$ is equal to the volume flow rate divided by the cross-sectional area A.
(b) Find the velocity ratio V_{m2}/V_{m1}.

Solution: For part (a), first consider the inlet area. Here

$$\begin{aligned}\overline{V_1} &= \frac{1}{A_1}\int u\,dA_1 = \frac{1}{2bW}\int_{-b}^{b} V_1(y)\,W\,dy \\ &= \frac{2}{2bW}\int_0^b V_{m1}\left[1-\left(\frac{y}{b}\right)^2\right] W\,dy\end{aligned}$$

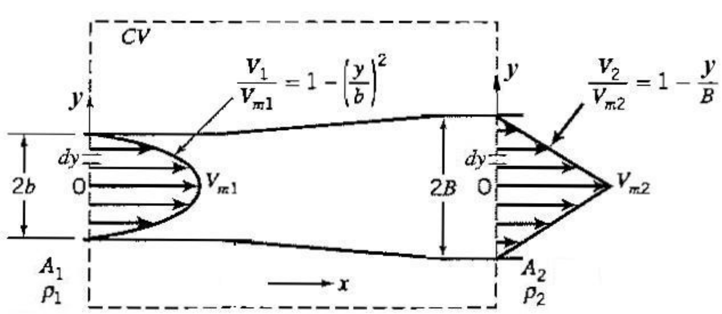


Figure 3.5: Two-dimensional duct showing control volume.

$$\text{So} \qquad \overline{V_1} \;=\; \frac{V_{m1}}{b}\int_0^b \left[1-\left(\frac{y}{b}\right)^2\right] dy \;=\; \tfrac{2}{3}V_{m1}$$

For the outlet area

$$\overline{V_2} \;=\; \frac{1}{A_1}\int u dA_1 \;=\; \frac{2}{2BW}\int_0^B V_{m2}\left(1-\frac{y}{B}\right) W dy \;=\; \tfrac{1}{2}V_{m1}$$

For part (b), we use mass conservation. The application is very similar to that given in Example 5.2, in that the continuity equation reduces to that given in equation 3.1, where

$$-\int_{A_1} \rho_1 V_1 dA \;+\; \int_{A_2} \rho_2 V_2 dA \;=\; 0$$

To evaluate the integrals over areas A_1 and A_2 we use symmetry, so that

$$-2\int_0^b (\rho_1 V_1)\, W dy + 2\int_0^B (\rho_2 V_2)\, W dy \;=\; 0$$

Since the densities are uniform over the inlet and outlet areas,

$$\begin{aligned}
2\rho_1 \int_0^b V_{m1}\left(1-\left(\frac{y}{b}\right)^2\right) dy \;&=\; 2\rho_2\int_0^B V_{m2}\left(1-\frac{y}{B}\right) dy \\
\rho_1 V_{m1}\left[y-\frac{y^3}{3b^2}\right]_0^b \;&=\; \rho_2 V_{m2}\left[y-\frac{y^2}{2B}\right]_0^B \\
\rho_1 V_{m1}\frac{2b}{3} \;&=\; \rho_2 V_{m2}\frac{B}{2}
\end{aligned}$$

Finally

$$\frac{V_{m2}}{V_{m1}} \;=\; \frac{\rho_1}{\rho_2}\frac{4b}{3B}$$

Example 3.6: Mass conservation for a moving piston in a cylinder

A leakproof piston moves with velocity V into a cylinder filled with liquid of density ρ, as shown in Figure 3.6. The cylinder has a cross-sectional area A_c, and the spout has an exit cross-sectional area A_s. Find U, the velocity at the exit from the spout, at any instant of time. The flow may be assumed to be one-dimensional. **Solution:** We select a control

volume not containing the piston or cylinder but only the spout. Since the spout is always full of fluid, the mass of fluid inside this control volume is not changing with time, and the flow is steady for this control volume. Therefore

$$\frac{\partial}{\partial t}\int \rho\, dv \;=\; 0$$

We see that there is a mass influx over the left face of the control volume, and there is an outflux over the right face, so that

$$\int \mathbf{n}\cdot\rho\mathbf{V} dA \;=\; \int -\mathbf{i}\cdot\rho V\mathbf{i} dA_c + \int \mathbf{i}\cdot\rho U\mathbf{i} dA_s \;=\; -\rho V A_c + \rho U A_s \;=\; 0$$

Hence,

$$U \;=\; \frac{A_c}{A_s}V$$

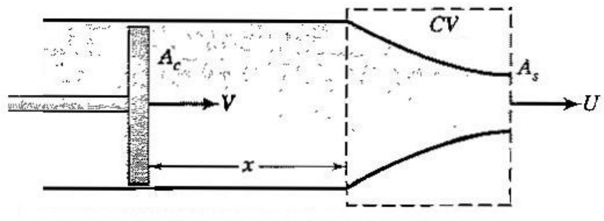


Figure 3.6: Control volume for an unsteady piston flow.

Example 3.7: Momentum balance in steady, one-dimensional flow

In Section 3.4, we considered the continuity equation applied to the diverging duct flow shown in Figure 3.3. We now find the x-component of the force exerted by the duct on the fluid, as in Figure 3.7. The flow is taken to be inviscid, steady and horizontal, and the pressures and densities are constant over areas A_1 and A_2.

Solution: Consider the x-component of the momentum equation. This equation is found by taking the dot product of equation 3.20 with the unit vector in the x-direction, $\mathbf{i}$. For steady, inviscid flow

$$\mathbf{i}\cdot\int(\mathbf{n}\cdot\rho\mathbf{V})\,\mathbf{V}\,dA \;=\; -\mathbf{i}\cdot\int\mathbf{n}\,p\,dA+\mathbf{i}\cdot\int\rho\mathbf{g}\,dV+\mathbf{i}\cdot\mathbf{R_{ext}}$$

so that

$$\int(\mathbf{n}\cdot\rho\mathbf{V})\,\mathbf{i}\cdot\mathbf{V}\,dA \;=\; -\int\mathbf{i}\cdot\mathbf{n}\,p\,dA+0+R^x_{ext}$$

where R^x_{ext} is the x-component of the force exerted by the duct on the fluid, and it was taken to be positive in the x-direction (the actual direction will come out as part of the solution, so that if we find that F_x is negative, it means it actually points in the negative x-direction).

The integrals are over the entire surface of the control volume. Areas A_1 and A_2 are the only places where the pressure is not atmospheric, so

$$-\int\mathbf{i}\cdot\mathbf{n}\,p\,dA \;=\; -\int -p_{1g}\,dA_1-\int +p_{2g}\,dA_2 \;=\; \int p_{1g}\,dA_1-\int p_{2g}\,dA_2$$

Areas A_1 and A_2 are the only places where mass is entering or leaving the control volume, so

$$\int(\mathbf{n}\cdot\rho\mathbf{V})\,\mathbf{i}\cdot\mathbf{V}\,dA \;=\; \int(-\rho_1V_1)(+V_1)\;dA_1+\int(\rho_2V_2)(+V_2)\;dA_2$$

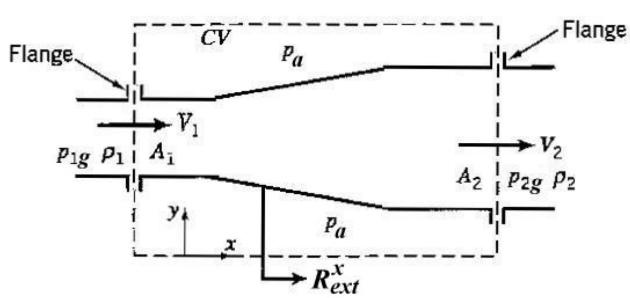


Figure 3.7: Flow through a diverging duct.

$$= \quad -\int \rho V_1^2 \, dA_1 + \int \rho V_2^2 \, dA_2$$

Therefore

$$R_{ext}^x = -\int p_{1g} \, dA_1 + \int p_{2g} \, dA_2 - \int \rho_1 V_1^2 \, dA_1 + \int \rho_2 V_2^2 \, dA_2$$

For a one-dimensional flow, this simplifies to

$$R_{ext}^x = -\left(p_{1g} A_1 - p_{2g} A_2\right) + \rho_2 V_2^2 A_2 - \rho_1 V_1^2 A_1$$

which is the same result given by equation 3.13.

Example 3.8: Momentum balance in steady, two-dimensional flow

In Section 3.4, we considered the continuity equation applied to the diverging duct flow shown in Figure 3.5. We now find the x-component of the force exerted by the duct on the fluid, as in Figure 3.8. The flow is steady, two-dimensional flow, and the duct has a width W. The pressure outside the duct is atmospheric everywhere, and over the inlet and outlet areas the gauge pressures are p_{1g} and p_{2g}, and the densities are ρ_1 and ρ_2, respectively. The pressures and densities are uniform over A_1 and A_2.

Solution: If we ignore gravity and friction, the only forces acting on the fluid will be forces due to pressure differences, and the force exerted by the duct on the fluid, $\mathbf{R_{ext}}$. We begin by finding R_{ext}^x, the x-component of the force exerted by the duct on the fluid. The x-component of the momentum equation (equation 3.20) is given by

$$\int (\mathbf{n} \cdot \rho \mathbf{V}) \, \mathbf{i} \cdot \mathbf{V} \, dA \;=\; p_{1g} A_1 - p_{2g} A_2 + R_{ext}^x$$

so that

$$\begin{aligned} R_{ext}^x &= p_{2g} A_1 - p_{1g} A_2 - \int \rho_1 V_1^2 \, dA_1 + \int \rho_2 V_2^2 \, dA_2 \\ &= p_{2g} A_1 - p_{1g} A_2 - 2\rho_1 W \int_0^b V_1^2 \, dy + 2\rho_2 W \int_0^B V_2^2 \, dy \\ &= p_{2g} A_1 - p_{1g} A_2 - \tfrac{16}{15} \rho_1 V_{m1}^2 W b + \tfrac{2}{3} \rho_2 V_{m2}^2 W B \end{aligned}$$

What about the y-direction? We see that the momentum only changes in the x-direction, and that the forces due to pressure differences only act in the x-direction. So the y-component of the force exerted by the duct on the fluid must be zero.

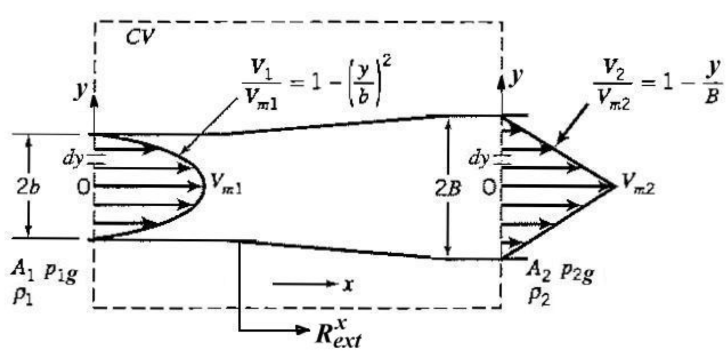


Figure 3.8: Two-dimensional duct showing control volume.

Example 3.9: Lift and drag on an airfoil

Consider an airfoil of span b placed in a wind tunnel of height h, as shown in Figure 3.9. The flow is steady and of constant density, and the airfoil develops a lift force and a drag force. The lift force F_L is defined as the force on the airfoil normal to the direction of the incoming flow, and the drag force F_D is defined as the force in the direction of the incoming flow. The incoming flow velocity is uniform across the wind tunnel with a magnitude V_1, but downstream the velocity varies across the tunnel in the y-direction. The velocity in the wake of the airfoil is less than V_1, and therefore, by conservation of mass, the velocity outside the wake must be greater than V_1. There is also a pressure difference, so that p_2 is less than p_1, but since the streamlines at stations 1 and 2 are parallel, the pressures are constant across the wind tunnel (we will ignore gravity). Find the lift and drag forces.

Solution: We start with the x-momentum balance for the control volume shown in Figure 3.9 to find R_{ext}^x, the x-component of the force exerted by the airfoil on the fluid. If viscous forces on the surface of the control volume can be neglected,

$$\int (\mathbf{n}\cdot\rho\mathbf{V})\,\mathbf{i}\cdot\mathbf{V}\,dA \;=\; -\int \mathbf{i}\cdot\mathbf{n}p\,dA + R_{ext}^x$$

Hence

$$-\rho V_1^2 bh + 2\int_0^{h/2} \rho V_2^2 b\,dy \;=\; (p_1 - p_2)\,bh + R_{ext}^x$$

For the velocity distribution shown,

$$R_{ext}^x \;=\; -(p_1 - p_2)\,bh - \rho V_1^2 bh + \tfrac{1}{3}\rho V_{2m}^2 bh$$

R_{ext}^x is the force exerted by the airfoil on the fluid, and so the force exerted by the fluid on the airfoil is $-R_{ext}^x$, and therefore $F_D = -R_{ext}^x$. That is,

$$F_D \;=\; (p_1 - p_2)\,bh + \rho V_1^2 bh - \tfrac{1}{3}\rho V_{2m}^2 bh$$

We can show that $V_{2m} = 2V_1$ by using the continuity equation. In addition, we can make the drag force nondimensional by dividing through by $\frac{1}{2}\rho V_1^2 bh$ (we recognize this as the upstream dynamic pressure multiplied by the cross-sectional of the tunnel). Therefore

$$\frac{F_D}{\frac{1}{2}\rho V_1^2 bh} \;=\; \frac{(p_1 - p_2)}{\frac{1}{2}\rho V_1^2} - \frac{2}{3}$$

Non-dimensionalizing has cleaned up the final expression, and it has also revealed the presence of a pressure coefficient on the right hand side, similar to that introduced in Section 4.3.1, as well as a new nondimensional parameter called a *drag coefficient*, C_D', on the

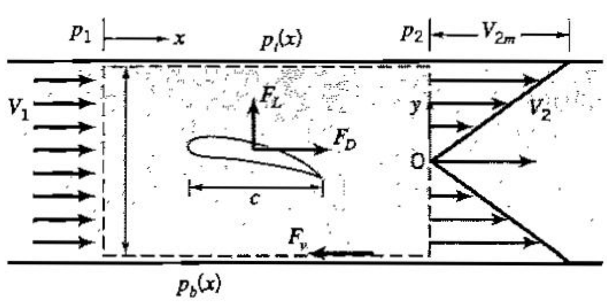


Figure 3.9: Lift and drag on an airfoil in a windtunnel.

left hand side, defined by

$$C'_D = \frac{F_D}{\frac{1}{2}\rho V_1^2 bh}$$

In the usual form of the drag coefficient, the plan area of the wing is used instead of the cross-sectional area of the tunnel. That is, for a rectangular wing,

$$C_D \equiv \frac{F_D}{\frac{1}{2}\rho V_1^2 bc} \tag{3.3}$$

where c is the chord length of the airfoil (the distance between its leading and trailing edges).

To find the lift force, we start by finding R^y_{ext}, the y-component of the force exerted by the airfoil on the fluid using the y-momentum balance. That is,

$$\int (\mathbf{n}\cdot\rho\mathbf{V})\,\mathbf{j}\cdot\mathbf{V}\,dA = -\int \mathbf{j}\cdot\mathbf{n}p\,dA + R^y_{ext}$$

Since there is no flow in the y-direction

$$0 = \int_b p_b\, b\,dx - \int_t p_t\, b\,dx + R^y_{ext}$$

where p_b and p_t are the pressure distributions over the bottom and top faces of the control volume. R^y_{ext} is the force exerted by the airfoil on the fluid, and so the force exerted by the fluid on the airfoil is $-R^y_{ext}$, and therefore $F_L = -R^y_{ext}$. That is,

$$F_L = b\left(\int_b p_b\,dx - \int_t p_t\,dx\right)$$

Therefore, the lift force can be found by measuring the pressure distributions on the upper and lower tunnel walls. When we divide through by $\frac{1}{2}\rho V_1^2 bh$, we obtain

$$\frac{F_L}{\frac{1}{2}\rho V_1^2 bh} = \frac{1}{\frac{1}{2}\rho V_1^2 h}\left(\int_b p_b\,dx - \int_t p_t\,dx\right)$$

We now have a nondimensional *lift coefficient* C'_L on the left hand side. In its more usual form, it is defined using the plan area of the wing so that

$$C_L \equiv \frac{F_L}{\frac{1}{2}\rho V_1^2 bc} \tag{3.4}$$

Example 3.10: Unsteady flow and moving control volumes

A steady jet of water having a velocity V_j hits a deflector which is moving to the right at a constant velocity V_d (see Figure 3.10). The deflector turns the flow through an angle $\pi - \theta$. Find $\mathbf{F}$, the force exerted by the fluid on the deflector. Assume that the effects of gravity and friction can be neglected.

Solution: The first thing to note is that the problem is unsteady for a stationary observer. This complicates the analysis considerably since we would need to use the unsteady form of the continuity and momentum equations, and Bernoulli's equation cannot be used. If the observer moves with the deflector, however, the problem becomes steady and these complications are avoided. A velocity transformation using a constant translating velocity has no effect on the forces acting on a system. That is, the forces are the same whether a motion is viewed in a stationary coordinate system or one that is moving at a constant velocity. The control volume for this steady flow is shown in Figure 3.11.

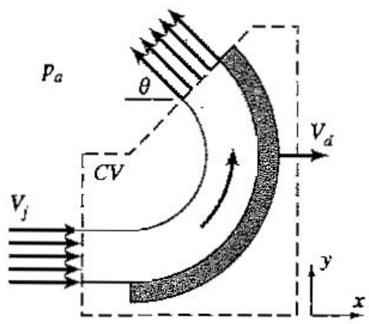


Figure 3.10: Moving deflector: unsteady flow.

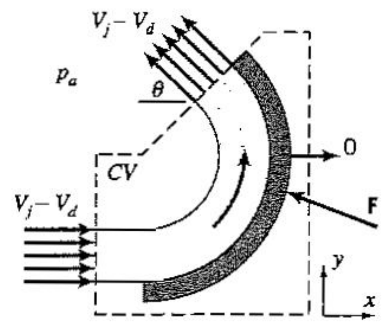


Figure 3.11: Moving deflector: steady flow.

Along the surface of the jet, we see that the pressure is constant and equal to the atmospheric pressure. The pressure is also constant across the jet over the inlet and outlet areas, since the streamlines are parallel and the jet is in free fall. Since the flow in this framework is steady and there is no friction, Bernoulli's equation applied along any streamline that starts at the inlet and finishes at the outlet indicates that the magnitude of the inlet and outlet velocities are equal. The deflector changes the velocity direction, but not its magnitude. Then, from the continuity equation, we know that the cross-sectional area of the jet, A, must remain constant.

The momentum equation gives

$$-\mathbf{F} - \int p\mathbf{n}\, dA \;=\; \int (\mathbf{n} \cdot \rho\mathbf{V})\, \mathbf{V}\, dA$$

where $+\mathbf{F}$ is the force exerted by the fluid on the deflector, and so $-\mathbf{F}$ is the force exerted on the fluid by the deflector. The pressure is constant everywhere, so that $\int p\mathbf{n}dA = 0$ for the control volume shown. Over the inlet, the velocity is $(V_j - V_d)\,\mathbf{i}$, and $\mathbf{n} = -\mathbf{i}$. Over the outlet the velocity magnitude is the same, but its direction is different, so that the outlet velocity is $(V_j - V_d)\,(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$, and $\mathbf{n} = -\cos\theta\mathbf{i} + \sin\theta\mathbf{j}$. Hence,

$$\begin{aligned}
-\mathbf{F} &= -\rho\,(V_j - V_d)^2\,\mathbf{i}A + \rho\,(V_j - V_d)^2\,(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j})\,A \\
&= -\rho A\,(V_j - V_d)^2\,[(1 + \cos\theta)\,\mathbf{i} - \sin\theta\mathbf{j}] \\
\text{and so:}\qquad \mathbf{F} &= \rho A\,(V_j - V_d)^2\,[(1 + \cos\theta)\,\mathbf{i} - \sin\theta\mathbf{j}]
\end{aligned}$$

We can check this answer by taking the limit when $V_j = V_d$, and the limit when $\theta = \pi$. In both cases, $\mathbf{F} = 0$, as expected.

Example 3.11: Drag force on a cylinder

Consider the flow shown in Figure 3.12. The reduced velocity region downstream of the cylinder is called the *wake* (see figure 4.3). This flow was first considered in Example 3.3, where mass conservation was used to show that $V_2 = \frac{4}{3}V_1$. Here, we find the forces that act on the cylinder. The presence of the cylinder changes the pressure and momentum of the fluid, because the cylinder exerts a force on the fluid, R_{ext}. Hence, the force exerted by the fluid on the cylinder is equal to $-R_{ext}$, and this force F_D is the *drag* force. We will now find F_D, assuming that forces due to viscous stresses and gravity are negligible, and that the pressure is constant over the inlet and outlet areas.

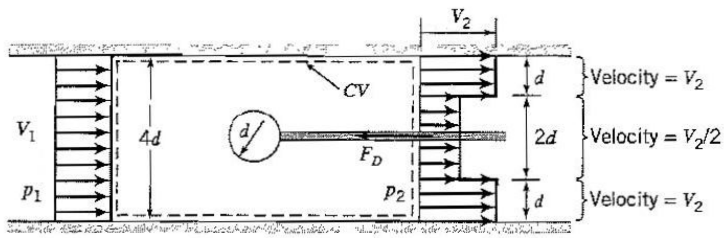


Figure 3.12: A cylinder in a wind tunnel.

Solution: To find the drag force F_D, we need to find R_{ext}, which is the force by the cylinder on the fluid. As usual, we will assume R_{ext} acts in the positive x-direction. It is important to understand how R_{ext} makes an appearance in the momentum equation for the fluid. In our choice of control volume, the cylinder is inside the volume, and the surface of the volume "cuts" through the support holding the cylinder in place. In effect, it is the support that applies the force to the fluid inside the control volume.

The resultant force acting on the fluid is then given by R_{ext} plus the force due to pressure differences. In finding the rate of change of momentum, we treat the downstream flow as two separate one-dimensional flows and add the result. Newton's second law applied to the fluid in the control volume then gives,

$$R_{ext} + p_1\,(4\ell d) - p_2\,(4\ell d) = \rho V_2^2\,(2\ell d) + \rho\left(\tfrac{1}{2}V_2\right)^2(2\ell d) - \rho V_1^2\,(4\ell d)$$

With $F_D = -R_{ext}$, and $V_2 = \frac{4}{3}V_1$ (from Example 3.3), we obtain:

$$F_D = 4\,(p_1 - p_2)\,\ell d - \tfrac{4}{9}\rho V_1^2 \ell d \tag{3.5}$$

Example 3.12: Converging duct

A steady flow of air of constant density ρ passes through a converging duct of constant width w, as shown in Figure 3.13. Find the average pressure p_{av} acting on the side walls, given that viscous and gravity forces may be neglected. The continuity equation for the control volume shown in the figure gives

$$2hw\,V_1 = hw\,V_2$$

so that

$$V_2 = 2V_1$$

The x-component momentum equation gives

$$p_1\,(2hw) - p_2\,(hw) - F_{px} = \rho V_2^2\,(hw) - \rho V_1^2\,(2hw)$$

where F_{px} is the x-component of the force due to the pressure acting on the side walls. The total force due to pressure on the side wall, F_p, is given by

$$F_p = 2p_{av}(Lw) = 2p_{av}\frac{h}{2\sin\theta}w$$

and so

$$F_{px} = F_p \sin\theta = p_{av}hw$$

Hence

$$2p_1 - p_2 - p_{av} = \rho V_2^2 - 2\rho V_1^2$$

and so, by using the result that $V_2 = 2V_1$

$$p_{av} = 2p_1 - p_2 - 2\rho V_1^2$$

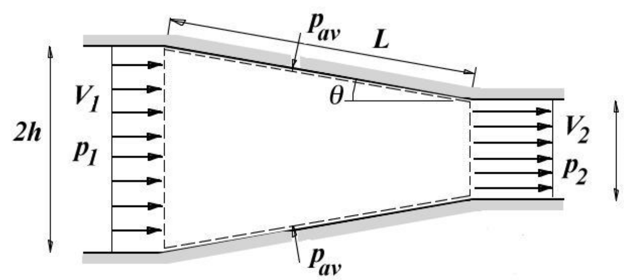


Figure 3.13: Control volume for the flow through a converging duct.

Example 3.13: Control volume selection

Consider the steady flow of constant density air through the sudden expansion shown in Figure 3.14. We aim to find the force exerted by the fluid on the duct, F_D.[1] Suppose that at the entry and exit from the control volume CV_1 (stations 1 and 2, respectively), the flow is approximately uniform so that the assumption of one-dimensional flow can be made. However, inside the control volume the flow is very complex (see, for example, Figure 3.15) and there are energy losses. We assume that at the entry to and exit from the control volume the pressures are approximately uniform across the flow, and at the exit it is equal to atmospheric pressure. In addition, at the entrance to the sudden expansion, the flow is approximately parallel, so that the pressure is approximately equal to the pressure in the recirculation zones (we will show why this is so in Section 4.2.2), and therefore we can assume that the pressure acting at section s-s is uniform across the flow.

We use two different control volumes to illustrate how the control volume selection governs the information that can be obtained. First, we use a control volume that encloses the duct, and cuts through the walls of the duct at an upstream section (CV_1 in Figure 3.14). Where the control volume cuts through the walls, there will be a reaction force R_{ext} acting on the fluid, so that the force exerted by the fluid on the duct is given by $F_D = -R_{ext}$. If we use gauge pressure in the x-component momentum equation, we obtain

$$R_{ext} + p_{1g}A_1 = \rho V_2^2 A_2 - \rho V_1^2 A_1$$

since the only place where the pressure is not atmospheric is at station 1. There is no viscous force acting on this control volume. Outside the duct there is no fluid motion, and there are clearly no viscous stresses acting. Over the parts of the control volume surface that lie inside the duct, there are no velocity gradients, and again there are no shearing stresses acting. For this control volume, viscous forces do not play a role. By using the continuity equation ($V_1A_1 = V_2A_2$) we obtain, since $F_D = -R_{ext}$,

$$F_D = p_{1g}A_1 + \rho V_1^2 A_1 \left(1 - \frac{A_1}{A_2}\right) \tag{3.6}$$

If we know the geometry of the duct, we can determine F_D by measuring the gauge pressure upstream, and either the inlet or the outlet velocity.

Second, we choose a control volume that coincides with the inside surface of the sudden expansion (CV_2 in Figure 3.14). There is now no reaction force acting on the fluid inside the control volume since the control volume does not cut the walls of the duct. The pressure distribution along the horizontal surfaces of the control volume is unknown, but the resulting force is in the vertical direction so it does not enter the x-momentum equation. However,

[1]This problem was adapted from an example given in the book *Engineering Fluid Mechanics* by Alan Mironer, published by McGraw-Hill, 1979.

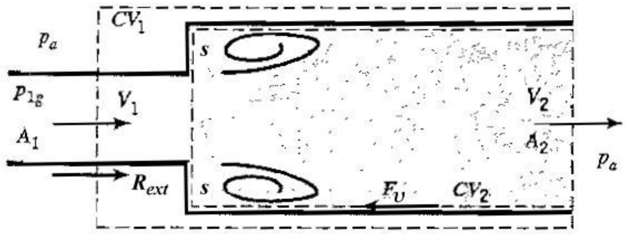


Figure 3.14: Alternative control volumes for the flow through a sudden expansion. The regions of separated flow are shown as recirculating eddies.

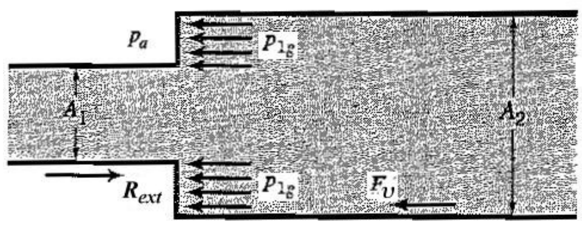


Figure 3.15: Free-body diagram for the sudden expansion.

the viscous shearing stresses along the horizontal surfaces give rise to a horizontal frictional force on the fluid, $-F_v$, which must be included (the sign conforms to what is shown in Figure 3.14: we expect it to be a retarding force for the fluid, but its actual direction will come out of the analysis). The x-component momentum equation becomes

$$-F_v + p_1 A_2 - p_2 A_2 = \rho V_2^2 A_2 - \rho V_1^2 A_1$$

By using the continuity equation and gauge pressures (so that $p_{1g} = p_1 - p_2$, since $p_2 = p_a$)

$$F_v = p_{1g} A_2 + \rho V_1^2 A_1 \left(1 - \frac{A_1}{A_2}\right) \tag{3.7}$$

We see that F_v is positive since $A_2 > A_1$, and therefore it will act in the direction shown in the figure. If the duct is relatively short, or if the expansion ratio A_2/A_1 is large, F_v is usually small compared to the force due to pressure differences, and it can be neglected. The gauge pressure at the upstream location p_{1g} can then be found by measuring either V_1 or V_2.

Finally, we can draw the free-body diagram for the duct showing the forces acting in the horizontal direction (Figure 3.15). When the viscous force F_v is negligible,

$$F_D = -p_{1g}\left(A_2 - A_1\right).$$

The same result can be obtained by combining equations 3.6 and 3.7 for $F_v = 0$.

Problems

3.1 Air at $60°F$ at atmospheric pressure flows through an air conditioning duct measuring $6\,in.$ by $12\,in.$ with a volume flow rate of $5\,ft^3/s$. Find:
(a) The mass flow rate.
(b) The average velocity.

3.2 Water at $20°C$ flows through a $10\,cm$ pipe with a volume flow rate of $0.5\,m^3/s$. Find:
(a) The mass flow rate.
(b) The average velocity.
(c) The momentum flow rate.

3.3 The largest artery in the body is the one that supplies blood to the legs. As it comes down the trunk of the body, it splits into a Y-junction, as shown in Figure P3.3. Blood with specific gravity of 1.05 is pumped into the junction at a speed $V_1 = 1.5\,m/s$. The diameter of the entrance flow is $d_1 = 20\,mm$, and for the exit flow $d_2 = 15\,mm$ and $d_3 = 12\,mm$. If the mass flow rates at stations 2 and 3 are equal, find V_2 and V_3.

3.4 Air enters an air conditioning duct measuring $1\,ft$ by $2\,ft$ with a volume flow rate of $1000\,ft^3/min$. The duct supplies three classrooms through ducts that are $8\,in.$ by $15\,in.$ Find the average velocity in each duct.

3.5 In the hovercraft shown in Figure P3.5, air enters through a $1.2\,m$ diameter fan at a rate of $135\,m^3/s$. It leaves through a circular exit, $2\,m$ diameter and $10\,cm$ high. Find the average velocity at the entry and the exit.

3.6 Water flows radially toward the drain in a sink, as shown in Figure P3.6. At a radius of $50\,mm$, the velocity of the water is uniform at $120\,mm/s$, and the water depth is $15\,mm$. Determine the average velocity of the water in the $30\,mm$ diameter drain pipe.

3.7 A circular pool, $15\,m$ diameter, is to be filled to a depth of $3\,m$. Determine the inlet flow in m^3/s and gpm if the pool is to be filled in $2\,hrs$. Find the number of $2\,in.$ diameter hoses required if the water velocity is not to exceed $10\,ft/s$.

3.8 A tank $10\,ft$ in diameter and $6\,ft$ high is being filled with water at a rate of $0.3\,ft^3/s$. If it has a leak where it loses water at rate of $30\,gpm$, how long will it take to fill the tank if it was initially half full?

3.9 Is the tank shown in Figure P3.9 filling or emptying? At what rate is the water level rising or falling? Assume that the density is constant. All inflow and outflow velocities are steady and constant over their respective areas.

3.10 A gas obeying the ideal gas law flows steadily in a horizontal pipe of constant diameter between two sections, 1 and 2. If the flow is isothermal and the pressure ratio $p_2/p_1 = \frac{1}{2}$, find the velocity ratio V_2/V_1.

3.11 A gas obeying the ideal gas law enters a compressor at atmospheric pressure and

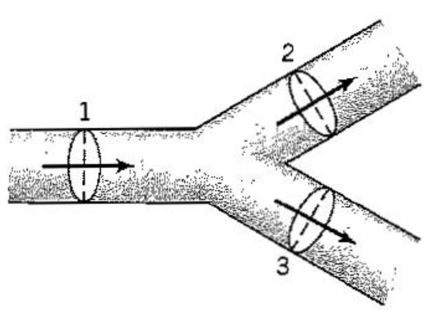


Figure P3.3:

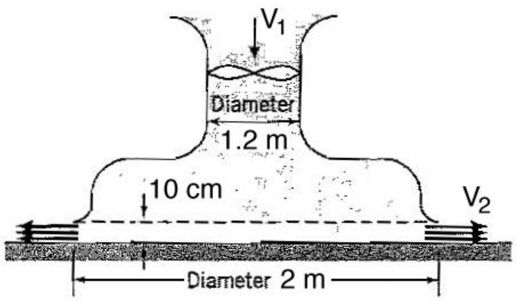


Figure P3.5:

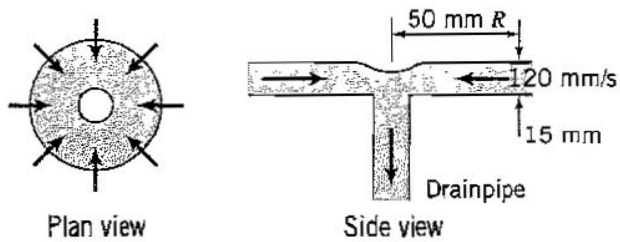


Figure P3.6:

$20\,C$, with a volume flow rate of $1\,m^3/s$. Find the volume flow rate leaving the compressor if the temperature and pressure at the exit are $80°C$ and $200\,bar$, respectively.

3.12 For the vessel shown in Figure P3.12, the flow is steady, with constant density, and it may be assumed to be one-dimensional over the entry and exit planes. Neglect forces due to gravity, and assume the pressure outside the box is atmospheric. Find the mass flow rate and the volume flow rate out of area A_3, and find the x- and y-components of the force exerted on the box by the fluid, in terms of ρ, A_1, θ and V_1.

3.13 A rectangular duct of width w and height h carries a flow of air of density ρ and viscosity μ. The velocity profile is parabolic, so that

$$\frac{U}{U_m} = 1 - \left(\frac{2y}{h}\right)^2$$

where U_m is the maximum velocity, which occurs on the centerline where $y = 0$.

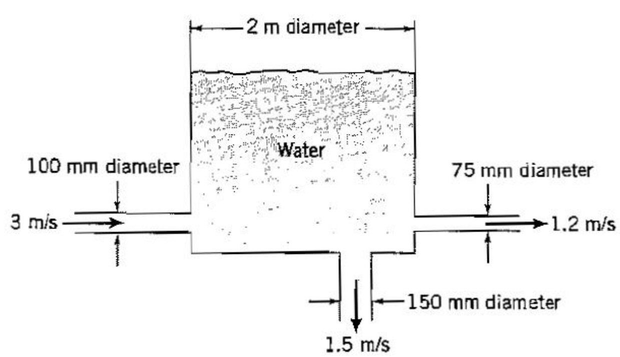


Figure P3.9

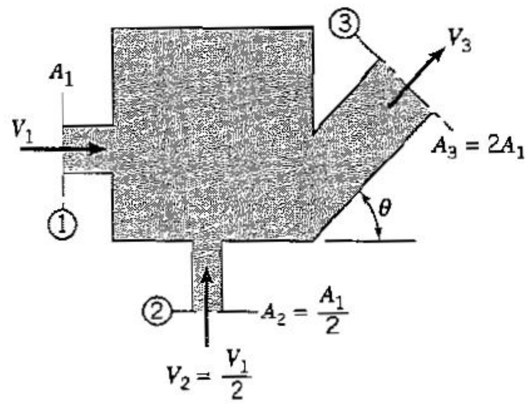


Figure P3.12

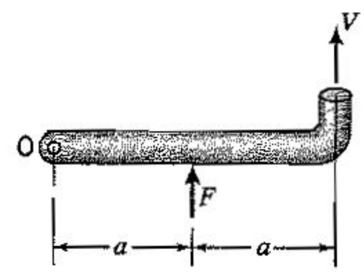


Figure P3.14

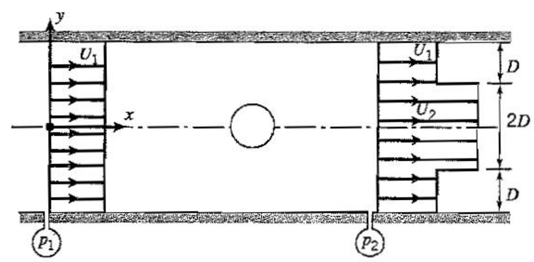


Figure P3.15

(a) Plot the shear stress as a function of y/h, given that $h = 10$ mm, $w = 200$ mm, $U_m = 1$ m/s, the air temperature is 20C, and the pressure is atmospheric.
(b) Find the area-averaged velocity $\overline{U}$ (need to integrate).
(c) Find the volume flow rate and the mass flow rate through the duct.
(d) Find the Reynolds number of the flow based on the average velocity and the height of the duct.
(e) Find the skin friction coefficient $C_f = \tau_w/(\frac{1}{2}\rho\overline{U}^2)$, where τ_w is the shear stress at the wall of the duct.

3.14 Find the force F required to prevent rotation of the pipe shown in Figure P3.14 about the vertical axis located at the point O. The pipe diameter is D, and the fluid density is ρ.

3.15 A porous circular cylinder, of diameter D is placed in a uniform rectangular wind tunnel section of height $4D$ and width W, as shown in Figure P3.15. The cylinder spans the width of the tunnel. An air volume flow rate of $\dot{q}$ per unit width issues from the cylinder. The flow field is steady and has a constant density. The pressures p_1 and p_2 are uniform across the entry and exit areas, and the velocity profiles are as shown. A force F is required to hold the cylinder from moving in the x-direction. Find U_2 in terms of $\dot{q}$, U_1 and D. Then find F.

3.16 A jet of cross-sectional area A_1 steadily issues fluid of density ρ at a velocity V_1, into a duct of area $A_2 = 5A_1$, as shown in Figure P3.16. The pressures at sections A and B are uniform across the duct. The surrounding flow in the duct has the same density ρ and a velocity $V_2 = \frac{1}{2}V_1$. The flow mixes thoroughly, and by section B the flow is approximately uniform across the duct area. Find the average velocity of flow at Section B in terms of V_1,

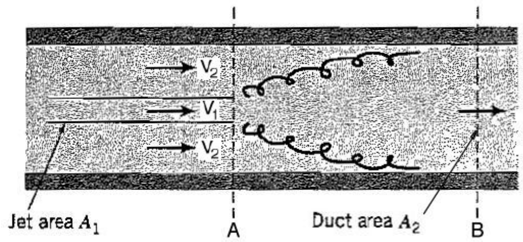


Figure P3.16

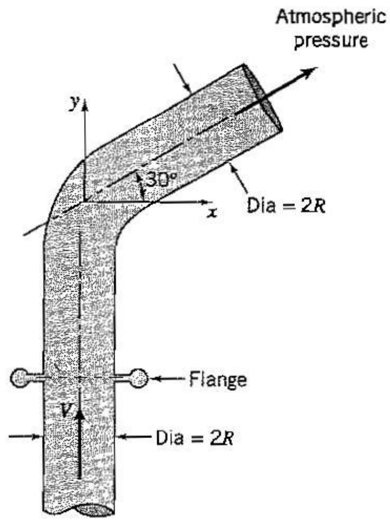


Figure P3.18

and the pressure difference between sections A and B in terms of ρ and V_1.

3.17 Water at $60\,F$ enters a $6\,in.$ diameter pipe at a rate of $3000\,gpm$. The pipe makes a 180° turn. Find the rate of change of momentum of the fluid. If the pressure in the pipe remains constant at $60\,psi$, what is the magnitude and direction of the force required to hold the pipe bend?

3.18 Air of constant density ρ flows at a constant velocity V through a horizontal pipe bend, as shown in Figure P3.18, and exits to the atmosphere. The pipe is circular and has an internal radius R. A force $\mathbf{F}$ must be applied at the flange to keep the bend in place. Find $\mathbf{F}$ in terms of ρ, R, and V. Neglect any pressure changes, and assume that the flow is one-dimensional.

3.19 An effectively two-dimensional jet of water impinges on a wedge as shown in Figure P3.19. The wedge is supported at its apex such that the lower surface remains horizontal. If the thicknesses t_2 and t_3 are equal and $U_2 = U_3 = 2U_1$, find the angle for which the magnitude of the x- and y-components of the reaction force at the apex are equal. Neglect gravity. The pressure is atmospheric everywhere.

3.20 A steady water jet of density ρ is split in half as it encounters a wedge of included angle 2α and constant width w, as shown in Figure P3.20. For the velocity distributions given in the diagram, find the force required to hold the wedge in place in terms of U_1, h, α, and the density of water ρ, assuming that the viscous effects and the weight of the water

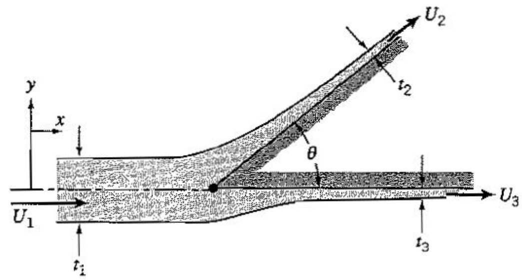


Figure P3.19

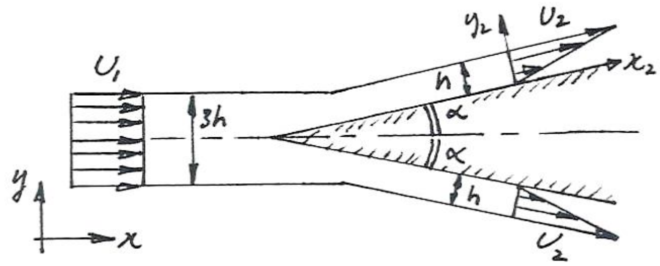


Figure P3.20

and wedge are negligible and the pressure is uniform everywhere.

3.21 Air is flowing through a duct of square cross-section of height h, as shown in Figure P3.21. An orifice plate with a square hole measuring $\frac{1}{2}h$ by $\frac{1}{2}h$ is placed on the centerline. The plate experiences a drag force F_D. The air density ρ is constant. Far upstream, the pressure and velocity are uniform and equal to p_1 and U_1, respectively. Far downstream, the pressure is p_2 and the velocity distribution is as shown. A manometer measures the pressure difference, and shows a deflection of Δ. The manometer fluid has a density ρ_m. The flow is steady, and friction can be neglected. Find the maximum velocity at the outlet U_2 in terms of U_1, and the drag force F_D in terms of h, ρ, ρ_m, Δ, g and U_1.

3.22 Figure P3.22 shows a two-dimensional flow of a constant density fluid between parallel plates a distance $2h$ apart. The velocity distribution is given by $V = V_m\left(1 - y^2/h^2\right)$, where V_m is the maximum velocity. Find the average velocity and the mass flow rate.

3.23 Two rectangular air-conditioning ducts, of constant width W (into the page), meet at right angles as shown in Figure P3.23. The flow is steady and the density is constant.

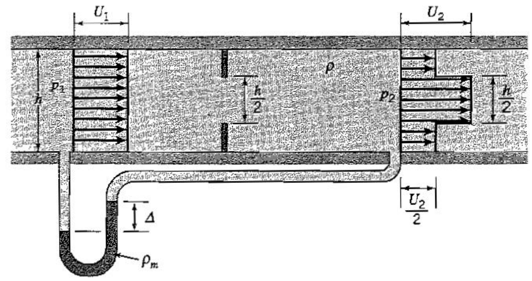


Figure P3.21

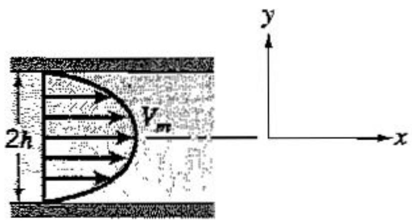


Figure P3.22

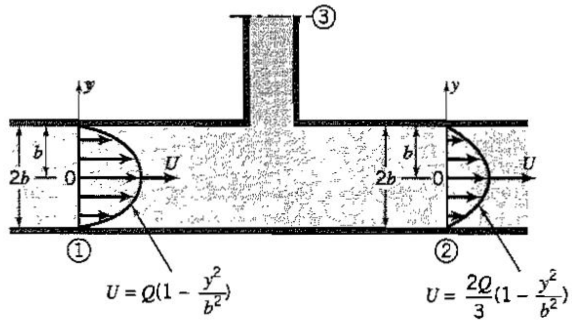


Figure P3.23

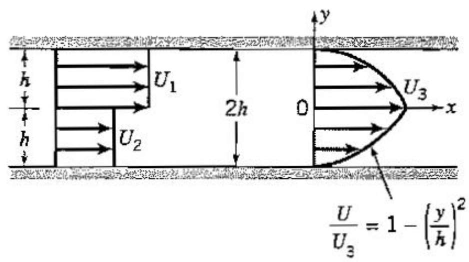


Figure P3.24

All velocities are normal to the exit and entrance areas. The velocity profiles at stations 1 and 2 are parabolic. Find the mass flow rate at station 3. Is it in or out?

3.24 In the rectangular duct shown in Figure P3.24, two parallel streams of a constant density gas enter on the left with constant velocities U_1 and U_2. After mixing, the gas exits on the right with a parabolic profile and a maximum value of U_3. Find U_3 in terms of U_1 and U_2.

3.25 Figure P3.25 shows a sketch of a simple carburetor. The air enters from the left with a uniform velocity, and flows through a contraction where fuel-rich air of the same density enters at a rate of $\dot{q}\, m^3/s$. The mixture then exits on the right with a triangular velocity profile as shown. The flow is steady, and the cross-sectional areas at stations 1 and 2 are rectangular of constant width W. Assume that all velocities are normal to their respective areas. Find $\dot{q}$ when $U_2 = 2U_1$.

3.26 A two-dimensional duct of constant width carries a steady flow of constant density fluid (density ρ), as shown in Figure P3.26. At the entrance, the velocity s constant over

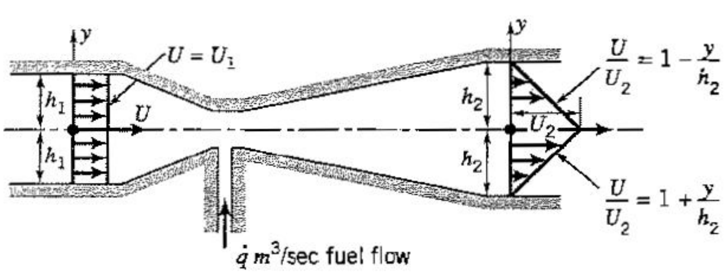


Figure P3.25

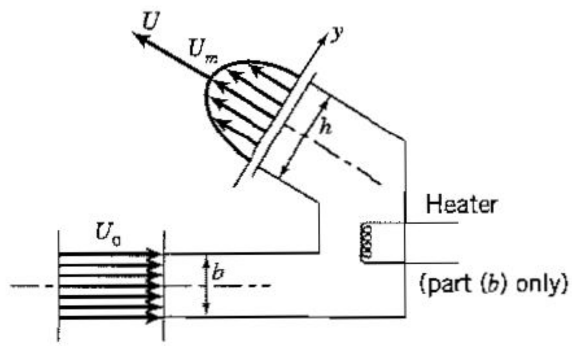


Figure P3.26

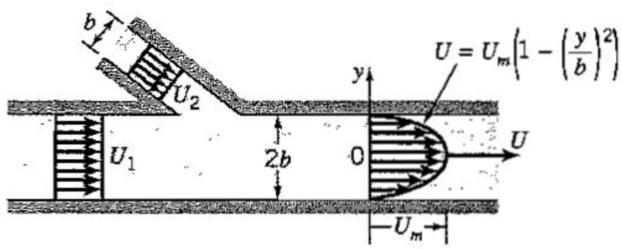


Figure P3.27

the area and equal to U_0. At the exit, the velocity profile is parabolic according to $U = U_m\left(1 - (2y/h)^2\right)$.
(a) Find U_m as a function of U_0, b and h.
(b) Will U_m increase or decrease if a heater is inserted into the duct as shown?

3.27 A constant density fluid flows steadily through a duct of width W, as shown in Figure P3.27. Find U_m in terms of U_1 and U_2. What is the momentum flux passing through the exit of the duct?

3.28 For the rectangular duct of width W shown in Figure P3.28, the flow is steady and the density is constant. Find U_m in terms of U_1, a and b, and find the y-component of the momentum flux leaving the duct.

3.29 For the constant density flow in the rectangular duct of width W shown in Figure P3.29, find U_m in terms of a, b and U_1, and the (vector) momentum flux leaving the duct.

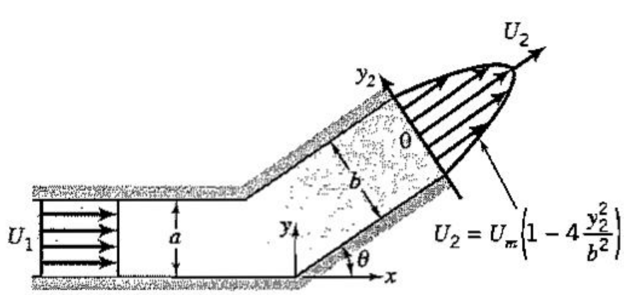


Figure P3.28

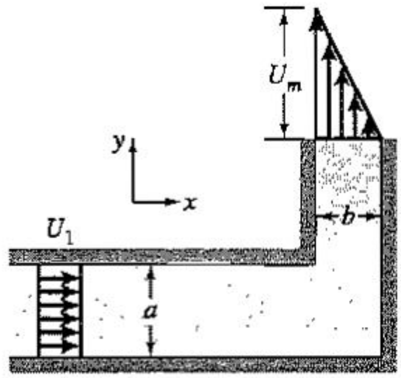


Figure P3.29

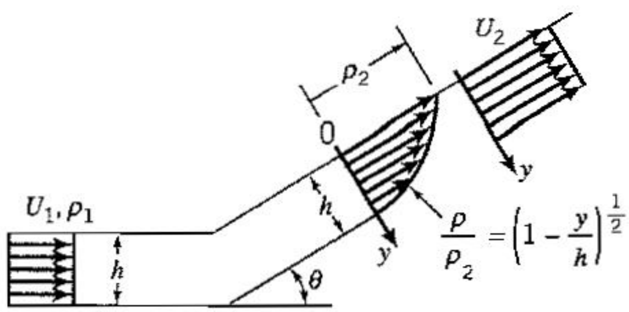


Figure P3.30

3.30 A fluid enters the rectangular duct shown in Figure P3.30 with constant velocity U_1 and density ρ_1. At the exit plane, the fluid velocity is U_2 and the density has a profile described by a square root relationship, with a maximum value of ρ_2. Find ρ_2 in terms of ρ_1 when $U_1 = U_2$. Also find the net vector momentum flux leaving the duct.

3.31 A fluid flows through a two-dimensional duct of width W, as shown in Figure P3.31. At the entrance to the box (face 1), the flow is one-dimensional and the fluid has density ρ_1 and velocity U_1. At the exit (face 2), the fluid has a uniform density ρ_2 but the velocity varies across the duct as shown. Find the rate at which the average density inside the box is changing with time.

3.32 The wake of a body is approximated by a linear profile as sketched in Figure P3.32.

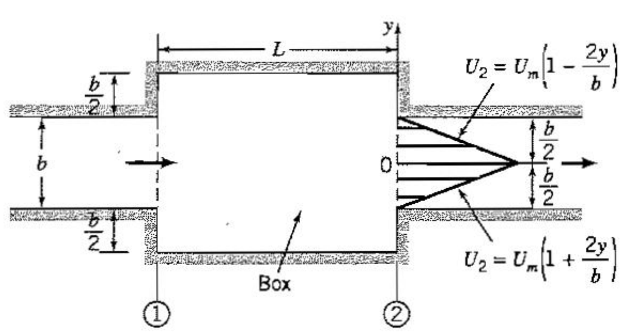


Figure P3.31

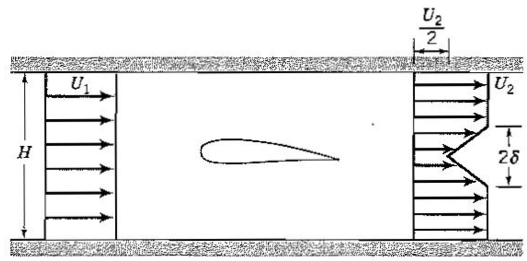


Figure P3.32

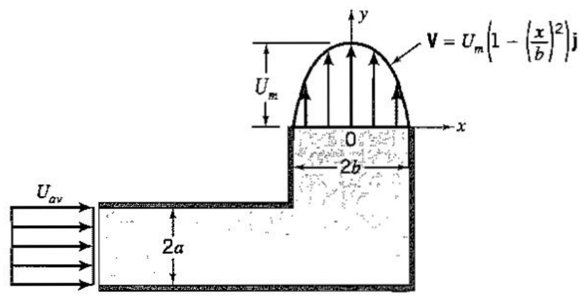


Figure P3.33

The flow is incompressible, steady and two-dimensional. Outside of the wake region the flow is inviscid and the velocity is U_2. The upstream velocity, U_1, is uniform.
(a) Find U_2/U_1 as a function of δ/H.
(b) Find the pressure coefficient $(p_2 - p_1)/(\frac{1}{2}\rho U_1^2)$ as a function of δ/H.
(c) Find a nondimensional drag coefficient $F_D/(\frac{1}{2}\rho U_1^2 H)$ as a function of δ/H, where F_D is the drag force per unit body width.

3.33 Air of constant density flows steadily through the rectangular duct of width W shown in Figure P3.33. At the entrance, the velocity is constant across the area and equal to U_{av}. The velocity at the exit has a parabolic distribution across the duct, with a maximum value U_m. The pressure is constant everywhere. By using the continuity equation, find the ratio b/a such that $U_m = U_{av}$. Then find the force $\mathbf{F}$ exerted by the flow on the duct, assuming that the wall friction is negligible.

3.34 A steady, constant density flow of air with a uniform velocity U_1 is sucked into a two-dimensional Venturi duct of width w and enters the rest of the duct at Station 2 with the velocity profile shown in Figure P3.34. Find:
(a) U_2 in terms of U_1.
(b) The force required to hold the duct in terms of U_1, a, w, θ, p_{2g} (the gauge pressure at Station 2), and the air density ρ.

3.35 A cylinder is held in a two-dimensional duct of constant width W and height H, as shown in Figure P3.35. As a result, the downstream velocity distribution becomes as shown. The density ρ is constant, and the flow is steady. Find U_2 in terms of U_1, and find the force exerted by the fluid on the cylinder in terms of ρ, U_1, W and H. Neglect frictional effects and assume that $p_1 = p_2$.

3.36 A propeller is placed in a constant area circular duct of diameter D, as shown in Figure P3.36. The flow is steady and has a constant density ρ. The pressures p_1 and p_2 are uniform across the entry and exit areas, and the velocity profiles are as shown.

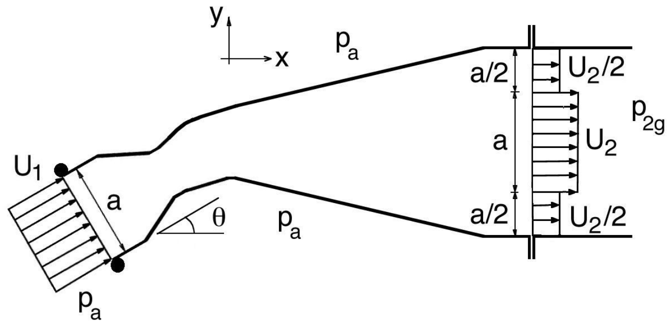


Figure P3.34

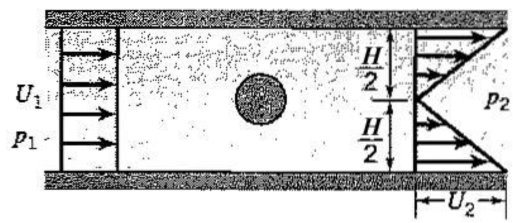
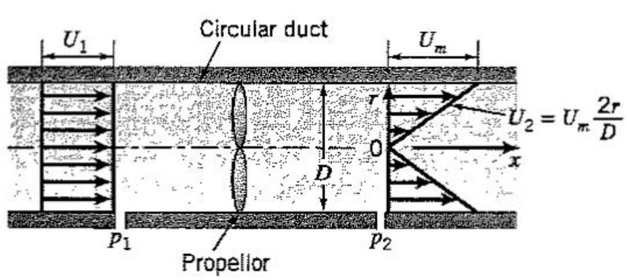


Figure P3.35

(a) Find U_m in terms of U_1.
(b) Find the thrust T exerted by the propeller on the fluid in terms of U_1, ρ, D, p_1 and p_2.

3.37 Air of constant density ρ enters a duct of width W and height H_1 with a uniform velocity V_1. The top wall diverges (as shown in Figure P3.37) so that the pressure remains uniform everywhere. Downstream, on the top and bottom wall, the velocity profiles are given by $V/V_1 = (y/\delta)^{1/4}$. Given that $H_2 = 1.1H_1$, find δ in terms of H_1, and find the magnitude and direction of the force F exerted by the air on the duct in terms of a nondimensional force coefficient $F/\rho V_1^2 H_1 W$ (ignore viscous stresses).

3.38 A fluid of constant density ρ enters a duct of width W and height h_1 with a parabolic velocity profile with a maximum value of V_1, as shown in Figure P3.38. At the exit plane the duct has height h_2 and the flow has a parabolic velocity profile with a maximum value of V_2. The pressures at the entry and exit stations are p_1 and p_2, respectively, and they are uniform across the duct.
(a) Find V_2 in terms of V_1, h_1 and h_2.
(b) Find the magnitude and direction of the horizontal force F exerted by the fluid on the



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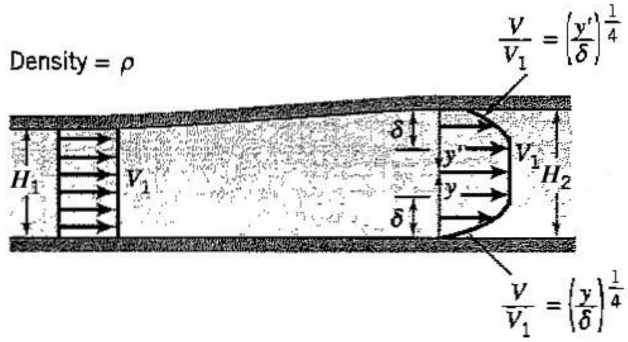

Figure P3.37

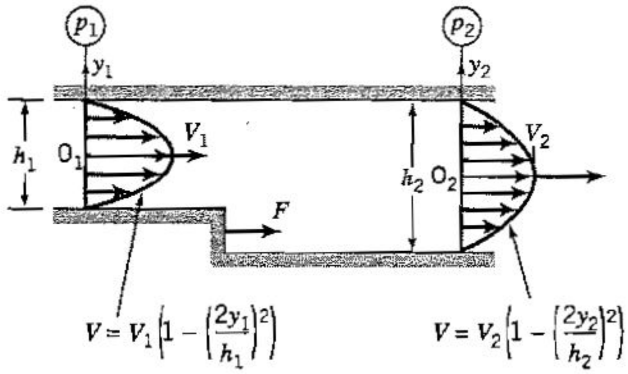

Figure P3.38

step in terms of ρ, V_1, W, p_1 and p_2, h_1 and h_2. Ignore friction. Note that at the point where the flow separates off the step, the flow streamlines can be assumed to be parallel.

3.39 Two fences are placed inside a horizontal duct of height H and width W, as shown in Figure P3.39. Air of constant density ρ flows steadily from left to right, and the velocity upstream of the fences (station 1) is constant across the area and equal to U_{av}. The velocity downstream of the fences (station 2) has a parabolic distribution across the duct with a maximum value U_m. The pressures at stations 1 and 2 are p_1 and p_2, respectively, and they are constant across the duct area.
(a) Find the ratio U_m/U_{av}.
(b) Find the force F acting on the fences, assuming that the wall friction is negligible. Express the result in terms of the nondimensional drag coefficient $C_D = F/\left(\frac{1}{2}\rho U_1^2 HW\right)$, and the nondimensional pressure coefficient

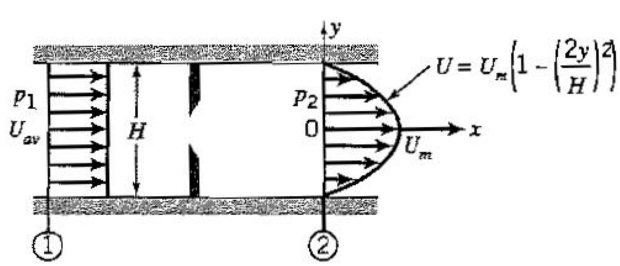

Figure P3.39

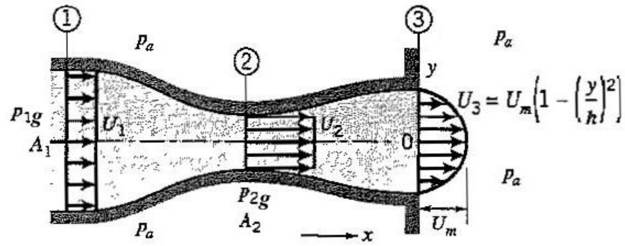

Figure P3.40

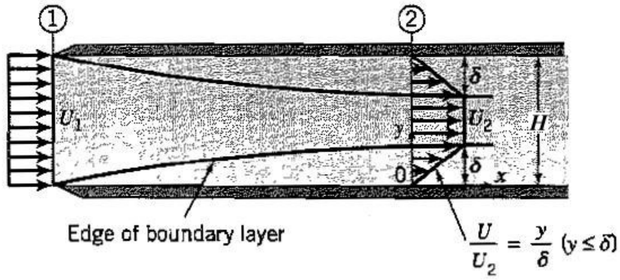

Figure P3.41

$C_p = (p_1 - p_2)/\left(\frac{1}{2}\rho U_1^2\right)$.

3.40 Air of constant density ρ flows steadily into a rectangular duct of constant width W, as shown in Figure P3.40. There is inflow over area $A_1 = 3hW$ and outflow over area $A_3 = 2hW$. Area $A_2 = hW$. Across A_1 and A_2 the velocities are constant and equal to U_1, and U_2, respectively, as shown. Across A_3 the velocity distribution is parabolic with a maximum value of U_m. Over area A_1 the gauge pressure p_{1g}, over area A_2 the gauge pressure is p_{2g}, and over area A_3 the pressure is atmospheric. There are no losses. ajsCan't apply Bernoulli here!
(a) Find the ratios U_2/U_1, and U_m/U_1.
(b) Find the pressure difference $p_{1g} - p_{2g}$ in terms of ρ and U_1.
(c) Find the force F acting on the fluid (magnitude and sign) between stations 1 and 3, in terms of ρ, U_1 and p_{1g}.

3.41 The entrance region of a parallel, rectangular duct flow is shown in Figure P3.41. The duct has a width W and a height H, where $W \gg H$. The fluid density ρ is constant, and the flow is steady. The velocity variation in the boundary layer of thickness δ at station 2 is assumed to be linear, and the pressure at any cross-section is uniform.
(a) Using the continuity equation, show that $U_1/U_2 = 1 - \delta/H$.
(b) Find the pressure coefficient $C_p = (p_1 - p_2)/\left(\frac{1}{2}\rho U_1^2\right)$.
(c) Show that

$$\frac{F_v}{\frac{1}{2}\rho U_1^2 WH} = 1 - \frac{U_2^2}{U_1^2}\left(1 - \frac{8\delta}{3H}\right)$$

where F_v is the total viscous force acting on the walls of the duct.

3.42 A steady jet of water of density ρ hits a deflector plate of constant width w inclined at an angle α, as shown in Figure P3.42. For the velocity distributions given in the figure, find the horizontal component of the force required to hold the deflector plate in place in terms of ρ, U_m, w, b, and α, assuming that viscous and gravity effects are negligible and the pressure is uniform everywhere.

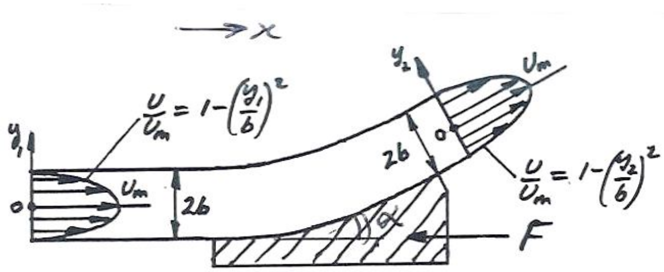

Figure P3.42

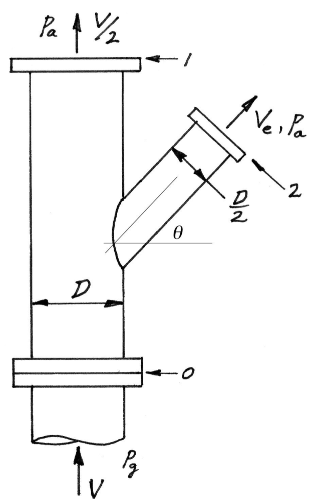

Figure P3.43.

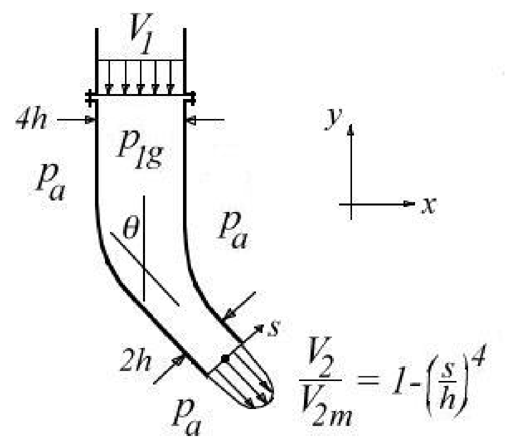

Figure P3.44

3.43 A circular pipe carries a steady flow of air of constant density ρ. The flow exits to atmosphere in two places through circular exits, labeled 1 and 2 in Figure P3.43. At the entrance (labeled 0 in the figure), the gauge pressure is p_g, the velocity is V, and the pipe diameter is D. At exit 1, the diameter is D and the velocity is $V/2$. At exit 2, the diameter is $D/2$. Assume one-dimensional, frictionless flow, and ignore gravity.
(a) Find the velocity at exit 2 in terms of V.
(b) Find the magnitude and direction of the force required to hold the duct in place.
Show your control volumes and all your working clearly, state all your assumptions, and indicate your coordinate system.

3.44 A rectangular duct of constant width w carries a steady air flow of constant density ρ as shown in Figure P3.44. The velocity V_1 is constant across the inlet area, and the outflow velocity V_2 has a velocity profile described by a quartic function, with a maximum value of V_{2m}. The flow exits to atmospheric pressure, and the gauge pressure at the inlet is p_{1g}. Ignore viscous effects and gravity.
(a) Find V_{2m} in terms of V_1.
(b) Find the vector force required to hold the duct in place in terms of ρ, V_1, h, w and θ, when $p_{1g} = \frac{1}{2}\rho V_1^2$. Express the answer in nondimensional form.

3.45 A steady flow of air of constant density ρ issues from two exit areas in a two-dimensional duct of width w and height $2h$, as shown in Figure P3.45, and for each exit plane the velocity distribution is parabolic with a maximum value of V_2. The inflow to the

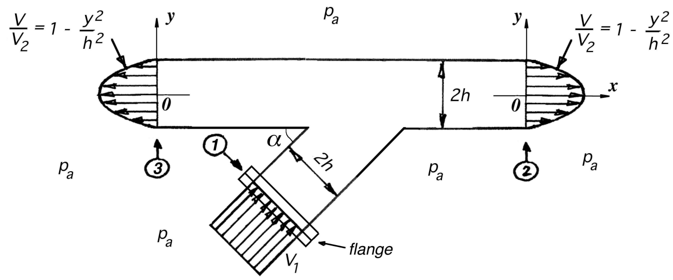

Figure P3.45

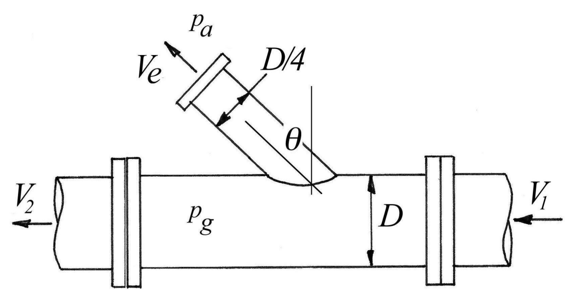

Figure P3.46

duct has a uniform velocity distribution of velocity of magnitude V_1, and it makes an angle α with the outlet duct. The pressure is atmospheric at the exit, and the gauge pressure at the inlet is p_{1g}. Ignore the weight of the fluid.
(a) Find the ratio V_2/V_1 (this is a number).
(b) Find the force (x- and y-components) required to hold the duct at the connecting flange in terms of h, w, ρ, V_1, α, and p_{1g}.

3.46 A circular pipe of diameter D carries a steady flow of air of constant density ρ and a constant gauge pressure of p_g. A portion of the flow exits to atmosphere through a circular exit of diameter $D/4$ at a velocity $V_e = 4V_1$, as shown in Figure P3.46. Assume one-dimensional flow and ignore gravity. Show your control volumes and all your working clearly, state all your assumptions, and indicate your coordinate system.
(a) Find V_2 in terms of V_1.
(b) Find the magnitude and direction of the force required to hold the length of duct between the two flanges in place in terms of ρ, V_1, D and θ.

3.47 Air flows steadily through a bend of width w, as shown in Figure P3.47. Gravity and viscous forces are not important, and the air density ρ is constant. Show all your working.
(a) Find the ratio V_2/V_1 (this is a number).
(b) Find the resultant force required to hold the bend in place in terms of ρ, V_1, h, w, p_{1g}, p_{2g}, and θ.

3.48 A horizontal steady jet of water of density ρ, height h, width w and velocity V hits a flat plate mounted on a wheeled cart. The jet hits the flat plate at right angles. The cart starts moving and accelerates until it reaches a constant velocity $V/2$. Find the magnitude and direction of the frictional force acting on the cart when it reaches its terminal speed, in terms of ρ, h, w, and V.

3.49 A steady jet of air of constant density ρ issues from a two-dimensional duct of width

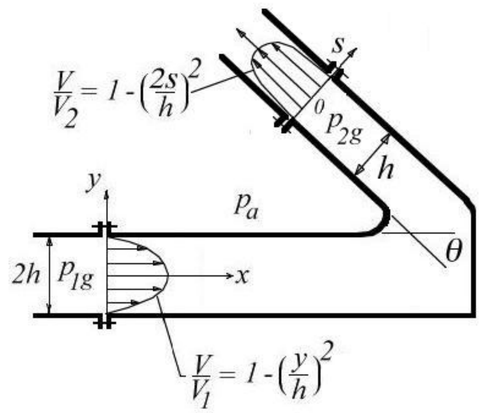

Figure P3.47

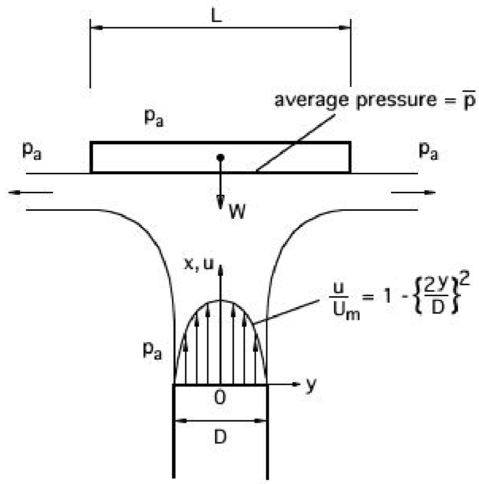

Figure P3.49

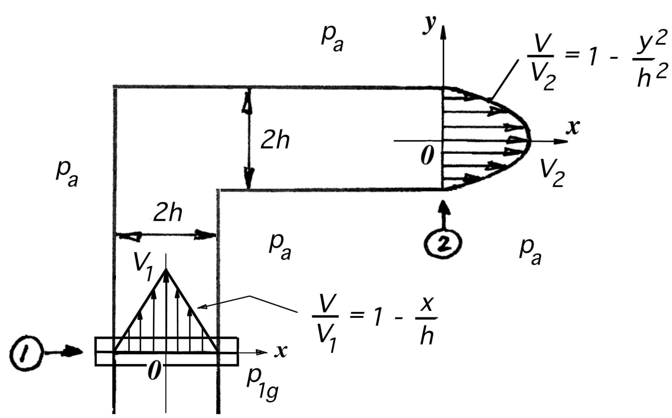

Figure P3.50

D and depth a (into the page), as shown in Figure P3.49, with velocity distribution that is parabolic. The jet supports a plate of width L, depth a (into the page) and weight W, and the air is deflected horizontally. The side of the plate facing the jet experiences an average pressure p, and the pressure is atmospheric everywhere else. Ignore the weight of the fluid.
(a) Find the average pressure p in terms of L, D, ρ, and U_m.
(b) Find the weight of the plate W in terms of L, D, a, ρ, and U_m.

3.50 A steady flow of air of constant density ρ issues from a two-dimensional duct of width w and height $2h$, as shown in Figure P3.50, with a velocity distribution that is parabolic with a maximum value of V_2. The inflow to the duct has a symmetrical, triangular distribution of velocity with a maximum value of V_1. The pressure is atmospheric at the exit, and the gauge pressure at the inlet is p_{1g}. Ignore the weight of the fluid.
(a) Find the ratio V_2/V_1 (this is a number).
(b) Find the force (x- and y-components) required to hold the duct at the connecting flange in terms of h, w, ρ, V_1, and p_{1g}.

3.51 A fluid of constant density ρ flows over a flat plate of length L and width W, as

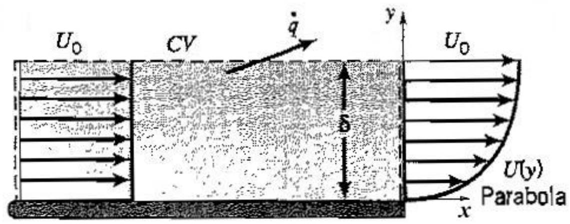

Figure P3.51

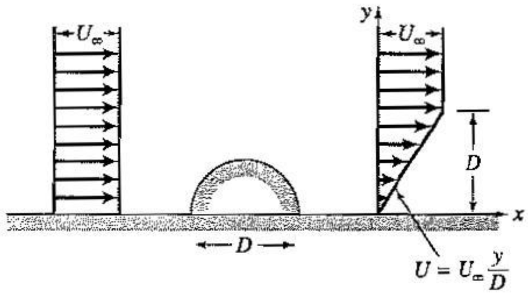

Figure P3.52

shown in Figure P3.51. At the leading edge of the plate the velocity is uniform and equal to U_0. A boundary layer forms on the plate so that at the trailing edge the velocity profile is parabolic. Find:
(a) The volume flow rate $\dot{q}$ leaving the top surface of the control volume where $y = \delta$.
(b) The x-component of the momentum flux leaving the control volume through the same surface.

3.52 A model of a two-dimensional semi-circular hut was put in a wind tunnel, and the downstream velocity profile was found to be as shown in Figure P3.52. Here, U_∞ is the freestream velocity, ρ is the air density, and D is the hut diameter. Assume that viscous effects and pressure variations can be neglected.
(a) Draw the flow pattern over the hut (remember that continuity must be satisfied).
(b) Find the average velocity in the y-direction over the area located at $y = D$.
(c) Find the nondimensional force coefficient C_D, where $C_D = F/\left(\frac{1}{2}\rho U_\infty^2 D\right)$, and F is the x-component of the force acting on the hut per unit width.

3.53 A cylinder of length W is located near a plane wall, as shown in Figure P3.53. The incoming flow has a uniform velocity U_∞, and the downstream flow has a linear velocity profile $U = U_\infty y/H$. Assuming steady, constant density, constant pressure flow, find:
(a) The average velocity in the y-direction over the area located at $y = H$.
(b) The x-component of the force exerted on the cylinder by the fluid. Neglect viscous forces.

3.54 A fluid of constant density ρ flows steadily over a cylinder that is located far from any solid boundary, as shown in Figure P3.54. Upstream, the flow has a uniform velocity U_1, and downstream the velocity distribution in the wake is triangular with a maximum value of U_1, as shown below. The pressure is atmospheric everywhere. For the control volume shown, the x-component of the velocity is uniform on every surface of the control volume and equal to U_1.
(a) Find the average value of the velocity normal to surfaces 3 and 4.

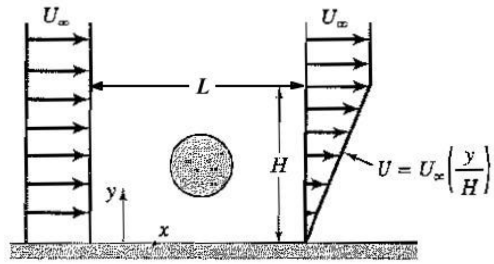

Figure P3.53

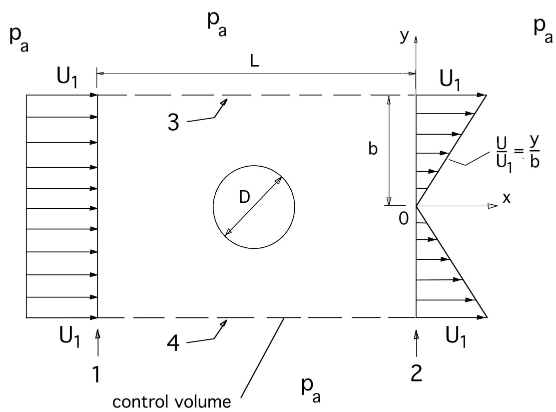

Figure P3.54

(b) Find the drag force acting on the cylinder (that is, the force exerted by the fluid on the cylinder).

3.55 A layer of fluid of kinematic viscosity ν and depth h flows down a plane inclined at an angle θ to the horizontal so that the flow is laminar with a velocity profile described by

$$\frac{u}{U_e} = \sin\left(\frac{\pi y}{2h}\right)$$

where U_e is the velocity at the free surface (see Figure P3.55). Use a control volume analysis to find U_e in terms of ν, g, h and θ.

3.56 A square sled, where each side is of length L, rides on thin layer of oil of thickness h and viscosity μ, as shown in Figure P3.56. The sled carries a tank of water of density ρ, as well as a circular water jet of diameter d, inclined at an angle α to the horizontal. If the

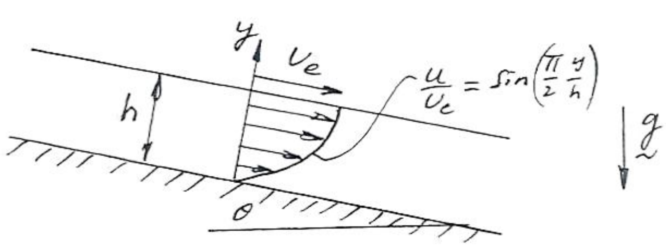

Figure P3.55

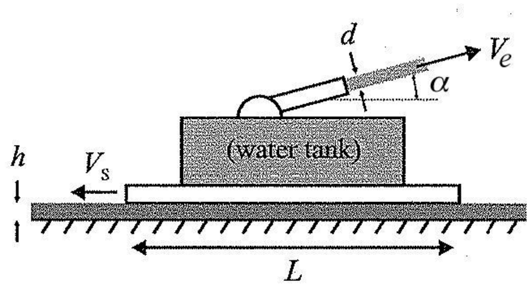

Figure P3.56

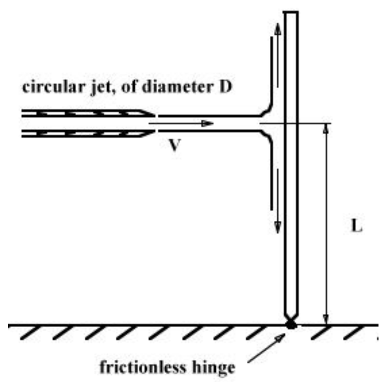

Figure P3.57

sled moves at a constant velocity V_s, and the exit velocity of the jet is V_e (relative to the cart), find h in terms of the other variables.

3.57 A circular water jet of diameter D, velocity V, and density ρ impinges on a vertical plate supported by a frictionless hinge a distance L above the hinge, as shown in Figure P3.57. Find the direction and magnitude of the moment that needs to be exerted around the hinge to maintain the plate in a vertical position. Note that $D << L$.

3.58 A horizontal jet of water, of constant velocity U_j strikes a deflector such that the jet direction is smoothly changed, as shown in Figure P3.58. Find the ratio of the vertical and horizontal components of the force required to hold the deflector stationary in terms of the angle θ. Neglect gravitational effects. Does this force ratio change when the deflector moves to the right with velocity U_b?

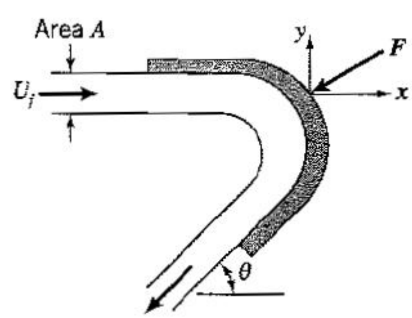

Figure P3.58

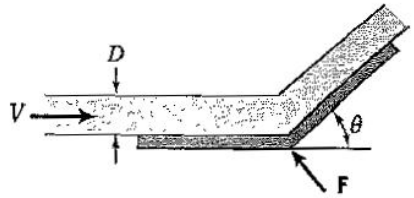

Figure P3.59

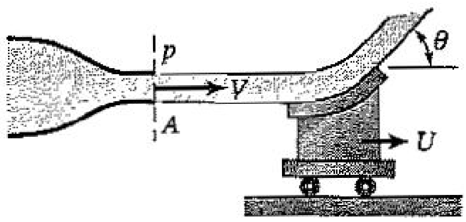

Figure P3.60

3.59 A horizontal jet of air of width W strikes a stationary scoop with a velocity V, as indicated in Figure P3.59. If the jet height h remains constant as the air flows over the plate surface, find:
(a) The force **F** required to hold the plate stationary.
(b) The change in **F** when the plate moves to the right at a constant speed $V/2$.

3.60 Water from a stationary nozzle impinges on a moving vane with a turning angle of $\theta = 60°$, as shown in Figure P3.60. The vane moves at a constant speed $U = 10\,m/s$, and the jet exit velocity is $V = 30\,m/s$. The nozzle has an exit area of $0.005\,m^2$. Find the force exerted by the fluid on the vane required to keep U constant.

3.61 Water of density ρ flows steadily through a smooth contraction of width w, as shown in Figure P3.61, and exits to atmospheric pressure. A Pitot tube in the jet is connected to a manometer where the other end is open to atmospheric pressure. The manometer fluid has the same density as water. The jet of water strikes a vane that turns the fluid an angle θ.
(a) Find the ratio V_2/V_1 (this is a number), given that the gauge pressure upstream of the contraction is $12\rho V_1^2$.
(b) Find V_2 in m/s when $h = 1\,m$.
(c) Find the resultant force required to hold the vane in place in terms of ρ, V_2, H_2, w and θ. Gravity and viscous forces are not important.
(d) If the vane now moves at a constant velocity $V_2/2$ to the right, how does the resultant force required to hold the vane change?

3.62 A snow plow mounted on a truck clears a path $12\,ft$ wide through heavy wet snow. The snow is $8\,in$ deep and its density is $10\,lb_m/ft^3$. The truck travels at $20\,mph$, and the plow is set at an angle of 45° from the direction of travel and 45° above the horizontal, as shown in Figure P3.62. The snow is therefore discharged from the plow at an angle of 45° from the direction of travel and 45° above the horizontal. Find the force required to push the plow.[2]

3.63 A pump submerged in water contained in a cart ejects the water into the atmo-

[2] Adapted from Fox & McDonald, *Introduction to Fluid Mechanics*, 4th ed., John Wiley & Sons, 1992.

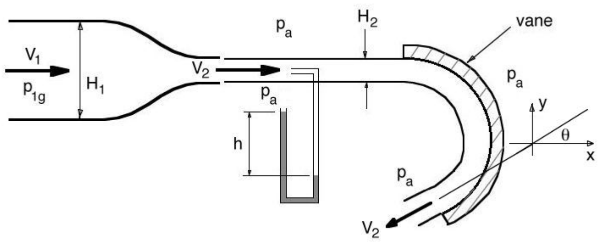

Figure P3.61

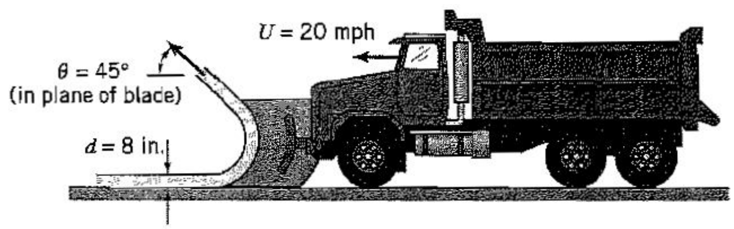

Figure P3.62

sphere, as shown in Figure P3.63. The flow area leaving the ejector is $0.01\,m^2$, and the exit of the ejector is at the same height as the top edge of the side of the cart.
(a) If the ejection flow velocity is $3\,m/s$, the flow will be returned to the cart. Find F, the force necessary to restrain the cart.
(b) If the ejection flow velocity is $4\,m/s$, the flow will just clear the side of the cart. Find F, the force necessary to restrain the cart.
(c) Find L, the minimum distance between the ejection exit and the side of the cart for the flow in part (b).

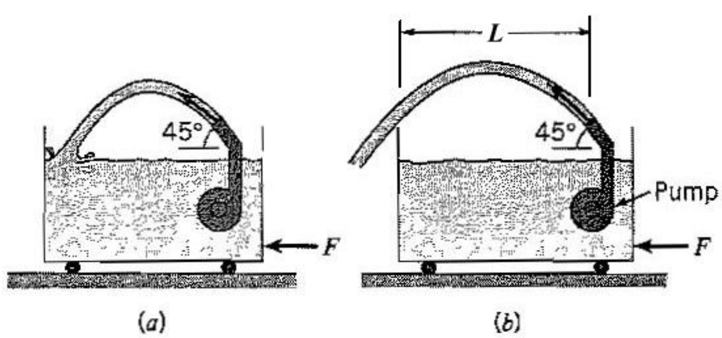

Figure P3.63

Chapter 4

Kinematics and Bernoulli's Equation

4.1 Study Guide

- What is meant by the term "steady, one-dimensional flow?"
- Give the definitions of a streamline, and a pathline. Under what condition are they the same?
- Write down Bernoulli's equation. Under what conditions does this relationship hold?
- Explain the terms "total pressure" and "dynamic pressure". What does a Pitot tube measure? What does a Pitot-static tube measure?
- To find the velocity of a flow using a Pitot probe, what measurements need to be made?
- Draw a sketch illustrating a simple stagnation point in a uniform flow. What is the stagnation pressure? Why do you think it is sometimes called the "total" pressure?

4.2 Worked Examples

Example 4.1: Flow in a jet

Consider a tank draining through a small orifice, where the orifice outlet points up at an angle θ, as in Figure 4.1. The magnitude of the exit velocity is still $V_e = \sqrt{2gH}$. As the jet issues into the atmosphere, the vertical component of the velocity, w, decreases under the action of gravity. At the top of the jet trajectory $w = 0$, and then becomes negative. The horizontal component of the fluid velocity, u, remains constant throughout the trajectory when air friction is neglected, because the only force acting is due to gravity. Find z_m, the maximum height to which the jet rises.

Solution: Consider a streamline that starts at the exit and follows the path of the jet. The pressure everywhere outside the jet is atmospheric, and since there are no pressure gradients across the jet (it is in free fall), the pressure inside the jet is also atmospheric. If there are no losses,

$$\tfrac{1}{2}V_e^2 = \tfrac{1}{2}V^2 + gz = \text{constant}$$

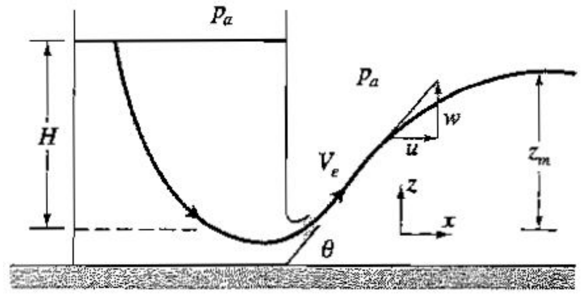

Figure 4.1: Tank draining with exit pointing up at angle θ.

Now $V^2 = u^2 + w^2$, and for the exit velocity, $V_e^2 = u_e^2 + w_e^2$. Since the horizontal component of the fluid velocity remains constant, $u_e = u = \text{constant}$ (there are no forces acting on the fluid in this direction). Bernoulli's equation reduces to

$$\tfrac{1}{2}w_e^2 = \tfrac{1}{2}w^2 + gz \tag{4.1}$$

The highest point of the trajectory is given by point where $z = z_m$ and $w = 0$. That is,

$$gz_m = \tfrac{1}{2}w_e^2$$

Since $w_e = V_e \sin\theta = \sqrt{2gH}\sin\theta$,

$$gz_m = \tfrac{1}{2}\left(2gH\sin^2\theta\right)$$

and

$$z_m = H\sin^2\theta$$

We can check this answer by taking limits. For $\theta = 90°$, $z_m = H$, and for $\theta = 0°$, $z_m = 0$, as expected.

Example 4.2: Forces exerted by an exiting jet

A tank sits on a balance scale to measure the vertical force F_z and F_x. The tank has an opening near the bottom that points up at an angle θ to the horizontal (Figure 4.2). The water level is kept constant so that W, the weight of the tank and its contents, is constant and the flow is steady. We will assume that there are no losses, and that the fluid is of constant density ρ.
(a) What is F_z?
(b) What is F_x?
(c) Find W_w, the weight of water in flight between the jet exit and the point of maximum jet height.

Solution: For part (a) we use the control volume labeled CV_1. The z-momentum leaving the control volume is given by the product of the mass flow rate $\rho V_e A_e$ and the z-component of the velocity $V_e \sin\theta$, where V_e is the velocity and A_e is the cross-sectional area of the jet at the exit. The momentum equation in the z-direction then gives

$$-W + F_z = \rho V_e A_e \left(V_e \sin\theta\right)$$

Hence,

$$F_z = W + \rho V_e A_e \left(V_e \sin\theta\right) \tag{4.2}$$

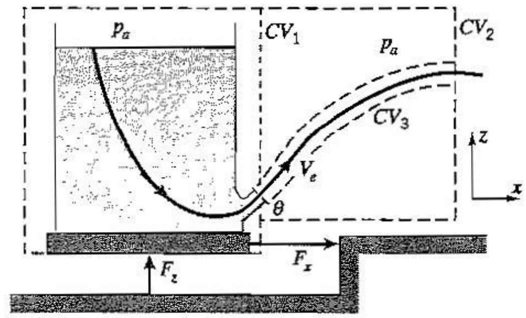

Figure 4.2: Reactive forces acting when a tank drains.

For part (b), we can use either control volume CV_1 or control volume CV_2, since the horizontal component of the momentum does not change in the x-direction (no forces act in that direction). The x-momentum leaving the control volume is given by the product of the mass flow rate $\rho V_e A_e$ and the x-component of the velocity $V_e \cos\theta$, so for CV_1 or CV_2

$$F_x = \rho V_e A_e \left(V_e \cos\theta\right) \tag{4.3}$$

For part (c), we can use either control volume CV_2 or control volume CV_3. For CV_2, the momentum equation in the z-direction is

$$-W - W_w + F_z \;=\; 0$$

From equation 4.2,

$$W_w = F_z - W = \rho V_e A_e \left(V_e \sin\theta\right) \tag{4.4}$$

For control volume CV_3, the momentum equation in the z-direction becomes

$$-W_w \;=\; -\rho V_e A_e \left(V_e \sin\theta\right)$$

so that

$$W_w = \rho V_e A_e \left(V_e \sin\theta\right)$$

which is the same result obtained using CV_2 (equation 4.4). That is, the z-momentum leaving CV_1 is the same as the weight of the water in flight.

Example 4.3: Jet on a cart

A pipe of cross-sectional area A is connected by a flange to a large, pressurized tank which supplies air of constant density ρ to a jet of cross-sectional area $\frac{1}{4}A$, as shown in Figure 4.3. The tank sits on a cart equipped with frictionless wheels so that it can roll freely, and the jet exits to atmosphere with a velocity V. Find:
(a) The gauge pressure p_g in the pipe at the flange;
(b) The force holding the pipe to the tank at the flange, F_f; and
(c) The tension in the string holding the cart, T.

Solution: For part (a), we will assume that the flow is quasi-steady (it is a large tank), and that there are no losses downstream of the flange, so that Bernoulli's equation can be used between a point in the pipe at the location of the flange, where the velocity is V_1, and a point in the exit plane of the jet, where the pressure is atmospheric. Hence

$$p_g + \tfrac{1}{2}\rho V_1^2 = \tfrac{1}{2}\rho V^2$$

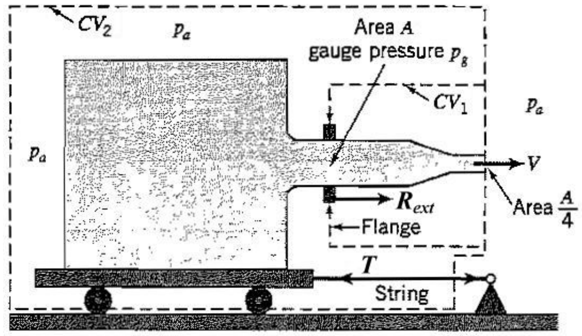

Figure 4.3: Jet on a cart, held in place by a string.

From the continuity equation we have

$$V_1 A = \tfrac{1}{4} V$$

so that

$$p_g = \tfrac{15}{32}\rho V^2$$

For part (b), we use control volume $CV1$ and apply the x-momentum equation. Let R_{ext} be the x-component of the force exerted by the pipe on the fluid. Therefore

$$R_{ext} + p_g A \;=\; \tfrac{1}{4}\rho V^2 A - \rho V_1^2 A \;=\; \tfrac{1}{4}\rho V^2 A - \tfrac{1}{16}\rho V^2 A$$

so that

$$R_{ext} \;=\; -p_g A + \tfrac{3}{16}\rho V^2 A \;=\; -\tfrac{15}{32}\rho V^2 A + \tfrac{3}{16}\rho V^2 A \;=\; -\tfrac{9}{32}\rho V^2 A$$

R_{ext} is the force exerted by the pipe on the fluid. Therefore the force exerted by the fluid on the pipe is $-R_{ext}$, so F_f, the force holding the pipe to the tank at the flange is $F_D = -(-R_{ext}) = R_{ext}$.

For part (c), we use control volume $CV2$ and apply the x-momentum equation. Let F_c be the x-component of the force exerted by the cart on the fluid. Therefore

$$F_c \;=\; \tfrac{1}{4}\rho V^2 A$$

Hence the force exerted by the cart on the fluid is $-F_c$, and T, the tension in the string holding the cart , is $T = -(-F_c) = F_c$.

Problems

4.1 Consider the velocity field given by $\mathbf{V} = x\mathbf{i} + y\mathbf{j}$. Find the general equation for the streamlines. Sketch the flow field.

4.2 Consider the velocity field given by $\mathbf{V} = x\mathbf{i} + y\mathbf{j}$. Find the general equation for the pathlines. Compare with the results obtained in the previous problem.

4.3 Consider the velocity field given by $\mathbf{V} = x^{-1}\mathbf{i} + x^{-2}\mathbf{j}$. Find an equation for the streamline passing through the point $[1, 2]$.

4.4 Consider the velocity field given by $\mathbf{V} = x^2\mathbf{i} - xy\mathbf{j}$. Find an equation for the streamline passing through the point $[2, 1]$. How long does it take a particle of fluid to move from this point to the point where $x = 4$?

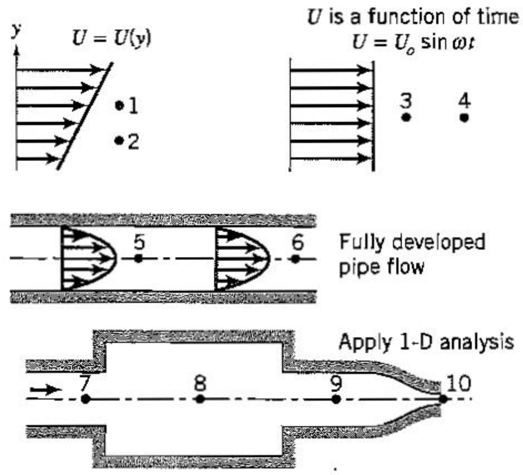

Figure P4.5

4.5 For the flows shown in Figure P4.5, give reasons why Bernoulli's equation can or cannot be used between points:
(a) 1 and 2;
(b) 3 and 4;
(c) 5 and 6;
(d) 7 and 8;
(e) 8 and 9;
(f) 9 and 10.

4.6 For the duct flow shown in Figure P4.6, the pressure at the exit (point 4) is atmospheric, the density ρ is constant, and the duct is of constant width w.
(a) Sketch the flow pattern.
(b) Can you use Bernoulli's equation between points 2 and 4? Why?
(c) Are the pressures at points 2 and 3 equal? Why?
(d) Find the gauge pressure at point 2 in terms of ρ and V_1.
(e) Find the gauge pressure at point 1 in terms of ρ and V_1.

4.7 Fluid passes through a fan placed in a duct of constant area, as shown in Figure P4.7. The density is constant.
(a) Is the volume flow rate at station 1 equal to that at station 2? Why?
(b) Can Bernoulli's equation be applied between stations 1 and 2? Why?

4.8 Consider a constant density gas flowing steadily through a smooth, circular, horizontal contraction. Calculate the velocity and pressure as the radius decreases to half its original value. State all your assumptions.

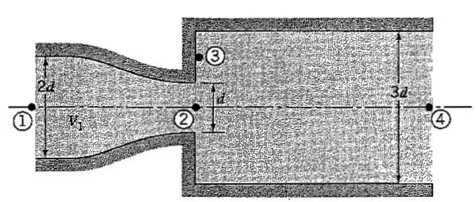

Figure P4.6

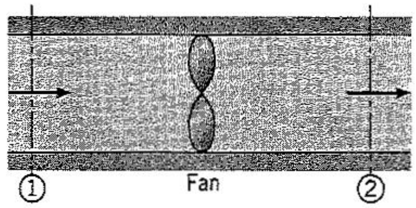

Figure P4.7

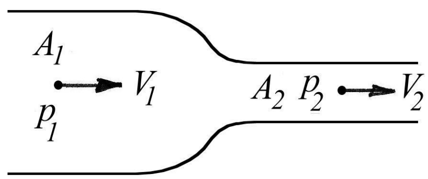

Figure P4.14

4.9 Air flows through a smooth contraction so that the velocity increase by a factor of 5. What is the change in pressure? Express the answer in non-dimensional form. List all your assumptions.

4.10 A Venturi tube is a tube that has a decreasing and then increasing diameter, and it is designed to have very small losses. Assume one-dimensional flow, and neglect gravity.

(i) What is the maximum included angle on the expansion to avoid separation?

(ii) If losses can be neglected, find the change in pressure at the narrowest point in terms of the diameter ratio, the upstream velocity, and the fluid density. Express the answer in non-dimensional form.

4.11 A Pitot tube is used to measure the total pressure in a wind tunnel. If the total pressure is $1200\,Pa$, and the local static pressure is $100\,Pa$, what is the wind tunnel speed if the air density ρ is $1.2\,kg/m^3$? What assumptions did you make?

4.12 Consider a constant density gas flowing steadily over an airfoil. Far upstream the velocity is V_0. Halfway along the top surface, the velocity has increased to $2V_0$. Halfway along the bottom surface, the velocity has decreased to $V_0/2$. Find the pressure difference between the top and bottom surface at this location. State all your assumptions.

4.13 A wing is tested in a wind tunnel at a speed of $10\,m/s$. The pressure in the wind tunnel is atmospheric. Find the gauge pressure at the stagnation point if the air density $\rho = 1.2\,kg/m^3$. What is the static (gauge) pressure at the point where the velocity on the wing is $15\,m/s$? What assumptions did you make?

4.14 A wind tunnel carrying air of density ρ has two pressure taps in its walls to measure the static pressure, as shown in Figure P4.14. One tap is located upstream of the contraction where the cross-sectional area is A_1, the velocity is V_1, and the pressure is p_1, and the other tap is located in the working section where the area is A_2, the velocity is V_2 and the pressure is p_2. If $A_1 \gg A_2$, find V_2 in terms of ρ, p_1 and p_2. Show your working and state all your assumptions.

4.15 An airplane is moving at a speed of $250\,mph$ at an elevation of $12,000\,ft$. By using Bernoulli's equation, find the pressure at the stagnation point, and at a point on the upper

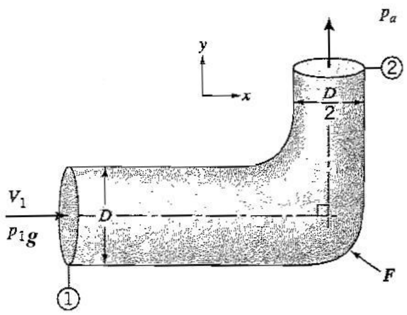

Figure P4.18

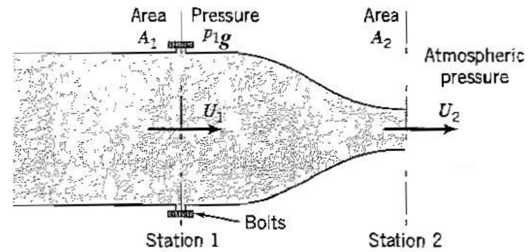

Figure P4.19

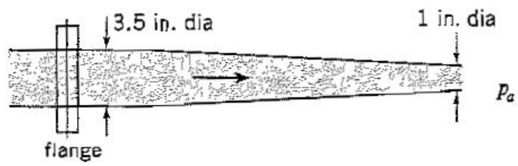

Figure P4.20

surface of the wing where the local velocity is $350\,mph$. Assume a Standard Atmosphere (Table Appendix-C.6).

4.16 An airplane is moving through still air at $60\,m/s$. At some point on the wing, the air pressure is $-1200\,N/m^2$ gauge. If the density of air is $0.8\,kg/m^3$, find the velocity of the flow at this point. Carefully list the assumptions you have made in your analysis. Express your answer in terms of the nondimensional pressure coefficient C_p.

4.17 Water flows steadily at a rate of $0.6 ft^3/s$ through a horizontal cone-shaped contraction the diameter of which decreases from $4.0\,in.$ to $3.0\,in.$ over a length of $1.2\,ft$. Assuming that conditions are uniform over any cross section, find the rate of change of pressure with distance in the direction of the flow at the section $0.6\,ft$ from the end of the contraction.

4.18 Consider the steady flow of air with no losses through the circular duct shown in Figure P4.18. The air has constant density ρ, and the duct exits to atmospheric pressure. Assume one-dimensional flow.
(a) Find p_{1g}, the gauge pressure at station 1, in terms of ρ and V_1.
(b) Find the direction of the force $\mathbf{F}$ that is necessary to hold the duct in place.

4.19 A one-dimensional air flow of constant density ρ exits steadily into the ambient atmosphere from the contraction shown in Figure P4.19. The contraction is bolted onto a constant area duct at station 1, and the area ratio $A_1/A_2 = 4$. If the total force in the bolts is F_x, find $F_x/\rho U_1^2 A_1$. Show all your work, and state your assumptions clearly. Would your analysis hold if the flow direction was reversed?

4.20 Neglecting friction, find the axial force produced at the flange when water discharges at at $200\,gpm$ into atmospheric pressure from the circular nozzle shown in Figure P4.20.

4.21 Air of constant density ρ flows steadily through the circular pipe of radius R with no losses at a velocity V, as shown in Figure P4.21. Find the vector force $\mathbf{F}$, exerted at the flange required to hold the pipe in place, in terms of ρ, R, V, and θ. Ignore gravity. For this problem, the nozzle is not attached to the exit.

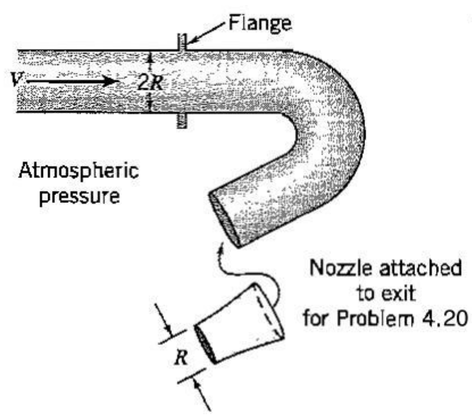

Figure P4.21

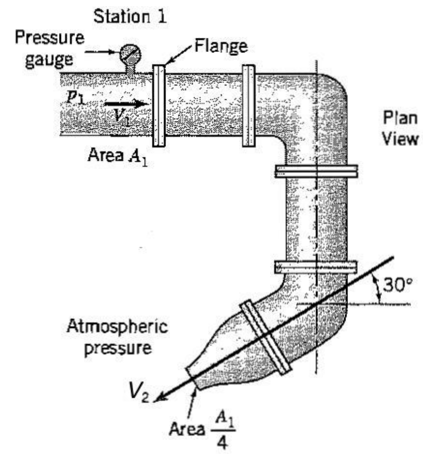

Figure P4.23

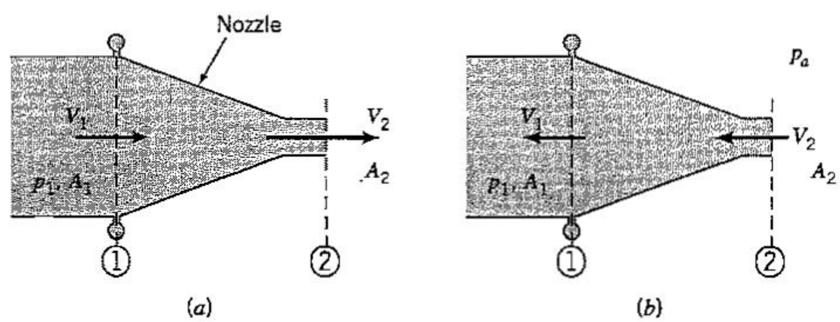

Figure P4.24

4.22 A nozzle with an exit radius of $\frac{1}{2}R$ is attached to the exit of the pipe bend described in the previous problem. Find the new vector force $\mathbf{F}'$, exerted at the flange required to hold the pipe in place, in terms of ρ, R, V, and θ. Ignore gravity, and neglect losses.

4.23 Air of constant density ρ flows steadily through the horizontal pipe system shown in Figure P4.23. The air flow exits to atmosphere through a 4:1 contraction. Assume one-dimensional flow, neglect losses, and neglect the weight of the piping.
(a) Find the gauge pressure measured by the gauge near station 1 in terms of ρ and V_1.
(b) Find the resultant force acting in the flange bolts at station 1.

4.24 An incompressible fluid of density ρ flows smoothly and steadily from left to right through the nozzle shown in Figure P4.24(a). The pressure outside the nozzle is equal to atmospheric pressure, and the flow may be assumed to be one-dimensional. Determine the magnitude and the direction of the force exerted by the fluid on the nozzle in terms of the density ρ, the inlet velocity V_1, and the inlet area A_1, given that the pressures and velocities at stations 1 and 2 are uniform across the areas A_1 and A_2, and that $A_1/A_2 = 4$.

4.25 Consider the previous problem in the case where the flow is from right to left [Figure P4.25(b)]. What additional information (beyond the density ρ, the inlet velocity V_1, and the inlet area A_1) would you require to determine the force exerted by the fluid on the nozzle with this new flow direction?

4.26 Water in an open cylindrical tank 10 ft in diameter and 6 ft deep drains through

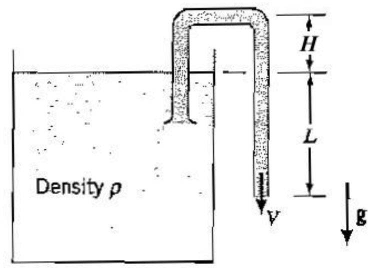

Figure P4.30

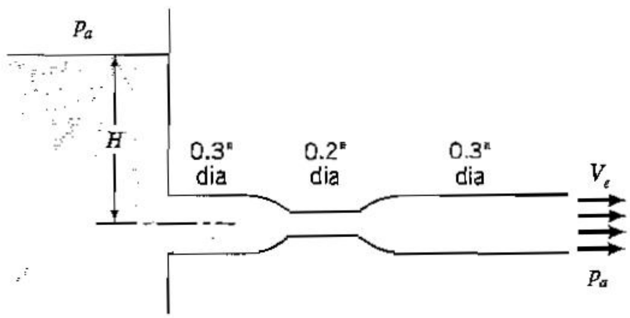

Figure P4.31

a $2\,in$ diameter nozzle in the bottom of the tank. Neglecting friction and the unsteadiness of the flow, find the volume of water discharged in $20\,s$, given that the pressure at the exit is atmospheric.

4.27 Water flows into a large circular tank at a rate of $\dot{q}\,m^3/s$ that is open to the atmosphere. Water also leaves through a smooth circular nozzle of diameter d a distance h below the free surface. Determine the height h for the flow to be independent of time.

4.28 A tube of constant cross-sectional area A is used to siphon water of density ρ from a large open tank to a point a distance H below the water level in the tank.
(i) Find the maximum volume flow rate in terms of ρ, g and H.
(ii) Find the dynamic pressure at the exit. Show your working and state all your assumptions.

4.29 A siphon is used to empty a large tank. The exit of the siphon is located a distance h below the surface of the water, and points up at an angle θ to the horizontal. The water is allowed to stream out of the exit without loss. Find the maximum vertical distance the water reaches.

4.30 Given the siphon arrangement shown in Figure P4.30, find the exit velocity of the siphon assuming no losses. What limit exists on the maximum value of L if the siphon is to continue to function? What limit exists on the value of H?

4.31 Water exits from a reservoir as shown in Figure P4.31. As H increases, the exit velocity increases until a critical elevation is reached and cavitation occurs. Find this value of H. Assume uniform flow and no losses, and that the vapor pressure is $0.25\,psia$.

4.32 A tube of constant area A_t is used as a siphon, and it draws fluid of constant density ρ without loss from an infinitely large reservoir, as shown in Figure P4.32. The fluid exits

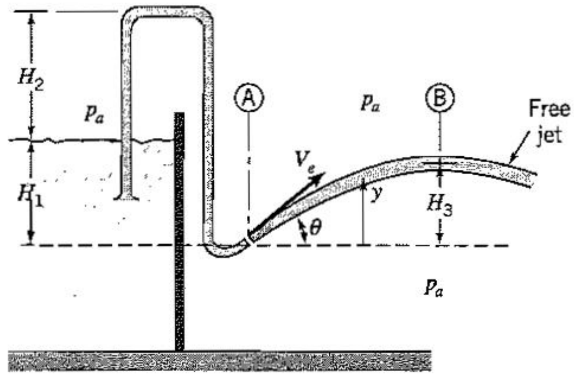

Figure P4.32

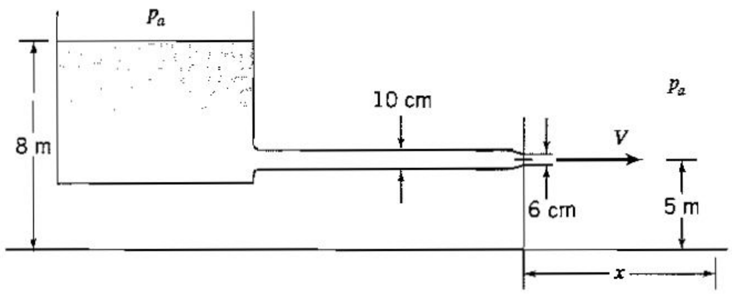

Figure P4.34:

with velocity V_e, at an angle θ to the horizontal, at a distance H_1 below the surface of the reservoir.
(a) Explain what the practical restrictions are on the maximum height of the siphon tube, H_2.
(b) Express the ratio of the cross-sectional area of the jet to the cross-sectional area of the tube as a function of H_1 and height y.
(c) Find H_3, the maximum height reached by the jet, as a function of H_1 and θ.
(d) Find the volume of water contained in the jet between stations A and B.

4.33 Water leaves a small hole in the vertical side of a bucket in a continuous, initially horizontal jet.
(a) If the head of water above the hole is $1.25\,m$, what is the jet velocity at exit?
(b) If the jet strikes the ground at a point situated $2.21\,m$ horizontally from the hole, and the ground is $1\,m$ below the hole, recalculate the jet velocity at exit. Why might this result be different from that obtained in part (a)?

4.34 Water flows from a large open reservoir and discharges into air at atmospheric pressure through a circular, horizontal pipe fitted with a nozzle, as shown in Figure P4.34. Subsequently, it strikes the ground a distance x downstream from the nozzle. Neglecting losses, find
(a) The velocity at the nozzle exit;
(b) The velocity and pressure in the pipe near the nozzle;
(c) The distance x.

4.35 A jet issues without loss from a hole in a water tank that is open to the atmosphere. The hole is located a distance H below the free surface, and the jet issues at an angle θ to

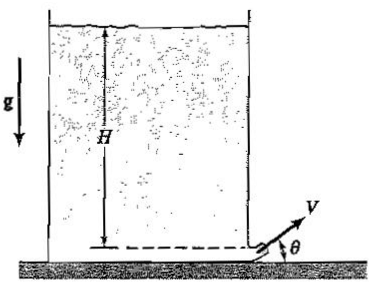

Figure P4.36

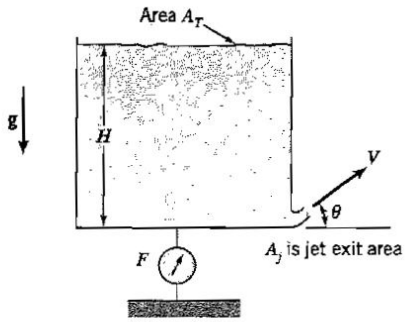

Figure P4.37

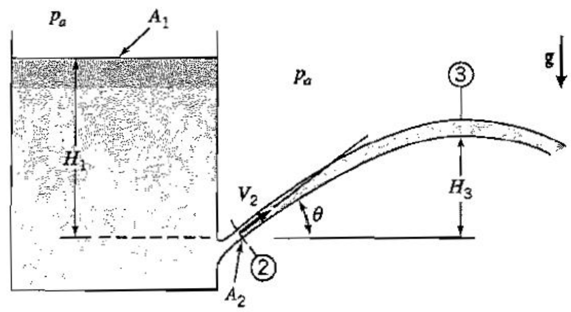

Figure P4.38

the horizontal direction, and it has an exit area of A. Express the maximum height of the jet as a function of θ and H (which is held constant). What is the the horizontal component of the force exerted on the tank by the jet? Ignore losses.

4.36 A jet of constant density fluid rises into the atmosphere, without loss, from the bottom of a large tank, as shown in Figure P4.36.
(a) Find the exit velocity V in terms of g and H. State all assumptions clearly.
(b) Find the horizontal component of the force the jet exerts on the tank in terms of ρ, V_1, θ and the exit area A.
(c) As the jet rises, its vertical velocity decreases but its horizontal velocity remains constant. Find the maximum height of the jet in terms of H and θ.

4.37 A jet of constant density fluid rises without loss into the atmosphere at an angle θ from a large tank, as shown in Figure P4.37. The exit is located very near the bottom, and the change in water depth H with time may be neglected.
(a) What is the maximum height, relative to the tank bottom, to which the jet rises?
(b) What is the vertical force F required to support the tank? The weight of the tank and its contents is W.

4.38 A large tank issues a jet of water from an orifice at a depth H below the surface, as shown in Figure P4.38. The orifice has an exit area A_2 and it points upwards at an angle of θ to the horizontal. Assuming that $A_1 \gg A_2$, and that there are no losses,
(a) Find the maximum height of the jet ($= H_3$) in terms of H_1 and θ.
(b) Find the volume of water in flight between the jet exit and the point of maximum jet height in terms of A_2, H_1 and θ.

4.39 Water of density ρ issues from a spigot of circular cross-section into the atmosphere, as shown in Figure P4.39. At the flange, the velocity is V. The diameter decreases from D

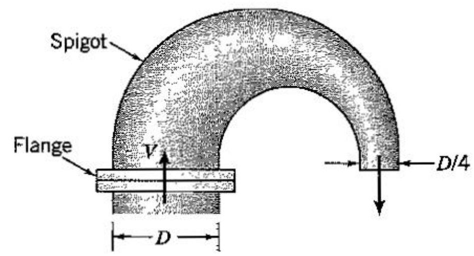

Figure P4.39

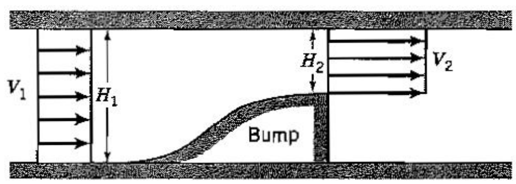

Figure P4.41

at the flange to $\frac{1}{4}D$ at the exit. Ignoring losses,
(a) Find the gauge pressure at the flange in terms of ρ and V.
(b) Find the magnitude and direction of the force exerted by the water on the spigot in terms of ρ, D and V (ignore the weight of water contained in the spigot).

4.40 For a constant density fluid flowing in a duct, show that the change in total pressure (= static pressure + dynamic pressure) after a sudden enlargement is

$$\frac{1}{2}\rho U_1^2\left(1-\frac{U_2}{U_1}\right)^2$$

where U_1 is the velocity upstream, and U_2 the velocity well downstream of the sudden change in the cross-section. Note that at the point where the fluid enters the enlargement, the streamlines are nearly parallel. Explain why the velocity U_2 must be taken to be well downstream of the sudden enlargement and support your explanation with a sketch of the flow pattern.

4.41 Consider the steady flow of a fluid of constant density in a duct of constant width w. The fluid flows smoothly up a bump, as shown in Figure P4.41, and then separates such that the streamlines are initially straight and parallel. Determine the magnitude and the direction of the horizontal component of the force exerted by the fluid on the bump in terms of the density ρ, the inlet velocity V_1, the inlet height H_1 and the width w, given that the pressures and velocities at stations 1 and 2 are uniform across the heights H_1 and H_2, and that $H_1/H_2 = 2$.

4.42 A fluid of density ρ_1 flows through a circular nozzle as shown in Figure P4.42. The pressure difference between sections 1 and 2 is measured using a manometer filled with a liquid of density ρ_2. Find the pressure difference $p_1 - p_2$ and the area ratio A_1/A_2 in terms of $z_1 - z_2$, D, ρ_1, ρ_2, and V_1.

4.43 Air of constant density ρ_a flows steadily through a circular pipe of diameter D, which is downstream of a frictionless nozzle of diameter d, as shown in Figure P4.43. Assume one-dimensional flow. If the manometer reads a deflection of h, find the velocity V at the nozzle exit in terms of h, D, d, ρ_a and the density of the manometer fluid ρ_m.

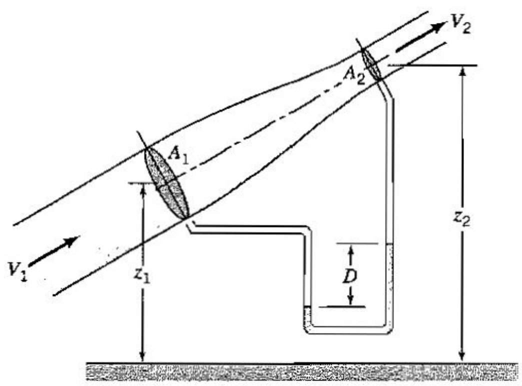

Figure P4.42:

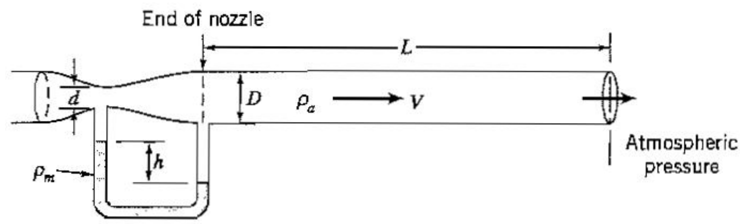

Figure P4.43

4.44 A fluid flows steadily from left to right through the duct shown in Figure P4.44. Another fluid of a different density enters from a second duct at right angles to the first. The two fluids mix together and at station 3 the resultant fluid has a uniform composition. This mixed fluid then exits without loss to atmosphere through a contraction with no further change in density. At stations 1, 2, 3 and 4 the flow properties and parameters are constant over their respective areas. Find the pressure p_1 in terms of ρ_1, A_1 and U_1. Ignore the effect of gravity. Assume $U_2 = 2U_1$, $\rho_2 = 3\rho_1$, $\rho_3 = 2\rho_1$, $\rho_3 = \rho_4$, $A_2 = \frac{1}{4}A_1$, $A_1 = A_3$, and $A_4 = \frac{1}{4}A_1$.

4.45 Two gases of density ρ_1 and ρ_3 are being mixed in the device shown in Figure P4.45. The device is of height d and constant width. Gas of density ρ_1 and velocity U_1 enters from the left, and it is mixed with gas of density ρ_3 and velocity U_3 entering through two ducts each of size $\frac{1}{4}d$. The mixing is complete at station 2, so that both the velocity U_3 and density ρ_3 are uniform across the duct. The mixture then accelerates smoothly through a contraction to exit at atmospheric pressure with a velocity U_4 and an unchanged density

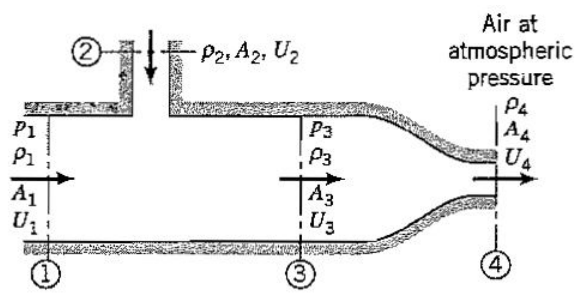

Figure P4.44

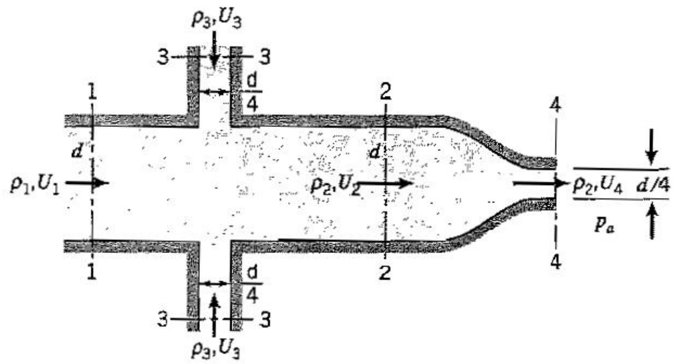

Figure P4.45

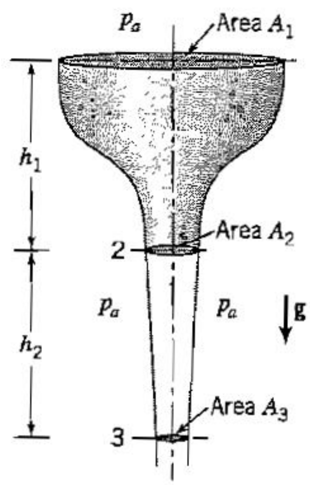

Figure P4.46

($\rho_4 = \rho_2$). You are given that $\rho_3 = 2\rho_1$, and $U_3 = 3U_1 = U_4$. Find the ratio ρ_2/ρ_1, and the pressure coefficient C_p, where

$$C_p = \frac{p_1 - p_2}{\frac{1}{2}\rho_1 U_1^2}$$

4.46 Water issues steadily without loss from the smooth, circular funnel shown in Figure P4.46 under the action of gravity.
(a) Find the area ratio A_3/A_2 in terms of h_1 and h_2.
(b) By using the momentum equation, find the volume of fluid contained in the jet between stations 2 and 3 in terms of h_1, h_2 and A_2. Assume that $A_1 \gg A_2$.

4.47 A circular hovercraft, of weight Mg, hovers a distance h above the ground, as shown in Figure P4.47. Far from the inlet the air is at atmospheric pressure and may be considered stationary. The air density remains constant throughout. The expansion downstream of the fan occurs without loss, and the exit streamlines are parallel to the ground. Find h in terms of the inlet velocity V_i, the diameter of the fan d, the diameter of the exit plane D, the density ρ and the weight Mg. Assume one-dimensional flow over the entry and exit areas.

4.48 An axisymmetric body of cross-sectional area a moves steadily down a tube of cross-sectional area A, which is filled with a fluid of constant density ρ, as shown in Figure P4.48. The flow over the main part of the body is streamlined, but the flow separates over the rear

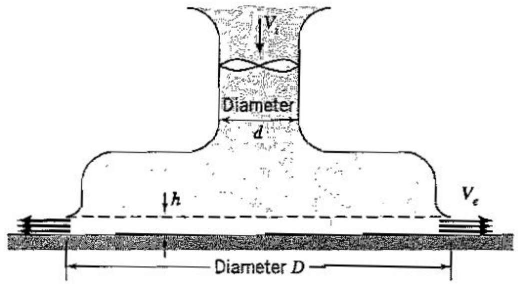

Figure P4.47

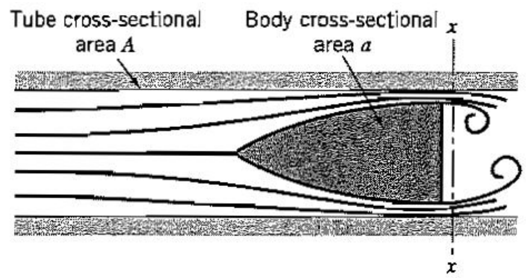

Figure P4.48

part of the body such that the pressure over the section x-x immediately downstream of the base is uniform. Neglecting viscous shear forces at the wall, show that the velocity of the body is given by

$$V = \left(\frac{A-a}{a}\right)\sqrt{\frac{2F}{\rho A}}$$

where F is the force necessary to keep the body moving at a constant speed.

4.49 Air flows steadily through a tee-piece of width w, as shown in Figure P4.49. Gravity is not important, and the air density ρ is constant. The flow exits to atmospheric pressure p_a. If the resultant force acting on the tee-piece is zero:
(a) Find the ratios V_2/V_1 and V_3/V_1 (these are numbers).
(b) find the gauge pressure p_{ig} at the flange in terms of ρ, V_1, h, and w.

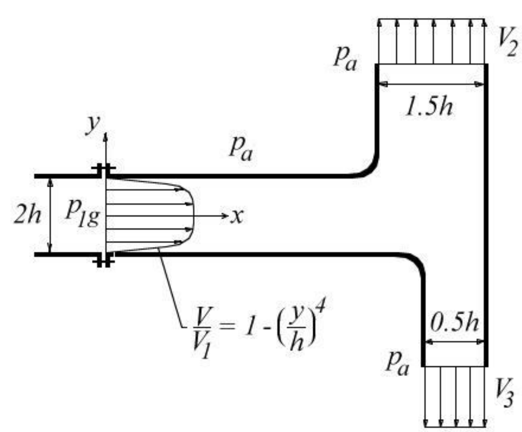

Figure P4.49

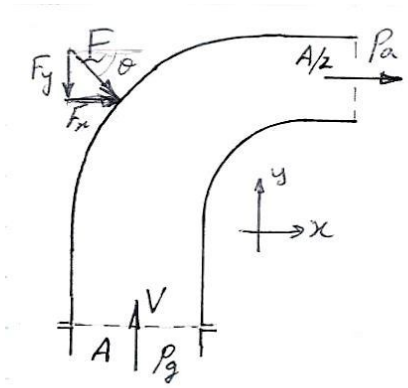

Figure P4.50

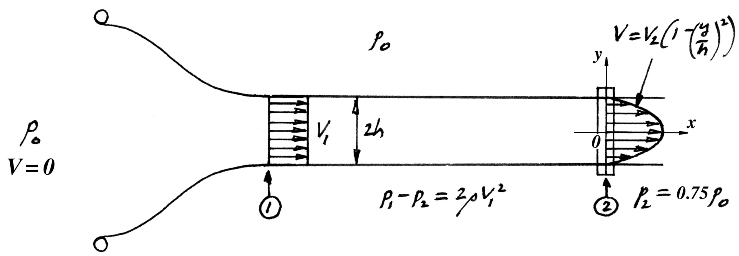

Figure P4.51

4.50 A duct carries a steady flow of air of constant density ρ and exits to atmosphere, as shown in Figure P4.50. At its entrance the gauge pressure is p_g, the velocity is V, and the area is A. At its exit the area is $A/2$. Assume one-dimensional, frictionless flow, and ignore gravity.
(a) Find the velocity at the exit in terms of V.
(b) Find p_g in terms of ρ and V.
(c) Find the magnitude and direction of the force required to hold the duct in place.
Show your control volumes and all your working clearly, state all your assumptions, and indicate your coordinate system.

4.51 A steady flow of air of constant density ρ is drawn into a duct of constant width w and height $2h$, as shown in Figure P4.51. Viscous effects and energy losses are important only downstream of point 1. The effects of gravity can be neglected. For the velocity distributions and pressures given in the figure:
(a) Show that the pressure $p_0 = 10\rho V_1^2$, given that $p_2 = 0.75p_0$, and $p_1 - p_2 = 2\rho V_1^2$.
(b) Show that the velocity $V_2 = 3V_1/2$.
(c) Show that

$$\frac{F}{\frac{1}{2}\rho V_1^2 hw} = -\frac{26}{5}$$

where F is the horizontal component of the force required to hold the duct (including the contraction) in place at point 2.

4.52 A water deflector is held onto a pipe of area A_1 at a flange. The deflector has two exits, as shown in Figure P4.52, which exit to atmospheric pressure. The flow is steady, one-dimensional, and the water has a constant density ρ. With both exits open, velocity $V_2 = V_1$, and area $A_2 = A_1/2$. Ignore the effects of gravity. Show your control volumes and

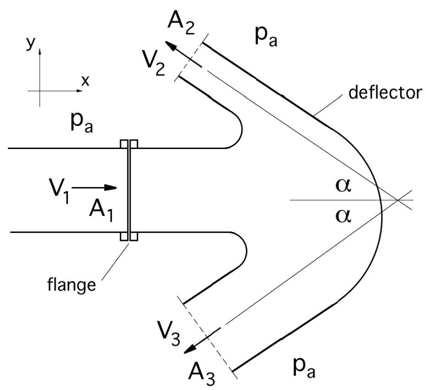

Figure P4.52

all your working clearly, state all your assumptions, and indicate your coordinate system.
(a) Find the velocity V_3 in terms of V_1, given that $A_3 = A_1$.
(b) Find the magnitude and direction of the x-component of the force at the flange (station 1) required to hold the deflector onto the pipe, in terms of V_1, A_1, ρ, α, and p_{1g} (the gauge pressure at station 1).
(c) Similarly, find the magnitude and direction of the y-component of the force at the flange required to hold the deflector onto the pipe.
(d) If the exit at station 3 was closed, find p_{1g} in terms of V_1 and ρ. Ignore losses.

4.53 A flange connects a pipe section of inlet area A_1, as shown in Figure P4.53. The pipe section reduces its area smoothly, and the air flow exits to atmospheric pressure though an area A_2. The exiting jet impacts a baffle placed normal to the incoming flow. The flow is steady, one-dimensional, and the air has a constant density ρ. The gauge pressure at station 1 is p_{1g}. Ignore the effects of gravity. Show your control volumes and all your working clearly, state all your assumptions, and indicate your coordinate system.
(a) Find the magnitude and direction of the force F_b required to hold the baffle in place in terms of ρ, V_1, A_1, and A_2.
(b) Find p_{1g} in terms of ρ, V_1, A_1, and A_2. Ignore losses.
(c) Find the magnitude and direction of the x-component of the force at the flange (station 1) required to hold the pipe section in place, in terms of ρ, V_1, A_1, A_2, and α.
(d) Similarly, find the magnitude and direction of the y-component of the force at the flange required to hold the pipe section in place, in terms of ρ, V_1, A_1, A_2, and α.

4.54 A flange connects a pipe section of inlet area A, as shown in Figure P4.54. The pipe section splits smoothly into two separate streams that both exit to atmospheric pressure though areas $A/4$. The flow is steady, one-dimensional, and the air has a constant density ρ. The gauge pressure at station 1 is p_{1g}. Ignore the effects of gravity. Show your control volumes and all your working clearly, and state all your assumptions.
(a) Find p_{1g} in terms of ρ and V_1. Ignore losses.
(b) Find the magnitude and direction of the x-component of the force at the flange (station 1) required to hold the pipe section in place, in terms of ρ, V_1, A, and θ.
(c) Similarly, find the magnitude and direction of the y-component of the force at the flange required to hold the pipe section in place, in terms of ρ, V_1, A, and θ.

4.55 A spray head steadily discharges a fluid of constant density ρ symmetrically over two side branches, as shown in Figure P4.55. You can neglect gravity, and the fluid viscosity.

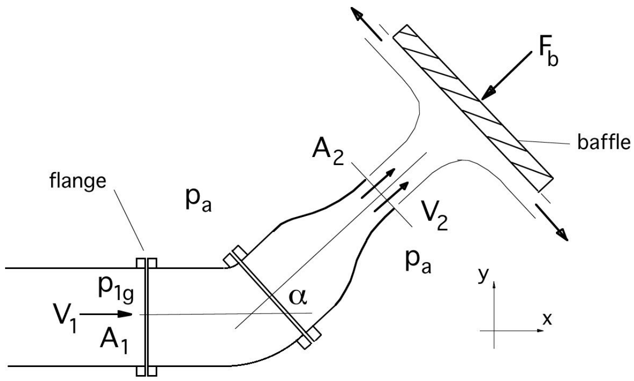

Figure P4.53

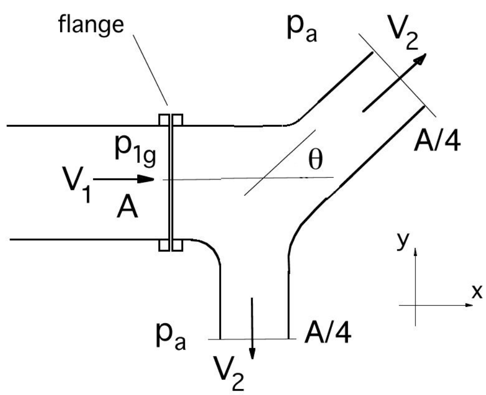

Figure P4.54

(a) Find p_{1g}, the gauge pressure at station 1, in terms of ρ, V_1, A_1 and A_2.
(b) Find the resultant force (magnitude and direction) required to hold the spray head onto the duct, in terms of ρ, θ, V_1, A_1 and A_2.
(c) Express the answer to part (b) as a non-dimensional force coefficient.

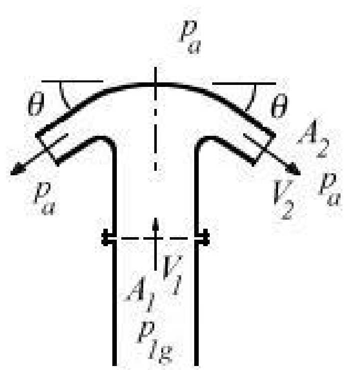

Figure P4.55

Chapter 5

Differential Equations of Motion

5.1 Study Guide

- State the definition of the total derivative. Define all symbols and notations, and describe the meaning of all terms in words.
- Write down the x-component of the acceleration following a fluid particle given that $\mathbf{V} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$, where $\mathbf{V}$ is the velocity and $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the unit vectors in a Cartesian coordinate system.
- State the continuity equation in differential form. Define all symbols and notations, and describe the meaning of all terms in words.
- Write down the vector differential form of the momentum equation for an inviscid, incompressible fluid. Define all symbols and notations, and describe the meaning of all terms in words.
- When is a flow steady? Incompressible?
- Write down the integral and differential forms of the continuity equation for:
  (a) Steady flow.
  (b) Constant density flow.

5.2 Worked Examples

Example 5.1: Streamlines and pathlines in steady flow

Consider the flow field given by

$$\mathbf{V} = x\mathbf{i} - y\mathbf{j}$$

This flow is steady (independent of time) and two-dimensional (it depends on two space coordinates, x and y).
(a) Describe the velocity field.
(b) Find the shape of the streamlines.
(c) Find the shape of the pathlines.

Solution: For part (a), the vector velocity makes an angle θ with the x-axis, so that

$$\tan\theta = \frac{v}{u} = -\frac{y}{x}$$

Along the x-axis, $y = 0$, and $\theta = 0°$ (or 180°), and $\mathbf{V}$ has the same sign and magnitude as x, so that the velocity points directly along the x-axis and increases in magnitude with distance from the origin. Along the y-axis, $x = 0$, and $\theta = 90°$ (or 270°). Here, $\mathbf{V}$ has the opposite sign but the same magnitude as y, so that the velocity points along the negative y-axis and increases in magnitude with distance from the origin. At $y = x$, $\theta = -45°$, and so forth. Hence, the entire velocity field can be built up by successively finding the velocity direction and magnitude at all points in the flow. This procedure will show that the flow field represents a stagnation point flow, as illustrated in Figure 4.8.

For part (b), the shape of the streamlines is given by the solution of:

$$\frac{dy}{dx} = \frac{v}{u} = -\frac{y}{x}$$

The variables can be separated and integrated to give

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

That is

$$\ln y = -\ln x + \text{ constant}$$

This can be written as

$$xy = C$$

where C is a constant. That is, the streamlines are hyperbolae in the x-y plane (see Figure 4.8).

For part (c), we can find the pathlines by using:

$$u = \frac{dx}{dt} = x, \quad \text{and} \quad v = \frac{dy}{dt} = -y$$

That is,

$$\frac{dx}{x} = -\frac{dy}{y}$$

Multiplying through by xy gives

$$y\,dx + x\,dy = d\,(xy) = 0$$

The equation to the pathline is then

$$xy \;=\; C$$

the same result obtained for the equation to the streamline. This is expected, since streamlines and pathlines are identical in steady flow.

Example 5.2: Streamlines and pathlines in unsteady flow

Consider the flow field given by

$$\mathbf{V} = x\mathbf{i} + yt\mathbf{j}$$

This flow is unsteady (it depends on time) and two-dimensional (it depends on two space coordinates, x and y). Find the shape of
(a) The streamline and;
(b) The pathline passing through the point $[1, 1]$ at time $t = 0$.

Solution: For part (a), the shape of the streamlines is given by the solution of

$$\frac{dy}{dx} = \frac{v}{u} = \frac{yt}{x}$$

The variables can be separated and integrated to give

$$\ln y = t \ln x + \text{constant}$$

This can be written as

$$y = C_1 x^t$$

where C_1 is a constant. For the streamline passing through the point $[1,1]$ at $t = 0$, $C_1 = 1$, so for this particular streamline,

$$y = x^t \tag{5.1}$$

For part (b), to find the pathlines, we use the fact that $u = dx/dt$, and $v = dy/dt$. For this problem,

$$\frac{dx}{dt} = x, \quad \text{and} \quad \frac{dy}{dt} = yt$$

Integration gives

$$\begin{aligned} x &= C_2 e^t \\ \text{and} \qquad y &= C_3\, e^{t^2/2} \end{aligned}$$

where C_2 and C_3 are constants. For the particle located at the point $[1,1]$ at $t = 0$, $C_2 = C_3 = 1$, so for this particular pathline

$$\begin{aligned} x &= e^t \\ \text{and} \qquad y &= e^{t^2/2} \end{aligned}$$

Eliminating time gives

$$2 \ln y = (\ln x)^2 \tag{5.2}$$

Note that the results given in Equations 5.1 and 5.2 are different because in an unsteady flow streamlines and pathlines do not coincide.

Example 5.3: Rate of change of density

Given the Eulerian, Cartesian velocity field $\mathbf{V} = 3\mathbf{i} + 2x\mathbf{j}$, and a density field described by $\rho = 4y^2$, find the rate of change of density following a fluid particle.

Solution: We need to find the total derivative of the density, that is,

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}$$

For these particular velocity and density fields, $w = 0$, and

$$\frac{\partial \rho}{\partial t} = 0$$

$$u\frac{\partial \rho}{\partial x} = 3\frac{\partial 4y^2}{\partial x} = 0$$

$$v\frac{\partial \rho}{\partial y} = 2x\frac{\partial 4y^2}{\partial y} = 16xy$$

Therefore,

$$\frac{D\rho}{Dt} = 16xy$$

Example 5.4: Acceleration of a fluid particle

Given the Eulerian, Cartesian velocity field $\mathbf{V} = 2t\mathbf{i} + xz\mathbf{j} - t^2y\mathbf{k}$, find the acceleration following a fluid particle.

Solution: We need to find the total derivative of the velocity, that is,

$$\frac{D\mathbf{V}}{Dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}$$

For this particular velocity field,

$$\begin{aligned}
\frac{\partial \mathbf{V}}{\partial t} &= \frac{\partial(2t)}{\partial t}\mathbf{i} + \frac{\partial(xz)}{\partial t}\mathbf{j} + \frac{\partial(-t^2y)}{\partial t}\mathbf{k} \;=\; 2\mathbf{i} + 0 - 2ty\mathbf{k} \\
u\frac{\partial \mathbf{V}}{\partial x} &= 2t\left[\frac{\partial(2t)}{\partial x}\mathbf{i} + \frac{\partial(xz)}{\partial x}\mathbf{j} + \frac{\partial(-t^2y)}{\partial x}\mathbf{k}\right] \;=\; 2t\left[0 + z\mathbf{j} - 0\right] \;=\; 2tz\mathbf{j} \\
v\frac{\partial \mathbf{V}}{\partial y} &= xz\left[\frac{\partial(2t)}{\partial y}\mathbf{i} + \frac{\partial(xz)}{\partial y}\mathbf{j} + \frac{\partial(-t^2y)}{\partial y}\mathbf{k}\right] \;=\; xz\left[0 + 0 - t^2\mathbf{k}\right] \;=\; -xzt^2\mathbf{k} \\
w\frac{\partial \mathbf{V}}{\partial z} &= -t^2y\left[\frac{\partial(2t)}{\partial z}\mathbf{i} + \frac{\partial(xz)}{\partial z}\mathbf{j} + \frac{\partial(-t^2y)}{\partial z}\mathbf{k}\right] \;=\; -t^2y\left[0 + x\mathbf{j} - 0\right] \;=\; -xyt^2\mathbf{j}
\end{aligned}$$

Finally,

$$\frac{D\mathbf{V}}{Dt} = 2\mathbf{i} - 2ty\mathbf{k} + 2tz\mathbf{j} - xzt^2\mathbf{k} - xyt^2\mathbf{j} \;=\; 2\mathbf{i} + \left(2tz - xyt^2\right)\mathbf{j} - \left(2ty + xzt^2\right)\mathbf{k}$$

Example 5.5: Incompressibility

Determine if the Eulerian velocity field $\mathbf{V} = 2x\mathbf{i} + t^2\mathbf{j}$ is incompressible.

Solution: A velocity field is incompressible if $\nabla \cdot \mathbf{V} = 0$. For the Cartesian velocity field given here,

$$\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\partial(2x)}{\partial x} + \frac{\partial(t^2)}{\partial y} = 2$$

and the flow field is therefore not incompressible.

Example 5.6: Euler equation

Find the x-component of the acceleration of an inviscid fluid under the action of a pressure gradient $\nabla p = x^2\mathbf{i} + 2z\mathbf{j}$. The x-direction is horizontal.

Solution: The flow of an inviscid fluid is described by the Euler equation (equation 5.10). That is,

$$\rho\frac{D\mathbf{V}}{Dt} = -\nabla p + \rho\mathbf{g}$$

We take the x-component by forming the dot product with the unit vector $\mathbf{i}$, so that

$$\rho\frac{D(\mathbf{i}\cdot\mathbf{V})}{Dt} = -\mathbf{i}\cdot\nabla p + \mathbf{i}\cdot\rho\mathbf{g}$$

That is,

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 0$$

Hence, for the flow field given here, the x-component of the acceleration is

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial x^2}{\partial x} = -\frac{2x}{\rho}$$

Example 5.7: Navier-Stokes equation

Consider the steady flow of a viscous fluid in a long horizontal duct of height $2h$, where $\mathbf{V} = \left(1 - (y/h)^2\right)\mathbf{i}$. Find the corresponding pressure gradient.

Solution: The flow of a viscous fluid is described by the Navier-Stokes equation (equation 5.19). That is,

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{V}$$

The acceleration for this particular flow is given by

$$\frac{D\mathbf{V}}{Dt} = \frac{D}{Dt}\left(1 - \left(\frac{y}{h}\right)^2\right)\mathbf{i} = 0$$

Since the channel is horizontal, the Navier-Stokes equation reduces to

$$0 = -\nabla p + \mu \nabla^2 \mathbf{V}$$

The viscous term becomes

$$\begin{aligned}
\mu \nabla^2 \mathbf{V} &= \mu \nabla^2 \left(1 - \left(\frac{y}{h}\right)^2\right)\mathbf{i} \\
&= \mu \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(1 - \left(\frac{y}{h}\right)^2\right)\mathbf{i} \;=\; -\frac{2\mu}{h^2}\mathbf{i}
\end{aligned}$$

and therefore

$$\nabla p = -\frac{2\mu}{h^2}\mathbf{i}$$

The pressure gradient acts only in the x-direction, so that

$$\frac{\partial p}{\partial x} = \frac{dp}{dx} = -\frac{2\mu}{h^2}$$

and we see that the pressure drops linearly in the streamwise direction. This is an example of a *fully-developed* flow where the velocity profile does not change in the flow direction, and the acceleration term goes to zero. Other examples of fully-developed flows will be considered in Chapter 8.

Example 5.8: Eulerian velocity field

An Eulerian, Cartesian velocity field is given by $\mathbf{V} = ax^2\mathbf{i} - 2axy\mathbf{j}$.
(a) Is it one-, two-, or three-dimensional?
(b) Is it steady or unsteady?
(c) Is it incompressible?
(d) Find the slope of the streamline passing through the point $[1, -1]$.

Solution: For parts (a) and (b), we see that the velocity field is described by two space coordinates (x and y), and it does not depend on time, so that it is two-dimensional and steady.

For part (c), we have

$$\nabla \cdot \mathbf{V} \;=\; \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \;=\; \frac{\partial (ax^2)}{\partial x} + \frac{\partial (-2axy)}{\partial y} \;=\; 2ax - 2ax \;=\; 0$$

so the flow field is incompressible.

For part (d), we know from the definition of a streamline that its slope in the $[x,y]$-plane is given by the angle α, where

$$\tan\alpha = \frac{v}{u} = \frac{-2axy}{ax^2} = \frac{-2y}{x} = 2$$

Therefore, at the point $[1,-1]$ the streamline makes an angle of 63.4° with the x-axis.

Problems

5.1 For a velocity field described by $\mathbf{V} = 2x^2\mathbf{i} - zy\mathbf{k}$, is the flow two- or three-dimensional? Incompressible?

5.2 For an Eulerian flow field described by $u = 2xyt$, $v = y^3x/3$, $w = 0$, find the slope of the streamline passing through the point $[2,4]$ at $t = 2$.

5.3 Find the angle the streamline makes with the x-axis at the point [-1, 0.5] for the velocity field described by $V = -xy\mathbf{i} + 2y^2\mathbf{j}$

5.4 A Cartesian velocity field is defined by $\mathbf{V} = 2x\mathbf{i} + 5yz^2\mathbf{j} - t^3\mathbf{k}$. Find the divergence of the velocity field. Why is this an important quantity in fluid mechanics?

5.5 Is the flow field $\mathbf{V} = x\mathbf{i}$ and $\rho = x$ physically realizable?

5.6 For the flow field given in Cartesian coordinates by $u = y^2$, $v = 2x$, $w = yt$:
(a) Is the flow one-, two-, or three-dimensional?
(b) What is the x-component of the acceleration following a fluid particle?
(c) What is the angle the streamline makes in the x-y plane at the point $y = x = 1$?

5.7 For an Eulerian flow field described by $u = 2xyt$, $v = y^3x/3$, $w = 0$:
(a) Find the rate of change of density following a fluid particle as a function of x, y and t.
(b) Find the x-component of the acceleration as a function of x, y and t.

5.8 A velocity field is described (in Cartesian coordinates) by $u = 2-x^3/3$, $v = x^2y-zt$, $w = 0$.
(a) Write down the y-component of the acceleration of a fluid particle (in the Eulerian system) for this flow field.
(b) Is this flow field incompressible?

5.9 For the velocity field given by $\mathbf{V} = 6x\mathbf{i} - 2yz\mathbf{j} + 3\mathbf{k}$, determine where the flow field is incompressible.

5.10 Consider the following velocity field (in Cartesian coordinates): $u = xt + 2y$, $v = xt^2 - yt$, $w = 0$. Is this flow incompressible?

5.11 Is the flow field

$$\mathbf{V} = (2x^2 - xy + z^2)\mathbf{i} + \left(x^2 - 4xy + y^2\right)\mathbf{j} + \left(-2xy - yz + y^2\right)\mathbf{k}$$

compressible or incompressible?

5.12 A flow field is described in Cartesian coordinates by

$$\mathbf{V} = \left(2x^2 + 6z^2x\right)\mathbf{i} + \left(y^2 - 4xy\right)\mathbf{j} - \left(2z^3 + 2yz\right)\mathbf{k}$$

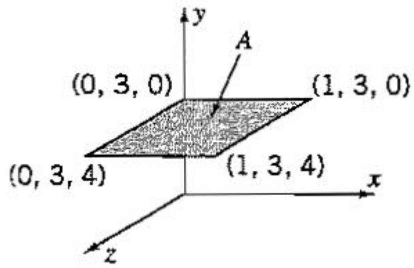

Figure P5.16

Is it incompressible?

5.13 A flow field is described (in Cartesian coordinates) by

$$u = \frac{4x}{t}, \qquad v = \frac{y^2}{t}, \qquad w = 0, \qquad \rho = 3 + \frac{t}{x}$$

Find the rate of change of density following a fluid particle two different ways. Is this flow field possible?

5.14 For an Eulerian flow field described by $u = 2x$, $v = 16\,(y + x)$, $w = 0$, $\rho = at^2 + xy$, find the rate of change of density of a particle of fluid with respect to time t in two different ways. Is this flow field possible?

5.15 For an Eulerian flow field described by $u = 2xt$, $v = y^2x/2$, $w = 0$,
(a) How many dimensions does the flow field have?
(b) Find the rate of change of density (per unit mass) following a fluid particle as a function of x, y and t.
(c) Find the x-component of the acceleration as a function of x, y and t.

5.16 In Cartesian coordinates, a particular velocity field is defined by $\mathbf{V} = -2x^2\mathbf{i} + 4xy\mathbf{j} + 3\mathbf{k}$.
(a) Is this flow field compressible or incompressible?
(b) Find the acceleration of the fluid at the point (1,3,0)
(c) Find the volume flux passing through area A shown in Figure P5.16.
(d) What are the dimensions of volume flux?

5.17 For the Cartesian velocity field $\mathbf{V} = 2x^2y\mathbf{i} + 3\mathbf{j} + 4y\mathbf{k}$:
(a) Find $\frac{1}{\rho}\frac{D\rho}{Dt}$.
(b) Find the rate of change of velocity following a fluid particle.

5.18 A Cartesian velocity field is defined by $\mathbf{V} = 3x^2\mathbf{i} + 4zxt\mathbf{k}$.
(a) Is the flow field steady?
(b) Is the flow field two- or three-dimensional?
(c) Find the rate of change of velocity following a fluid particle.
(d) Is the flow field incompressible?

5.19 For an Eulerian flow field described by $u = 2xyt$, $v = y^3x/3$, $w = 0$:
(a) Is this flow one-, two-, or three-dimensional?
(b) Is this flow steady?
(c) Is this flow incompressible?
(d) Find the x-component of the acceleration vector.

5.20 For the flow field described by $\mathbf{V} = 2xy\mathbf{i} - 3y^2\mathbf{j}$.
(a) Is the flow field incompressible?
(b) Is the flow field steady?
(c) Is the flow field two- or three-dimensional?
(d) Find the angle the streamline makes that passes through the point $(3, -2)$.
(e) Find the acceleration of the flow field.

5.21 For the flow field described by $u = 2$, $v = yz^2t$, $w = -z^3t/3$.
(a) Is this flow one-, two-, or three-dimensional?
(b) Is this flow steady?
(c) Is this flow incompressible?
(d) Find the z-component of the acceleration vector.

5.22 For an Eulerian flow field described by $u = xyz$, $v = t^2$, $w = 3$:
(a) Is this flow one-, two-, or three-dimensional?
(b) Is this flow steady?
(c) Is this flow incompressible?
(d) Find the x-component of the acceleration vector.

5.23 A velocity field is described by $\mathbf{V} = 2xyz\mathbf{i} - y^2z\mathbf{j}$.
(a) Is the flow field one-, two- or three-dimensional?
(b) Is the flow field steady?
(c) Find the acceleration at the point $[1, -1, 1]$.
(d) Find the slope of the streamline passing through the point $[1, -1, 1]$.

5.24 An Eulerian flow field is described in Cartesian coordinates by $\mathbf{V} = 4\mathbf{i} + xz\mathbf{j} + 5y^3t\mathbf{k}$.
(a) Is it compressible?
(b) Is it steady?
(c) Is the flow one-, two- or three-dimensional?
(d) Find the y-component of the acceleration.
(e) Find the y-component of the pressure gradient if the fluid is inviscid and gravity can be neglected.

5.25 For the flow field described by $u = 2 - x^3/3$, $v = x^2y - zt$, $w = 0$.
(a) Is this flow two- or three-dimensional?
(b) Write down the x-component of the acceleration.
(c) Is this flow field incompressible?

5.26 A fluid flow is described (in Cartesian coordinates) by $u = x^2$, $v = 4xz$.
(a) Is this flow two-dimensional or three-dimensional?
(b) Is this flow field steady or unsteady?
(c) Find the simplest form of the z-component of velocity if the flow is incompressible.

5.27 For the flow field given in Cartesian coordinates by $u = 0, v = x^2z, w = z^3$:
(a) Is the flow compressible?
(b) What is the y-component of the acceleration following a fluid particle?

5.28 An Eulerian velocity field in Cartesian coordinates is given by $u = x^2y$, $v = -xy^2$, $w = 2xy$.
(a) Is the flow field two- or three-dimensional?
(b) Is this flow field compressible or incompressible?
(c) Is this flow field rotational or irrotational?

5.29 For the flow field given in Cartesian coordinates by $u = 2xt + y$, $v = -2yt$, $w = 0$:

(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow steady?
(c) Is the flow compressible?
(d) What is the vector acceleration following a fluid particle?

5.30 A fluid flow is described in Cartesian coordinates by $u = 2xyz$, $v = -y^2z$, $w = 0$.
(a) Is the flow field two- or three-dimensional?
(b) Is this flow field compressible or incompressible?
(c) What is the x-component of the acceleration?
(d) What is the angle a streamline makes in the x-y plane at the point $[1, 1]$.

5.31 An Eulerian velocity field in Cartesian coordinates is given by $u = 2y - 3x$, $v = 3y + 2xz$, $w = 0$:
(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow steady?
(c) Is the flow compressible or incompressible?
(d) What is the vector acceleration following a fluid particle?
(e) If viscous and gravitational forces are negligible, use the Navier-Stokes equation to find the pressure gradient vector.

5.32 An Eulerian velocity field in Cartesian coordinates is given by $u = x$, $v = -2y$, $w = z$.
(a) Is the flow field two- or three-dimensional?
(b) Is this flow field compressible or incompressible?
(c) Find the rate of change of velocity following a fluid particle.

5.33 For the Eulerian flow field given in Cartesian coordinates by $u = 3y4zx$, $v = 0$, $w = 2z^2$:
(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow steady?
(c) Is the flow compressible or incompressible?
(d) What is the y-component of the acceleration following a fluid particle?
(e) Find the viscous force per unit volume ($= \mu\nabla^2\mathbf{V}$).

5.34 An Eulerian velocity field in Cartesian coordinates is given by $u = ax^2y$, $v = bxy^2$, $w = xyz$.
(a) Is the flow field two- or three-dimensional?
(b) Find values for a and b for which the flow field is incompressible.
(c) Find the rate of change of the y-component of velocity following a fluid particle.

5.35 For the Eulerian flow field given in Cartesian coordinates by $u = -2x^2$, $v = 2z^3$, $w = 4xz$:
(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow steady?
(c) Is the flow compressible or incompressible?
(d) What is the rate of change of velocity following a fluid particle?
(e) If this flow is inviscid, and gravity is not important, find ∇p as a function of $[x,z]$.

5.36 An Eulerian velocity vector field is described by $\mathbf{V} = 2x\mathbf{i} - y^2\mathbf{j}\, m/s$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x- and y-directions, respectively.
(a) Find the angle the streamline makes at the point (1,1).
(b) Find the equation describing the streamline passing through the point (1,1).
(c) If the flow is inviscid and has constant density, find the change in pressure between

points (1,1) and $(e, \frac{2}{3})$, where $e = 2.71828...$ Ignore gravity, and use $\rho = 1.2\,kg/m^3$.

5.37 An Eulerian velocity vector field is described by $\mathbf{V} = 2x\mathbf{i} - y^2\mathbf{j} - 3\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x-, y- and z-directions, respectively.
(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow compressible or incompressible?
(c) Does the the density field $\rho = 2x/y$ satisfy the continuity equation?
(d) What is the acceleration following a fluid particle?

5.38 An Eulerian velocity vector field is described by $\mathbf{V} = 2\mathbf{i} + yz^2t\mathbf{j} - \frac{z^3t}{3}\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x-, y- and z-directions, respectively.
(a) Is this flow one-, two-, or three-dimensional?
(b) Is this flow steady?
(c) Is the flow incompressible or compressible?
(d) Find the z-component of the acceleration vector.

5.39 An Eulerian velocity vector field is described by $\mathbf{V} = 2x^2y\mathbf{i} - 2xy^2\mathbf{j} - 4xy\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x-, y- and z-directions, respectively.
(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow compressible or incompressible?
(c) What is the x-component of the acceleration following a fluid particle?
(d) Bonus question: Is the flow irrotational?

5.40 An Eulerian velocity vector field is described by $\mathbf{V} = \mathbf{i} + 2z\mathbf{j} - 3x^2\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x-, y- and z-directions, respectively.
(i) Is the flow incompressible or compressible?
(ii) Find the rate of change of velocity following a fluid particle.
(iii) Find the viscous force per unit volume (that is, the viscous term in the Navier-Stokes equation).

5.41 An Eulerian velocity vector field is described by $\mathbf{V} = 3xz\mathbf{j} + y\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x-, y- and z-directions, respectively.
(a) Is the flow one-, two- or three-dimensional?
(b) Is the flow compressible or incompressible?
(c) What is the acceleration following a fluid particle?
(d) If gravity and viscous forces can be neglected, what is the pressure gradient?

5.42 An Eulerian velocity vector field is described by $\mathbf{V} = 2y\mathbf{i} - 3z^2\mathbf{j} + t^2\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the x-, y- and z-directions, respectively.
(a) Is this flow one-, two-, or three-dimensional?
(b) Is this flow steady?
(c) Is the flow incompressible or compressible?
(d) Find the x-component of the acceleration vector.
(e) Find the x-component of the vorticity vector.

5.43 For a two-dimensional, incompressible flow, the x-component of velocity is given by $u = xy^2$. Find the simplest y-component of the velocity that will satisfy the continuity equation.

5.44 Find the y-component of velocity of an incompressible two-dimensional flow if the x-component is given by $u = 15 - 2xy$. Along the x-axis, $v = 0$.

5.45 A two-dimensional flow field has an x-component of velocity given in Cartesian

coordinates by $u = 2x - 3y$.
(a) Find v, the y-component of velocity, if the flow is incompressible and $v = 0$ when $x = 0$.
(b) If the flow follows the Bernoulli equation, find an expression for the pressure distribution as a function of x and y, given that the pressure is p_0 at the stagnation point.

5.46 The velocity in a one-dimensional compressible flow is given by $u = 10x^2$. Find the most general variation of the density with x.

5.47 The x-, y- and z-components of a velocity field are given by $u = ax + by + cz$, $v = dx + ey + fz$, and $w = gx + hy + jz$. Find the relationship among the coefficients a through j if the flow field is incompressible.

5.48 For a flow in the xy-plane, the y-component of velocity is given by $v = y^2 - 2x + 2y$. Find a possible x-component for steady, incompressible flow. Is it also valid for unsteady, incompressible flow? Why?

5.49 The x-component of velocity in a steady, incompressible flow field in the xy-plane is $u = A/x$. Find the simplest y-component of velocity for this flow field.

5.50 The flow of an incompressible fluid in cylindrical coordinates is given by

$$u_\theta = \left(1 + \frac{4}{r^2}\right)\sin\theta - \frac{1}{r}$$

Find u_r if $u_r = 0$ at $r = 2$ for all θ. The flow does not depend on z.

5.51 An air bearing is constructed from a circular disk that issues air from many small holes in its lower surface.
(a) Find an expression for the radial velocity under the bearing, assuming that the flow is uniform, steady and incompressible.
(b) The bearing floats $1.5\,mm$ above the table and the air flows through the bearing with an average velocity of $2\,m/s$. If the bearing is $1\,m$ in diameter, find the magnitude and location of the maximum radial acceleration experienced by a fluid particle in the gap.

5.52 Show that the velocity distribution in the linear Couette flow illustrated in Figure 1.14 is an exact solution to the incompressible Navier-Stokes equation (the pressure is constant everywhere). Find all the components of the viscous stress for this flow.

Chapter 6

Irrotational, Incompressible Flows

6.1 Study Guide

For an irrotational flow field:

- The flow can be described completely by the scalar velocity potential ϕ, instead of the vector velocity $\mathbf{V}$.
- The function ϕ can be found by solving Laplace's equation, $\nabla^2\phi = 0$.
- Laplace's equation is a *linear* equation, so that solutions for complex flow fields can be constructed from the linear addition of the solutions for simpler flow fields.
- The velocity field can be found by taking the gradient of the velocity potential, that is, $\mathbf{V} = \nabla\phi$.
- The pressure can be found using Bernoulli's equation, as long as the flow is also incompressible and steady.

For a two-dimensional, incompressible flow field:

- The flow can be described completely by the scalar stream function ψ, instead of the vector velocity $\mathbf{V}$.
- Lines of constant ψ are streamlines.
- If the flow is also irrotational, the function ψ can be found by solving Laplace's equation, $\nabla^2\psi = 0$.

6.2 Worked Examples

Example 6.1: Stream functions and velocity potentials

Show that $\phi = x^3 - 3xy^2$ is a valid velocity potential, and that it describes an incompressible flow field. Determine the corresponding stream function. Find the stagnation points, and the pressure distribution, given that the flow is steady.

Solution: To be a valid velocity potential, ϕ must satisfy the condition of irrotationality. For this two-dimensional, Cartesian flow field, we have

$$u = \frac{\partial \phi}{\partial x} \quad \text{and} \quad v = \frac{\partial \phi}{\partial y}$$

Therefore

$$u = 3x^2 - 3y^2 \quad \text{and} \quad v = -6xy \tag{6.1}$$

The curl of the velocity is given by

$$\nabla \times \mathbf{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k} = (-6y + 6y)\, \mathbf{k} = 0$$

and so ϕ is a valid velocity potential.

The divergence of the velocity field is given by

$$\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 6x - 6x = 0$$

and so the flow field is incompressible.

From the definition of the stream function

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}$$

Using the result given in equation 6.1, we obtain by integration

$$\begin{aligned} \psi &= 3x^2 y - y^3 + f(x) + C_1 \\ \text{and} \qquad \psi &= 3x^2 y + g(y) + C_2 \end{aligned}$$

where C_1 and C_2 are constants of integration, and f and g are unknown functions of x and y, respectively. By comparing the two results for ψ, we get

$$\psi = 3x^2 y - y^3 + C$$

The constant is arbitrary (we are only interested in the derivatives of ψ), so we can set it to zero by choosing $\psi = 0$ at the origin. Finally,

$$\psi = 3x^2 y - y^3$$

The stagnation points can be found by locating the points where $u = v = 0$. For the velocity field given by equation 6.1, this happens only at the origin, so there is only one stagnation point, at $[0, 0]$.

The pressure distribution is given by Bernoulli's equation, so that

$$p + \tfrac{1}{2}\left(u^2 + v^2\right) = p_0$$

where p_0 is a constant for the entire flow field since it is irrotational. Hence

$$p - p_0 = 9\left(x^2 + y^2\right)^2$$

Example 6.2: Solutions obtained by superposition

We have demonstrated that it is possible to generate some interesting two-dimensional flows by superposition of some basic building blocks such as uniform flow, sources, and sinks. In the same way, we can add vortices, and generate even more flow patterns. Unfortunately, the procedure is rather tedious, especially when a large number of elements are used. To reduce the effort involved in generating such flows, it is possible to use the "Ideal Flow Machine" available on the web at *http://www.engapplets.vt.edu/*. By using this resource
(a) Generate the streamline patterns for a source and a sink of equal strength $\dot{q} = Ua$, first separated by a distance a, and then by a distance $2a$ in the direction of a uniform flow of strength U.
(b) Repeat this example with the axis joining the source and sink placed at right angles to the uniform flow.
(c) For the case in part (a), add a vortex of strength $\Gamma = Ua$, located halfway between the source and the sink, and then repeat using a vortex of strength $\Gamma = 2Ua$.

Example 6.3: Lift

Consider a wing traveling at a velocity V, with a span equal to $20c$, where c is the chord length. Given a lift coefficient of 2.0 at a zero angle of attack, find the strength of the bound vortex, and the velocity at which the trailing vortices move downward under their own induced velocity field.

Solution: For a two-dimensional wing, the Joukowski lift law (equation 6.42) gives

$$F_L = \rho U_\infty \Gamma_K$$

If we assume that the wing has a sufficiently large wingspan so that the two-dimensional lift estimate is reasonable, we have (since F_L is the force per unit span)

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 c} = \frac{\rho V \Gamma_K}{\frac{1}{2}\rho V^2 c} = 2$$

Therefore,

$$\Gamma_K = Vc$$

If the trailing vortices are approximated as a pair of line vortices of infinite extent, the downward propagation velocity is given by equation 6.44, so that

$$u_p = \frac{\Gamma}{\pi s}$$

Hence,

$$u_p = \frac{Vc}{\pi 20c} = \frac{V}{20\pi}$$

so that

$$\frac{u_p}{V} = \frac{1}{20\pi}$$

Problems

6.1 Define vorticity in terms of the vector velocity field. How is the "rotation" of a fluid particle related to its vorticity? In Cartesian coordinates, write down the general form of the z-component of vorticity. What is the condition on the vector velocity field such that a flow is irrotational?

6.2 For a certain incompressible two-dimensional flow, the stream function, $\psi(x, y)$ is prescribed. Is the continuity equation satisfied?

6.3 If $u = -Ae^{-ky}\cos kx$ and $v = -Ae^{-ky}\sin kx$, find the stream function. Is this flow rotational, or irrotational?

6.4 An inviscid flow is bounded by a wavy wall at $y = H$ and a plane wall at $y = 0$. The stream function is $\psi = A\left(e^{-ky} - e^{ky}\right)\sin kx$, where A and k are constants.
(a) Does the flow satisfy the continuity equation?
(b) Is the flow rotational or irrotational?
(c) Find the pressure distribution on the plane wall surface, given that $p = 0$ at $[0, 0]$.

6.5 An inviscid flow is bounded by a wavy wall at $y = H$ and a plane wall at $y = 0$. The stream function is $\psi = A\left(e^{-ky} - e^{ky}\right)\sin kx + By^2$, where A, B, and k are constants.
(a) Does the flow satisfy the continuity equation?
(b) Is the flow rotational or irrotational?
(c) Find the pressure distribution on the plane wall surface, given that $p = 0$ at $[0, 0]$.

6.6 For the flow defined by the stream function $\psi = V_0 y$:
(a) Plot the streamlines.
(b) Find the x and y components of the velocity at any point.
(c) Find the volume flow rate per unit width flowing between the streamlines $y = 1$ and $y = 2$.

6.7 Find the stream function for a parallel flow of uniform velocity V_0 making an angle α with the x-axis.

6.8 A certain flow field is described by the stream function $\psi = xy$.
(a) Sketch the flow field.
(b) Find the x and y velocity components at $[0, 0]$, $[1, 1]$, $[\infty, 0]$, and $[4, 1]$.
(c) Find the volume flow rate per unit width flowing between the streamlines passing through points $[0, 0]$ and $[1, 1]$, and points $[1, 2]$ and $[5, 3]$.

6.9 Express the stream function $\psi = 3x^2y - y^3$ in cylindrical coordinates (note that $\sin^3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta$). Sketch the streamlines, and find the magnitude of the velocity at any point.

6.10 For the previous four problems, find the velocity potential, and sketch lines of constant ϕ.

6.11 The velocity potential for a steady flow field is given by $x^2 - y^2$. Find the equation for the streamlines.

6.12 The velocity components of a steady flow field are $u = 2cxy$ and $v = c(a^2 + x^2 + y^2)$. Is the flow incompressible? Is it rotational or irrotational? Find the velocity potential and the stream function.

6.13 The velocity potential for a certain flow is given in cylindrical coordinates by $Cr^2\cos 2\theta$, where C is a constant. Show that this represents the flow in a right-angle corner. If the velocity at $r = 1\,m$, $\theta = 0$ is $-10\,m/s$, find the velocity at $r = 2\,m$, $\theta = \pi/4$.

6.14 A fluid flows along a flat surface parallel to the x-direction. The velocity u varies linearly with y, the distance from the wall, so that $u = ky$.
(a) Find the stream function for this flow.

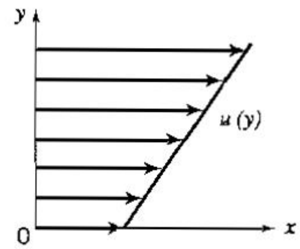

Figure P6.15:

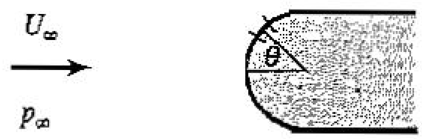

Figure P6.20:

(b) Is this flow irrotational?

6.15 Consider the parallel two-dimensional flow shown in Figure P6.15. Is the flow irrotational? Find the stream function, given that $u = 1.5\,m/s$ at $y = 0$, and $u = 4\,m/s$ at $y = 1.2\,m$.

6.16 Given $u_r = 1/r$ and $u_\theta = 1/r$, find the stream function ψ, and sketch the flow field.

6.17 An irrotational, incompressible velocity field is described by the velocity potential $\phi = A\theta$ $(A > 0)$.
(a) Sketch lines of constant ϕ.
(b) Find the velocity components u_r and u_θ at any point.
(c) Find ψ, and sketch a few streamlines.

6.18 An irrotational, incompressible velocity field is described by the stream function $\psi = \frac{2}{3}r^{3/2}\sin\frac{3}{2}\theta$.
(a) Plot the streamline $\psi = 0$.
(b) Find the velocity at the points defined by the cylindrical coordinates $[2, \frac{\pi}{3}]$ and $[3, \frac{\pi}{6}]$.
(c) Find the velocity potential ϕ.

6.19 Consider the two-dimensional flow of an inviscid, incompressible fluid described by the superposition of a parallel flow of velocity V_0, a source of strength q, and a sink of strength $-q$, separated by a distance b in the direction of the parallel flow, the source being upstream of the sink.
(a) Find the resultant stream function and velocity potential.
(b) Sketch the streamline pattern.
(c) Find the location of the upstream stagnation point relative to the source.

6.20 A static pressure probe is constructed with a semi-cylindrical nose, as shown in Figure P6.20. Where should a pressure tap be located so that it reads the same static pressure as that found far from the probe in a uniform flow?

6.21 A two-dimensional source is placed in a uniform flow of velocity $2\,m/s$ in the x-direction. The volume flow rate issuing from the source is $4\,m^3/s$ per meter.
(a) Find the location of the stagnation point.

(b) Sketch the body shape passing through the stagnation point.
(c) Find the width of the body.
(d) Find the maximum and minimum pressures on the body when the pressure in the uniform flow is atmospheric. The fluid is air and its temperature is $20°C$.

6.22 Repeat the previous problem using the Java-based potential flow solver available at: *http://www.engapplets.vt.edu*, "The Ideal Flow Mapper."

6.23 Using the potential flow solver available at: *http://www.engapplets.vt.edu* ("The Ideal Flow Mapper"), place two sinks and one source along the x-direction, each of strength $1\,m^3/s$ per meter, separated from each other by a distance of $2\,m$.
(a) Plot the streamlines.
(b) Locate the stagnation points.
(c) Vary the strength of the sinks and source (keeping their relative strengths equal) until the stagnation points are a distance of $4\,m$ apart.

6.24 Using the potential flow solver available at: *http://www.engapplets.vt.edu* ("The Ideal Flow Mapper"), place two sources and two sinks alternately along the x-direction, spaced $1\,m$ apart, each of strength $4\,m^3/s$ per meter. Add a a uniform flow of velocity $2\,m/s$ in the x-direction.
(a) Plot the streamlines.
(b) Vary the strength of the sources and sinks until the streamline defining a closed body has a major axis that is twice the minor axis.

6.25 Using the potential flow solver available at: *http://www.engapplets.vt.edu* ("The Ideal Flow Mapper"), place a doublet of strength $-8\,m^3/s$ at the center of the field. Add a a uniform flow of velocity $2\,m/s$ in the x-direction. Place a clockwise vortex at the center of the field. Find the strength of the vortex that will cause the two stagnation points to coincide.

6.26 Using the potential flow solver available at: *http://www.engapplets.vt.edu* ("The Ideal Flow Mapper"), place a clockwise and a counterclockwise vortex of strength $10\,m^3/s$ along the x-direction separated by a distance of $4\,m$.
(a) Find the velocity u_c at the point halfway between them, and check the result against equation 6.44.
(b) Add a vertical velocity equal to $0.5u_c$ and find the location of the stagnation points.

Chapter 7

Dimensional Analysis

7.1 Study Guide

- What does "dimensional homogeneity" mean?
- Remember: $\Pi = N - r$. That is, the number of dimensionless groups (Π-products) is equal to the number of dimensional parameters (N, which includes the output and all input parameters) minus the rank of the matrix of dimensions.
- In deriving the dimensionless groups, remember that they must be independent (that is, any one parameter cannot be formed by a combination of the other parameters). An easy way to check is to make sure that each dimensionless group contains a parameter that is not contained by any other group.
- For two flows to be dynamically similar, all of the dimensionless groups must have the same value.

7.2 Worked Examples

Example 7.1: Vortex shedding

When the wind blows over a chimney, vortices are shed into the wake (see Figures 7.1 and 9.11, and Section 9.5). The frequency of vortex shedding f depends on the chimney diameter D, its length L, the wind velocity V and the kinematic viscosity of air ν.
(a) Express the nondimensional shedding frequency in terms of its dependence on the other nondimensional groups.
(b) If a 1/10th scale model were to be tested in a wind tunnel and full dynamic similarity was required
(i) What air velocity would be necessary in the wind tunnel compared to the wind velocity experienced by the full scale chimney?
(ii) What shedding frequency would be observed in the wind tunnel compared to the shedding frequency generated by the full scale chimney?

Solution: We are given that

$$f = \phi\left(D, L, V, \nu\right)$$

so that $N = 5$.

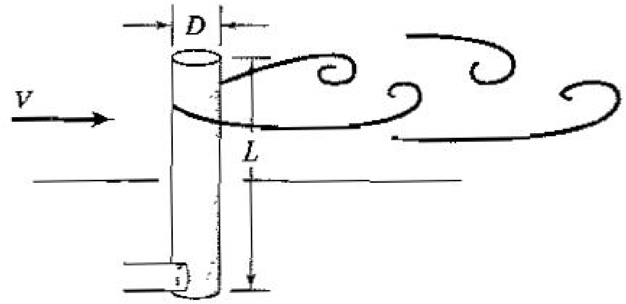

Figure 7.1: Vortex shedding from a chimney.

For part (a), we write down the matrix of dimensions:

	f	D	L	V	ν
L	0	1	1	1	2
T	-1	0	0	-1	-1

The rank of the largest determinant is 2, so we need to find $N - r = 3$ independent dimensionless groups. Two obvious Π-groups are the Reynolds number and the length-to-diameter ratio. We can also make the frequency nondimensional by using the diameter and the velocity. Hence

$$\frac{fD}{V} = \phi'\left(\frac{VD}{\nu}, \frac{L}{D}\right)$$

The ratio fD/V is called the Strouhal number, and it always makes an appearance in unsteady problems with a dominant frequency (see Section 9.5).

For part (b), to achieve dynamic similarity the Π-products in the model tests and the full-scale flow must be equal. That is,

$$\left(\frac{VD}{\nu}\right)_m = \left(\frac{VD}{\nu}\right)_p, \qquad \left(\frac{fD}{V}\right)_m = \left(\frac{fD}{V}\right)_p, \qquad \left(\frac{L}{D}\right)_m = \left(\frac{L}{D}\right)_p$$

where the subscripts m and p indicate the model and "prototype" or full-scale values, respectively. Starting with the Reynolds number, similarity requires that

$$\frac{V_m D_m}{\nu_m} = \frac{V_p D_p}{\nu_p}, \quad or \quad \frac{V_m}{V_p} = \frac{D_p}{D_m}\frac{\nu_m}{\nu_p}$$

Since $\nu_m = \nu_p$ (the fluid is air in both cases), and $D_p = 10D_m$, we have

$$V_m = 10V_p$$

The Strouhal number similarity gives

$$\frac{f_m D_m}{V_m} = \frac{f_p D_p}{V_p}, \quad or \quad \frac{f_m}{f_p} = \frac{D_p}{D_m}\frac{V_m}{V_p}$$

Since $V_m = 10V_p$, and $D_p = 10D_m$, we obtain

$$f_m = 100 f_p$$

To have a dynamically similar model, therefore, we will need to run the wind tunnel at a speed 10 times greater than the natural wind speed, and we expect to see a shedding frequency 100 times greater than for the full-scale chimney.

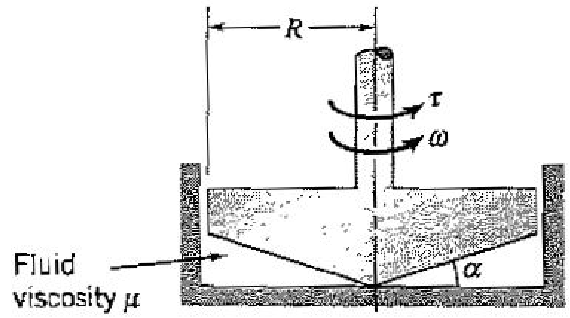

Figure 7.2: A cone and plate viscometer.

Example 7.2: Viscometer

A cone and plate viscometer consists of a cone with a very small angle α which rotates above a flat surface, as shown in Figure 7.2. The torque required to spin the cone at a constant speed is a direct measure of the viscous resistance, which is how this device can be used to find the fluid viscosity. We see that the torque T is a function of the radius R, the cone angle α, the fluid viscosity μ, and the angular velocity ω.
(a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups.
(b) If α and R are kept constant, how will the torque change if both the viscosity and the angular velocity are doubled?

Solution: We are given that

$$T = \phi(R, \alpha, \mu, \omega)$$

So that $N = 5$. The dimensions of torque are force $\times$ distance, that is, $MLT^{-2}L = ML^2T^{-2}$, and the dimensions of viscosity are stress over velocity gradient, that is, $MLT^{-2}L^{-2} \times (LT^{-1}L^{-1})^{-1} = ML^{-1}T^{-1}$.

For part (a), we write down the matrix of dimensions

	T	R	α	μ	ω
M	1	0	0	1	0
L	2	1	0	-1	0
T	-2	0	0	-1	-1

The rank of the largest determinant is 3, so we need to find 2 independent dimensionless groups. One dimensionless number is given by the angle α. The second will be a combination of the other parameters so that the torque becomes nondimensional . Hence,

$$\frac{T}{\mu\omega R^3} = \phi'(\alpha)$$

For part (b), if α is constant, the parameter $T/(\mu\omega R^3)$ must also remain constant to maintain full similarity. So when μ and ω are both doubled, the torque T will increase by a factor of 4.

Example 7.3: Draining tank

A large water tank slowly empties through a small hole under the action of gravity. The flow is steady, and the volume flow rate $\dot{q}$ depends on the exit velocity U, the gravitational

acceleration g, the depth of the water h, and the diameter of the nozzle D.
(a) By using dimensional analysis, find the nondimensional groups that govern the behavior of the nondimensional flow rate.
(b) A test is to be made on a 1/4th scale model. If the test is designed to ensure full dynamic similarity, what is the ratio of model volume flow rate to prototype volume flow rate?
(c) If you now decide that, instead of the volume flow rate, you are interested in the mass flow rate $\dot{m}$ (the density must therefore be included in the dimensional analysis), will the number of nondimensional groups change?
(d) If you later discover that the mass flow rate $\dot{m}$ depends on the viscosity μ and the surface tension σ (dimensions of force per unit length), in addition to ρ, U, g, h, and D, find all the relevant nondimensional groups.

Solution: For part (a), we are given that

$$\dot{q} = \phi(U, g, h, D)$$

so that $N = 5$. The dimensions of volume flow rate are L^3T^{-1}, and the matrix of dimensions becomes

	$\dot{q}$	U	g	h	D
L	3	1	1	1	1
T	-1	-1	-2	0	0

The rank of the largest determinant is 2, so we need to find 3 independent dimensionless groups. By inspection, one dimensionless number is given by the ratio of lengths D/h. The second will be the nondimensional volume flow rate, for example, $\dot{q}/UD^2$, and the third will be a Froude number, $U/\sqrt{gh}$. Hence,

$$\frac{\dot{q}}{UD^2} = \phi'\left(\frac{D}{h}, \frac{U}{\sqrt{gh}}\right)$$

For part (b) dynamic similarity requires that all the nondimensional parameters take the same values in the model and prototype. Therefore

$$\frac{\dot{q}_m}{U_m D_m^2} = \frac{\dot{q}_p}{U_p D_p^2}$$

Since $D_p/D_m = 4$, we have

$$\frac{\dot{q}_m}{\dot{q}_p} = \frac{U_m}{16U_p}$$

We also require

$$\frac{U_m}{\sqrt{gh_m}} = \frac{U_p}{\sqrt{gh_p}}$$

Since $h_p/h_m = 4$,

$$\frac{U_m}{U_p} = \frac{\sqrt{gh_m}}{\sqrt{gh_p}} = \frac{1}{2}$$

and therefore, for dynamic similarity,

$$\frac{\dot{q}_m}{\dot{q}_p} = \frac{1}{32}$$

For part (c), we are given that

$$\dot{m} = \phi(U, g, h, D, \rho)$$

so that $N = 6$. The dimensions of mass flow rate are MT^{-1}, and the matrix of dimensions becomes

	$\dot{m}$	U	g	h	D	ρ
M	1	0	0	0	0	1
L	0	1	1	1	1	-3
T	-1	-1	-2	0	0	0

The rank of the largest determinant is 3, so we will still have 3 independent dimensionless groups. By inspection we find that they are $\dot{m}/\rho UD^2$, D/h, and $U/\sqrt{gh}$.

For part (d), we are given that

$$\dot{m} = \phi\left(U, g, h, D, \rho, \mu, \sigma\right).$$

so that $N = 8$. The dimensions of the surface tension are force per unit length, MT^{-2}, and the matrix of dimensions becomes

	$\dot{m}$	U	g	h	D	ρ	μ	σ
M	0	0	0	0	0	1	1	1
L	3	1	1	1	1	-3	-1	0
T	-1	-1	-2	0	0	0	-1	-2

The rank of the largest determinant is 3, so we will need to find 5 independent dimensionless groups. We already have three: the nondimensional mass flow rate $\dot{m}/\rho UD^2$, the ratio of lengths D/h and the Froude number, $U/\sqrt{gh}$. The fourth Π-product will be a Reynolds number, $\rho Ud/\mu$, and the fifth will be a nondimensional surface tension such as $\sigma/\rho U^2 D$. Hence

$$\frac{\dot{m}}{\rho UD^2} = \phi'\left(\frac{D}{h}, \frac{U}{\sqrt{gh}}, \frac{\rho UD}{\mu}, \frac{\sigma}{\rho U^2 D}\right)$$

Example 7.4: Nuclear explosion

Imagine you were given a film of the first atomic bomb explosion in New Mexico. In the movie, there are some images of trucks and other objects that provide a length scale, so that you can plot the radius of the fireball r as a function of time t. Can you estimate E, the energy released by the explosion?

Solution: The most difficult part is choosing the shortest, correct list of variables. If we had some insight, and some luck, we might suppose that

$$r = f_b\left(E, t, \rho\right)$$

where ρ is the ambient density before the explosion takes place. Energy has the dimensions of a force $\times$ a length, that is, ML^2T^{-2}, and the matrix of dimensions is

	r	E	t	ρ
M	0	1	0	1
L	1	2	0	-3
T	0	-2	1	0

We see that $N = 4$, the rank of the matrix is 3, and therefore the number of dimensionless groups is 1. So

$$\Pi_1 = \frac{r^5\rho}{t^2 E}$$

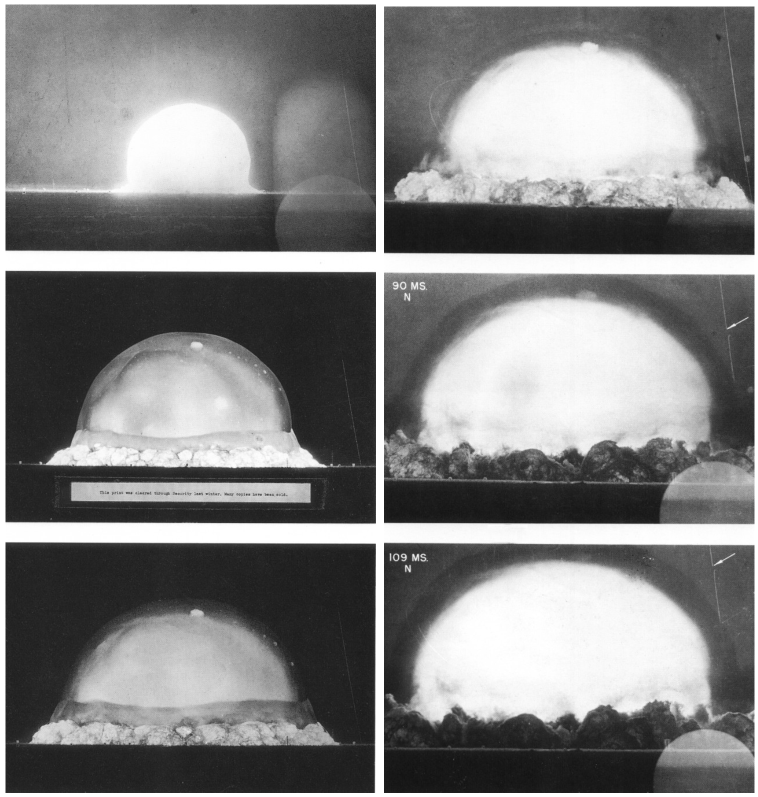

Figure 7.3: The first nuclear explosion in history took place in New Mexico, on July 16 1945, 5:29:45 A.M., at the Alamogordo Test Range, on the Jornada del Muerto (Journey of Death) desert, in the test named Trinity. Left: Trinity at 6, 16, and 34 ms. Right: 53, 90, and 109 ms. At 6 ms, the fireball is about 100 m in diameter. Photos by Berlyn Brixner, LANL.

or, better,

$$\Pi_1{}' = \frac{r}{t^{2/5}}\left(\frac{\rho}{E}\right)^{1/5}$$

The constant $\Pi_1{}'$ must be found by experiment. For example, we can use film of another explosion of conventional explosives where the energy release is known to find $\Pi_1{}'$. This experiment can also be used to check the analysis: if r varies as $t^{2/5}$, as predicted, then our analysis is substantiated and the film of the atomic explosion contains all the information required to find the energy released by the bomb.

This analysis was first performed by the British scientist Sir Geoffrey Ingram Taylor at a time when information on the explosive power of atomic bombs was highly classified. He used films of the explosion, such as the one illustrated in Figure 7.3. When he tried to publish his findings, which gave a very good estimate of the actual energy released, he found his paper immediately classified at such a level that even he was not allowed to read it.

Problems

7.1 The velocity of propagation c of surface waves in a shallow channel is assumed to depend on the depth of the liquid h, the density ρ, and the acceleration due to gravity g.

By means of dimensional analysis, simplify this problem and express this dependence in nondimensional terms.

7.2 The period of a pendulum T is assumed to depend only on the mass m, the length of the pendulum ℓ, the acceleration due to gravity g, and the angle of swing θ. By means of dimensional analysis, simplify this problem and express this dependence in nondimensional terms.

7.3 The sound pressure level p' generated by a fan is found to depend only on the fan rotational speed ω, the fan diameter D, the air density ρ, and the speed of sound a. Express this dependence in nondimensional terms.

7.4 The power input to a water pump, P, depends on its efficiency η, its discharge (volume flow rate) $\dot{q}$, the pressure increase Δp across the pump, the density of the liquid ρ, and the diameter of the impeller D. Express this dependence in nondimensional terms.

7.5 The thrust T of a marine propeller is assumed to depend only on its diameter D, the fluid density ρ, the viscosity μ, the revolutions per unit time ω, and its forward velocity V. Express this dependence in nondimensional terms.

7.6 In Section 1.6.3, it was argued that the viscosity must depend on the average molecular speed $\bar{v}$, the number density ρ and the mean free path ℓ. Express this dependence in nondimensional terms.

7.7 A ship $100\,m$ long moves in fresh water at $15°C$. Find the kinematic viscosity of a fluid suitable for use with a model $5\,m$ long, if it is required to match the Reynolds number and Froude number. Comment on the feasibility of this requirement.

7.8 The height h to which a column of liquid will rise in a small-bore tube due to surface tension is a function of the density of the liquid ρ, the radius of the tube r, the acceleration due to gravity g, and the surface tension of the liquid σ (see Section 1.9.3).
(a) Express this dependence in nondimensional terms.
(b) If the capillary rise for liquid A is $25\,mm$ in a tube of radius $0.5\,mm$, what will be the rise for a liquid B having the same surface tension but four times the density of liquid A in a tube of radius $0.25\,mm$ a dynamically similar system? Also find r_B.

7.9 A chimney $100\,ft$ high is being forced to vibrate at a frequency f in a wind of $20\,ft/s$ by vortices that are shed in its wake. This phenomenon depends on the fluid density ρ and viscosity μ and the chimney material modulus of elasticity E, where $E =$ stress/strain. A model is constructed which is geometrically similar to the chimney in every way and is $10\,ft$ high. The mass per unit length, m, of chimney model can be adjusted by attaching dummy masses inside without affecting its elastic behavior. Gravity is not involved in this problem.
(a) Using dimensional analysis derive all relevant nondimensional groups. Try to use physically meaningful nondimensional groups wherever possible. Make sure these groups are independent.
(b) If the full-scale chimney is made from steel with a modulus of elasticity $E = 30 \times 10^6\,lb_f/in^2$, find the necessary modulus of elasticity so as to simulate the correct conditions at the model scale.
(c) At what frequency would you expect the full scale chimney to vibrate if the model vibrates at $5\,Hz$?

7.10 Two cylinders of equal length are concentric: the outer one is fixed and the inner one can rotate with an angular speed ω. A viscous incompressible fluid fills the gap between

them.
(a) Using dimensional analysis, derive an expression for the torque T required to maintain constant-speed rotation of the inner cylinder if this torque depends only on the diameters and length of the cylinders, the viscosity and density of the fluid, and the angular speed of the inner cylinder.
(b) To test a full-scale prototype, a half-scale model is built. The fluid used in the prototype is also to be used in the model experiment. If the prototype angular speed is ω_p and the prototype torque is T_p

(i) At what angular speed must the model be run to obtain full dynamical similarity?
(ii) How is the model torque related to the prototype torque?

7.11 The power P required to drive a propeller is known to depend on the diameter of the propeller D, the density of fluid ρ, the speed of sound a, the angular velocity of the propeller ω, the freestream velocity V, and the viscosity of the fluid μ.
(a) How many dimensionless groups characterize this problem?
(b) If the effects of viscosity are neglected, and if the speed of sound is not an important variable, express the relationship between power and the other variables in nondimensional form.
(c) A one-half scale model of a propeller is built, and it uses P_m horsepower when running at a speed ω_m. If the full-scale propeller in the same fluid runs at $\omega_m/2$, what is its power consumption in terms of P_m if the functional dependence found in part (b) holds? What freestream velocity should be used for the model test?

7.12 The torque T required to rotate a disk of diameter D with angular velocity ω in a fluid is a function of the density ρ and the viscosity μ of the fluid (torque has units of work).
(a) Find a nondimensional relationship between these quantities.
(b) Calculate the angular velocity and the torque required to drive a 750 mm diameter disc rotating in air if it requires a torque of 1.2 Nm to rotate a similar disc of 230 mm diameter in water at a corresponding speed of 1500 rpm. Assume the temperature is 15°C.

7.13 Consider geometrically similar animals of different linear size, L. Assume that the distance an animal can jump, H, is a function of L, the average density ρ, the average muscle stress σ (that is, muscle force over leg cross-sectional area), and the gravitational constant g.
(a) Find a nondimensional functional expression for H.
(b) In Swift's *Gulliver's Travels*, the Lilliputians were a race of very small people, tiny compared to Gulliver's size. If the Lilliputians had the same ρ and σ as Gulliver, what conclusions can you draw on dimensional grounds? Would they jump higher than Gulliver? The same height as Gulliver? Lower than Gulliver?

7.14 A simple carburetor is sketched in Figure P7.14. Fuel is fed from a reservoir (maintained at a constant level) through a tube so as to discharge into the airstream through an opening where the tube area is a. At this point, the flow area for the air is A and the fuel level is a distance L higher. Let the density and mass flow rate for the air and fuel be ρ_a, $\dot{m}_a$ and ρ_f, $\dot{m}_f$ respectively.
(a) Assuming the flows are inviscid, perform a dimensional analysis to determine the fuel-air ratio, $\dot{m}_f/\dot{m}_a$ as a function of the other dimensionless parameters.
(b) Analyze the problem dynamically and determine a specific relationship between $\dot{m}_f/\dot{m}_a$ and the relevant variables.

7.15 A golf ball manufacturer wants to study the effects of dimple size on the performance of a golf ball. A model ball four times the size of a regular ball is installed in a wind tunnel.
(a) What parameters must be controlled to model the golf ball performance?

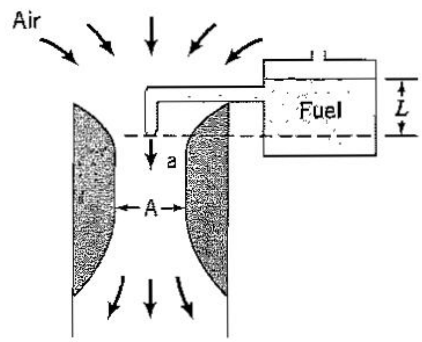

Figure P7.14

(b) What should be the speed of the wind tunnel to simulate a golf ball speed of 200 ft/s?
(c) What rotational speed must be used if the regular ball rotates at 60 revolutions per second?

7.16 When a river flows at a velocity V past a circular pylon of diameter D, vortices are shed at a frequency f. It is known that f is also a function of the water density ρ and viscosity μ, and the acceleration due to gravity, g.
(a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups.
(b) A test is to be performed on a 1/4th scale model. If previous tests had shown that viscosity is not important, what velocity must be used to obtain dynamic similarity, and what shedding frequency would you expect to see?

7.17 A propeller of diameter d develops thrust T when operating at N revolutions per minute with a forward velocity V in air of density ρ.
(a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups. Try to choose groups that look familiar.
(b) The single propeller described above is to be replaced by a pair of two propellers of the same shape operating at the same forward velocity and together producing the same thrust in air with the same density. Use the concepts of dynamic similarity to determine the diameter d_2 and the rotational speed N_2 of each of the propellers.
(c) What change in power, if any, is required?

7.18 The lift force F on a high-speed vehicle is a function of its length L, velocity V, diameter D, and angle of attack α (the angle the chord line makes with the flow direction), as well as the density ρ and speed of sound a of air (the speed of sound of air is only a function of temperature).
(a) Express the nondimensional lift force in terms of its dependence on the other nondimensional groups.
(b) If a 1/10th scale model were to be tested in a wind tunnel at the same pressure and temperature (that is, the same sound speed) as encountered in the flight of the full-scale vehicle, and full dynamic similarity was required

(i) What air velocity would be necessary in the wind tunnel compared to the velocity of the full-scale vehicle?

(ii) What would be the lift force acting on the model compared to the lift force acting on the vehicle in flight?

7.19 The drag of a golf ball F_D depends on its velocity V, its diameter D, its spin rate ω (commonly measured in radians/sec, or revolutions per minute), the air density ρ and

viscosity μ, and the speed of sound a.
(a) Express the nondimensional drag force in terms of its dependence on the other nondimensional groups.
(b) If it was decided that the speed of sound was not important, how would this change the dimensional analysis? Under these conditions, if an experiment was carried out in standard air at a velocity of $2V$, what diameter and spin rate would be required to be dynamically similar to an experiment at a velocity of V?
(c) Design an experiment to investigate the influence of the speed of sound on the drag. Start by considering the requirements for dynamic similarity, and then explain how the influence of sound speed on the drag could be isolated by an experiment. Be as specific as you can.

7.20 Consider a hydraulic jump in a viscous fluid.
(a) Carry out a dimensional analysis of a hydraulic jump that includes the effects of viscosity.
(b) Find the minimum number of nondimensional parameters describing this flow, and write the functional relationship governing the ratio of downstream to upstream depth.
(c) If a 1/4th scale model of a hydraulic jump was to be tested in the laboratory
(i) What is the ratio of incoming flow velocities for the model and the full-scale required for dynamic similarity.
(ii) What is the ratio of kinematic viscosities required for the model and the full-scale?

7.21 A large water tank empties slowly through a small hole under the action of gravity. The flow is steady, and the mass flow rate $\dot{m}$ depends on the exit velocity V, the gravitational acceleration g, the depth of the water h, the diameter of the nozzle D, the viscosity μ, and the surface tension σ (force/unit length).
(a) Express the nondimensional mass flow rate in terms of its dependence on the other nondimensional groups.
(b) If an experiment was carried out using the same fluid (water) in a 1/5th scale model
(i) What is the ratio of the model mass flow rate to prototype mass flow rate that would be needed to obtain dynamical similarity.
(ii) Do you anticipate any difficulties in obtaining full dynamical similarity?

7.22 Tests on a model propeller in a wind tunnel at sea level (air density $\rho = 1.2\,kg/m^3$) gave the following results for the thrust at given forward velocities:

V (m/s)	0	10	20	30
Thrust (N)	300	278	211	100

The propeller diameter was $0.8\,m$ and it was spun at $2000\,rpm$.
(a) Using dimensional analysis find the nondimensional parameters which govern this observed behavior.
(b) Using the experimental data given in the table find the thrust generated by a geometrically similar propeller of diameter $3\,m$, spinning at $1500\,rpm$ at a forward velocity of $45\,m/s$, while operating at an altitude where the density is half that at sea level. You may interpolate from tabulated values.

7.23 In testing the aerodynamic characteristics of golf balls, the scientific officers at the USGA collected the experimental data given below. The diameter is always D, the roughness height is k, the air density is ρ $(= 1.2\,kg/m^3)$, the freestream velocity is V, the lift force L is in Newtons, and the rate of spin ω is measured in revolutions per second.
(a) How many nondimensional parameters describe this problem?
(b) Express all the experimental data in nondimensional form and plot the data in this form.

(c) By comparing the results from Balls 1 and 2, what can you say about the effect of roughness?
(d) By comparing the results from Balls 1 and 3, can you say that the experiments were dynamically similar?

ω	20	40	60	80
L	-1.8	+0.9	+7.2	+18

Ball 1: $D = 42.7\,mm$, $k = 0$, $V = 100\,m/s$

ω	5	15	25	35
L	-0.23	-0.23	+0.68	+2.7

Ball 2: $D = 42.7\,mm$, $k = 1\,mm$, $V = 50\,m/s$

ω	1	2	3	4
L	-0.87	-0.29	+1.7	+5.8

Ball 3: $D = 171\,mm$, $k = 4\,mm$, $V = 20\,m/s$

7.24 The resistance of a sea-going ship is due to wave-making and viscous drag, and it may be expressed in functional form as

$$F_D = f\left(V, L, B, \rho, \mu, g\right)$$

where F_D is the drag force, V is the ship speed, L is its length, B is its width, ρ and μ are the sea water density and viscosity, and g is the gravitational constant.
(a) Find the nondimensional parameters that describe the problem.
(b) If we are to test a model of the ship, what are the requirements for dynamic similarity?
(c) We are going to test a 1/25th scale model of a $100\,m$ long ship. If the maximum velocity of the full-scale ship is $10\,m/s$, what should the maximum speed of the model be? What should the kinematic viscosity of the model test fluid be compared to the kinematic viscosity of sea water?

7.25 The drag force F_D acting on a ship depends on the forward speed V, the fluid density ρ and viscosity μ, gravity, its length L and width B and the average roughness height k.
(a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups.
(b) If the ship moved from fresh water to salt water, where the density and viscosity are both increased by 10%, how would its drag force change at fixed speed?

7.26 The drag D of a ship's hull moving through water depends on its speed V, its width W, length L and depth of immersion H, the water density ρ and viscosity μ, and the gravitational acceleration g.
(a) Use dimensional analysis to express this information in terms of a functional dependence on nondimensional groups. Try to choose groups that look familiar.
(b) The full-scale ship will have a velocity V_1 and it will operate in sea water with a kinematic viscosity ν_1.
(i) If the wave pattern produced by a 1/30th scale model is to be similar to that observed on the full-scale ship, what must be the model test velocity V_2?
(ii) To obtain full dynamic similarity, what must be the kinematic viscosity ν_2 of the test fluid?

7.27 A 1/10th scale model of a windmill is placed in a wind tunnel. The power $\dot{W}_s$ depends on the number of blades n, the wind velocity V, the diameter of the blades D, the density of the air ρ, and the frequency of rotation ω.

(a) Express the non-dimensional power as a function of the other nondimensional groups. Show all your working.
(b) If the experiment was carried out at the same velocity as the full-scale prototype, what is the ratio of the model rotation frequency to prototype rotation frequency that would be needed to obtain dynamical similarity? What would be the ratio of model power to prototype power?

7.28 The lift force (F) produced by a fly depends on the wing beat frequency (f), its forward velocity (V), the density (ρ) and viscosity (μ) of the fluid, the length or "span" (S), and width or "chord" (c) of the wings, and the Youngs modulus of the wing material (E, dimensions of stress).
(a) Express the non-dimensional lift force as a function of the other non-dimensional groups.
(b) A fly is observed to travel at 1 m/s when it beats its wings at 120 Hz. If a dynamically similar robotic fly was built 100 times larger than a real fly, what would the forward velocity and wing beat frequency be if it was tested in silicon oil with a density equal to that of water, but with a viscosity 50 times that of water?

7.29 The power P required to turn a ship will depend on its velocity V, its length L, its width B, the water density ρ and viscosity μ, the rate of turning ω, and the acceleration due to gravity g.
(a) Express the non-dimensional power in terms of its dependence on the other non-dimensional groups. Show all your working.
(b) It is proposed to test a model ship at 1/100th scale. What scaled velocity, kinematic viscosity, and turning rate would be required for the test to be dynamically similar? Do you foresee any possible difficulties in performing this test?

7.30 The drag force (F) produced by an eel swimming on the surface of a lake depends on the frequency of the tail beat (f), the peak-to-peak amplitude of the tail movement (d), its forward velocity (V), the density (ρ) and viscosity (μ) of the fluid, its length (L), and the acceleration due to gravity (g).
(a) Express the non-dimensional drag force as a function of the other non-dimensional groups. Try to use groups that are in common use.
(b) An eel is observed to travel at $1\, m/s$ in water when it beats its tail at $1\, Hz$. What would the forward velocity and tail beat frequency of a dynamically similar robotic eel that is 3 times larger than a real eel? What would the kinematic viscosity of the test fluid need to be (compared to water) to maintain dynamic similarity?

7.31 A waterfall designed for the Ground Zero memorial consists of a sheet of water (density ρ, viscosity μ, surface tension σ) of thickness h and velocity V passing over a short ramp inclined at an angle α to the horizontal. As the sheet falls under the action of gravity (acceleration g) , it breaks up into a number of ribbons of width w.
(a) Express the non-dimensional ribbon width in terms of its dependence on the other non-dimensional groups (surface tension has the dimensions of force/unit length). Show all your working.
(b) It is proposed to test a model waterfall at 1/10th scale. If water is used in the model test, what scaled velocity would be required for the model test to be dynamically similar to the full scale? Explore all possibilities. Do you foresee any difficulties in performing this test?

7.32 The pressure difference across a pump (Δp) is a function of the volume flow rate through the machine ($\dot{q}$), the size of the machine, denoted by, for example, the diameter of the rotor (D), the rotational speed (N) usually measured in rpm or revolutions per second, and the density (ρ) and viscosity (μ) of the fluid.

(a) Express the non-dimensional pressure drop as a function of the other non-dimensional groups.
(b) Tests on a model water pump are performed at one-half scale. To achieve dynamic similarity in the model test, what would the ratio of the model to full-scale prototype rotational speeds need to be? What would the ratio of the volume flow rates need to be? Water is used in the test and the prototype.

7.33 The support for a Pitot probe on an aircraft is a circular cylinder of diameter D inclined at an angle α to the local velocity direction. When the aircraft flies at a speed V, the support sheds vortices at a frequency f. The shedding frequency depends on D, α, V, the fluid density ρ and viscosity μ, and the speed of sound a.
(a) Express the non-dimensional frequency in terms of its dependence on the other non-dimensional groups. Show all your working.
(b) It is proposed to test a model support at 1/5th scale. If air were used in the model test, what scaled velocity would be required for the model test to be dynamically similar to the full scale? Explore all possibilities. Do you foresee any difficulties in performing this test?

7.34 The drag force experienced by a commercial transport (such as a 787) in cruise depends on its surface area (A), its velocity (V), the speed of sound (a), the density (ρ) and viscosity (μ) of the fluid, and its angle of attack (α).
(a) Express the non-dimensional drag as a function of the other non-dimensional groups.
(b) Tests on a model 787 are performed at 1/40th scale. Assume that the speed of sound in the wind tunnel is one-half the speed of sound of the full-scale aircraft. To achieve dynamic similarity in the model test, what would the wind tunnel speed need to be in the model compared to the full-scale prototype? What would the kinematic viscosity of the wind tunnel fluid be compared to air?

7.35 The drag force F acting on a bullet of diameter D and length L is described in terms of its velocity V, the fluid density ρ and viscosity μ, the speed of sound a, and γ (the ratio of the specific heat at constant pressure to the specific heat at constant volume).
(a) Express the nondimensional drag force in terms of its dependence on the other non-dimensional groups. Show all your working.
(b) It is proposed to test a model bullet at 1/2 scale. If air were used in the model test (as in the full scale prototype), what velocity would be required for the model test to be dynamically similar to the full scale? Explore all possibilities. Do you foresee any difficulties in performing this test?

7.36 The power or energy per unit time (P) generated by a windmill depends on the number of blades (N), the length (L) and chord or width (C) of each blade, the blade angle of attack (α), the frequency of rotation (ω), the wind velocity (V), and the density (ρ) and viscosity (μ) of the air.
(a) Express the non-dimensional power as a function of the other non-dimensional groups.
(b) Tests on a model windmill are performed in a wind tunnel at 1/5th scale.
(i) Find the speed of the flow in the wind tunnel necessary to achieve full similarity relative to that experienced by the full-scale windmill.
(ii) What would be the power generated by the model compared to the full-scale prototype?
(iii) What would the frequency of rotation be of the model compared to the full scale?

7.37 Particles are injected into water flowing through a pipe. N, the number of particles per unit volume in the flow at any given station downstream of the point of injection, depends on the distance from the point of injection x, the particle diameter d, the density of the particles ρ_p, the density of water ρ, the viscosity of the water μ, the pipe diameter

D, the average flow velocity $\overline{V}$, and the acceleration due to gravity g.
(a) Express the non-dimensional number of particles per unit volume in terms of its dependence on the other non-dimensional groups. Show all your working.
(b) It is proposed to test a model pipe at 1/4 scale. If the test fluid used in the model test were not known, what velocity would be required for the model test to be dynamically similar to the full scale? What would the kinematic viscosity of the test fluid need to be compared to water to obtain full dynamic similarity?

7.38 The distance s a rotating ball will skip on the surface of water depends on its initial velocity V, initial height above the water h, initial angular velocity ω, the ball diameter d, the gravitational acceleration g, and the coefficient of restitution of the water surface e (the ratio of incoming and rebound velocities).
(a) Express the non-dimensional distance as a function of the other non-dimensional groups.
(b) Tests on a model ball are performed at $4\times$ full scale. To achieve dynamic similarity in the model test, what would the initial velocity need to be in the model test compared to the full-scale? What would the initial rotation rate need to be in the model test compared to the full-scale?

7.39 The distance s a golf ball carries depends on its initial velocity V and angular velocity ω, its diameter d, the air density ρ and dynamic viscosity μ, and the average depth of the dimples k.
(a) Express the non-dimensional distance as a function of the other non-dimensional groups.
(b) Tests on a model golf ball are performed in a wind tunnel at $4\times$ full scale. To achieve dynamic similarity in the model test, what would the wind tunnel speed need to be in the model test compared to the full-scale? What would the rotation rate need to be in the model test compared to the full-scale?

7.40 The thrust T developed by a marine propeller depends on the flow velocity V, the density of the water ρ, the propeller diameter D, its rate of rotation ω, the absolute pressure at the depth of the propeller p, and the viscosity μ.
(a) Express the non-dimensional thrust as a function of the other non-dimensional groups.
(b) Tests on a model propeller are performed in water at half full scale. Assume that the Reynolds number is large enough so that the effects of viscosity can be neglected. To achieve dynamic similarity in the model test:

(ii) What would the flow velocity need to be in the model test compared to the full-scale, assuming the absolute pressures are matched?

(i) What would the rate of rotation need to be in the model test compared to the full-scale?

7.41 The pressure difference Δp produced by a water pump, and the power P required to operate it, each depend on the size of the pump, measured by the diameter D of the impeller, the volume flow rate $\dot{q}$, the rate of rotation ω, the water density ρ and dynamic viscosity μ.
(a) Express the non-dimensional pressure difference and power as separate functions of the other non-dimensional groups.
(b) Tests on a model pump are performed at $0.5\times$ full scale, at a rotation rate that is $2\times$ the full-scale value. To achieve dynamic similarity in the model test: (i) what would the volume flow rate of the water need to be in the model test compared to the full-scale? (ii) What would the pressure difference be compared to the full scale? (iii) What would the power consumption be relative to the full scale?

7.42 The power P developed by the helical flagellum of a bacteria swimming in water, as shown in Figure P7.42, depends on the helix angle θ, the flagellum diameter d, the helix

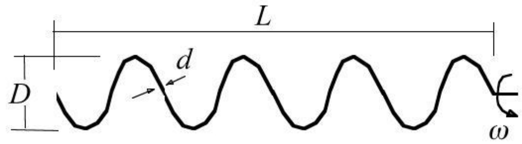

Figure P7.42

diameter D, its length L, its rate of rotation ω, the fluid viscosity μ, and the stiffness of the helix as measured by the product of Young's modulus and the moment of inertia EI (Pa.m^4).
(a) Express the non-dimensional power as a function of the other non-dimensional groups.
(b) Tests on a model flagellum are performed in a fluid with a viscosity 1000 times larger than water, at a model scale 100 times larger than the organism. The stiffness EI of the model is 10^9 times greater than in the organism. To achieve dynamic similarity in the model test:
(i) What would the rate of rotation need to be in the model test compared to the organism?
(ii) What power would be be required to drive the model compared to the organism?

7.43 Wind blowing past a flag causes it to flutter in the breeze. The frequency of this fluttering, ω, is assumed to be a function of the wind speed V, the air density ρ, the acceleration due to gravity g, the length of the flag ℓ, and the area density ρ_A (with dimensions of mass per unit area) of the flag material. It is desired to predict the flutter frequency of a large $12\,m$ flag in a $10\,m/s$ wind. To do this a model flag with $\ell = 1.2\,m$ is to be tested in a wind tunnel.
(a) Express the non-dimensional frequency as a function of the other non-dimensional groups.
(b) Determine the required area density of the model flag material if the large flag has $\rho_A = 1\,kg/m^2$.
(c) What wind tunnel velocity is required for testing the model?
(d) If the model flag flutters at $6\,Hz$, predict the frequency for the large flag.

7.44 The pressure difference Δp required to drive the flow of water in a needle depends on the water viscosity μ, the surface tension σ (force per unit length), the needle diameter D, the flow velocity V, and the speed of sound in air a.
(a) Express the non-dimensional pressure difference as a function of the other non-dimensional groups.
(b) Tests on a model needle are performed at twice full scale. Assume that the effects of compressibility can be neglected. To achieve dynamic similarity in the model test:
(i) What would the flow velocity need to be in the model test compared to the full-scale?
(ii) What would the pressure difference need to be in the model test compared to the full-scale?

Chapter 8

Viscous Internal Flows

8.1 Study Guide

- What is the Moody Diagram?
- Explain briefly why a faucet acts to control the flow rate.
- What is a reasonable upper limit on the Reynolds number for laminar pipe flow?
- At what Reynolds number would you expect fully developed pipe flow to become turbulent? Write down the definition of the Reynolds number for pipe flow and explain your notation.
- Compare the velocity profile, the wall shear stress, and the pressure drop per unit length for laminar and turbulent pipe flow (draw some diagrams to illustrate your answer).
- In Section 8.9, it is stated that squeezing a hose at its exit will only increase the exit velocity for $fL/D > 1$. By using the energy equation, demonstrate the accuracy of this statement.

8.2 Worked Examples

Example 8.1: Pipe flow with friction

Consider a large tank filled with water draining through a long pipe of diameter D and length $L = 100D$, located near the bottom of the tank (Figure 8.1). The entrance to the pipe is square-edged, and there is a ball valve to control the flow rate. The valve is wide

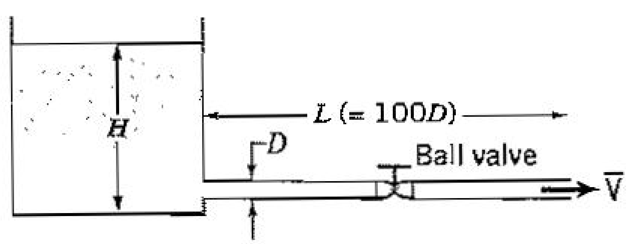

Figure 8.1: Tank draining through a pipe, with losses.

open, and the pipe is horizontal. Find an expression for the average exit velocity $\overline{V}$. The Reynolds number of the pipe flow is 2000, so that the flow is laminar. The flow can be assumed to be steady and adiabatic.

Solution: For laminar flow, $\alpha = 2$. The loss coefficient K for a square-edged entrance is 0.5, and at the exit, all the kinetic energy of the flow is lost, so that the loss coefficient at this point is 1.0 (see Table 8.2). The ball valve has an equivalent length $L_e/D = 3$ (Table 8.3). At this Reynolds number, the friction factor $f = 0.032$ (Figure 8.7). The one-dimensional energy equation (equation 8.30), starting at a point on the free surface of the tank and ending at the exit of the pipe, gives

$$\left(\frac{p_a}{\rho} + 0 + gH\right) - \left(\frac{p_a}{\rho} + \alpha\frac{\overline{V}^2}{2} + 0\right) = gh_\ell$$

so that

$$\begin{aligned} gH &= \alpha\frac{\overline{V}^2}{2} + K\frac{\overline{V}^2}{2} + f\frac{L_e}{D}\frac{\overline{V}^2}{2} + f\frac{L}{D}\frac{V^2}{2} \\ &= \tfrac{1}{2}\overline{V}^2\left(2 + 0.5 + 1.0 + 0.032 \times 3 + 0.032 \times 100\right) \end{aligned}$$

Therefore

$$\overline{V} = 0.542\sqrt{gH} = \sqrt{0.294gH}$$

We see that the exit velocity found here is very much less than the ideal value for frictionless flow ($= \sqrt{2gH}$).

Example 8.2: Pipe flow with friction

Figure 8.2 shows water at $60°F$ flowing in a cast iron pipe. The pressure drop $p_1 - p_2 = 3500\,lb_f/ft^2$, the height difference $z_2 - z_1 = 30ft$, the length $L = 150ft$, and the diameter $D = 3\,in$. Find the volume discharge rate $\dot{q}$. Ignore minor losses, and assume that the flow is turbulent.

Solution: To find $\dot{q}$, we need to know the average velocity in the pipe $\overline{V}$. The one-dimensional energy equation gives:

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{\overline{V}^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{\overline{V}^2}{2} + gz_2\right) = gh_\ell$$

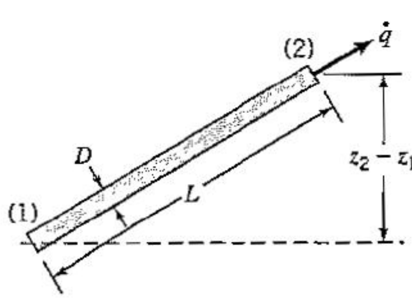

Figure 8.2: Water flowing uphill in a smooth pipe.

For turbulent flow, $\alpha_2 = \alpha_1 = 1.0$, and

$$f \frac{L}{D}\frac{\overline{V}^2}{2} = \frac{p_1 - p_2}{\rho} + g\,(z_1 - z_2)$$

To find $\overline{V}$, we need to know the friction factor. But the friction factor depends on the Reynolds number, and without knowing $\overline{V}$, we cannot find the Reynolds number. However, if the Reynolds number is large enough, the friction factor for a rough pipe is independent of Reynolds number (see Figure 8.7). From Table 8.1, we see that cast iron pipes typically have $k = 0.00085\,ft$, so that $k/D = 0.0034$. For large Reynolds numbers, the Moody diagram then gives $f \approx 0.027$. Therefore

$$\begin{aligned}\overline{V}^2 &= \frac{2}{f}\frac{D}{L}\left[\frac{p_1 - p_2}{\rho} + g\,(z_1 - z_2)\right] \\ &= \frac{2}{0.027}\frac{0.25}{150}\left(\frac{3500\,lb_f}{1.938\,slug/ft^3} - 32.2\,ft/s^2 \times 30\,ft\right)\end{aligned}$$

and so

$$\overline{V} = 10.2\,ft/s$$

Hence

$$\dot{q} = \tfrac{\pi}{4} D^2 \overline{V} = 0.50\,ft^3/s$$

Before we accept this answer, we need to check that the Reynolds number is high enough for the pipe to be fully rough. That is,

$$Re = \frac{\overline{V} D}{\nu} = \frac{10.2 \times 0.25}{1.21 \times 10^{-5}} = 211,000$$

The Moody diagram shows that at this Reynolds number the flow is not yet fully rough, and the friction factor is a little higher than we assumed — closer to 0.028 than 0.027. To obtain a more accurate answer for the volume flow rate, we would need to iterate. When we use $f = 0.028$, we find that $\overline{V} = 10.0\,ft/s$, $\dot{q} = 0.49\,ft^3/s$, and $Re = 207,000$. The corresponding friction factor is again 0.028, so this second value of $\dot{q}$ is accurate enough.

Example 8.3: Pipe flow with shaft work

Consider the previous example, where a $20\,hp$ pump is now placed halfway along the pipe. We will assume that the pressure drop remains the same at $3500\,lb_f/ft^2$, and the friction factor is 0.028. Find the volume flow rate through the pipe.

Solution: The energy equation becomes

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V}^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V}^2}{2} + g z_2\right) = g h_\ell - \frac{\dot{W}_{shaft}}{\dot{m}}$$

where $\dot{W}_{shaft}$ is positive since the pump does work *on* the fluid. That is,

$$\frac{p_1 - p_2}{\rho} + g\,(z_1 - z_2) = f \frac{L}{D}\frac{\overline{V}^2}{2} - \frac{\dot{W}_{shaft}}{\dot{m}}$$

$\dot{W}_{shaft} = 20\,hp$, so that

$$\frac{\dot{W}_{shaft}}{\dot{m}} = \frac{20 \times 550}{1.938 \times \frac{\pi}{4} D^2 \overline{V}}\,ft^2/s^2 = \frac{115,630}{\overline{V}}\,ft^2/s^2$$

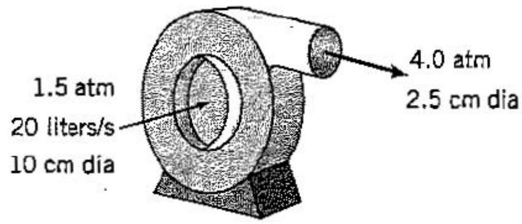

Figure 8.3: Control volume for the pump.

where $\overline{V}$ is measured in ft/s. The energy equation becomes

$$1806 - 966 = 8.4\overline{V}^2 - \frac{115,630}{\overline{V}}$$

so that

$$\overline{V}^3 - 100\overline{V} - 13765 = 0$$

This equation has one physically meaningful solution, where $\overline{V} = 25.4\,ft/s$. Finally, the volume flow rate is given by

$$\dot{q} \;=\; \tfrac{\pi}{4}D^2\overline{V} \;=\; 1.25\,ft^3/s$$

By comparing this value with the result from the previous example, we see that the addition of a $20\,hp$ pump has more than doubled the volume flow rate.

Example 8.4: Power required to drive a pump

A pump delivers $20\,l/s$ (liters per second) of water at $5°C$, increasing the pressure from $1.5\,atm$ to $4.0\,atm$ (see Figure 8.3). The inlet diameter is $10\,cm$, and the outlet diameter is $2.5\,cm$. The inlet and outlet are approximately at the same height, and the change in internal energy is negligible. The flow can be assumed to be one-dimensional. If there is no heat transfer to the fluid, and there is no work done by viscous forces:
(a) Find the power required to drive the pump, $\dot{W}_{pump}$.
(b) Find the change in enthalpy of the water.

Solution: For part (a), we apply the one-dimensional energy equation (equation 8.30) to the control volume shown in Figure 8.3. We have $\dot{Q} = 0$ and $\dot{W}' = \dot{W}_{shaft} = \dot{W}_{pump}$, where $\dot{W}_{shaft}$ is the work done on the water by the pump. Hence,

$$\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} + \tfrac{1}{2}V_2^2 - \tfrac{1}{2}V_1^2 = \frac{\dot{W}_{shaft}}{\dot{m}} = \frac{\dot{W}_{pump}}{\dot{m}}$$

To find $\dot{W}_{pump}$, we need to find the mass flow rate $\dot{m}$, V_1, and V_2. The volume flow rate $\dot{q} = 20\,l/s$, and so

$$\dot{m} = \rho\dot{q} = \frac{1000\,kg/m^3 \times 20\,l/s}{10^3\,l/m^3} = 20\,kg/s$$

Also,

$$V_1 = \frac{\dot{q}}{A_1} = \frac{20\,l/s}{10^3\,l/m^3} \times \frac{1}{\frac{\pi}{4}(0.1)^2\,m^2} = 2.55\,m/s$$

and

$$V_2 = \frac{\dot{q}}{A_2} = \frac{20\,l/s}{10^3\,l/m^3} \times \frac{1}{\frac{\pi}{4}(0.025)^2\,m^2} = 40.7\,m/s$$

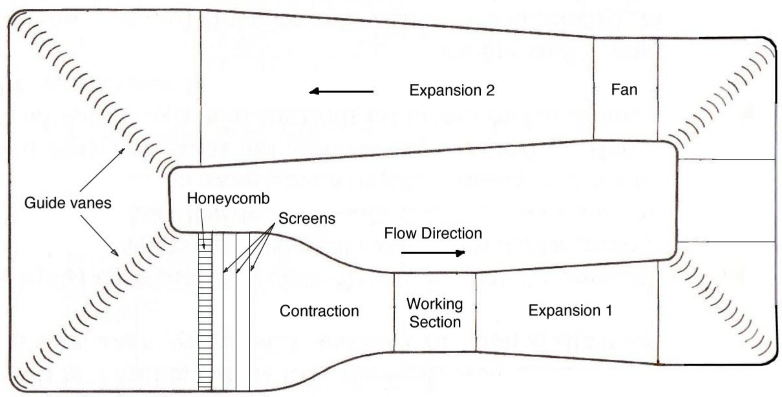

Figure 8.4: Closed-loop wind tunnel.

With $p_1 = 151,988\,Pa$ and $p_2 = 405,300\,Pa$, we obtain

$$\begin{aligned} \dot{W}_{pump} &= 20\,kg/s\left[\frac{(405,300 - 151,988)\,Pa}{1000\,kg/m^3} + \frac{1}{2}\left(40.7^2 - 2.55^2\right)\,m^2/s^2\right] \\ &= 21,566\,watt \end{aligned}$$

This is equivalent to $21,566/746\,hp = 28.9\,hp$

For part (b), we use the definition of enthalpy, $h = \hat{u} + p/\rho$. That is,

$$\begin{aligned} h_2 - h_1 &= \left(\hat{u}_2 + \frac{p_2}{\rho_2}\right) - \left(\hat{u}_1 + \frac{p_1}{\rho_1}\right) \\ &= \frac{p_2 - p_1}{\rho} = \frac{(405,300 - 151,988)\,Pa}{1000\,kg/m^3} = 253\,m^2/s^2 \end{aligned}$$

Example 8.5: Wind tunnel

Consider a closed-loop wind tunnel, as shown in Figure 8.4, driven by a $60\,kW$ fan with an efficiency of 80% at its top speed. The losses in the system are dominated by the minor losses in the bends and the screens that are used to improve the flow quality. Each bend is equipped with guide vanes (see Figure 8.12), and they each have a loss coefficient $K_b = 2$. There are three screens, each with $K_s = 1$, and one honeycomb flow straightener with $K_h = 0.3$. The expansion downstream of the working section is in two parts. Each expansion increases the area by a factor of 3, and each has a loss coefficient $K_e = 0.7$ based on the maximum velocity. The contraction upstream of the working section decreases the area by a factor of 9, and it has a loss coefficient $K_c = 0.2$ based on the maximum velocity. Find the maximum flow velocity in the working section, which has a cross-sectional area of $0.7\,m^2$, assuming that there are no major losses and there is no heat transfer.

Solution: We apply the energy equation around the circuit, starting and ending in the working section where the velocity is $\overline{V}$. Hence

$$0 = gh_{\ell,min} - \frac{\dot{W}_{shaft}}{\dot{m}}$$

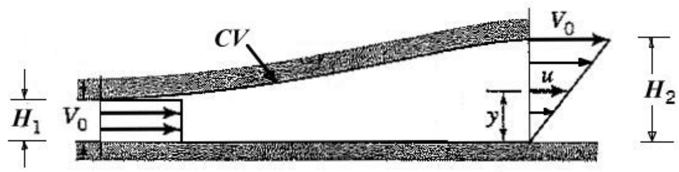

Figure 8.5: Control volume for diverging duct.

where $\dot{W}_{shaft}$ is positive since the fan does work on the fluid. That is,

$$\frac{\dot{W}_{shaft}}{\dot{m}} = K_e \frac{\overline{V}^2}{2} + (2K_b + K_e)\frac{(\overline{V}/3)^2}{2} + (2K_b + 3K_s + K_h)\frac{(\overline{V}/9)^2}{2} + K_c \frac{\overline{V}^2}{2}$$

With $\rho = 1.2\, kg/m^3$, we obtain

$$\dot{W}_{shaft} \;=\; 1.2 \times 0.7\, \overline{V} \left[K_e + K_c + (2K_b + K_e)\,\frac{1}{9} + (2K_b + 3K_s + K_h)\,\frac{1}{81} \right] \frac{\overline{V}^2}{2}$$

where $\overline{V}$ is measured in m/s. With $\dot{W}_{shaft} = 60 \times 0.8\, kW$, we have

$$\overline{V} = 42\, m/s$$

Example 8.6: Energy equation applied to duct flow

An incompressible fluid flows steadily through a duct, as shown in Figure 8.5. The duct has a width W. The velocity at entrance to the duct is uniform and equal to V_0, while the velocity at the exit varies linearly to its maximum value, which is equal to V_0. Find the the net enthalpy transport out of the control volume in terms of ρ, V_0, $\dot{Q}$, W and H_1, where $\dot{Q}$ is the rate at which heat is transferred to the fluid.

Solution: Since the flow is steady and there is no shaft work (we will ignore the work done by viscous forces), the energy equation (equation 3.25) becomes:

$$\int (\mathbf{n} \cdot \rho \mathbf{V}) \left(\hat{u} + \frac{p}{\rho} + \tfrac{1}{2} V^2 + gz \right) dA \;=\; \dot{Q}$$

Ignoring potential energy changes due to changes in height, and introducing the enthalpy $h = \hat{u} + p/\rho$, we have

$$\int (\mathbf{n} \cdot \rho \mathbf{V}) \left(h + \tfrac{1}{2} V^2 \right) dA \;=\; \dot{Q}$$

That is,

$$\int (\mathbf{n} \cdot \rho \mathbf{V})\, h\, dA \;=\; \text{net enthalpy flux} \;=\; -\int (\mathbf{n} \cdot \rho \mathbf{V})\, \tfrac{1}{2} V^2\, dA + \dot{Q}$$

The net rate of enthalpy transport out of the duct is given by the net rate of transport of kinetic energy out of the control volume plus the net rate of heat transfer to the fluid in the control volume. Since the entrance and exit areas are the only places where there is a flux, the net enthalpy flux is given by

$$\begin{aligned} & -\left(-\rho V_0\right) \tfrac{1}{2} V_0^2 W H_1 - \int_0^{H_2} \left(\rho V_0 \left(\frac{y}{H_2} \right) \right) \tfrac{1}{2} V_0^2 \left(\frac{y}{H_2} \right)^2 W dy + \dot{Q} \\ = \;& \tfrac{1}{2} \rho V_0^3 W H_1 - \tfrac{1}{2} \rho V_0^3 W \int_0^{H_2} \left(\frac{y}{H_2} \right)^3 dy + \dot{Q} \\ = \;& \tfrac{1}{2} \rho V_0^3 W H_1 - \tfrac{1}{8} \rho V_0^3 W H_2 + \dot{Q} \end{aligned}$$

We can relate H_1 to H_2 by using the continuity equation, where, for steady flow

$$\int \mathbf{n} \cdot \rho \mathbf{V} \, dA \; = \; 0$$

(equation 3.11). That is,

$$-\rho V_0 W H_1 + \int_0^{H_2} \rho V_0 \left(\frac{y}{H_2} \right) W dy \; = \; 0$$

so that

$$H_1 \; = \; \tfrac{1}{2} H_2$$

Finally,

$$\text{net enthalpy flux} = \tfrac{1}{4} \rho V_0^3 W H_1 + \dot{Q}$$

Problems

8.1 Calculate the Reynolds number for pipe flow with diameter $12\,mm$, average velocity $50\,mm/s$, and kinematic viscosity $10^{-6}\,m^2/s$. Will the flow be laminar or turbulent?

8.2 Would you expect the flow of water in an industrial quality pipe of diameter $0.010\,m$ to be laminar or turbulent, when the average velocity was $1\,m/s$, and the kinematic viscosity was $10^{-6}\,m^2/s$?

8.3 What is the likelihood that a flow with an average velocity of $0.15\,ft/s$ in a $6\,in.$ water pipe is laminar? Find the speed at which the flow will always be laminar.

8.4 If the critical Reynolds number for a river is 2000 based on average velocity and depth, what is the maximum speed for laminar flow in a river $10\,ft$ deep? $2\,ft$ deep? Do you expect any river flow to be laminar?.

8.5 Find the Reynolds number for water at $15°C$ flowing in the following conduits, indicating in each case whether you expect the flow to be laminar or turbulent.
(a) A tube of $6\,mm$ diameter with an average velocity of $10\,cm/s$.
(b) A pipe of $20\,cm$ diameter with an average velocity of $1\,m/s$.
(c) A tube of $2\,m$ diameter with an average velocity of $3\,m/s$.

8.6 A given pipe first carries water and then carries air at $15°C$ and atmospheric pressure. What is the ratio of the mass discharge rates and the volume discharge rates, if the friction factors were the same for these two flows?

8.7 Two horizontal pipes of the same length and relative roughness carry air and water, respectively, at $60°F$. The velocities are such that the Reynolds numbers and pressure drops for each pipe are the same. Find the ratio of the average air velocity to water velocity.

8.8 Water flows steadily through a smooth, circular, horizontal pipe. The discharge rate is $1.5\,ft^3/s$ and the pipe diameter is $6\,in.$ Find the difference in the pressure between two points $400\,ft$ apart if the water temperature is $60°F$.

8.9 A 2000-gallon swimming pool is to be filled with a $0.75\,in.$ diameter garden hose. If the supply pressure is $60\,psig$, find the time required to fill the pool. The hose is $100\,ft$ long, with a friction factor equal to 0.02.

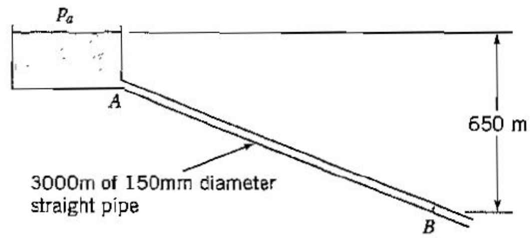

Figure P8.15

8.10 A hole in the bottom of a large open tank discharges water to the atmosphere. If the exit velocity in the absence of losses is V_e, find the loss coefficient for the hole if the actual velocity is $V_e/2$. Assume turbulent flow.

8.11 Air enters a duct at a speed of $100\,m/s$ and leaves it at $200\,m/s$. If no heat is added to the air and no work is done by the air, what is the change in temperature of the air as it passes through the duct?

8.12 Air enters a machine at $373^\circ K$ with a speed of $200\,m/s$ and leaves it at $293^\circ C$. If the flow is adiabatic, and the work output by the machine is $10^5\,N \cdot m/kg$, what is the exit air speed? What is the exit speed when the machine is delivering no work?

8.13 Two jets of air with the same mass flow rate mix thoroughly before entering a large closed tank. One jet is at $400^\circ K$ with a speed of $100\,m/s$ and the other is at $200^\circ K$ with a speed of $300\,m/s$. If no heat is added to the air, and there is no work done, what is the temperature of the air in the tank?

8.14 A basketball is pumped up isothermally using air that is originally at $20^\circ C$ and $1.05 \times 10^5\,Pa$. As a result, the air is compressed to 20% of its original volume. If the mass of air is $0.1\,kg$, find
(a) The final pressure;
(b) The work required;
(c) The changes in internal energy and enthalpy.

8.15 Figure P8.15 shows a pipeline through which water flows at a rate of $0.07\,m^3/s$. If the friction factor for the pipe is 0.04, and the loss coefficient for the pipe entrance at point A is 1.0, calculate the pressure at point B.

8.16 Water flows from a large reservoir down a straight pipe of $2500\,m$ length and $0.2\,m$ diameter. At the end of the pipe, at a point $500\,m$ below the surface of the reservoir, the water exits to atmosphere with a velocity V_e. If the friction factor for the pipe is 0.03, and the loss coefficient for the pipe entrance is 1.0, calculate V_e.

8.17 Figure P8.17 shows a pipeline connecting two reservoirs through which water flows at a rate of $\dot{q}\,m^3/s$. The pipe is $700\,m$ long, it has a diameter of $50\,mm$, and it is straight. If the friction factor for the pipe is 0.001 and the loss coefficients for the pipe entrance and exits are 0.5 and 1.0 respectively, find $\dot{q}$, given that the difference in height between the surfaces of the two reservoirs is $100\,m$.

8.18 A pump is capable of delivering a gauge pressure p_1, when pumping water of density ρ. At the exit of the pump, where the diameter of the exit pipe is D and the velocity is V, there is a valve with a loss coefficient of 0.6, as shown in Figure P8.18. At a distance $100D$ downstream, the diameter of the pipe smoothly reduces to $D/2$. At a further distance $100D$

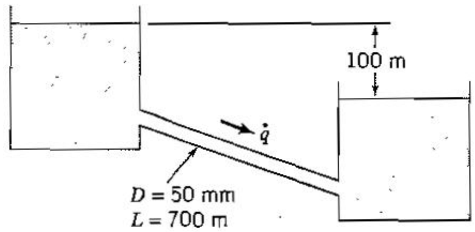

Figure P8.17

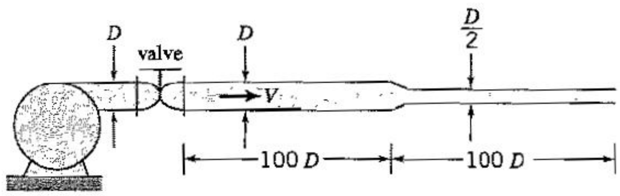

Figure P8.18

downstream, the flow exits to atmosphere. The pipes are horizontal. Calculate the pressure p_1 in terms of the density ρ and V, when the friction factor f is taken to be constant and equal to 0.01 everywhere.

8.19 Figure P8.19 shows a pipeline through which water flows at a rate of $0.06\,m^3/s$. The pipe is $3000\,m$ long, it has a diameter of $120\,mm$, and it is straight. If the friction factor for the pipe is 0.03 and the loss coefficient for the pipe entrance is 1.0, calculate H, the depth of the reservoir, given that the pipe exits to atmosphere.

8.20 A water tank of constant depth H, open to the atmosphere, is connected to the piping system shown in Figure P8.20. After a length of pipe L of diameter D, the diameter decreases smoothly to a value of $D/2$ and then continues on for another length L before exiting to atmosphere. The friction factor f is the same for all piping. C_{D1} and C_{D2} are the loss coefficients for the entry and exit. Calculate the depth of the tank required to produce an exit velocity of V.

8.21 A small gap is left between a window and the windowsill, as shown in Figure P8.21. The gap is $0.15\,mm$ by $30\,mm$, and the width of the window is $1\,m$. The pressure difference across the window is $60\,Pa$. Estimate the average velocity and the volume flow rate through the gap. The air temperature is $0°C$. Ignore minor losses.

8.22 For a constant mass flow rate, and a constant friction factor, show that the pressure drop in a pipe due to friction varies inversely with the pipe diameter to the fifth power.

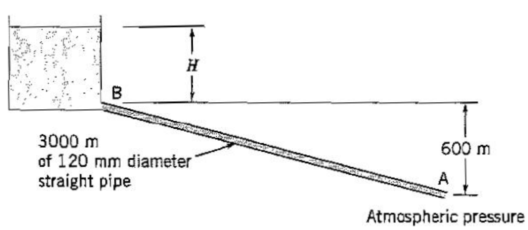

Figure P8.19

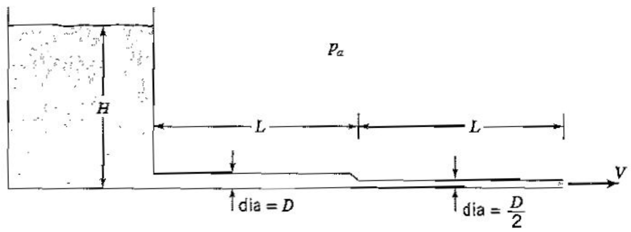

Figure P8.20

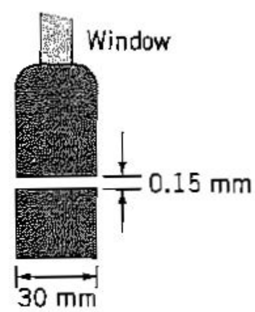

Figure P8.21:

8.23 (a) Simplify the Navier Stokes equation for steady, fully developed flow in the x-direction between two infinite parallel plates, where one plate is held stationary and the other is moving at a constant velocity U_0. The pressure is constant everywhere, and you can neglect gravity.
(b) Find the equation for the velocity profile, given that the gap between the plates is h.

8.24 In fully developed two-dimensional channel flow of height $2h$, the velocity profile is described by

$$\frac{u}{\overline{V}} = \frac{3}{2}\left(1 - \left(\frac{y}{h}\right)^2\right)$$

where y is the distance measured from the centerline, and $\overline{V}$ is the average velocity in the channel. Express the viscous stress at the wall (that is, at $y = \pm h$) in terms of the fluid viscosity μ, $\overline{V}$ and h.

8.25 The laminar flow of a fluid with viscosity μ and density ρ in a circular pipe of radius R has a velocity distribution described by

$$\frac{U}{U_{CL}} = 1 - \left(\frac{r}{R}\right)^2$$

where U_{CL} is the velocity on the centerline where $r = 0$.
(a) Using a control volume analysis, find τ_w, the shear stress at the wall, in terms of R and the pressure gradient dp/dx.
(b) Using the expression for the velocity profile, find τ_w, the shear stress at the wall, in terms of μ, R, and U_{CL}.
(c) Find the average velocity $\overline{U}$ in terms of U_{CL}.
(d) Using the results from parts (a)-(c), show that

$$f = \frac{\left(\frac{dp}{dx}\right) D}{\frac{1}{2}\rho\overline{U}^2} = \frac{64}{Re}$$

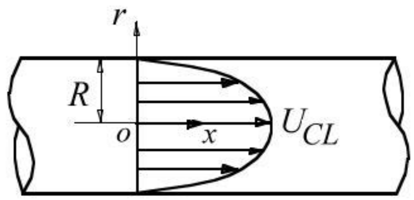

Figure P8.25

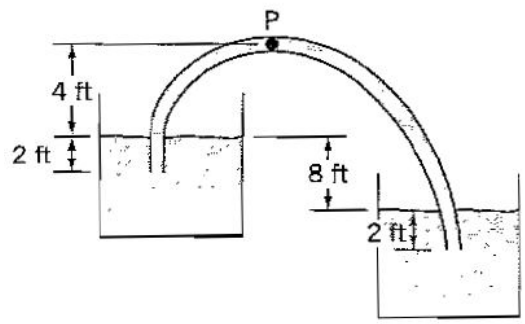

Figure P8.27:

where Re is the Reynolds number based on the diameter D.

8.26 A $0.5\,hp$ fan is to be used to supply air to a class room at $60°F$ through a smooth air-conditioning duct measuring $6\,in.$ by $12\,in.$ by $50\,ft$ long. Find the volume flow rate, and the pressure just downstream of the fan. Use the concept of hydraulic diameter. State all your other assumptions.

8.27 Water at $60°F$ is siphoned between two tanks as shown in Figure P8.27. The connecting tube is $2.0\,in.$ diameter smooth plastic hose $20\,ft$ long. Find the volume flow rate and the pressure at point P, which is $8\,ft$ from the entrance to the tube. Ignore minor losses. (To begin, assume a friction factor, then iterate.)

8.28 Due to corrosion and scaling, the roughness height of a pipe k increases with its years in service t, varying approximately as

$$k = k_0 + \varepsilon t,$$

where k_0 is the roughness of the new pipe. For a cast iron pipe, $k_0 \approx 0.26\,mm$, and $\varepsilon \approx 0.00001\,m$ per year. Estimate the discharge (volume flow rate) of water through a $20\,cm$ diameter cast iron pipe $500\,m$ long as a function of time over a 20-year period. Assume the pressure drop is constant and equal to $150\,kPa$.[1]

8.29 Oil of kinematic viscosity $\nu = 4 \times 10^{-4} ft^2/s$ at room temperature flows through an inclined tube of $0.5\,in.$ diameter. Find the angle α the tube makes with the horizontal if the pressure inside the tube is constant along its length and the flow rate is $5\,ft^2/hr$. The flow is laminar.

8.30 Water at $60°F$ flows at a rate of $300\,ft^3/s$ through a rectangular open channel that slopes down at an angle α, as shown in Figure P8.30. The depth of the water is $5\,ft$, and the width of the channel is $8\,ft$. The channel is made of concrete which has a friction factor

[1] Adapted from John & Haberman, *Introduction to Fluid Mechanics*, Prentice Hall, 1988.

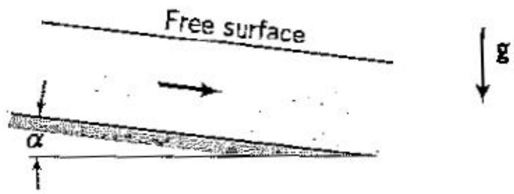

Figure P8.30:

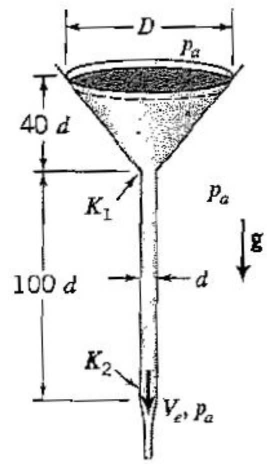

Figure P8.33

$f = 0.02$. Find the angle α.

8.31 You have a choice between a $3\,ft$ or a $2\,ft$ diameter steel pipe to transport 2000 gallons per minute of water to a power plant. The larger diameter pipe costs more but the losses are smaller. What is the relative decrease in head loss if you choose the larger diameter pipe? The kinematic viscosity is $1 \times 10^{-5}\,ft^2/s$.

8.32 A farmer must pump at least 100 gallons of water per minute from a dam to a field, located $25\,ft$ above the dam and $2000\,ft$ away. She has a $10\,hp$ pump, which is approximately 80% efficient. Find the minimum size of smooth plastic piping she needs to buy, given that the piping is sized by the half-inch. Use $\nu = 10^{-5} ft^{/}s$. Ignore minor losses.[2]

8.33 Under the action of gravity, water of density ρ passes steadily through a circular funnel into a vertical pipe of diameter d, and then exits from the pipe, falling freely under gravity, as shown in Figure P8.33. Atmospheric pressure acts everywhere outside the funnel and pipe. The entrance to the pipe has a loss coefficient $K_1 = 0.5$, the pipe has a friction factor $f = 0.01$, and the exit from the pipe has a loss coefficient $K_2 = 1.0$. The kinetic energy coefficient $\alpha = 1$. Find the outlet velocity in terms of g and d. You may assume that $D \gg d$.

8.34 Determine the horsepower required to pump water vertically through $300\,ft$ of $1.25\,in.$ diameter smooth plastic piping at a volume flow rate of $0.1\,ft^3/s$.

8.35 Water flows with a volume flow rate of $100\,liters/s$ from a large reservoir through a pipe $100\,m$ long and $10\,cm$ diameter which has a turbomachine near the exit. The outlet

[2]Adapted from Potter & Foss *Fluid Mechanics*, Great Lakes Press, Inc., 1982.

is $10\,m$ below the level of the reservoir, and it is at atmospheric pressure. If the friction factor of the pipe is 0.025, find the power added to, or provided by the turbomachine. Is it a pump or a turbine? Neglect minor losses.

8.36 Repeat the previous problem for a volume flow rate of $10\,liters/s$.

8.37 Water at $10°C$ is pumped from a large reservoir through a $500\,m$ long, $50\,mm$ diameter plastic pipe to a point $100\,m$ above the level of the reservoir at a rate of $0.015m^3/s$. If the outlet pressure must exceed $10^6\,Pa$, find the minimum power required. Neglect minor losses.

8.38 The laminar flow of water in a small pipe is given by

$$\frac{u}{U_{max}} = 1 - \left(\frac{r}{R}\right)^2$$

where $R = 0.5\,cm$ is the pipe radius, and $U_{max} = 20\,cm/s$. Assume the temperature is $15°C$.
(a) Find the wall shear stress.
(b) Find the pressure drop in a $10\,cm$ length of the pipe.
(c) Verify that the flow is indeed laminar.

8.39 The velocity distribution in a fully developed laminar pipe flow is given by

$$\frac{u}{U_{CL}} = 1 - \left(\frac{r}{R}\right)^2$$

where U_{CL} is the velocity at the centerline, and R is the pipe radius. The fluid density is ρ, and its viscosity is μ.
(a) Find the average velocity $\overline{V}$.
(b) Write down the Reynolds number Re based on average velocity and pipe diameter. At what approximate value of this Reynolds number would you expect the flow to become turbulent? Why is this value only approximate?
(c) Assume that the stress/strain rate relationship for the fluid is Newtonian. Find the wall shear stress τ_w in terms of μ, R and U_{CL}. Express the local skin friction coeffient C_f in terms of the Reynolds number Re.

8.40 A fully developed two-dimensional duct flow of width W and height D has a parabolic velocity profile, as shown in Figure P8.40.
(a) If $w \gg D$, show that the wall shear stress τ_w is related to the pressure gradient dp/dx according to $dp/dx = -2\tau_w/D$.
(b) Express the pressure gradient in terms of the velocity on the centerline, the fluid viscosity μ, and the duct height D.
(c) How does the z-component of vorticity vary with y?

8.41 Water of density ρ and viscosity μ flows steadily vertically down a circular tube of diameter D, as shown in Figure P8.41. The flow is fully developed, and it has a parabolic velocity profile given by

$$\frac{u}{U_c} = 1 - \left(\frac{2r}{D}\right)^2$$

where the maximum velocity is U_c and the other notation is given in the figure.
(a) Find the shear stress acting at the wall in terms of the density ρ, the gravitational constant g and the diameter D.
(b) Express the kinematic viscosity in terms of D, U_c and g.

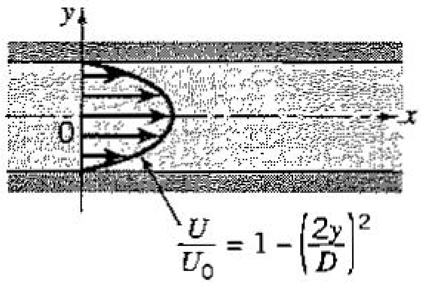

Figure P8.40

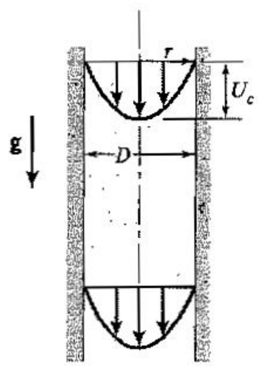

Figure P8.41

8.42 A thin layer of water of depth h, flows down a plane inclined at an angle θ to the horizontal, as shown in Figure P8.42. The flow is laminar, and fully developed (velocity does not change in the x-direction).
(a) Find a theoretical expression for the velocity profile and for the volume flow rate per unit width. Use the boundary condition that at the edge of the layer the shear stress is zero.
(b) If the flow is turbulent and the surface fully rough, find the volume flow rate in terms of h, g, and f.

8.43 Consider the steady, laminar, fully developed flow of water of depth h down a plane inclined at an angle θ to the horizontal, as shown in Figure P8.43. The velocity profile is quadratic, as shown, and the velocity at the free surface is U_e. Show that $U_e = gh^2 \sin\theta / 2\nu$, where ν is the kinematic viscosity of water.

8.44 As shown in Figure P8.44, water flows from a large tank of depth $2\,m$ through a square-edged entrance with a loss coefficient $K_1 = 0.5$ to a pump that delivers a mass flow rate $\dot{m} = 2.83\,kg/s$ through a circular pipe of diameter $D = 0.03\,m$, a length of $30\,m$, and a friction factor of 0.01. The pipe exits to atmospheric pressure with an exit loss coefficient of

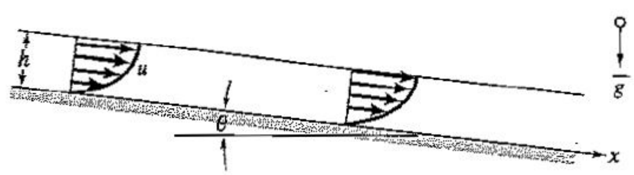

Figure P8.42

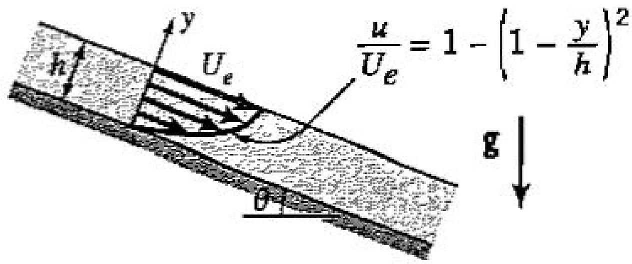

Figure P8.43

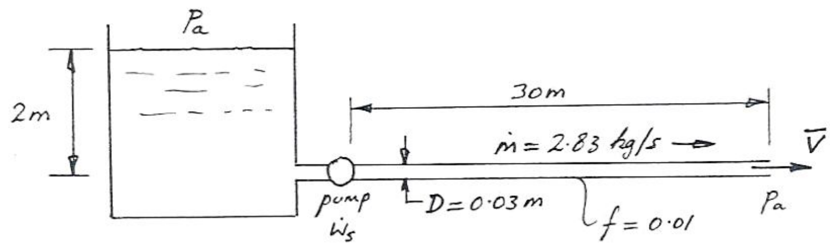

Figure P8.44

$K_2 = 1.0$. Given that the density of water is $1000\,kg/m^3$, and the kinetic energy coefficient $\alpha = 1$, find the power required to drive the pump, $\dot{W}_{shaft}$.

8.45 A water tank of constant depth H, open to the atmosphere, is connected to the piping system, as shown in Figure P8.45. After a length of pipe L of diameter D, the diameter decreases smoothly to a value of $D/2$ and then continues on for another length L before exiting to atmosphere. The flow is turbulent and the friction factor f is the same for all piping. C_{D1} and C_{D2} are the loss coefficients for the entry and exit. Calculate the depth of the tank required to produce a mean exit velocity of V.

8.46 At a ski resort, water of density ρ flows through a pipe of diameter D and turbulent flow friction factor f for a distance L from a large reservoir with a free surface elevation of $1500\,m$ to a snow making machine at an elevation of $1400\,m$, as shown in Figure P8.46. K_1 and K_2 are the loss coefficients for the entry and exit, respectively, and along the way the flow passes through 5 bends, each with a loss coefficient of K_3.
(a) Determine the exit velocity if $D = 0.1\,m$, $L = 100m$, $f = 0.01$, $K_1 = K_2 = K_3 = 1.0$.
(b) Compare the computed exit velocity to the exit velocity if there were no losses.

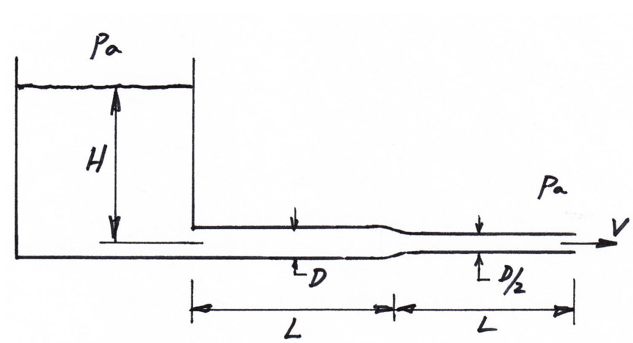

Figure P8.45

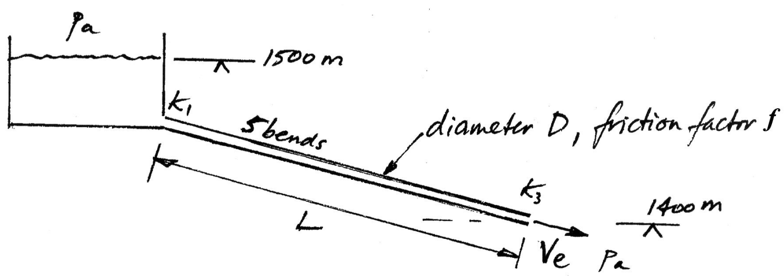

Figure P8.46

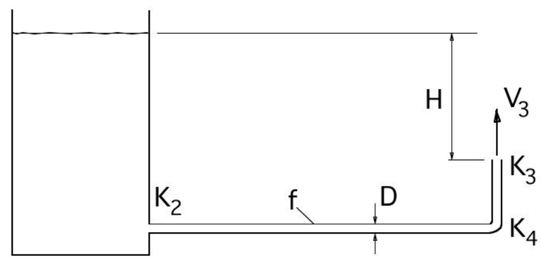

Figure P8.47

(c) If a pump was put in the system to double the exit velocity, what is its required minimum power ($\rho = 1000\, kg/m^3$)?

8.47 Water of density ρ flows from a large tank through a pipe of diameter D, length L, and friction factor f, as shown in Figure P8.47. K_2 and K_3 are the loss coefficients for the entry and exit, respectively, and towards the end of the pipe there is one bend with a loss coefficient of K_4. The flow in the pipe is turbulent. The exit from the pipe is a distance H below the level of the water in the tank.
(a) Determine the exit velocity V_3 if $D = 0.05\, m$, $L = 10m$, $f = 0.01$, $K_2 = K_3 = K_4 = 1.0$, and $H = 1\, m$.
(b) Under the conditions given in (a), find the height h to which the jet of water issuing from the pipe rises above the exit.
(c) If a pump was put in the system to increase h by a factor of 10, what is its required minimum power ($\rho = 1000\, kg/m^3$)?

8.48 Water of density ρ ($= 1000\, kg/m^3$) flows from a large tank through a pipe of diameter D, total length L, and friction factor f, as shown in Figure P8.48. The loss coefficients for the entry and exit are K_1 and K_3, respectively, and along the length of the pipe there are four bends each with a loss coefficient of K_2. The flow in the pipe is turbulent. The entrance to the pipe is a distance H below the level of the water in the tank.
(a) Determine the exit velocity V_3 if $D = 0.025\, m$, $L = 10\, m$, $f = 0.01$, $K_1 = K_2 = K_3 = 1.0$, and $H = 2\, m$.
(b) Under the conditions given in (a), find the gauge pressure at the point 2 along the pipe (p_{2g}), where point 2 is at located at a distance $L/2$ along the pipe, at a height $2H$ above the entrance to the pipe.
(c) If a pump was put in the system to double the flow rate, what is its required minimum power?

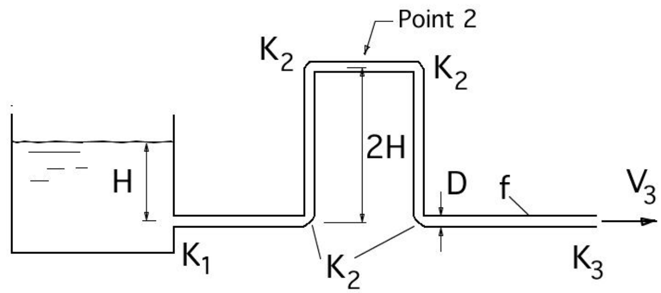

Figure P8.48

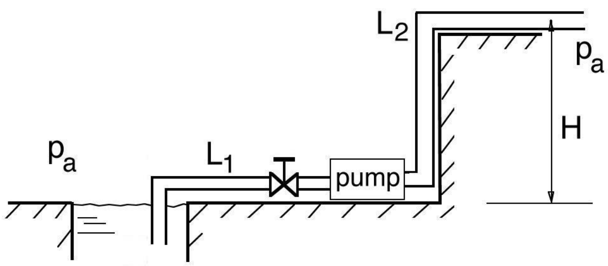

Figure P8.49

8.49 Water of density $1000\,kg/m^3$ is pumped from a sump tank up a distance of H before exiting to atmospheric pressure, as shown in Figure P8.49. The entrance and exit have a loss coefficient of $K = 1$. Each bend also has a loss coefficient of K, but the valve has a loss coefficient of $2K$. The pipe has a diameter D, $L_1 = L_2 = 50D$, and the friction factor $f = 0.01$. Find the power expended by the pump if the volume flow rate is $40\,liters/s$, given that $D = 50\,mm$, and $H = 2\,m$. Assume that the flow in the pipe is turbulent.

8.50 A large tank feeds a straight pipe that delivers water to a turbine, and then exits to atmosphere, as shown in Figure P8.50. The friction factor $f = 0.01$, the length $L = 100$ m, the diameter of the pipe $D = 0.1$ m, the loss coefficients $K_1 = K_2 = 1$, the water density $\rho = 1000$ kg/m^3, the water viscosity $\mu = 15 \times 10^{-3}$ N.s/m^2, the bulk velocity in the pipe $\overline{V} = 2$ m/s, and the turbine develops 500 W.
(a) Do you expect the flow in the pipe to be turbulent or laminar?
(b) Find the height H.

8.51 A pump delivers water through the piping system which exits to atmosphere, as shown in Figure P8.51. The friction factor $f = 0.01$, the overall length of the pipe $L = 10$ m, the diameter of the pipe $D = 0.1$ m, the loss coefficients $K_1 = K_2 = 1$, the water density $\rho = 1000$ kg/m^3, the bulk velocity in the pipe $\overline{V} = 4$ m/s, $H = 5$ m, and the pump develops 750 W. Find the gauge pressure at the inlet to the pump, p_{ig}.

8.52 A large tank feeds a siphon as shown in the figure. For the flow in the siphon, the friction factor $f = 0.01$, the length of the siphon $L = 1$ m, and its diameter $D = 0.01$ m, the loss coefficients $K_1 = K_2 = K_3 = K_4 = 1$, and the water density $\rho = 1000$ kg/m^3.
(a) If there were no losses, what is the exit velocity of the siphon?
(b) Assuming the flow in the siphon to be turbulent, and taking into account all losses, what is the actual exit velocity of the siphon?

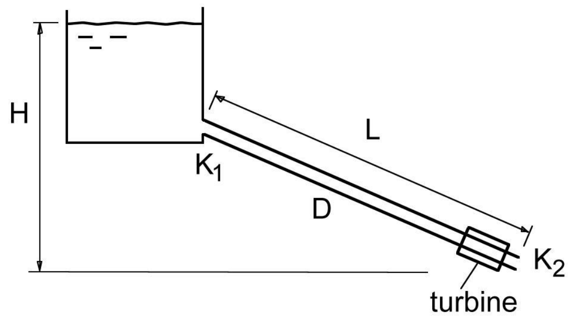

Figure P8.50

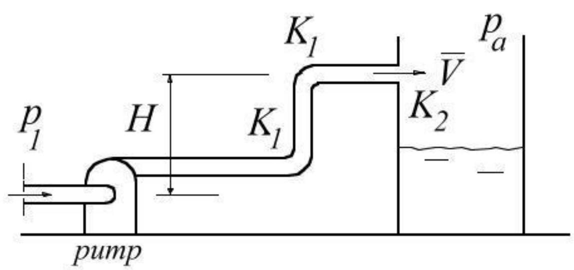

Figure P8.51

(c) If a pump was added somewhere in the siphon, and taking into account all losses, what would be the output power of the pump if the exit velocity was seen to double?

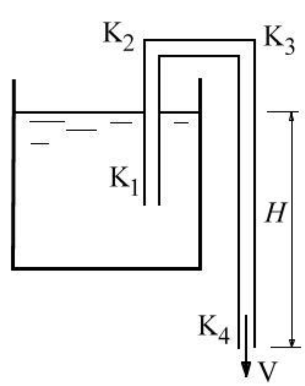

Figure P8.52

Chapter 9

Viscous External Flows

9.1 Study Guide

- Describe what is meant by the term "boundary layer." Illustrate your answer using a diagram.

- A laminar boundary layer is observed to grow on a flat plate of width w and length L such that the pressure and the freestream velocity U_e is constant everywhere. The total skin friction coefficient C_F is defined according to

$$C_F = \frac{F}{\frac{1}{2}\rho U_e^2 w L}$$

  Here, F is the total frictional force acting on one side of the plate, and ρ is the fluid density. By using the momentum equation and the continuity equation, find the relationship between C_F and the momentum thickness θ for an arbitrary velocity profile. Indicate all your assumptions.

- What is the definition of the displacement thickness? Momentum thickness? Shape factor? Give physical interpretations for these parameters.

- Give brief physical explanations and interpretations of the following terms:
  (a) The 99% boundary layer thickness.
  (b) The boundary layer displacement thickness δ^*.
  (c) The boundary layer momentum thickness θ.

- Describe the differences between laminar and turbulent boundary layers in terms of:
  (a) The velocity profile.
  (b) The frictional drag.
  (c) The behavior in adverse pressure gradients (that is, the tendency to separate).

- Consider the differences between a laminar and a turbulent boundary layer in a zero pressure gradient. If the boundary layer thickness was the same: (a) Sketch the velocity profile for each flow. (b) Which flow has the higher wall shear stress? Why? (c) How do these observations explain why a dimpled golf ball can travel further than a smooth golf ball?

- Draw the flow around a typical power plant smoke stack. Identify the laminar and turbulent boundary layers, the points of separation, the separated flow, the vortex shedding, and the freestream flow.

- Consider a laminar and a turbulent boundary layer at the same Reynolds number.
  (a) How do the velocity profiles compare?
  (b) Which boundary layer grows faster?
  (c) Which boundary layer has the larger ratio of displacement thickness to momentum thickness (δ^*/θ)?
  (d) When subjected to the same adverse pressure gradient, which boundary layer will separate sooner?

9.2 Worked Examples

Example 9.1: Boundary layer flow

A flat plate 10 ft long is immersed in water at $60°F$, flowing parallel to the plate at 20 ft/s. The boundary layer is initially laminar, then transitions to turbulent flow at $Re_x = 10^5$.
(a) Find the approximate boundary layer thickness at $x = 5\,ft$ and $x = 10\,ft$, where x is measured from the leading edge.
(b) Find the total drag coefficient C_F. (c) Find the total drag per unit width of the plate if the water covers both sides.

Solution: First, we need to know where transition is likely to occur. The boundary layer is laminar for $Re_x < 10^5$, so that

$$x < \frac{\nu \times 10^5}{U_e} = \frac{1.21}{20}\,ft = 0.73\,in.$$

We see that the region of laminar flow is very short, and we can assume that the boundary layer is turbulent from the leading edge on.

For part (a), we use the power law approximation for turbulent boundary layer given by equation 9.21. For $x = 5\,ft$

$$Re_x = \frac{xU_e}{\nu} = \frac{5 \times 20}{1.21 \times 10^{-5}} = 8.26 \times 10^6$$

and

$$\frac{\delta}{x} = \frac{0.37}{Re_x^{0.2}} = 0.0153$$

so that

$$\delta = 0.0765\,ft = 0.92\,in.$$

For $x = 10, ft$, $Re_x = 16.5 \times 10^6$, and $\delta = 0.133\,ft = 1.60\,in$. Note how thin the boundary layer is compared to the development length x.

For part (b), the total drag coefficient is given by equation 9.20, so that

$$C_F = \frac{0.074}{Re_L^{0.2}}$$

With $L = 10\,ft$, $Re_L = 16.5 \times 10^6$, and $C_F = 0.00266$.

For part (c), we have a total viscous force acting on the plate equal to $2F_v$, where F_v is the force acting on one surface. From the definition of the total drag coefficient, the total drag on the plate per unit width is given by

$$\frac{2F_v}{W} = \rho U_e^2 L C_F = 1.938\,\frac{slugs}{ft^3} \times 400\,\frac{ft^2}{s^2} \times 10\,ft \times 0.00266\,lb_f = 20.6\,\frac{lb_f}{ft}$$

Example 9.2: Vortex shedding

In an exposed location, telephone wires will "sing" when the wind blows across them. Find the frequency of the note when the wind velocity is 30 mph, and the wire diameter is 0.25 $in.$ For air, we assume that $\nu = 15 \times 10^{-6}\, m^2/s$.

Solution: First we need to know the Reynolds number, Re, where

$$\begin{aligned} Re &= \frac{VD}{\nu} \\ &= \frac{\left(30\frac{mi}{hr} \times 5280\frac{ft}{mi} \times 12\frac{in.}{ft} \times 0.0254\frac{m}{in.} \times \frac{1}{3600}\frac{hr}{s}\right)\left(0.25in. \times 0.0254\frac{m}{in.}\right)}{15 \times 10^{-6}\frac{m^2}{s}} \\ &= 2774. \end{aligned}$$

From Figure 9.12, we see that at this Reynolds number the Strouhal number is approximately equal to 0.21. That is,

$$St = \frac{fD}{V} = 0.21$$

so that

$$\begin{aligned} f &= \frac{0.21V}{D}\, Hz \\ &= \frac{\left(0.21 \times 30\frac{mi}{hr} \times 5280\frac{ft}{mi} \times 12\frac{in.}{ft} \times 0.0254\frac{m}{in.} \times \frac{1}{3600}\frac{hr}{s}\right)}{0.25in. \times 0.0254\frac{m}{in.}}\, Hz \\ &= 444\, Hz, \end{aligned}$$

which is very close to the note middle A (above middle C) (= 440 Hz).

Example 9.3: Drag of a submarine model

A submarine has the shape of an 8:1 ellipsoid. Find the power required to maintain a velocity of 20 ft/s when it is fully submerged under water. The sea water temperature is $68°F$. The submarine has a frontal area of 50 ft^2, and a drag coefficient $C_D = 0.15$.

Solution: The drag coefficient is given by

$$C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

(equations 3.3 and 7.12), where A is the cross-sectional area of the body, and F_D is the drag force acting on the body. Hence

$$F_D = \tfrac{1}{2}\rho V^2 A\, C_D$$

The density of sea water at $68°F$ is $1,025\, kg/m^3$, that is, $1.989\, slug/ft^3$ (Table 1.2). Therefore

$$F_D = \tfrac{1}{2} \times 1.989\, slug/ft^3 \times 400\, ft^2/s^2 \times 50\, ft^2 \times 0.15\, lb_f = 2983\, lb_f$$

The power required to overcome the drag is the work done per unit time, that is, the drag force times the velocity of motion. Therefore

$$\begin{aligned} \text{power required} &= F_D \times V = 2,983 \times 20\, ft \cdot lb_f/s = 59,700\, ft \cdot lb_f/s \\ &= 59,700\, ft \cdot lb_f/s\, \frac{1.341\, hp}{737.5\, ft \cdot lb_f/s} = 108\, hp \end{aligned}$$

Problems

9.1 The flow over a leaf is being studied in a wind tunnel. If the wind blows at 8 mph, and the leaf is aligned with the flow, is the boundary layer laminar or turbulent?

9.2 For steady, fully developed flow of a constant property fluid in a two-dimensional, rectangular duct of height $2h$ the velocity profile is parabolic. Find the ratio of the displacement thickness δ^* to the half-height h (assume you can put the freestream velocity equal to the maximum velocity, and h equal to the boundary layer thickness).

9.3 For the boundary layer velocity profile given by:

$$\frac{u}{U_e} = \frac{y}{\delta} \qquad (y \leq \delta)$$

Find the wall shear stress, the skin friction coefficient, the displacement thickness, and the momentum thickness.

9.4 (a) A laminar pipe flow is described by a parabolic velocity profile. The fluid has viscosity μ and a density ρ. The coordinate y is measured from the centerline of the pipe. The velocity profile is described by

$$\frac{u}{\overline{\overline{V}}} = 1\left[1 - \left(\frac{y}{R}\right)^2\right]$$

where R is the pipe radius and $\overline{V}$ is the average velocity. Find the viscous stress at the wall τ_w and express it non-dimensionally in terms of a skin friction coefficient and the Reynolds number based on diameter and average velocity.
(b) Given the velocity profile in part (b), calculate the displacement thickness and the momentum thickness, using the centerline velocity instead of the freestream velocity.

9.5 Consider fully developed, steady flow of a constant density Newtonian fluid in a circular pipe of diameter D, as shown in Figure P9.5.
(a) Show that the pressure gradient $dp/dx = 4\tau_w/D$, where τ_w is the viscous shear stress at the wall.
(b) If the velocity distribution is triangular, find the average velocity $\overline{V}$ at any cross section, and express the skin friction coefficient

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho\overline{V}^2}$$

in terms of the Reynolds number based on $\overline{V}$.

9.6 A rectangular duct of height h mm and width w mm carries a flow of air of density

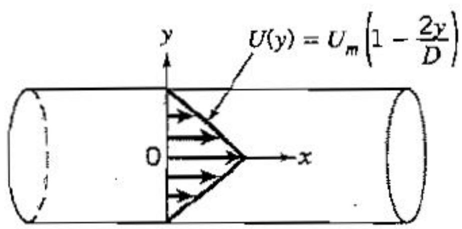

Figure P9.5

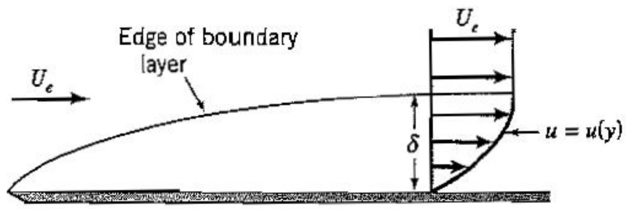

Figure P9.8

ρ, and viscosity μ. The velocity profile is quartic, so that

$$\frac{U}{U_m} = 1 - \left(\frac{2y}{h}\right)^4$$

where U_m m/s is the maximum velocity, which occurs on the centerline where $y = 0$.

(i) Find the skin friction coefficient $C_f = \tau_w/(\frac{1}{2}\rho {U_m}^2)$, where τ_w is the shear stress at the wall of the duct, in terms of μ, U_m and h.

(ii) Express C_f in terms of the Reynolds number based on U_m and h.

9.7 In laminar flow, the growth of the boundary layer thickness δ with distance downstream x is given by

$$\frac{\delta}{x} = \frac{K}{\sqrt{Re_x}}$$

where $Re_x = xU_e/\nu$, ν is the fluid kinematic viscosity, U_e is the freestream velocity, and K is a constant.
(a) Given that the velocity profile is described by $U/U_e = y/\delta$, find the growth of the displacement thickness, and the momentum thickness with downstream distance.
(b) What is the shape factor H?

9.8 A constant density, constant pressure laminar boundary layer is growing on flat plate of width W in a stream of velocity U_e, as shown in Figure P9.8. Show that the total drag force F acting on the plate is given by $F/\rho U_e^2 \delta W = (4 - \pi)/2\pi$ when the velocity profile is given by

$$\frac{u}{U_e} = \sin\left(\frac{\pi y}{2\delta}\right) \quad \text{for} \quad \frac{y}{\delta} \leq 1$$

9.9 For the boundary layer profile given in the previous problem, show that the displacement thickness $\delta^* = \delta(1 - 2/\pi)$.

9.10 A laminar boundary layer is formed on a flat plate of width w and length L in a zero pressure gradient. The fluid has viscosity μ and a density ρ. The coordinate x is measured from the leading edge, and the coordinate y is measured from the wall. The velocity profile is described by

$$\frac{U}{U_e} = a\left(\frac{y}{\delta}\right) + b\left(\frac{y}{\delta}\right)^2$$

where δ is the boundary layer thickness and U_e is the freestream velocity.
(a) Find the constants a and b by using the definition of the boundary layer thickness and given that

$$\tau_w = \frac{3\mu U_e}{2\delta}$$

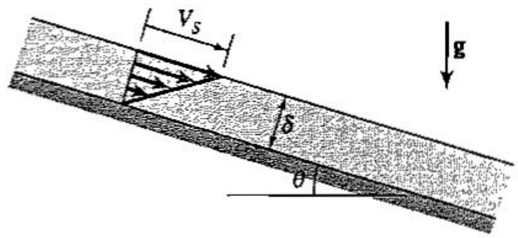

Figure P9.11

where τ_w is the shear stress at the wall.
(b) For the flow described in part (a), find the total viscous drag exerted on one side of the plate in terms of w, L, μ, ρ, c, and U_e, given that

$$\frac{\delta}{x} = \frac{c}{\sqrt{Re_x}}$$

where Re_x is the Reynolds number based on x.
(c) For the flow described in part (a), find the displacement thickness δ^*.

9.11 Consider the two-dimensional, laminar, fully developed flow of water of depth δ down a plane inclined at an angle θ to the horizontal, as shown in Figure P9.11. The velocity profile may be assumed to be linear, and the velocity at the free surface is V_s.
(a) Find the shear stress at the wall by
 (i) Using a control volume analysis.
 (ii) Using the given velocity profile.
(b) Express the skin friction coefficient C_f in terms of
 (i) the angle θ, and a Froude number based on δ and V_s.
 (ii) the Reynolds number based on δ and V_s.

9.12 For a turbulent boundary layer velocity profile given by $u/U_e = (y/\delta)^{1/9}$, where U_e is the freestream velocity and δ is the boundary layer thickness, calculate the displacement thickness, the momentum thickness, and the shape factor.

9.13 A flat plate $1\,m$ long and $0.5\,m$ wide is parallel to a flow of air at a temperature of $25°C$. The velocity of the air far from the plate is $20\,m/s$.
(a) Is the flow laminar or turbulent? (b) Find the maximum thickness of the boundary layer.
(c) Find the overall drag coefficient C_F, assuming that transition occurs at the leading edge.
(d) Find the total drag of the plate if the air covers both sides.

9.14 Repeat problem 9.13 for the case where the fluid is water at $15°C$, assuming the Reynolds number is kept constant by changing the freestream velocity.

9.15 Air at $80°F$ flows over a flat plate $6\,ft$ long and $3\,ft$ wide at a speed of $60\,ft/s$. Assume the transition Reynolds number is 5×10^5.
(a) At what distance from the leading edge does transition occur?
(b) Plot the local skin friction coefficient $C_f = \tau_w/(\frac{1}{2}\rho V^2)$ as a function of $Re_x = U_e x/\nu$.
(c) Find the total drag of the plate if the air covers both sides.

9.16 Find the ratio of the friction drags of the front and rear halves of a flat plate of total length ℓ if the boundary layer is turbulent from the leading edge and follows a one-seventh-power law velocity distribution.

9.17 For a particular turbulent boundary layer on a flat plate, the velocity profile is given by $u/U_e = (y/\delta)^{1/7}$, where u is the streamwise velocity, U_e is the freestream velocity and δ is the boundary layer thickness. Given that the boundary layer thickness grows as in equation 9.21:
(a) Find the distribution of the vertical velocity component $v(y)$.
(b) Calculate the angle the flow vector makes with the flat plate at $y/\delta = 0.05$, 0.2, and 0.8 at $Re_x = 10^6$.

9.18 A turbulent boundary layer on a flat plate of length L and width w has a velocity distribution described by

$$\frac{U}{U_e} = \left(\frac{y}{\delta}\right)^{1/7}$$

where U_e is the freestream velocity, y is the distance measured normal to the plate, and δ is boundary layer thickness.
(a) Using a control volume analysis, find k such that

$$\frac{F_D}{\rho U_e^2 w L} = k\frac{\delta}{L}$$

Here F_D is the total viscous force acting on the plate, and ρ is the fluid density. Assume that the pressure is constant everywhere.
(b) Using the result from part (a), show that

$$\frac{d\delta}{dx} = \frac{\tau_w}{k\rho U_e^2}$$

where τ_w is the wall shear stress.
(c) Hence find τ_w, given that

$$\frac{\delta}{x} = \frac{0.37}{Re_x^{0.2}}$$

9.19 Air enters a long two-dimensional duct of constant height h, as shown in Figure P9.19. Identical boundary layers develop on the top and bottom surfaces. In the core region, outside the boundary layers, the flow is inviscid and no losses occur. The flow is steady and the density is constant.
(a) Show that

$$\frac{U_2}{U_1} = \frac{h}{h - 2\delta^*}$$

where δ^* is the displacement thickness at station 2.
(b) Given the information in part (a), find the pressure drop between stations 1 and 2 along the central streamline.

9.20 In the previous problem, if you were given that velocity u at station 3 varied with y according to:

$$\frac{u}{U_m} = 1 - \left(\frac{2y}{h}\right)^2$$

(a) What is the distribution of the viscous stress, and what is its value at the wall?
(b) What is the distribution of the z-component of vorticity, and what is its value at the wall?

9.21 A laminar boundary layer velocity profile may be described approximately by $u/U_e = \sin(\pi y/2\delta)$. Find the shear stress at the wall, τ_w, and express the local skin friction coefficient $C_f = \tau_w/(\frac{1}{2}\rho U_e^2)$ in terms of a Reynolds number based on U_e and δ.

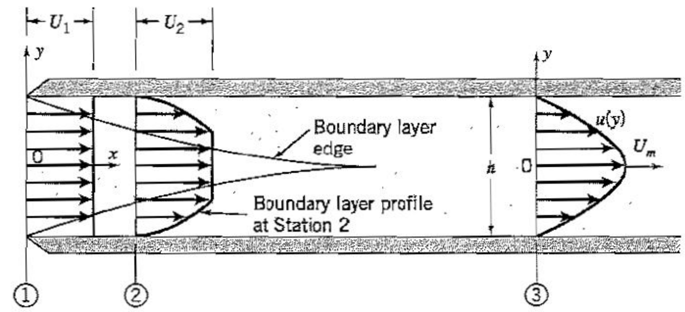

Figure P9.19

9.22 Consider the entrance section to a circular pipe of diameter D. The incoming velocity is constant over the area and equal to U_1, as shown in Figure P9.22. Downstream, however, a boundary layer grows and this causes the flow in the central region to accelerate. The flow has a constant density ρ. Find U_2/U_1 given that $\delta^* = D/16$ at station 2. (Note that $\delta << D$)

9.23 Assuming that the velocity profile in a laminar boundary layer of thickness δ is given by

$$\frac{u}{U_e} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^3$$

where u is the velocity at distance y from the surface and U_e is the freestream velocity (as shown in Figure P9.23), demonstrate that

$$\frac{\theta}{\delta} = \frac{31}{420} \quad \text{and} \quad C_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U_e^2} = \frac{4\nu}{U_e \delta}$$

where θ is the momentum thickness, τ_w is the viscous stress at the wall, C_f is the local skin friction coefficient at a distance x from the leading edge of the plate, ρ is the density and ν is the kinematic viscosity.

9.24 (a) For the laminar velocity profile given by

$$\frac{U}{U_e} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

where U_e is the freestream velocity and δ is the boundary layer thickness, find the displacement thickness δ^*, the momentum thickness θ, and the shape factor H.
(b) What is the relationship between the total drag force on a flat plate F_D, and the momentum thickness θ measured at the downstream end of the plate?

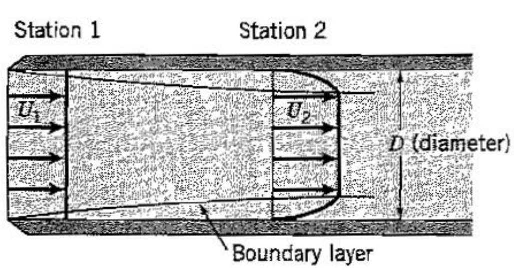

Figure P9.22

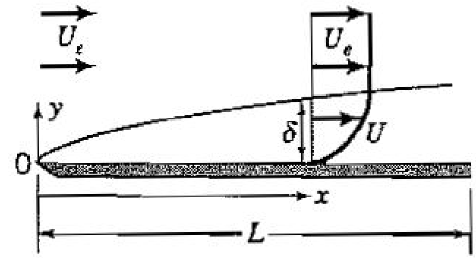

Figure P9.23

(c) Compare and contrast the general attributes of laminar and turbulent boundary layers on a flat plate (4 points).
(d) Compare and contrast the behavior of laminar and turbulent boundary layers in an adverse pressure gradient (4 points).

9.25 For two-dimensional, constant pressure, constant density, laminar flow over a flat plate, the boundary layer velocity profile is crudely described by the following relationship:

$$\frac{u}{U_e} = \frac{y}{\delta} \qquad \text{for} \quad y \leq \delta$$

where u is the velocity parallel to the surface, U_e is the (constant) freestream velocity, y is the distance normal to the surface, and δ is the boundary layer thickness.
(a) Evaluate $d\theta/dx$, where θ is the momentum thickness, and x is the distance along the surface measured from the leading edge of the plate, in terms of the boundary layer growth rate.
(b) Evaluate the skin friction coefficient C_f.
(c) Using the results from parts (a) and (b), show that θ grows as $\sqrt{x}$.

9.26 A laminar boundary layer is observed to grow on a flat plate such that the pressure is constant everywhere. If the boundary layer velocity profile is given by:

$$\frac{u}{U_e} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \qquad \text{for} \quad y \leq \delta$$

where u is the velocity at a distance y from the surface, δ is the boundary layer thickness and U_e is the freestream velocity.
(a) Show that $\theta/\delta = 0.139$, where θ is the momentum thickness.
(b) Find the viscous stress on the plate in terms of the viscosity, the boundary layer thickness, and the freestream velocity.

9.27 A laminar boundary layer is observed to grow on a flat plate such that the pressure is constant everywhere. If the boundary layer velocity profile is given by:

$$\frac{U}{U_e} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

(for $y \leq \delta$), where U is the velocity at a distance y from the surface, δ is the boundary layer thickness and U_e is the freestream velocity.
(a) Find the viscous stress on the plate τ_w (the local wall shear stress) in terms of the viscosity μ, the boundary layer thickness δ, and the freestream velocity U_e.
(b) Express the answer to part (a) in terms of non-dimensional parameters (that is, in terms of the skin friction coefficient and the Reynolds number based on δ).
(c) Find the ratio of the displacement thickness to the boundary layer thickness (this is a number).

9.28 (a) If the boundary layer velocity profile for $y \leq \delta$ is given by

$$\frac{U}{U_e} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

where U is the velocity at a distance y from the surface, δ is the boundary layer thickness and U_e is the freestream velocity.

(i) Find the ratio of the displacement thickness to the boundary layer thickness (this is a number).

(ii) Find the ratio of the momentum thickness to the boundary layer thickness (this is a number).

(b) Air enters a two-dimensional duct with a uniform velocity profile. As the boundary layers on the top and bottom walls grow with downstream distance, the velocity in the freestream tends to increase. However, if the walls diverged with downstream distance so that the freestream velocity remained constant, express the angle of divergence of the walls in terms of the boundary layer displacement thickness δ^* and the distance along the duct x.

(c) A laminar boundary layer is observed to grow on a flat plate of width w and length $x = L$ such that the pressure is constant everywhere. If the skin friction coefficient C_f is given by:

$$C_f = \frac{0.654}{\sqrt{Re_x}}$$

By integrating the shear stress on the surface, find the frictional force coefficient C_F as a function of the Reynolds number based on L, where C_F is defined according to:

$$C_F = \frac{F}{\frac{1}{2}\rho U_e^2 w L}$$

Here F is the total frictional force acting on one side of the plate, ρ is the fluid density, and U_e is the freestream velocity.

9.29 A ship 200 ft long with a wetted area of 5000 ft^2 moves at 25 ft/s. Find the friction drag, assuming that the ship surface may be modeled as a flat plate, and $\rho = 1.94\, slugs/ft^3$ and $\nu = 1.2 \times 10^{-5}\, ft^2/s$. What is the minimum power required to move the ship at this speed?

9.30 For the flow over a body completely immersed in a fluid, what is meant by the terms "pressure drag" and "viscous drag"? Using these terms, explain the difference between a "bluff" body and a "streamlined" body.

9.31 The vehicle shown in Figure P9.31 has a circular cross-section of diameter D, and it is traveling at a velocity U in air with a density of ρ and a viscosity μ. It has a total drag coefficient C_D, given by

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 \frac{\pi}{4} D^2}$$

where F_D is the total drag force. The skin friction coefficient is given by

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.37}{Re_x^{0.2}}$$

where τ_w is the viscous stress, $Re_x = xU/\nu$, and x is the distance along the vehicle surface. Find the ratio F_v/F_D in terms of C_D, the Reynolds number based on the length L, and the aspect ratio L/D, where F_v is the total viscous force exerted over the surface of the vehicle.

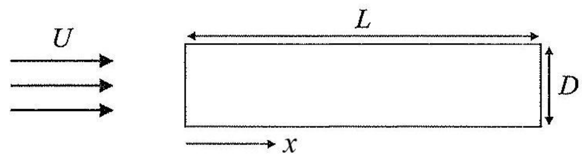

Figure P9.31

9.32 Explain why dimples on a golf ball help to reduce drag.

9.33 Pylons supporting a bridge over a fast-flowing river often have footings (the part of the pylon below and just above the water level) shaped like a wedge, both in the upstream and downstream direction. Why do you think this is done?

9.34 A disk 15 cm in diameter is placed in a wind tunnel at right angles to the incoming flow. The drag of the plate is found to be 3.2 N when the air velocity is 20 m/s and the air temperature is 25°C. Using this information, estimate the drag of a disk 40 cm in diameter in a water flow having a velocity 5 m/s at a temperature of 15°C.

9.35 A rectangular banner 2 ft high by 10 ft long is carried by marchers in a parade that is moving at 2 mph. A 30 mph wind is blowing. If the air density is $2.4 \times 10^{-3}\, slug/ft^3$, find the maximum force exerted on the banner by the wind. Estimate how many marchers it might take to hold the banner safely.

9.36 Air flows over a 60 ft high circular smoke stack at a uniform speed of 30 mph. Find the total force acting on the stack if its diameter is 6 ft and the air temperature is 70°F.

9.37 A 0.5 $in.$ diameter cable is strung between poles 120 ft apart. A 60 mph wind is blowing at right angles to the cable. Find the force acting on the cable due to the wind. The air temperature is 40°F.

9.38 A beach ball 20 cm in diameter traveling at a speed of 50 m/s in still air at 30°C is found to have a drag of 8 N.
(a) Find the velocity at which the drag of a 60 cm sphere immersed in water at 15°C can be found from the above data.
(b) What is the drag of the larger sphere at this velocity?

9.39 (a) Find the aerodynamics drag force on a car traveling at 75 mph if the drag coefficient is 0.4 and the frontal area is 24 ft^2.
(b) Find the maximum possible speed of a minivan, on the level with no wind, if it has a frontal area of 30 ft^2, a drag coefficient of 0.6 and a 120-horsepower engine that is 80% efficient. How will this speed change if there is a 30 mph headwind?

9.40 A spherical dust particle of density ρ_p and radius R falls at a constant velocity V in an atmosphere of density ρ_a under its own weight. Find V if $\rho_p/\rho_a = 1000$, $R = 0.5\, mm$, and the drag coefficient $C_D = F/(0.5\rho a V^2 \pi R^2) = 1$, where F is the drag force acting on the sphere. Ignore the buoyancy force.

9.41 A spherical rain drop of diameter D falls at its terminal velocity V in air of density ρ_a and viscosity μ_a. The raindrop has a drag coefficient given by

$$C_D = \frac{F_D}{\frac{1}{2}\rho_a V^2 \left(\frac{\pi}{4}D^2\right)} = \frac{24}{Re}$$

where F_D is the drag force acting on the drop and Re is the Reynolds number based on the drop diameter. What is V when $\rho_a = 1.2$ kg/m^3, $\mu_a = 18 \times 10^{-6}$ N.s/m^2, and the density of water $\rho_w = 1000$ kg/m^3? The volume of a sphere is given by $\pi D^3/6$. State all your assumptions.

9.42 A skydiver of mass 75 kg is falling freely, while experiencing a drag force due to air resistance. If her drag coefficient is 1.2, and her frontal area is 1 m^2, and assuming standard atmospheric conditions, find her terminal velocity at 10°C.

9.43 Find the terminal velocity of a parachutist, assuming that the parachute can be modeled as a semicircular cup of diameter 6 m. Use standard atmosphere properties of air corresponding to an altitude of 3000 m. The total mass of the person and parachute is 90 kg.

9.44 Find the terminal velocity of a steel sphere of density 7850 kg/m^3 and diameter 0.5 mm diameter, falling freely in SAE 30 motor oil, which has a density of 919 kg/m^3 and a viscosity of 0.04 $N \cdot s/m^2$.

9.45 A bracing wire used on a biplane to strengthen the wing assembly is found to vibrate at 5000 Hz. Estimate the speed of the airplane if the wire has a diameter of 1.2 mm. If the natural frequency of the wire is 500 Hz, at what speed will the wire vibration resonate?

9.46 An automobile radio aerial consists of three sections having diameters $\frac{1}{8}\,in.$, $\frac{3}{16}\,in.$, and $\frac{1}{4}\,in.$ Find the frequency of the vortex shedding when the car is traveling at 35 mph and 65 mph.

9.47 For problems 9.36 and 9.37 find the vortex shedding frequency. Estimate the wavelength between successive vortices.

Chapter 10

Open Channel Flow

10.1 Study Guide

- What is the propagation speed of a small planar disturbance in a shallow water basin?
- Write down the speed of a small amplitude gravity wave in shallow water. Use this result to describe qualitatively the formation of a breaking wave on a beach. How does the slope of the beach affect the formation of this wave?
- For open channel flow, name one mechanism by which the flow can become supercritical, and one mechanism by which the flow can become subcritical.
- Write down the definition of Froude number. Give two physical interpretations for its significance.
- Water flows over a rock in a river, and near the crest of the rock the water level is seen to go down. Is the upstream Froude number subcritical or supercritical? If downstream of the rock there is a hydraulic jump, what can you say about the Froude number downstream of the rock and before the hydraulic jump?

Summary of flow in a constriction without losses:

1. When $F < 1$ everywhere, the water level drops in the converging part, and rises in the diverging part. The Froude number first increases, and then decreases.

2. When $F > 1$ everywhere, the water levels rises in the converging part, and falls in the diverging part. The Froude number first decreases, and then increases.

3. $F = 1$ only at the throat.

4. The downstream solution is indeterminate when $F = 1$ at the throat. However, when there are no losses, only two possibilities exist: a supercritical solution and a subcritical solution. These solutions are independent of the upstream conditions, and depend only on the downstream conditions. In particular, they correspond to two special values for the downstream depth. If the downstream conditions require that the downstream depth be different from these special values, hydraulic jumps will appear and losses will occur.

5. Remember: hydraulic jumps can only occur at places where the hydraulic jump relationship is satisfied. That is, jumps occur where the Froude number is such that the change in height due to the jump is of the "right" value. Downstream of the jump, $F < 1$, and the water surface will continue to rise as the channel expands.

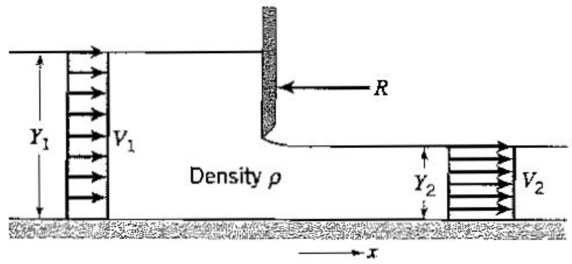

Figure 10.1: Partially opened sluice gate.

10.2 Worked Examples

Example 10.1: Flow under a sluice gate

Water flows steadily under a partially open gate of width W, as shown in Figure 10.1. The flow in the vicinity of the gate is complicated but at some distance upstream of the gate, the streamlines are initially straight and the water depth is Y_1. Some distance downstream of the gate, the streamlines are again straight and the water depth is $Y_2/Y_1 = 0.5$. If there are no frictional effects on the flow, that is, the water flows smoothly through the gate and there are no losses:
(a) What is F_1, the Froude number of the upstream flow?
(b) Show that the flow downstream of the gate is supercritical.

Solution: From mass conservation

$$V_1 Y_1 W = V_2 Y_2 W$$

It is generally a good idea to nondimensionalize the equations. Multiplying and dividing the left hand side of the continuity equation by $\sqrt{gY_1}$, and the right hand side by $\sqrt{gY_2}$, we obtain

$$\frac{V_1}{\sqrt{gY_1}} Y_1 \sqrt{gY_1} = \frac{V_2}{\sqrt{gY_2}} Y_2 \sqrt{gY_2}$$

That is,

$$F_1^2 = F_2^2 \left(\frac{Y_2}{Y_1}\right)^3 \tag{10.1}$$

To find F_1, we need to know something about F_2. Since there are no losses, we could use Bernoulli's equation along the surface streamline. This may seem somewhat of a stretch, in that the surface streamline takes two sharp corners and is in contact with a solid surface. However, if we assume that the losses are small, and the flow is steady, then along the surface streamline

$$\tfrac{1}{2}V_1^2 + gY_1 \; = \; \tfrac{1}{2}V_2^2 + gY_2$$

Non-dimensionalizing by dividing through by gY_1, we get

$$\tfrac{1}{2}\frac{V_1^2}{gY_1} + 1 \; = \; \tfrac{1}{2}\frac{V_2^2}{gY_2}\frac{Y_2}{Y_1} + \frac{Y_2}{Y_1}$$

That is,

$$\tfrac{1}{2}F_1^2 + 1 \; = \; \tfrac{1}{2}F_2^2\frac{Y_2}{Y_1} + \frac{Y_2}{Y_1} \; = \; \frac{Y_2}{Y_1}\left(\tfrac{1}{2}F_2^2 + 1\right)$$

Eliminating F_2 by using mass conservation (equation 10.1) gives

$$\tfrac{1}{2}F_1^2 + 1 \;=\; \frac{Y_2}{Y_1}\left[\tfrac{1}{2}F_1^2\left(\frac{Y_1}{Y_2}\right)^3 + 1\right]$$

and by collecting terms

$$F_1^2 \;=\; \frac{2 - 2\frac{Y_2}{Y_1}}{\left(\frac{Y_1}{Y_2}\right)^2 - 1}$$

Since $Y_2/Y_1 = 0.5$,

$$F_1 = \frac{1}{\sqrt{3}}$$

In addition, from equation 10.1

$$F_2 \;=\; F_1\left(\frac{Y_1}{Y_2}\right)^{3/2} \;=\; \sqrt{\frac{8}{3}}$$

so that the upstream Froude number is subcritical, and the downstream Froude number is supercritical.

Example 10.2: Force on a sluice gate

For the gate shown in Figure 10.1, find the force required to hold the gate fixed. Express the result nondimensionally.

Solution: The force R required to hold the gate fixed is shown in the figure as acting in the negative x-direction. Therefore the force exerted by the fluid on the gate is R, and the force acting on the fluid is $-R$. We also have the force due to differences in hydrostatic pressure, similar to that considered in the analysis of the hydraulic jump (Section 10.7). Therefore, the x-component momentum equation becomes, for one-dimensional flow,

$$-R + \frac{\rho g Y_1^2 W}{2} - \frac{\rho g Y_2^2 W}{2} \;=\; -\rho V_1^2 Y_1 W + \rho V_2^2 Y_2 W$$

where friction has been ignored. By dividing through by $\rho V_1^2 Y_1 W$, and by substituting for V_2 from the continuity equation, we obtain

$$-\frac{R}{\rho V_1^2 Y_1 W} + \frac{g Y_1^2}{2V_1^2 Y_1} - \frac{g Y_2^2}{2V_1^2 Y_1} \;=\; -1 + \frac{V_2^2 Y_2}{V_1^2 Y_1} \;=\; \frac{V_1^2 Y_1^2}{V_1^2 Y_1 Y_2}$$

Hence

$$\frac{R}{\frac{1}{2}\rho V_1^2 Y_1 W} = \frac{1}{F_1^2}\left(1 - \frac{Y_2^2}{Y_1^2}\right) + 2\left(1 - \frac{Y_1}{Y_2}\right)$$

Example 10.3: Flow over a bump

Water flows from left to right in an open channel of constant width, as shown in Figure 10.2. The flow becomes supercritical as it passes over a bump of height H. It remains supercritical for some distance downstream, reaching a maximum Froude number of 1.83 at station 3. It then becomes subcritical by means of a hydraulic jump.
(a) Find the nondimensional water depth at the throat (Y_2/Y_1).
(b) Find the nondimensional height of the bump ($= H/Y_1$).
(c) Find the nondimensional water depth before the hydraulic jump ($= Y_3/Y_1$).
(d) Find the nondimensional water depth after the hydraulic jump ($= Y_4/Y_1$).

Solution: For part (a), we use mass conservation:

$$V_1 Y_1 = V_2 Y_2$$

Non-dimensionalizing, as we have done a number of times, we obtain

$$F_1^2 = F_2^2 \left(\frac{Y_2}{Y_1}\right)^3 \tag{10.2}$$

We are given that $F_1 = 0.5$ and $F_2 = 1$, so that

$$\frac{Y_2}{Y_1} = 0.63$$

For part (b), Bernoulli's equation can be used along the surface streamline in the region where there are no losses. Between stations 1 and 2, therefore

$$\tfrac{1}{2}V_1^2 + gY_1 \;=\; \tfrac{1}{2}V_2^2 + g\left(Y_2 + H\right)$$

Non-dimensionalizing by dividing through by gY_1, we obtain

$$\tfrac{1}{2}F_1^2 + 1 \;=\; \tfrac{1}{2}F_2^2 \frac{Y_2}{Y_1} + \frac{Y_2}{Y_1} + \frac{H}{Y_1}$$

Hence

$$1.125 \;=\; \frac{3}{2}\frac{Y_2}{Y_1} + \frac{H}{Y_1}$$

and

$$\frac{H}{Y_1} \;=\; 0.180$$

For part (c) we can use mass conservation again, this time between stations 1 and 3:

$$F_1^2 = F_3^2 \left(\frac{Y_3}{Y_1}\right)^3$$

Since $F_1 = 0.5$ and $F_3 = 1.83$,

$$\frac{Y_3}{Y_1} = 0.421$$

For part (d), we use the hydraulic jump relationship (equation 10.9), so that

$$\frac{Y_4}{Y_3} \;=\; \frac{1}{2}\left(\sqrt{1 + 8F_3^2} - 1\right) \;=\; 2.136$$

S Hence

$$\frac{Y_4}{Y_1} \;=\; \frac{Y_4}{Y_3}\frac{Y_3}{Y_1} = 2.136 \times 0.421 \;=\; 0.899$$

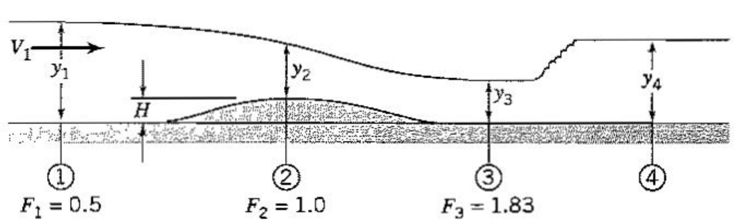

Figure 10.2: Flow over a bump.

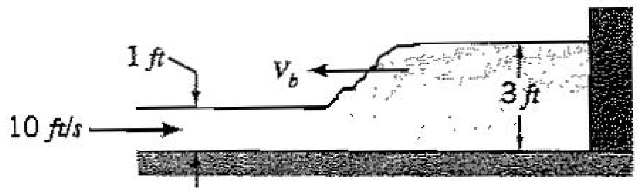

Figure 10.3: Bore traveling upstream in a river.

Example 10.4: Moving hydraulic jump

Water flows in a rectangular channel at a depth of $1\,ft$ and a velocity of $10\,ft/s$. When a gate is suddenly placed across the end of the channel, blocking the entire flow, a bore travels upstream with velocity V_b, as indicated in Figure 10.3. Find V_b when the depth of the water behind the bore is $3\,ft$.

Solution: To use the hydraulic jump relationship (equation 10.9), we need to move in a frame of reference where the flow is steady. This is accomplished by moving with the bore. Relative to the bore, the incoming water velocity is $V_b + 10\,ft/s$, and the incoming Froude number F_1 is given by

$$F_1 = \frac{V_b + 10}{\sqrt{32.2 \times 1}}$$

where V_b is in ft/s. From the hydraulic jump relationship (equation 10.9), we obtain

$$\frac{Y_2}{Y_1} = \frac{1}{2}\left(\sqrt{1 + 8F_1^2} - 1\right)$$

so that

$$8F_1^2 = \left(\frac{2Y_2}{Y_1} + 1\right)^2$$

Hence

$$8\,\frac{(V_b + 10)}{\sqrt{32.2}} = 48$$

and

$$V_b = 3.90\,ft/s$$

Problems

10.1 Calculate the Froude angle for a flow of depth $0.102\,m$ moving at $2\,m/s$.

10.2 Determine the minimum depth in a $3\,m$ wide rectangular channel if the flow is to be subcritical with a flowrate of $30\,m^3/s$.

10.3 Estimate the speed of a tsunami as it travels across the Pacific Ocean, given that the average depth of the water is about $2000\,m$. State your assumptions.

10.4 Consider a small amplitude gravity wave moving at a speed c_m from left to right into a shallow basin of depth y, as shown in Figure P10.4. The water in this basin is moving from right to left at speed U.
(a) Find the wave speed c_m in terms of y and U, stating all your approximations clearly.
(b) What happens when the Froude number based on y and U equals one?

10.5 To find the drift velocity δv for a small amplitude gravity wave, the linear analysis given in Section 10.2 is not sufficient. By repeating the same analysis without linearization, show that

$$\delta v \approx \frac{2\left(c^2 - gy\right)}{3c}$$

10.6 As an open channel flow enters a smooth constriction, the water level is observed to fall. What can you say about the upstream Froude number?

10.7 Water flows in a channel and passes over a bump on the channel floor. The water level is seen to decrease and stay low. What can you say about the upstream and downstream Froude numbers?

10.8 Consider the one-dimensional open channel flow shown in Figure P10.8.
(a) Using the continuity principle and Bernoulli's equation, show that

$$\frac{dy}{dx} = \frac{1}{F^2 - 1}\frac{dh}{dx}$$

where F is the local Froude number.
(b) Discuss the implications of this result for subcritical flow everywhere, supercritical flow everywhere, and for $F = 1$ at station 2.

10.9 A rectangular channel has a contraction that smoothly changes the width to a minimum width of $1.5\,ft$ (the throat). If the flow is critical at the throat, find the volume flow rate when the depth at the throat is $1.5\,ft$.

10.10 Water in an open channel of constant width flows over a bump of height $1\,ft$, as shown in Figure P10.10. What is the depth of the water Y_2? Assume uniform flow of constant width with no losses.

10.11 Water flows smoothly over a small bump in a channel of constant width, as shown in Figure P10.11. At any cross-section the velocity may be considered constant over the entire area. If V_1 is the velocity at entry, where the depth is Y_1, and V_2 is the velocity where the bump has its highest point, where the depth is Y_2. If $Y_1/Y_2 = 1.8$, find
(a) The Froude number of the flow at entry.
(b) The Froude number at the top of the bump.

10.12 Water in a two-dimensional channel flows smoothly over a submerged obstacle of height H as shown in Figure P10.12. The water depth at the peak of the obstacle is Y_2,

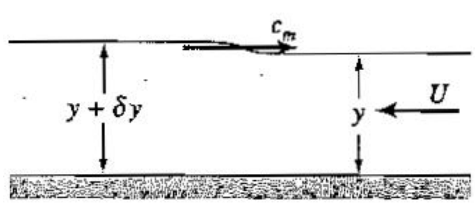

Figure P10.4

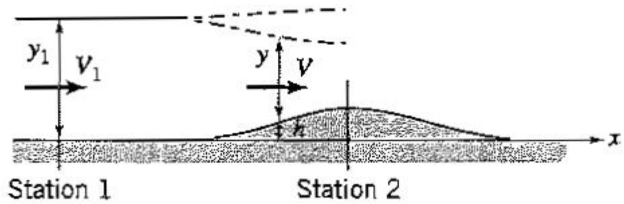

Figure P10.8

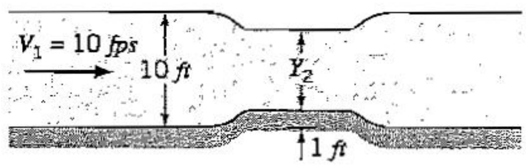

Figure P10.10

where $Y_2 = Y_1/3$.
(a) Find the value of the Froude number of the incoming flow, F_1, given that the Froude number is unity at the point where the depth is Y_2.
(b) Find the nondimensional height of the obstacle, H/Y_1.

10.13 Water in a two-dimensional channel flows over a bump, as shown in Figure P10.13. If $H_2/H_1 = 1/4$, find the Froude numbers at entry and exit, F_1 and F_2.

10.14 For the flow shown in Figure P10.14,
(a) Find the Froude number at station 1, where the water exits the tank.
(b) Find the water depth at station 2 in terms of h_1 and h_2, given that the Froude number at station 2 is one.

10.15 A smooth transition section connects two open channels of the same width, as shown in Figure P10.15. The water depth decreases so that the ratio of the downstream to upstream depths $Y_2/Y_1 = 0.5$. If the upstream Froude number $F_1 = 0.35$, determine the downstream Froude number F_2, and the ratio h/Y_1.

10.16 An exit of width W allows water to flow from a large tank into an open channel of the same width, as shown in Figure P10.16. The depth of water in the tank is maintained constant at H, which is large compared to h_1, where h_1 is the depth of the water at the exit. As the water flows smoothly over a bump of height b, the depth increases so that the ratio $h_3/h_1 = 4$. There are no losses anywhere. Show that the Froude number F_1 is supercritical. Use Bernoulli's equation and continuity to find the numerical value of F_1 and the ratio b/h_1.

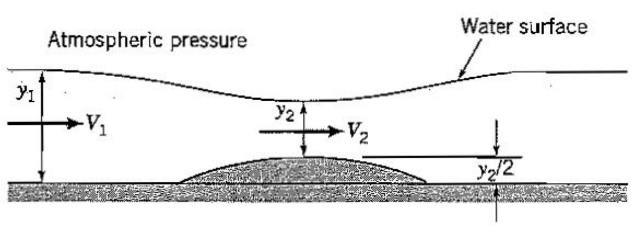

Figure P10.11

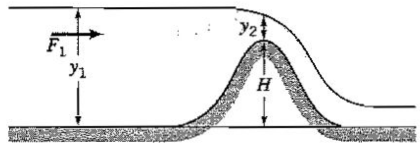

Figure P10.12

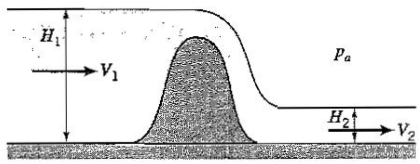

Figure P10.13

10.17 A smooth transition section connects two rectangular channels, as shown in Figure P10.17. The channel width increases from B_1 to B_2 and the water surface elevation is the same in each channel. If the upstream depth of flow is H_1, determine h, the amount the channel bed needs to be raised across the transition section to maintain the same surface elevation.

10.18 Water of density ρ flows in an open channel of constant width w with an initial depth of H_1. At some point, the bottom rises smoothly to a height H_3 and the depth of the water decreases to H_2, as shown in Figure P10.18. If the upstream Froude number $F_1 = 0.4$, and the downstream Froude number $F_2 = 1.0$:
(a) Find H_2/H_1 (this is a number).
(b) Find H_3/H_1 (this is a number).
(c) Find the force F_x exerted by the water on the step in the horizontal direction. Express the result in terms of the non-dimensional force coefficient using ρ, g, w, and H_2 (this is a number).
Show all your working, and state all your assumptions. Viscous forces can be neglected.

10.19 Water flows steadily with no losses in an open-channel flow that contracts in width, as shown in Figure P10.19. The velocity of the water at the upstream station is V_1, the Froude number is F_1, the width of the channel is W_1, and the depth is h_1. At the point of minimum cross-sectional area (the throat), the velocity is V_2, the Froude number $F_2 = 1$, the width of the channel is W_2, and the depth $h_2 = 3h_1/4$. Assume one-dimensional flow.
(a) Find F_1 (this is a number).
(b) Find W_2/W_1 (this is a number).
(c) Find the horizontal force exerted by the fluid on the contracting part of the channel in terms of the non-dimensional force coefficient $C_D = F/(\rho g W_1 h_1^2)$ (this is a number).

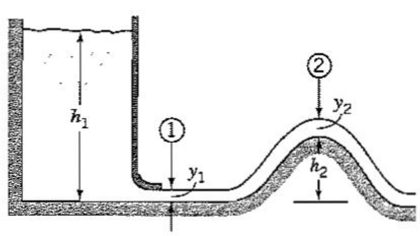

Figure P10.14

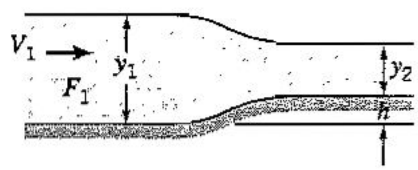

Figure P10.15

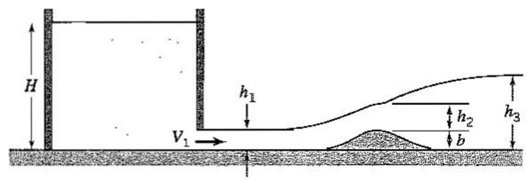

Figure P10.16

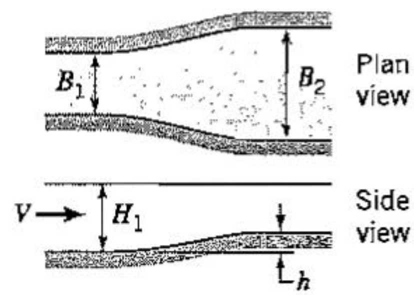

Figure P10.17

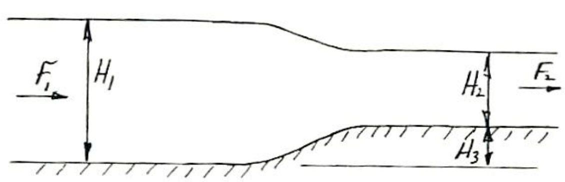

Figure P10.18

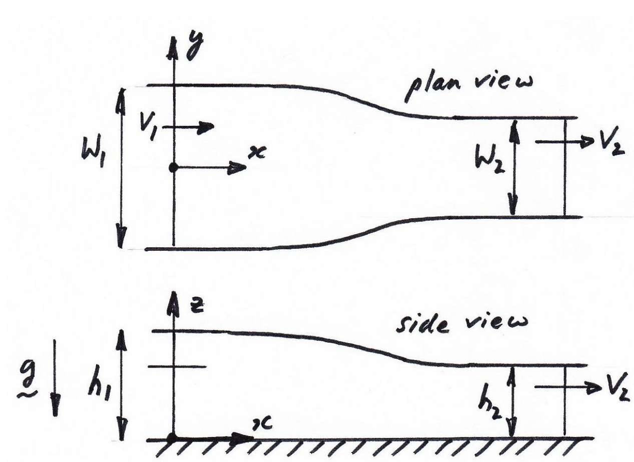

Figure P10.19

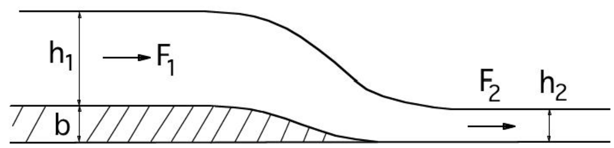

Figure P10.20

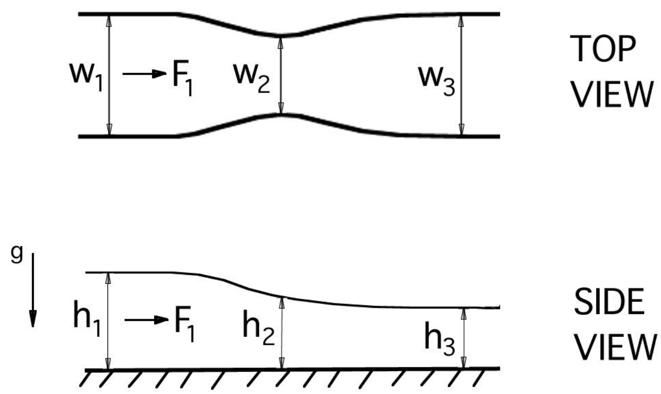

Figure P10.21

10.20 Water flows steadily and smoothly down a ramp of height b in an open-channel flow of constant width, as shown in Figure P10.20. The Froude number at the upstream station is subcritical and equal to F_1, and the depth at this station is h_1. Downstream, the Froude number is F_2, and it is found by experiment that surface waves make an angle of 30° to the flow direction. State all your assumptions.
(a) Find F_2 (this is a number).
(b) Find F_1 if $h_1/h_2 = 2$ (this is a number).
(c) Find b/h_1 (this is a number).

10.21 Water flows steadily and smoothly through an open-channel flow of varying width, as shown in Figure P10.21. The Froude number at the upstream station is subcritical and equal to F_1, and the width and depth at this station are w_1 and h_1, respectively. At the point of minimum channel width (the "throat"), the Froude number $F_2 = 1$, and the width and depth are w_2 and h_2. Downstream of the throat the channel expands to a constant width, and at this point the Froude number is F_3 and the width and depth of the flow are w_3 and h_3. Note that $h_2/h_1 = 3/4$, and $w_3 = w_1$. State all your assumptions.
(a) Find F_1 (this is a number).
(b) Find w_2/w_1 (this is a number).
(c) Show that h_1/h_3 is approximately equal to 2.37.

10.22 A smooth transition section connects two open channels of the same width, as shown in Figure P10.22. The water depth decreases so that the ratio of the downstream to upstream depths $Y_2/Y_1 = 1/4$. The upstream Froude number $F_1 = 0.3$.
(a) Determine the downstream Froude number F_2, and the ratio h/Y_1.

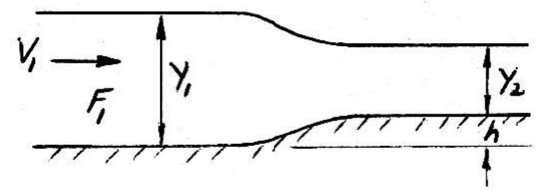

Figure P10.22

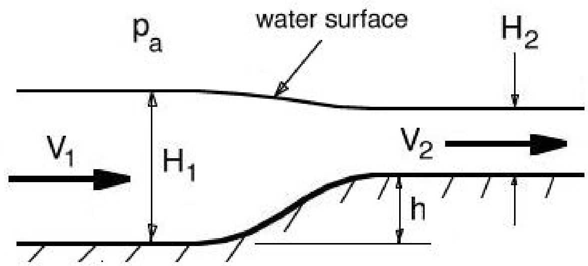

Figure P10.23

(b) Find the drag coefficient $F/(\rho V_1^2 w Y_1)$ where F is the force exerted by the fluid on the bump (this is a number).

10.23 Water flows steadily over a bump of height h, as shown in Figure P10.23. The width of the channel is constant. At position 1, the depth of the water is H_1 and its velocity is V_1, and at position 2, the depth of the water is H_2 and its velocity is V_2. The Froude number at position 1 is $F_1 = 0.2$.
(a) Find the Froude number at position 2 if $H_1/H_2 = 2$ (this is a number).
(b) Find the h/H_1 (this is a number).

10.24 A river of width w flows over a large rock of height h, as shown in Figure P10.24.
(a) By examining the water profile, is the flow upstream of the rock subcritical or supercritical?
(b) Find the Froude number upstream of the rock, given that $H_2/H_1 = 1.5$, assuming that there are no losses, and that the Froude number at the top of the rock $F_2 = 1$.
(c) Find the non-dimensional height of the rock h/H_1.
(d) Find the drag coefficient $F_D/\rho g w H_1^2$, where F_D is the drag force acting on the rock. Neglect friction on the river bottom.

10.25 Consider the steady, frictionless open channel flow of water through the constriction, as shown in Figure P10.25. At position 1, the depth is Y_1, the velocity is V_1 and the Froude number is $F_1 = V_1/\sqrt{gY_1}$. A similar notation is used at position 2 (the throat), and

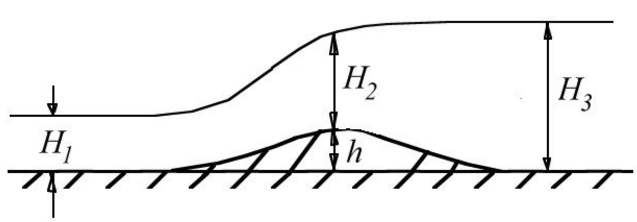

Figure P10.24

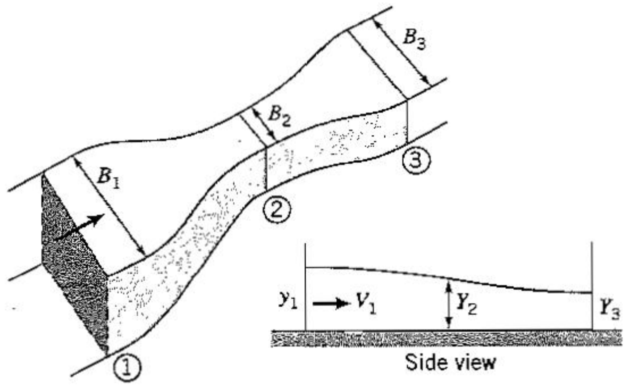

Figure P10.25

positions 3 and 4. You may assume one-dimensional flow.
(a) If $Y_1 > Y_2 > Y_3$, is F_1 supercritical, or subcritical? Also, is F_3 supercritical, or subcritical?
(b) If $Y_1 < Y_2$, and $Y_3 < Y_2$, is F_1 supercritical, or subcritical? Also, is F_3 supercritical, or subcritical?
(c) Given that $Y_2/Y_I = 0.75$, and that $Y_3 < Y_2$, find F_1, and B_2/B_1 (these answers are numbers).
(d) What is the smallest value of Y_2/Y_1 that can be achieved?

10.26 By combining the momentum and continuity equations (equations 10.6 and 10.7), obtain equation 10.8 where F_1 is the upstream Froude number, that is, $F_1 = U_1/\sqrt{gH_1}$.

10.27 By using the definition of the Bernoulli constant (equation 8.34), the continuity equation (equation 10.10), and the hydraulic jump relationship (equation 10.9), obtain equation 10.12 for a two-dimensional channel flow.

10.28 The water depth upstream of a stationary hydraulic jump is $1\,m$, while the depth after the jump is $2\,m$. Find the upstream and downstream velocities and Froude numbers.

10.29 The depth of water in a rectangular channel is $1.5\,ft$. The channel is $6\,ft$ wide and carries a volume flow rate of $200\,ft^3/s$. Find the water depth after a hydraulic jump.

10.30 Draw and label a hydraulic jump (in a channel of constant width).
(a) Indicate regions of supercritical and subcritical flow.
(b) What happens to Bernoulli's constant through the jump?
(c) Write down the relationship between the ratio of upstream and downstream water depths, and the upstream Froude number.
(d) Derive a relationship between the ratio of upstream and downstream water depths, and the downstream Froude number.
(e) A surge is moving at $5\,m/s$ into an estuary of depth $1\,m$ where the water is moving out to sea at $1\,m/s$. Find the water depth behind the surge.

10.31 A surge is moving at $10\,ft/s$ into a depth $2\,ft$ where the speed is zero. Find the water depth behind the surge.

10.32 A tidal bore is moving at $5\,m/s$ into a stagnant basin of depth $1\,m$. What is the depth of water behind the bore?

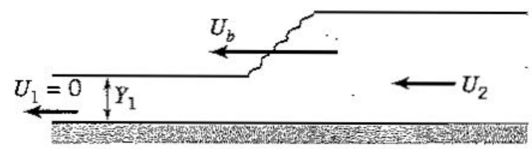

Figure P10.34

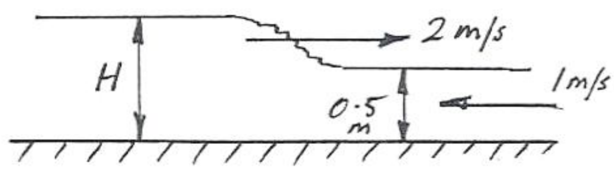

Figure P10.38

10.33 Consider a bore traveling at $6\,m/s$ into tidal basin of depth $1.5\,m$. Calculate the depth of the water behind the bore.

10.34 A bore is moving with velocity U_b into a stagnant tidal channel of depth Y_1, as shown in Figure P10.34. When $U_b = \sqrt{3gY_1}$, find U_2/U_b.

10.35 A surge is moving at $9\,ft/s$ into a basin of depth $1.4\,ft$ where the speed is zero. Find the water depth behind the surge and the drift velocity behind the surge (that is, the velocity downstream of the surge in a stationary frame of reference).

10.36 A river of depth $2\,m$ flows downstream at $1\,m/s$. A surge of depth H moves up the river at $5\,m/s$ relative to a stationary observer on the bank of the river. Find H.

10.37 A tidal bore is moving upstream into a river of constant width and of depth 1.5 m. The river is flowing at a speed of 2 m/s. Find the speed of the bore relative to a stationary observer if the height of the water downstream of the bore is 3 m.

10.38 A planar bore moves at a speed of $2\,m/s$ against a flow that has a speed of $1\,m/s$ and a depth of $0.5\,m$, as shown in Figure P10.38. Find H, the depth of the water behind the bore.

10.39 A surge of depth H moves into a stagnant tidal pool of depth $1\,m$ at a speed of $6\,m/s$ relative to a stationary observer on the bank of the pool. Find H, and then find the drift velocity behind the surge, relative to the stationary observer.

10.40 A surge is observed to enter a tidal channel that is moving at $1\,m/s$ against the surge. If the depths of the water upstream and downstream of the surge are $1\,m$ and $2\,m$, respectively, what is the speed of the surge relative to a stationary observer on the bank of the channel?

10.41 A tidal bore is moving upstream into a river of depth $1\,m$, and constant width, and the bore raises the water level by $1\,m$. The river is flowing downstream at a steady velocity of $2\,m/s$.

(i) Find the velocity of the bore relative to a stationary observer.

(ii) Find the velocity of the water downstream of the bore relative to a stationary observer.

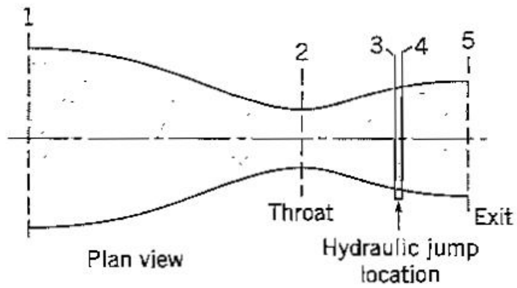

Figure P10.45

10.42 A bore (a moving hydraulic jump) travels into a stagnant basin where the depth is 1 m. The depth of the water behind the bore is 2 m.
(a) Find the speed of the bore relative to a stationary observer.
(b) Find the speed of the water behind the bore relative to a stationary observer.

10.43 A bore moving at $10\,m/s$ moves upstream into a river, which is moving downstream at $2\,m/s$. The river has a constant width and a depth of $2\,m$. Find the speed of the river flow downstream of the bore.

10.44 Show that the head loss in a hydraulic jump can be expressed as:

$$\text{head loss} \;=\; \frac{(Y_2 - Y_1)^3}{4Y_1Y_2}$$

where Y_1 and Y_2 are the water depths upstream and downstream of the jump, respectively. The head loss is the difference between the upstream and downstream values of the Bernoulli constant, and it has the dimensions of length.

10.45 Consider the steady open channel flow of water in the smooth constriction shown in the plan view in Figure P10.45. At position 1, the depth is Y_1 and the width is B_1. You may assume that B_1 is much larger than either B_2 or B_3, the widths of the channel at positions 2 and 3, respectively. A hydraulic jump is observed between positions 3 and 4 where the upstream depth $Y_3 = Y_1/3$. You may assume one-dimensional flow, and that $F_1 < 1$.
(a) Find Y_2, the depth at position 2, in terms of Y_1.
(b) Find F_3, the Froude number at position 3.
(c) Find Y_4, the depth just downstream of the hydraulic jump, in terms of Y_1.
(d) Find F_4, the Froude number just downstream of the hydraulic jump.
(e) Find F_5, the Froude number at the exit, given that $Y_5 = 1.1Y_4$.

10.46 Consider the steady open channel flow of water in the smooth constriction shown in plan view in Figure P10.46. At position 1, the depth is Y_1, the width is B_1 and the Froude number $F_1 = 4$. A similar notation is used at positions 2, 3 and 4. A hydraulic jump is observed just downstream of position 2 where the Froude number $F_2 = 2$. You may assume one-dimensional flow.
(a) Find Y_2, the depth at position 2, in terms of Y_1.
(b) Find Y_3, the depth just downstream of the hydraulic jump, in terms of Y_1.
(c) Find F_3, the Froude number just downstream of the hydraulic jump.
(d) Find F_4, the Froude number at the exit, given that $Y_3/Y_4 = 1.2$.
(e) Find B_4/B_1.

10.47 A two-dimensional channel flow flows smoothly over a small bump, as shown in

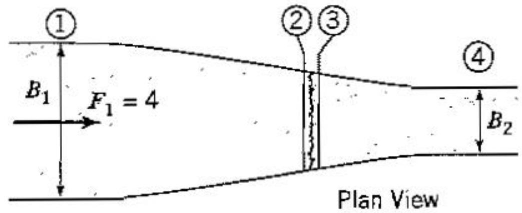

Figure P10.46

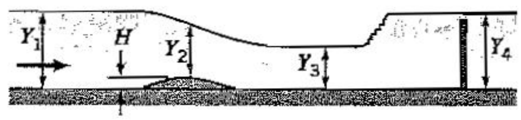

Figure P10.47

Figure P10.47.
(a) If the Froude number at station 2 equals unity ($F_2 = 1$), find Y_2/Y_1 in terms of F_1 and H/Y_1.
(b) If $Y_3/Y_1 = 0.5$, find F_3 in terms of F_1.
(c) If $F_3 = \sqrt{3}$, find Y_4/Y_1 such that a hydraulic jump is formed in front of the barrier.

10.48 Water flows steadily and smoothly over a bump of height b in an open-channel flow of constant width, as shown in Figure P10.48. The Froude number at the upstream station is F_1, and the depth is h_1. The point of minimum depth (the throat) occurs at the crest of the bump, where the Froude number is $F_2 = 1$, and the depth $h_2 = b = h_1/2$. A stationary hydraulic jump is located between points 3 and 4. Assume one-dimensional flow.
(a) Find F_1 (this is a number).
(b) Derive an equation for h_1/h_3 in terms of F_1. Do not attempt to solve.
(c) Find h_4/h_1 (this is a number).

10.49 A circular jet of water impinges on a flat plate as shown in Figure P10.49. The water spreads out equally in all directions and at station 1, the flow is essentially uniform over the depth Y_1. The jet exit velocity V is $10\,m/s$ and the exit diameter D is $10\,mm$. If you are given that $R = 8D$, and $Y_1 = D/4$
(a) Find the Froude number of the flow at station 1.
(b) If you are given that a circular hydraulic jump forms near station 1, estimate the depth of water after the hydraulic jump using the one-dimensional hydraulic jump relationship. Under what conditions will the accuracy of this estimate improve?

10.50 Consider the steady flow of water under the sluice gate shown in Figure P10.50. The velocity of the flow at sections 1, 2 and 3 is independent of depth, and the streamlines

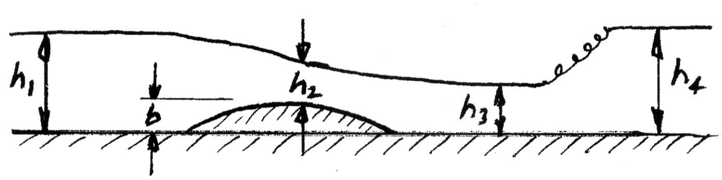

Figure P10.48

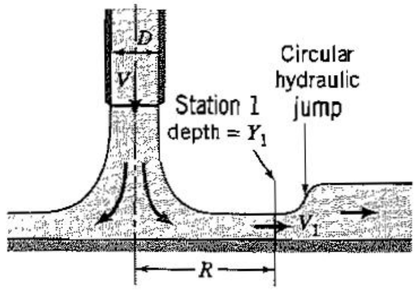

Figure P10.49

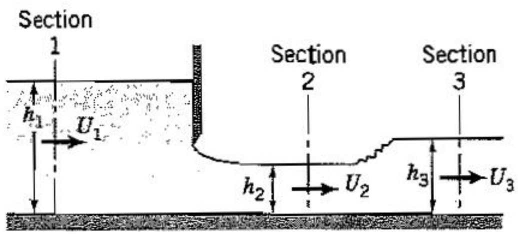

Figure P10.50

are parallel.
(a) Show that for $h_2 < h_1$ the flow at section 2 is always supercritical. Assume the flow between sections 1 and 2 occurs without loss.
(b) Between sections 2 and 3 a stationary hydraulic jump is formed. If $h_2/h_1 = 0.5$, find h_3/h_2.

10.51 Water flows in an open channel of constant width as shown in Figure P10.51. The upstream Froude number $F_1 = 0.5$. At the point where the water flows over a bump of height H, the Froude number equals one.
(a) Find the depth ratio Y_2/Y_1.
(b) Find the ratio H/Y_1.
(c) Given that a hydraulic jump occurs downstream of the bump such that $Y_4/Y_3 = 8$, find Y_3/Y_1.

10.52 A smooth transition section connects two rectangular channels. In the direction of flow, the channel width increases from B_1 to B_2 and the water surface elevation decreases so that the ratio of the downstream to upstream depths $Y_2/Y_1 = 0.5$.
(a) If the upstream Froude number $F_1 = 1.5$, find the downstream Froude number F_2, and the ratio B_2/B_1.
(b) If a stationary hydraulic jump occurs in the channel downstream of the expansion, determine Y_3, the depth downstream of the jump, in terms of Y_1.

10.53 Consider the steady open channel flow of water in the smooth constriction shown in

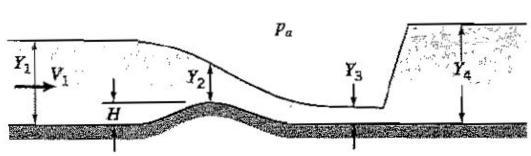

Figure P10.51

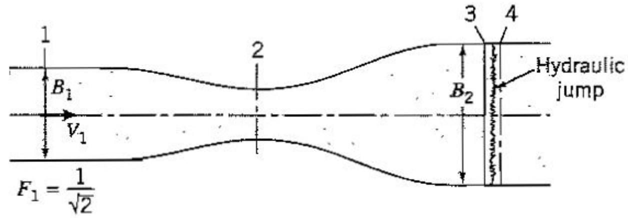

Figure P10.53

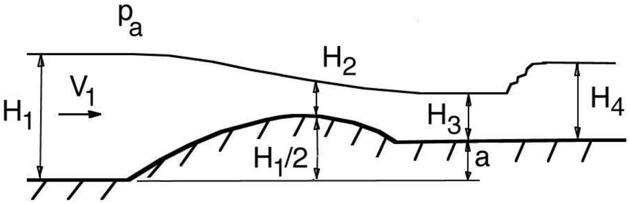

Figure P10.54

plan view in Figure P10.53. At position 1, the depth is Y_1, the width is B_1 and the Froude number is $F_1 = 1/\sqrt{2}$. A similar notation is used at positions 2, 3 and 4. A hydraulic jump is observed just downstream of position 3 where the depth $Y_3 = Y_1/4$. You may assume one-dimensional flow.
(a) Find F_3, the Froude number at position 3 (this is a number).
(b) Find B_3, the width of the channel at position 3 in terms of B_1.
(c) Find Y_4, the depth of the water at position 4, downstream of the hydraulic jump, in terms of Y_1.

10.54 Water flows steadily and smoothly over a bump in an open-channel flow of constant width, as shown in Figure P10.54. The Froude upstream of the bump is F_1, and at the point of minimum channel depth the Froude number $F_2 = 1$. Downstream of the bump the channel bottom has risen by an amount a, and the water has a depth of H_3. Further downstream, there is a stationary hydraulic jump where the downstream water depth is H_4. If $H_1 = 2H_2$ and $F_3 = 2$,
(a) Find F_1 (this is a number).
(b) Find H_3/H_1 (this is a number).
(c) Find a/H_1 (this is a number).
(d) Find H_4/H_1 (this is a number).

10.55 An oil boom of width w is placed in a river to catch oil drops on the surface of water of density ρ flowing in a shallow river, as shown in Figure P10.55.
(a) Find the Froude number upstream of the boom, given that $H_1/H_2 = 2$, assuming that there are no losses.
(b) Find the Froude number downstream of the boom.
(c) Find the drag coefficient $F_D/\rho g w H_1^2$, where F_D is the drag force acting on the boom. Neglect friction on the river bottom.
(d) Find the Froude number downstream of the hydraulic jump.

10.56 Water flows steadily in a smooth constriction, as shown in plan view in Figure P10.56. At position 1, where the depth of the water is Y_1 and the width of the channel is B_1, the Froude number is 0.5. A hydraulic jump is located between positions 3 and 4

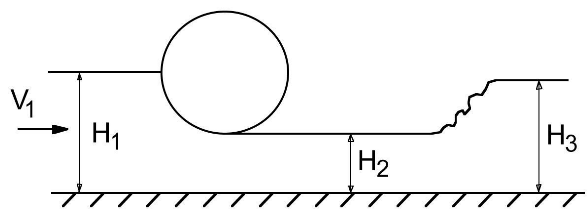

Figure P10.55

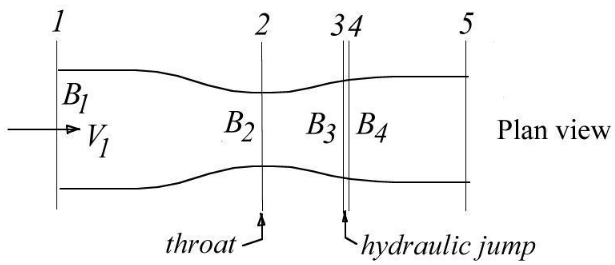

Figure P10.56

where the upstream depth is Y_3. Assume one-dimensional flow. Show all your working.
(a) Find the ratio of depths Y_2/Y_1, where Y_2 is the depth of the water at the throat.
(b) Find the ratio of channel widths B_2/B_1, where B_2 is the width of the channel at the throat.
(c) Find the Froude number F_3 just upstream of the hydraulic jump, where $Y_3/Y_1 = 0.375$.
(d) Find the Froude number F_4 just downstream of the hydraulic jump.

10.57 Water issues from a large reservoir in the form of a jet of width W and depth Y_1, as shown in Figure P10.57. The cross-sectional area of the tank A is much larger than the jet area WY_1, and the depth H is much larger than Y_1.
(a) Calculate the exit Froude number F_1, and show that $F_1 > 1$.
(b) Show that Y_2, the depth at the crest of the bump of height h, is given by the solution to

$$\left(\frac{Y_2}{Y_1}\right)^3 - \left(1 + \frac{F_1^2}{2} - \frac{h}{Y_1}\right)\left(\frac{Y_2}{Y_1}\right)^2 + \frac{F_1^2}{2} = 0$$

Carefully note all your assumptions.

10.58 Consider water flowing without friction through a contraction in an open channel.

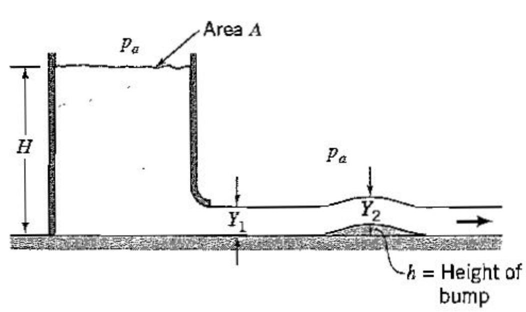

Figure P10.57

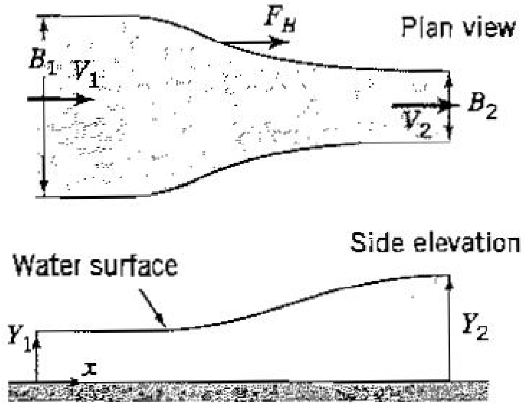

Figure P10.58

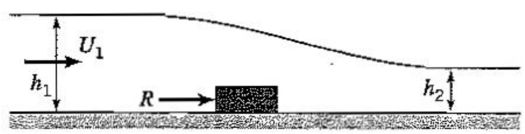

Figure P10.60

The channel and the surface profile are shown in Figure P10.58.
(a) Is the Froude number at station 1 subcritical or supercritical?
(b) Given that $Y_2/Y_1 = 2$, and $B_2/B_1 = 0.6$, find the Froude number at station 2 (this is a number).
(c) If there was a hydraulic jump downstream of station 2, what would be the height of the jump in terms of Y_1?

10.59 For the flow described in the previous problem, find the force F_H exerted by the water on the flume. Use the continuity equation and the momentum equation to find the nondimensional force coefficient $F_H/(\rho g B_1 Y_1^2)$ in terms of the incoming Froude number and the length ratios Y_2/Y_1, and B_2/B_1.

10.60 An open channel flow of constant width W flows over a small obstruction, as shown in Figure P10.60. Show that

$$\frac{R}{\rho U_1^2 W h_1} = \frac{h_1}{h_2} - 1 - \frac{1}{2F_1^2}\left(1 - \frac{h_2^2}{h_1^2}\right)$$

where R is the force exerted by the fluid on the obstruction, ρ is the density and F_1 is the Froude number of the incoming flow. Neglect viscous forces on the channel floor.

10.61 For the open-channel flow in the smooth contraction shown in plan view in Figure P10.61, the depth of the incoming flow is Y_1 and the depth of the exiting flow is Y_2. The flow attains the critical Froude number at the throat. If the upstream Froude number is much less than unity, show that $Y_2/Y_1 \approx 2/3$, and that

$$\frac{F_v}{\frac{1}{2}\rho g Y_1^2 B_1} = 1 - \frac{4}{3}\frac{B_2}{B_1}$$

where F_v is the force acting on the contraction walls, and B_1 and B_2 are the upstream and downstream channel widths, respectively. Ignore all viscous forces.

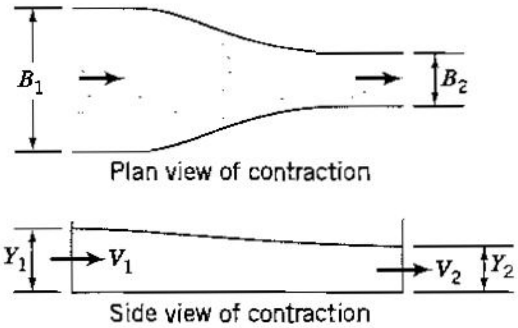

Figure P10.61

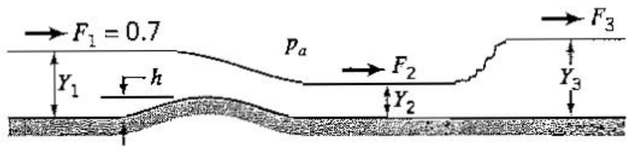

Figure P10.62

10.62 Water flows smoothly over a bump of height h in the bottom of a channel of constant width W, as shown in Figure P10.62. If the upstream depth is Y_1, the upstream Froude number F_1 is 0.7, and $Y_1/Y_2 = 2$, find
(a) The downstream Froude number F_2. Is it supercritical?
(b) The magnitude and direction of the horizontal force exerted by the fluid on the bump, in terms of g, W, h and Y_1.
(c) If the flow was now subcritical everywhere again find the force on the bump.
(d) If a hydraulic jump is located downstream of the bump (and F_1 is 0.7 again), calculate the Froude number of the flow downstream of the jump.

10.63 Water flows steadily in an open channel of width W, as shown in Figure P10.63. It passes smoothly over a bump of height h. Initially, the Froude number $F_1 = 0.5$, and the depth of the water is Y_1. The water depth decreases to a depth of Y_2 over the bump, and then continues to decrease downstream of the bump to a depth of Y_3.
(a) What is the Froude number Y_2?
(b) Use Bernoulli's equation and continuity to find the numerical value of Y_2/Y_1, and the numerical value of h/Y_1.
(c) If $Y_3/Y_1 = 0.422$, find the numerical value of the force coefficient $C_D = D/(\rho g W Y_1^2)$, where D is the horizontal force exerted by the fluid on the bump, ρ is the water density, and g is the gravitational acceleration. Ignore friction.

10.64 Consider the steady, smooth flow of water in an open channel of constant width W, as shown in Figure P10.64. A deflector plate causes the water to accelerate to a supercritical

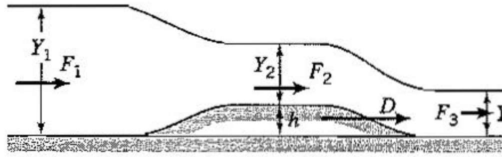

Figure P10.63

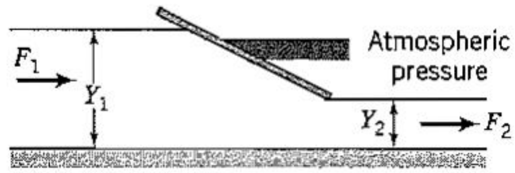

Figure P10.64

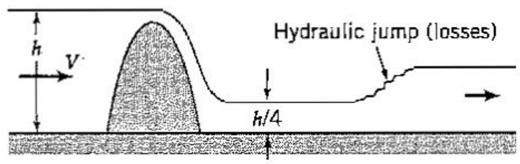

Figure P10.65

speed. At position 1 the depth is Y_1 and the Froude number $F_1 = 0.2$. At position 2 the depth is Y_2 and the Froude number $F_2 = 3.38$. Assume one-dimensional flow.
(a) Find the ratio Y_2/Y_1 two different ways (this is a number).
(b) Find the nondimensional ratio $2R/\rho g Y_1^2 W$, where R is the force exerted on the fluid by the deflector plate, in term of Y_2/Y_1, F_1 and F_2.

10.65 Water in an open-channel flow of constant width flows steadily over a dam as shown in Figure P10.65. The velocity of the water far upstream of the dam is V, the Froude number is F and the depth is h. Far downstream of the dam the depth is $h/4$.
(a) Find the horizontal force per unit width acting on the dam in terms of V, h, g and the density ρ. Ignore friction.
(b) There is a hydraulic jump located even further downstream. Find the depth of water downstream of the jump in terms of F and h.

10.66 Water in a two-dimensional open channel flows smoothly and steadily over a submerged obstacle of height H and width W as shown in Figure P10.66. The flow over the peak of the obstacle becomes critical at the point where the depth is Y_2, where $Y_2 = Y_1/3$.
(a) Find F_1, the Froude number of the incoming flow.
(b) Find H/Y_1, the nondimensional height of the obstacle.
(c) Find the resultant force acting on the obstacle in terms of ρ, g, W, and Y_1, given that $Y_3/Y_1 = 0.165$.

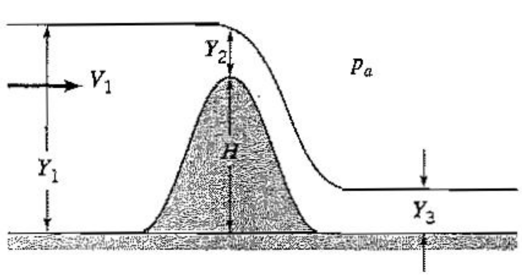

Figure P10.66

Chapter 11

Compressible Flow

11.1 Study Guide

- Weak pressure disturbances in a flowing fluid create a wave that makes an angle of α_M with the flow direction, where $\sin\alpha_M = 1/M$, where M is the Mach number .
- Define enthalpy and entropy in terms of other thermodynamic variables.
- Define C_p, C_v, γ, and express C_p and C_v in terms of R and γ for an ideal gas.
- What are the relationships between pressure an temperature, and pressure and density, for isentropic flow of an ideal gas?
- Write down the energy equation for one-dimensional, steady, adiabatic flow of a perfect gas in terms of the total temperature, T_0. What can you say about T_0?
- Study how isentropic flow behaves in converging and diverging flows at subsonic and supersonic flow (see Figure 11.8).
- What is choked flow?
- What happens to T_0 across a shock wave? What happens to temperature, density, pressure, entropy, total pressure?
- When is an oblique shock "strong" and when is it "weak"? Which is the usual case?
- What happens when the turning angle exceeds the maximum value ($\alpha > \alpha_{max}$) for a given Mach number? What is the Prandtl-Meyer function, and how is it used to find the flow properties in isentropic expansions and compressions?

11.2 Worked Examples

Example 11.1: Thermodynamic properties

When a fixed mass of air is heated from $20°C$ to $100°C$:
(a) What is the change in enthalpy?
(b) For a process at constant volume, what is the change in entropy?
(c) What is the change in entropy for a process at constant pressure?
(d) For an isentropic process, find the changes in density and pressure.
(e) Compare the isentropic speed of sound in air to its isothermal value. Assume that air behaves as a perfect gas.

Solution: For part (a), we have $C_p = 1004\, J/kg \cdot K$ (Table Appendix-C.10 and equation 11.11). From equation 11.7

$$h_2 - h_1 = C_p\,(T_2 - T_1) = 1004\,(100 - 20)\ J/kg = 80,320\, J/kg$$

For part (b), we use equation 11.16. Since the process is at constant volume, $\rho_2 = \rho_1$, and

$$s_2 - s_1 = C_v\ \ln\left(\frac{T_2}{T_1}\right) = \frac{C_p}{\gamma}\ \ln\left(\frac{T_2}{T_1}\right)$$

Therefore

$$s_2 - s_1 = \frac{1004}{1.4}\ \ln\left(\frac{100 + 273.15}{20 + 273.15}\right)\ J/kg \cdot K = 173\, J/kg \cdot K$$

For part (c), we use equation 11.17. Since the process is at constant pressure, $p_2 = p_1$, and

$$s_2 - s_1 = C_p\ \ln\frac{T_2}{T_1} = 1004\ \ln\left(\frac{100 + 273.15}{20 + 273.15}\right)\ J/kg \cdot K = 242.3\, J/kg \cdot K$$

For part (d), we use $20°C$ as the reference temperature in the isentropic relationships (equation 11.18). With $\gamma = 1.4$, we obtain

$$\frac{\rho_{100}}{\rho_{20}} = \left(\frac{T_{100}}{T_{20}}\right)^{2.5} = \left(\frac{100 + 273.15}{20 + 273.15}\right)^{2.5} = 1.828$$

and

$$\frac{p_{100}}{p_{20}} = \left(\frac{T_{100}}{T_{20}}\right)^{3.5} = \left(\frac{100 + 273.15}{20 + 273.15}\right)^{3.5} = 2.327$$

For part (e), the isentropic speed of sound is given by equation 11.20:

$$a_s = \sqrt{\gamma RT} = \sqrt{1.4 \times 287.03 \times (20 + 273.15)}\, m/s = 343.2\, m/s$$

The isothermal speed is given by

$$a = \sqrt{\left.\frac{\partial p}{\partial \rho}\right|_T}$$

For an ideal gas at constant temperature

$$\frac{p}{\rho} = RT = \text{ constant.}$$

By differentiating this relationship, we find that

$$\frac{dp}{p} - \frac{d\rho}{\rho} = 0$$

Therefore

$$\left.\frac{\partial p}{\partial \rho}\right|_T = a_T^2 = \frac{p}{\rho} = RT$$

That is,

$$a_T = \sqrt{RT} = \sqrt{287.03 \times (20 + 273.15)}\ m/s = 290.07\, m/s$$

When Newton attempted to compute the speed of sound, he wrongly assumed that the transmission of sound was an isothermal, rather than an isentropic phenomenon. We see that for air this error leads to an estimate that is about 18% too low.

Example 11.2: Normal shock relations

A normal shock is observed in an air flow at Mach 3 at $100^\circ K$. The density of the air is $0.8\,kg/m^3$. Find
(a) The stagnation temperature.
(b) The density, pressure, temperature, and Mach number downstream of the shock.
(c) The entropy rise across the shock.
To find the solutions, use the normal shock relations given here. You should then check the results using the compressible flow calculator available on the web at *http://www.engapplets.vt.edu/*.

Solution: For part (a), equation 11.25 (with $\gamma = 1.4$):

$$T_0 = 100 \times \left(1 + 0.2 \times 3^2\right) = 100 \times 2.8^\circ K = 280^\circ K$$

For part (b), we use equations 11.42, and 11.48 to 11.50. Hence,

$$M_2 = \frac{3^2 + 5}{7 \times 3^2 - 1} = 0.226$$

$$\rho_2 = \frac{0.8 \times 2.4 \times 3^2}{0.4 \times 3^2 + 2}\,kg/m^3 = 3.086\,kg/m^3$$

and

$$p_2 = \left]1 + \frac{2.8}{2.4}\left(3^2 - 1\right)\right]\,p_1 = 10.33 p_1$$

From the ideal gas law, $p_1 = 0.8 \times 287.03 \times 100\,Pa = 22,962\,Pa$, and so

$$p_2 = 237,202\,Pa$$

Finally,

$$T_2 = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} T_1 = 10.33 \times \frac{0.8}{3.086} \times 100^\circ K = 267.8^\circ K$$

For part (c), we use equations 11.51 and 11.52. Hence,

$$\frac{p_{02}}{p_{01}} = 10.33 \times \left(\frac{100}{267.8}\right)^{3.5} = 0.329$$

and

$$s_2 - s_1 = 287.03 \times \ln\left(\frac{1}{0.329}\right) J/kg \cdot K = 319.4\,J/kg \cdot K$$

Example 11.3: Unsteady shock motion

Consider a constant area duct with a normal shock wave moving through it (Figure 11.1). This situation occurs in a *shock tube* where two gases at different pressures are initially separated by a thin diaphragm. When the diaphragm is broken, the high pressure gas propagates into the low pressure gas and this pressure "wave" rapidly forms into a moving shock. This process is very similar to that which generates a moving hydraulic jump (see Section 10.9).

The pressure, temperature and velocity upstream of the shock are $p_1 = 75\,kPa$, $T_1 = 20^\circ C$, and $V_1 = 0\,m/s$. Downstream of the shock the pressure, temperature and velocity are $p_2 = 180\,kPa$, $T_2 = 97^\circ C$, and $V_2 = 280\,m/s$. Find the shock speed, and the Mach number of the upstream flow relative to an observer moving with the shock.

Solution: To a stationary observer, the flow is not steady: first, nothing would be happening, then a shock moves past, followed by a steady flow of gas. If we move with the shock,

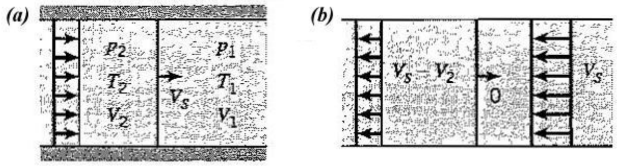

Figure 11.1: Moving normal shock in a constant area duct. (a) In a stationary frame of reference. (b) In a frame of reference moving with the shock.

however, the flow becomes steady. For a control volume moving with the shock, the steady one-dimensional continuity equation gives

$$-\rho_1 V_s A + (V_s - V_2)\, \rho_2 A \;=\; 0$$

where V_s is the speed of the shock relative to a stationary observer. Using the ideal gas law (equation 1.3)

$$-\frac{p_1}{p_2}\frac{T_2}{T_1} V_s + V_s - V_2 \;=\; 0$$

and so:

$$-\frac{80}{180}\frac{380}{293} V_s + V_s - 280 \;=\; 0$$

where V_s is in meters per second. Hence,

$$V_s = 661\, m/s$$

From Section 11.4.4, $a = \sqrt{\gamma R T}$, so that, relative to the moving shock, the upstream flow has a Mach number of

$$M \;=\; \frac{V_s}{\sqrt{\gamma R T_1}} \;=\; \frac{661}{\sqrt{1.4 \times 287.03293}} \;=\; 1.93$$

Example 11.4: Flow over a wedge

A supersonic airfoil with a symmetrical diamond-shaped cross-section is moving at Mach 3 in air at $100^\circ K$ (see Figure 11.2). Shocks form at the leading edge as the flow is turned over the front part of the airfoil, and also at the trailing edge where the flow is turned back in the freestream direction. At the shoulder, an expansion fan forms, and this will be discussed in Section 11.10. Here, we consider only the flow over the leading edge, which has a half-angle of 10° . The density of the air is $0.8\, kg/m^3$. Find
(a) the stagnation temperature.
(b) the angle the shock on the leading edge makes with the freestream (assuming the weak solution).
(c) the density, pressure, temperature, and Mach number downstream of the shock.
(d) the entropy rise across the shock.

Solution: For part (a), we use the definition of the total temperature (equation 11.25)

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

and we find that with $\gamma = 1.4$, $M = 3$ and $T = 100^\circ K$, the stagnation temperature $T_0 = 380^\circ K$.

For part (b) we find from Figure 11.15 that the shock angle for the weak solution is $\beta \approx 27^\circ$. A more accurate value can be found by iterating equation 11.64, but 27° is accurate enough for our purposes here.

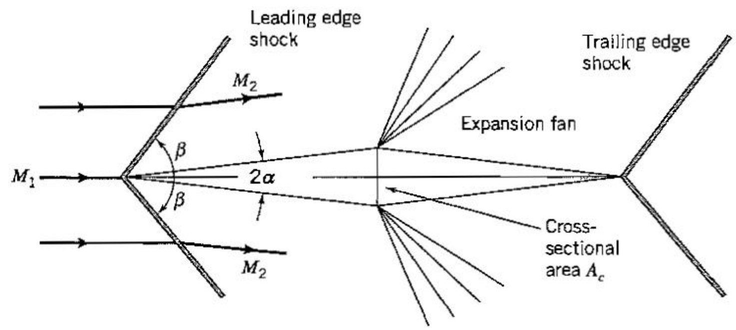

Figure 11.2: Diamond-shaped airfoil in a supersonic flow.

For part (c), we use equations 11.59 to 11.61, and the ideal gas law. With $M_1 = 3$ and $\beta = 27°$, $M_1^2 \sin^2 \beta = 1.855$, so that

1. Density: $\rho_2/\rho_1 = 1.624$, so that $\rho_2 = 1.299\, kg/m^3$.

2. Pressure: $p_2/p_1 = 1.997$. From the ideal gas law, $p{=}\rho_1 R T_1 = 0.8 \times 287.03 \times 100\, Pa = 22,962\, Pa$. Hence, $p_2 = 45,866\, Pa$.

3. Temperature: From the ideal gas law, $T_2 = p_2/(\rho_2 R) = 45,866/\ (287.03 \times 1.299)°K = 123°K$.

4. Mach number: From equation 11.61, we see that $M_2^2 \sin^2(\beta - \alpha) = 0.572$, so that $M_2 = 2.587$.

For part (d), we use equations 11.51 and 11.52. Hence, $p_{02}/p_{01} = 0.9676$, and $\Delta s = 9.444\, J/kg \cdot K = 9.444\, m^2/s^2 K$.

Example 11.5: Isentropic compression and expansion

In Example 11.4, we considered a diamond-shaped airfoil traveling in air flow at Mach 3 (see Figure 11.2). The deflection angle on the top and bottom of the wedge was 10°.
(a) If instead the compression was achieved by a series of Mach waves, so that the compression was isentropic, find the downstream Mach number and pressure.
(b) If instead of a compression, the flow experienced a 10° isentropic expansion, find the downstream Mach number and pressure.

Solution: For part (a) we find

(i) By interpolation from Table Appendix-C.19 $\nu_1 = 49.75°$. For an isentropic compression of 10°, $\nu_2 = 49.75° - 10° = 39.75°$. From the table, $M_2 = 2.527$ (compared to 2.587 for the same deflection by a single oblique shock).

(ii) Since the flow is isentropic, $p_{01} = p_{02}$, and equation 11.28 gives

$$\frac{p_2}{p_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}\right]^{\frac{\gamma}{\gamma-1}} \tag{11.1}$$

so that $p_2/p_1 = 2.062$ (compared to 1.997 for the same deflection by a single oblique shock).

For part (b) we find

(i) For an isentropic expansion of 10°, $\nu_2 = 49.75° + 10° = 59.75°$. From Table Appendix-C.19, $M_2 = 3.578$.

(ii) Equation 11.1 applies to an isentropic expansion as well, so that $p_2/p_1 = 0.786$.

The pressure rises through a compression and falls through an expansion, and the Mach number decreases through a compression and increases through an expansion.

Problems

11.1 What is the speed of a sound wave in air at $300°K$? What is the speed in helium at the same temperature? Can you explain why a person who has inhaled helium speaks with a high-pitched voice (do not try this yourself)?

11.2 Find the Mach number of an airplane traveling at $2000\, ft/s$ at altitudes of $5000\, ft$, $10,000\, ft$, $20,000\, ft$, and $30,000\, ft$. Assume a standard atmosphere (Table Appendix-C.6).

11.3 Methane (CH_4) at $20°C$ flows through a pipe at a speed of $400\, m/s$. For methane, $R = 518.3\, J/(kg \cdot K)$, $\gamma = 1.32$. Is the flow subsonic, sonic, or supersonic?

11.4 Find the stagnation pressure and temperature of air flowing at $100\, ft/s$ if the temperature in the freestream is $60°F$ and the pressure is atmospheric.

11.5 Find the stagnation pressure and temperature of air flowing at $200\, m/s$ if the pressure and temperature in the undisturbed flow field are $0.96 \times 10^5\, Pa$ and $10°C$, respectively.

11.6 Find the stagnation pressure and temperature of air flowing at $200\, m/s$ in a standard atmosphere at sea level, and at heights of $2000\, m$ and $10,000\, m$.

11.7 An airplane flies at a speed of $150\, m/s$ at an altitude of $500\, m$, where the temperature is $20°C$. The plane climbs to $12,000\, m$ where the temperature is $-56.5°C$ and levels off at a speed of $600\, m/s$. Calculate the Mach number for both cases.

11.8 The Lockheed SR-71 reconnaissance airplane is rumoured to fly at a Mach number of about 3.5 at an altitude of $90,000\, ft$. Estimate its flight speed under these conditions.

11.9 The National Transonic Facility (NTF) is a wind tunnel designed to operate at Mach and Reynolds numbers comparable to flight conditions. It uses nitrogen at cryogenic temperatures as the working fluid. A schlieren photograph taken in the NTF of a 1/20th scale model of a Concorde airplane (full-scale wingspan of $30\, m$) shows a Mach angle of $28°$ at a point where the temperature is $100°K$ and the pressure is $9,000\, Pa$. Find the local Mach number and the Reynolds number based on the model wingspan, given that the viscosity of nitrogen under these conditions is $7 \times 10^{-6} N \cdot s/m^2$. Compare with the conditions experienced by the full-scale airplane flying at $600\, m/s$ at an altitude of $20,000\, m$ in a standard atmosphere.

11.10 Show that the air temperature rise in $°K$ at the stagnation point of an aircraft is almost exactly (speed in $mph/100)^2$.

11.11 The working section of a transonic wind tunnel has a cross-sectional area $0.5\, m^2$. Upstream, where the cross-section area is $2\, m^2$, the pressure and temperature are $4 \times 10^5\, Pa$ and $5°C$, respectively. Find the pressure, density and temperature in the working section at the point where the Mach number is 0.8. Assume one-dimensional, isentropic flow.

11.12 Air at $290°K$ and $10^5\, Pa$ approaches a normal shock. The temperature downstream of the shock is $540°K$. Find:

(a) The velocity downstream of the shock.
(b) The pressure change across the shock, and compare it with that calculated for an isentropic flow with the same deceleration.

11.13 A Pitot tube is placed in a supersonic flow where the freestream temperature is $90^\circ K$ and the Mach number is 2.5. A normal shock forms in front of the probe. The probe indicates a stagnation pressure of $52 \times 10^3\,Pa$. If the stagnation temperature is $270^\circ K$, find the pressure, density, stagnation pressure and velocity of the flow upstream of the shock.

11.14 Air with a stagnation temperature of $700^\circ K$ is compressed by a normal shock. If the upstream Mach number is 3.0, find the velocity and temperature downstream of the shock, and the entropy change across the shock.

11.15 Find the maximum increase in density across a shock wave for a gas with $\gamma = 1.4$.

11.16 A blowdown wind tunnel is supplied by a large air reservoir where the pressure is constant at $1.014 \times 10^5\,Pa$ and the temperature is constant at $15^\circ C$. The air passes through a working section of area $0.04\,m^2$ and exits to a large vacuum vessel. Find the pressure, density, velocity and mass flow rate in the working section if the Mach number there is 4.0.

11.17 A rocket motor is designed to give $10,000\,N$ thrust at $10,000\,m$ altitude. The combustion chamber pressure and temperature are $2 \times 10^6\,Pa$ and $2800^\circ K$, respectively. The gases exit the combustion chamber through a Laval nozzle. Find the exit Mach number, and the cross-sectional areas of the exit and the throat of the nozzle. Assume the nozzle flow is isentropic and one-dimensional, and that the ratio of specific heats γ for the combustion gases is 1.32.

11.18 A blowdown wind tunnel exits to atmospheric pressure. In the working section where the cross-sectional area is $0.04\,m^2$, the flow has a Mach number of 3 and a pressure of $0.3 \times 10^5\,Pa$.
(a) What is the minimum stagnation pressure required?
(b) What is the minimum stagnation temperature required to avoid air condensation in the working section (the condensation temperature under these conditions is about $70^\circ K$)?
(c) What is the corresponding stagnation density under these conditions?
(d) What is the mass flow rate under these conditions?

11.19 Air flows through a converging and diverging nozzle with an area ratio (exit to throat) of 3.5. The upstream stagnation conditions are atmospheric, and the back pressure is maintained by a vacuum system. Find:
(a) The mass flow rate if the throat area is $500\,mm^2$.
(b) The range of back pressures for which a normal shock will occur within the nozzle.

11.20 A large reservoir maintains air at $6.8 \times 10^5\,Pa$ and $15^\circ C$. The air flows isentropically through a convergent and divergent nozzle to another large reservoir where the back pressure can be varied. The area of the throat is $25\,cm^2$ and the area of the nozzle exit is $100\,cm^2$. Find
(a) The maximum mass flow rate through the nozzle.
(b) The two values of the Mach number at the nozzle exit corresponding to this mass flow rate.
(c) The back pressures required to produce these Mach numbers.

11.21 An air flow with a Mach number of 2.0 passes through an oblique shock wave inclined at an angle of 45°. Find the flow deflection angle α.

11.22 An air flow with a Mach number of 8 is deflected by a wedge through an angle α. What is the maximum value of α for an attached oblique shock?

11.23 A supersonic flow passes over a symmetrical wedge of semi-angle $\alpha = 10°$. At the leading edge an attached shock of $\beta = 30°$ is observed. Find:
(a) The upstream Mach number.
(b) The downstream Mach number.
(c) The static pressure ratio across the shock.

11.24 Air having an initial Mach number of 2.4, a freestream static pressure of $10^5\,Pa$ and a static temperature of $270°K$ is deflected by a wedge through an angle of 10°. Find:
(a) The Mach number, pressure and temperature downstream of the shock.
(b) The change in entropy across the shock.

11.25 The shock in the previous problem "reflects" at the opposite wall, as shown in Figure P11.25. The condition on the second shock is that the flow is turned parallel to the wall, so that the flow deflection through the second shock must be 10°. Find:
(a) The Mach number, pressure and temperature downstream of the second shock.
(b) The change in entropy $s_3 - s_1$.
(c) The maximum wedge angle for the reflected shock to remain attached.

11.26 An air flow with an initial Mach number of 1.5 and initial pressure p_1 is expanded isentropically by passing through a deflection angle of 5°. Find the Mach number and pressure ratio after the deflection.

11.27 Air at Mach 3.0 is deflected through 20° by an oblique shock. What isentropic expansion turning angle is required to bring the flow back to
(a) The original Mach number.
(b) The original pressure?

11.28 A flat plate airfoil with a chord length of $1\,m$ and a width of $6\,m$ is required to generate a lift of $40,000\,N$ when flying in air at a Mach number of 2.0, a temperature of $-20°C$ and a pressure of $10^5\,Pa$. What is the required angle of attack? What is the wave drag at this angle of attack?

11.29 A symmetrical diamond-shaped airfoil is placed at an angle of attack of 2° in a flow at Mach 2 and static pressure of $2 \times 10^3\,Pa$. The half-angle at the leading and trailing edges is 3°. If its total surface area (top and bottom) is $4\,m^2$, find the forces due to lift and wave drag acting on the airfoil.

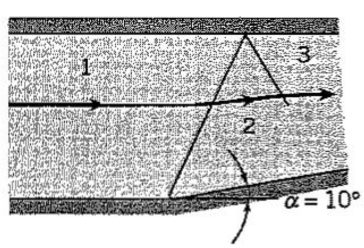

Figure P11.25

Chapter 12

Turbomachines

12.1 Worked Examples

These examples were adapted from Fox & Macdonald, *Introduction to Fluid Mechanics*, John Wiley & Sons, 4th ed., 1992.

Example 12.1: Axial-flow fan

An axial-flow fan operates at $1200\,rpm$. The blade tip diameter D_t is $1.1\,m$ and the hub diameter D_h is $0.8\,m$. The blade inlet and outlet angles are 30° and 60°, respectively. Inlet guide vanes give the absolute flow entering the first stage an angle of 30°. The fluid is air at atmospheric pressure and $15°C$, and it may be assumed that it is incompressible. The axial velocity of the flow does not change across the rotor. Assume the relative flow enters and leaves the rotor at the geometric blade angles and use properties at the mean blade radius for calculations.
(a) Sketch the rotor blade shapes.
(b) Draw the inlet velocity diagram.
(c) Find the volume flow rate.
(d) Draw the outlet velocity diagram.
(e) Calculate the minimum torque and power needed to drive the fan.

Solution: For parts (a) and (b), the blade shapes are as shown in Figure 12.1(a), and the corresponding inlet velocity diagram is shown in Figure 12.1(b).

For parts (c) and (d), the continuity equation gives

$$\rho V_{n1} A_1 = \rho V_{n2} A_2$$

or

$$\dot{q} = V_{n1} A_1 = V_{n2} A_2$$

Since $A_1 = A_2$, then $V_{n1} = V_{n2}$, and the outlet velocity diagram is as shown in Figure 12.2. To complete the velocity diagrams, and find the volume flow rate, we need to find U, V_{n1}, V_1, V_{t1}, V_{rb_1}, V_2, V_{t2}, and α_2. We start with U. At the mean blade radius,

$$U = \omega R_m$$

The mean blade radius $R_m = (D_t + D_h)/4$. Also,

$$\omega = 1200\,\frac{rev}{min} \times 2\pi\,\frac{rad}{rev} \times \frac{min}{60\,sec} = 126\,rad/s$$

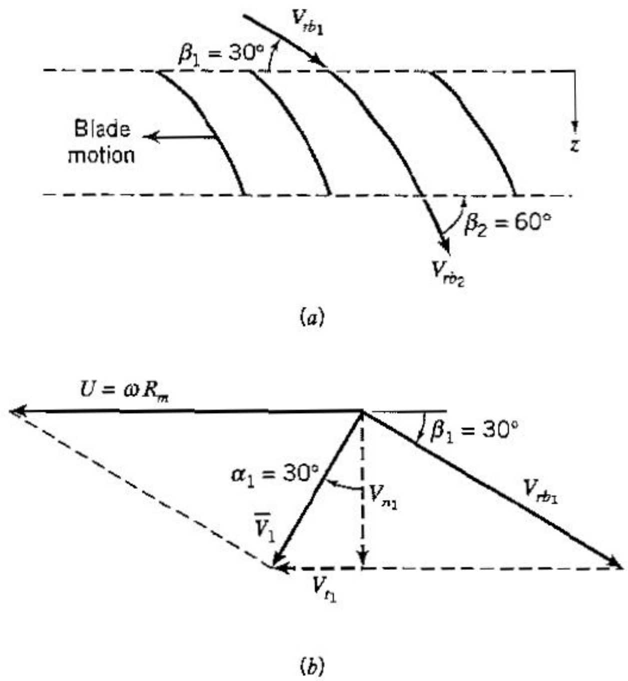

Figure 12.1: Blade shapes and inlet velocity diagram.

Therefore

$$U = \frac{(1.1 + 0.8)\ m}{4} \times 126 \frac{rad}{sec} = 59.7\, m/s$$

From the inlet velocity diagram,

$$U = V_{n1} \left(\tan \alpha_1 + \cot \beta_1 \right)$$

where $\alpha_1 = \beta_1 = 30°$, so that

$$V_{n1} = V_{n2} = 25.9\, m/s$$

Hence,

$$V_1 = \frac{V_{n1}}{\cos \alpha_1} = 29.9\, m/s$$

$$V_{t1} = V_1 \sin \alpha_1 = 15.0\, m/s$$

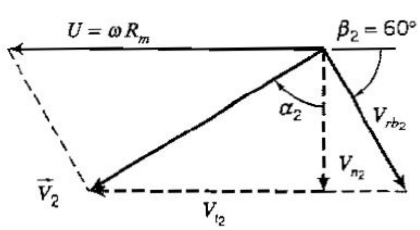

Figure 12.2: Outlet velocity diagram.

and

$$V_{rb_1} = \frac{V_{n1}}{\sin\beta_1} = 51.8\,m/s$$

The volume flow rate can now be found.

$$\dot{q} = V_{n1}A_1 = V_{n1}\tfrac{\pi}{4}\left(D_t^2 - D_h^2\right) = 11.6\,m^3/s$$

From the outlet velocity diagram

$$V_{t2} = U - V_{n2}\tan\left(90^\circ - \beta_2\right) = \left(59.7 - 25.9\tan 30^\circ\right)\,m/s = 44.7\,m/s$$

Also:

$$\tan\alpha_2 = \frac{V_{t2}}{V_{n2}}$$

so that

$$\alpha_2 = 59.9^\circ$$

and

$$V_2 = \frac{V_{n2}}{\cos\alpha_2} = 51.6\,m/s$$

For part (d), to find the torque, we use equation 12.6, with $r_1 = r_2 = R_m$ so that

$$T_{shaft} = \rho\dot{q}R_m\left(V_{t2} - V_{t1}\right)$$

Hence, using SI units throughout,

$$T_{shaft} = 1.2 \times 11.6 \times \frac{0.95}{2} \times (44.7 - 15.0)\;N\cdot m = 196\,N\cdot m$$

and the power required is given by equation 12.7, so that

$$\dot{W}_{shaft} = \omega T_{shaft} = 24.7\,kW$$

Example 12.2: Centrifugal pump

Water at 150 gpm enters a centrifugal pump impeller axially through a 1.25 $in.$ diameter inlet (see Figure 12.3). The inlet velocity is axial and uniform. The impeller outlet diameter is 4 $in.$ Flow leaves the impeller at 10 ft/s relative to the radial blades. The impeller speed is 3450 rpm. Determine
(a) The impeller exit width, b_2.
(b) The minimum torque input to the impeller.
(c) The minimum power required.

Solution: For part (a), the continuity equation gives

$$\rho V_1 A_1 = \rho V_2 A_2$$

or

$$\dot{q} = V_{rb_2} 2\pi R_2 b_2$$

so that

$$\begin{aligned} b_2 &= \frac{\dot{q}}{2\pi R_2 b_2 V_{rb_2}} = \frac{1}{2\pi} \times 150\frac{gal}{min} \times \frac{1}{2\,in.} \times \frac{s}{10\,ft} \times \frac{ft^3}{7.48\,gal} \times \frac{min}{60\,s} \times \frac{12\,in.}{ft} \\ &= 0.0319\,ft \quad \text{or} \quad 0.383\,in. \end{aligned}$$

For part (b), the torque is given by equation 12.6

$$T_{shaft} = \rho \dot{q} R_2 V_{t2}$$

since the axial inlet flow has no z-component of angular momentum. That is,

$$T_{shaft} = \rho \left(V_{rb_2} 2\pi R_2 b_2\right) R_2 \left(\omega R_2\right) = 2\pi \rho \omega R_2^3 b_2 V_{rb_2}$$

so that

$$\omega = 3450 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{min}{60\, sec} = 361\, rad/s$$

Hence, using BG units throughout,

$$T_{shaft} = 2\pi \times 1.94 \frac{slug}{ft^3} \times 361 \frac{rad}{s} \times \frac{2^3}{12^3} ft^3 \times 0.0319\, ft \times 10 \frac{ft}{s} = 6.50\, ft \cdot lb_f$$

For part (c), the power required is given by equation 12.7

$$\dot{W}_{shaft} = \omega T_{shaft} = 361 \frac{rad}{sec} \times \frac{6.50}{550} hp = 4.27\, hp$$

The actual input torque and power may be higher than the estimates found here because of energy losses in the system, which are not accounted for in the analysis.

Example 12.3: Windmill performance

Calculate the ideal efficiency, actual efficiency, and thrust for a Dutch windmill with $D = 26\, m$, $\omega = 20\, rpm$, $V_1 = 36\, km/hr$, and power output of $41\, kW$. Assume that $V_4/V_1 = 0.5$.

Solution: We have

$$\omega = 20 \frac{rev}{min} \times 2\pi \frac{rad}{rev} \times \frac{min}{60\, sec} = 2.09\, rad/s$$

The tip speed ratio is given by equation 12.39, so that

$$\chi = \frac{\omega R}{V_1} = \frac{2.09 \times 13 \times 3600}{36000} = 2.72$$

Figure 12.20 indicates that the ideal efficiency for this value of χ is about 0.53. The actual efficiency is given by

$$\eta_{actual} = \frac{P_{actual}}{P_{KEF}} = \frac{P_{actual}}{\frac{1}{2}\rho V^3 \frac{\pi}{4} D^2}$$

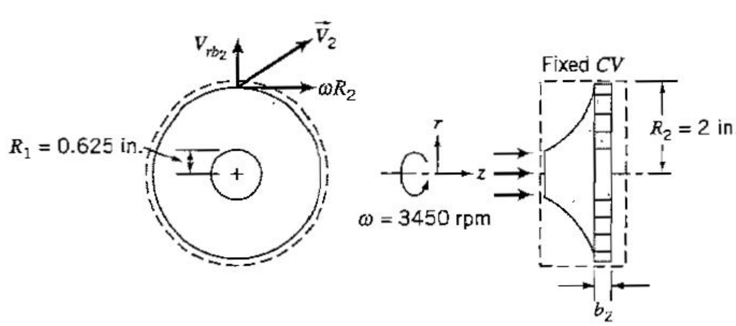

Figure 12.3: Centrifugal pump.

With $\rho = 1.2\, kg/m^3$,

$$\eta_{actual} = 0.129$$

which is only 24% of the ideal efficiency at this tip speed ratio.

The thrust is given by equation 12.36. With $V_4/V_1 = 0.5$, we have

$$F = \tfrac{1}{2}\rho V_1^2 \left(\frac{V_4^2}{V_1^2} - 1\right) \tfrac{\pi}{4} D^2 = -\tfrac{3\pi}{32}\rho V_1^2 D^2$$

That is,

$$F = -\tfrac{3\pi}{32} \times 1.2 \times 10^2 \times 26^2\, N = 23.9\, kN$$

The thrust is negative in the sense that it is opposite in direction to the thrust developed by a propeller

Problems

12.1 A hydraulic turbine generates $50,000\, hp$ at $75\, rpm$ with a head of $100\, ft$. Find the specific speed and use Figure 12.16 to determine what type of turbine is being used.

12.2 A hydraulic turbine operates at a specific speed of 100 (U.S. customary units) and delivers $500\, kW$ with a head of $10\, m$. What type of turbine is being used, and what is the rotational speed necessary to give the optimum efficiency?

12.3 When it operates at peak efficiency, a turbine delivers $25,000\, hp$ at a speed of $450\, rpm$ with a head of $4500\, ft$. What type of turbine is being used? Estimate the flow rate and the size of the machine, assuming there are no losses in the flow upstream of the turbine.

12.4 A Pelton wheel of $4\, m$ diameter operates with a head of $1000\, m$. Estimate the flow rate and power output of the machine, assuming that it operates at peak efficiency and there are no losses in the flow upstream of the turbine.

12.5 A Pelton wheel operates with a head of $400\, m$ at $350\, rpm$, and it is powered by a jet $12\, cm$ in diameter. Find the specific speed (U.S. customary units), and the wheel diameter, if there are no losses in the flow upstream of the turbine and it operates at peak efficiency.

12.6 A turbine of the Francis type operates with a head of $200\, m$ and flow rate of $3\, m^3/s$. The peak efficiency occurs at a head coefficient C_H of 9, a flow coefficient C_Q of 0.3, and a power coefficient C_P of 2.5 (as defined in Section 12.7). Estimate the size of the machine, the maximum power produced, and the speed.

12.7 A pump delivers $0.7\, ft^3/s$ of water against a head of $50\, ft$ at a speed of $1750\, rpm$. Find the specific speed.

12.8 A water pump delivers $0.25\, m^3/s$ against a head of $20\, m$ at a speed of $2400\, rpm$. Find the specific speed, in U.S. customary units.

12.9 For the previous two problems, determine the type of pump that would give the highest efficiency by using Figure 12.17.

12.10 A radial-flow pump is required to deliver $1000 gpm$ against a total head of $350\, ft$. Find the minimum practical speed using Figure 12.17.

12.11 An axial-flow pump runs at 1800 rpm against a head of 1200 ft. Find the flow rate delivered by the most efficient pump using Figure 12.17.

12.12 Axial-flow pumps are to be used to lift water a combined height of 150 ft at a rate of 30 ft^3/s. Each pump is designed to operate at 1800 rpm. Using Figure 12.17, find the number of pumps required if they are all operating at their maximum efficiency. Neglect losses in the piping.

12.13 Using the curves given in Figure 12.15 for a centrifugal water pump with an impeller of 32 $in.$ diameter, operating at 2000 hp find:
(a) The volume flow rate, the total head, and the efficiency.
(b) The specific speed.

12.14 For the pump described in the previous problem, the inlet is located a distance h_i above the surface of the reservoir, and there are no losses in the inlet piping. The temperature of the water is $50°F$. Find the maximum value of h_i for which cavitation will not be a problem.

12.15 Using Figure 12.18, find the effect of a 50% increase in the impeller diameter on the rotation rate and the volume flow rate of the centrifugal water pump described in Problem 13.13, if the total head and power input remain the same, and the pump continues to operate at its best efficiency.

12.16 Use Figure 12.18 to find the impeller diameter, power input and head for a pump of the same family, if it operates at 900 rpm with a volume flow rate of 5000 gpm at peak efficiency.

12.17 A basement sump pump provides a discharge of 12 gpm against a head of 15 ft. What is the minimum horsepower required to drive this pump, given that its efficiency is only 60%?

12.18 For a given pump, the diameter, the gauge pressures at the inlet and outlet are $-30 \times 10^3\, Pa$ and $200 \times 10^3\, Pa$, respectively. If the flow rate is $0.1\, m^3/s$, and the power required is 25 kW, find the efficiency. The intake and discharge are at the same height.

12.19 The impeller of a radial-flow turbine has an outer radius of 4 ft, turning at 200 rpm. The volume flow rate is 1000 ft^3/s, and it discharges in the axial direction. Neglect all losses. If the tangential component of the velocity at the inlet is 5 ft/s, find:
(a) The torque exerted on the impeller.
(b) The power developed by the machine.

12.20 The diameter of an axial-flow impeller is 2.8 m, and the inlet flow makes an angle of 10° with the circumferential direction. The volume flow rate is 24 m^3/s over an area of 8 m^2 with a head of 100 m, and it discharges in the axial direction without swirl. Find the speed of the runner. Neglect losses.

12.21 A centrifugal water pump with a 200 mm diameter impeller rotates at 1750 rpm. Its width is 20 mm and the blades are curved backwards so that $\beta_2 = 60°$. If the flow enters the pump in the axial direction, and the volume flow rate is 100 L/s, estimate the power input if the pump is 100% efficient.

12.22 A centrifugal pump is designed to have a discharge of 600 gpm against a head of 200 ft. The impeller outlet diameter is 12 $in.$, and its width is 0.5 $in.$. The outlet blade angle is 65°. What is the design speed and the minimum horsepower required to drive this pump?

12.23 A centrifugal pump has inner and outer impeller diameters of $0.5\,m$ and $1\,m$, respectively, and a width of $0.15\,m$. The outlet blade angle is $65°$. At $350\,rpm$, the volume flow rate is $4\,m^3/s$. Find:
(a) The exit blade angle so that the water enters the pump in the radial direction. (b) The minimum power required.

12.24 A propeller $1\,ft$ in diameter rotates at $1200\,rpm$ in water and absorbs $2\,hp$. Find the power coefficient, and estimate the power required to increase the speed to $1500\,rpm$ if the power coefficient remains constant.

12.25 A high-speed two-bladed wind turbine $35\,m$ in diameter operates at its peak efficiency in winds of $30\,km/hr$. Estimate the power generated, the rotor speed, and the wind velocity in the wake.

12.26 A propeller $2\,ft$ in diameter moves at $20\,ft/s$ in water and produces a thrust of $1000\,lb_f$. Find the ratio of the upstream to downstream velocities, and the efficiency.

12.27 An airplane flies at $140\,mph$ at sea level at $60°F$. The propeller diameter is $8\,ft$, and the slipstream has a velocity of $200\,mph$ relative to the airplane. Find:
(a) The propeller efficiency.
(b) The flow velocity in the plane of the propeller.
(c) The power input.
(d) The thrust of the propeller.
(e) The pressure difference across the propeller disk.

12.28 An airplane flies at $300\,km/h$ at sea level at $20°C$. The propeller diameter is $1.8\,m$, and the velocity of the air through the propeller is $360\,km/hr$. Find: (a) The slipstream velocity relative to the airplane.
(b) The thrust.
(c) The power input.
(d) The power output.
(e) The efficiency.
(f) The pressure difference across the propeller disk.

12.29 On a high-speed submarine the propeller diameters are limited to $15\,ft$, and their maximum rotational speed is set at $200\,rpm$ to avoid cavitation. If each propeller has an efficiency of 86% and it is limited to $10,000\,hp$, find the minimum number of propellers required to move at a speed of $35\,mph$, and the torque in each shaft.

12.30 A modern multi-bladed windmill design is adapted to work in a tidal flow. When the water flows at $5\,m/s$, find the maximum power generated for a diameter of $4\,m$. Calculate the tip speed, and determine if cavitation could be a problem.

12.31 An American farm windmill is used to pump water from a $100\ ft$ deep well through clean plastic pipe of $2\,in.$ diameter. If the rotor diameter is $6\,ft$, estimate the volume flow rate when the winds are blowing at $20\,mph$. Assume that the windmill is working at peak efficiency, and the pump is 90% efficient. What is the expected flow rate when the wind drops to $15\,mph$?

Chapter 13

Environmental Fluid Mechanics

Problems

13.1 Describe the flow of of air around a high-pressure center over Australia in terms of its direction of rotation, its vertical movement, and its motion relative to the center.

13.2 What information is typically available from a wind rose for a given locale?

13.3 Explain the difference between the atmospheric lapse rate and the adiabatic lapse rate.

13.4 How are the environmental (atmospheric) and the adiabatic lapse rate used to classify the degree of stability in the atmosphere?

13.5 Classify the stability of the atmosphere on the basis of the average temperature gradient for the following conditions:
(a) Temperature at ground level is $70°F$, temperature at $1500\,ft$ is $80°F$.
(b) Temperature at ground level is $70°F$, temperature at $2500\,ft$ is $60°F$.
(c) Temperature at ground level is $60°F$, temperature at $1900\,ft$ is $48°F$.

13.6 Classify the stability of the atmosphere on the basis of the average temperature gradient for the following conditions:
(a) Temperature at ground level is $24.6°C$, temperature at $2000\,m$ is $5°C$.
(b) Temperature at ground level is $30°C$, temperature at $500\,m$ is $20°C$.
(c) Temperature at ground level is $25°C$, temperature at $700\,m$ is $28°C$.

13.7 On a particular day, the wind speed is $2\,m/s$ at a height of $10\,m$. Estimate the wind speed at heights of $100\,m$ and $300\,m$ if the power law exponent on the velocity profile is 0.15 (level ground, little cover). Repeat the estimates for an exponent of 0.4 (urban area).

13.8 The wind speed over an urban area is $10\,m/s$ at a height of $10\,m$. Estimate the wind speed at a height of $30\,m$.

13.9 At a given location the ground-level temperature is $70°F$. At an elevation of $2000\,ft$, the temperature of the air is $65°F$.
(a) What is the maximum mixing depth, in feet, when the normal maximum surface temperature for that time is $90°F$?
(b) What if it was $84°F$?

13.10 At a given location the ground-level temperature is $18°C$, while the normal maximum surface temperature for that month is known to be $30°C$. At an elevation of $700\,m$, the temperature of the air is $15°C$.
(a) What is the maximum mixing depth, in meters?
(b) What is it if the temperature at $700\,m$ is $20°C$?

13.11 The atmospheric lapse rate on a particular day is constant in the lower part of the atmosphere. At ground level, the pressure is $1020\,mBar$ and the temperature is $15°C$. At a height z_1 the pressure and temperature are $975\,mBar$ and $11.5°C$. Determine the atmospheric temperature gradient, and the height z_1.

13.12 Find the terminal speed of spherical particles falling through atmospheric air at $20°C$. Particle densities of:
(a) $1.0 \times 10^{-3}\,g/mm^3$
(b) $2.0 \times 10^{-3}\,g/mm^3$
should be considered with particle diameters of 10, 100, and $1000\,\mu m$. Take note of the results given in Figure 9.14.

13.13 Find the terminal speed of spherical particles falling through water at $20°C$. Particle densities of:
(a) $1000\,kg/m^3$
(b) $2000\,kg/m^3$
should be considered with particle diameters of 10, 100, and $1000\,\mu m$. Take note of the results given in Figure 9.14.

13.14 Air at $20°C$ and atmospheric pressure flows horizontally through a collection chamber $3\,m$ high and $5\,m$ long. The air carries particles $70\,\mu m$ diameter, with a specific gravity of 1.5. What is the maximum air speed that can be used to ensure that all particles have settled out in the chamber?

13.15 Air at $60°F$ and atmospheric pressure flows horizontally at a speed of $1\,ft/s$ through a collection chamber $12\,ft$ high. The air carries particles $0.002\,in.$ diameter, with a specific gravity of 2.0. What is the minimum length of the chamber necessary to ensure that all particles are collected?

13.16 Air at $0°C$ and atmospheric pressure flows horizontally at a speed of $0.5\,m/s$ through a collection chamber $4\,m$ high and $20\,m$ long. The air carries particles with a density of $1600kg/m^3$. What is the minimum diameter of particles that will be collected in the chamber?

Answers to Selected Problems

Chapter 1

1.1	$400\,kg, 27.4\,slug, 882\,lb_m$
1.2	$11.39\,ft^3$, $0.821\,ft^3$, $1.027\,ft^3$, $0.001206\,ft^3$, $0.000169\,ft^3$
1.3	$1204\,lb_f$, $102.7\,lb_f$, $62.5\,lb_f$, $0.0764\,lb_f$, $0.00379\,lb_f$
1.4	$417\,N$, $69.5\,N$
1.5	19.33, 2.775, 1.027, 0.001398, 0.001682
1.6	$+2.20\%$, -25.6%, $+5.96\%$, -25.4%
1.7	(a) $6680\,m$, (b) $4855\,m$
1.8	Need solution
1.9	$34.8\,min$
1.10	$18,900\,psi$
1.11	$\Delta v = -4.79 \times 10^{-7}\,m^3$
1.12	(a) $1.002 \times 10^{-3}\,N \cdot s/m^2$, (b) 2.73, 0.789, (c) 0.42%
1.13	$21,886\,N$, $4920\,lb_f$
1.14	$0.645\,psi$, $4448\,Pa$
1.15	(a) $0.95\,psi$
1.16	$252,000\,N$, $56,650\,lb_f$
1.17	$51.0 \times 10^{18}\,kg$
1.18	$-0.158\,psi$
1.19	-1.1%
1.20	0.733
1.21	1.343
1.22	(a) 0.424, (b) 168×10^6
1.23	$\mu(V/h)\pi D^2/4$
1.24	$0.798\,N$
1.25	$10^{-3}\,kg/ms$
1.26	$2\pi r^2 \mu w L/(R-r)$
1.27	$(R_2 - R_1)\,T/\left(2\pi\mu R_1^3 L\right)$
1.28	$\mu = T\delta/(2\pi\omega R^3 H)$
1.29	$2\pi\mu U d_1/(d_2 - d_1)$
1.30	(a) $\pi\mu\omega L D^3/(4\epsilon)$, (b) $\pi\mu\omega^2 L D^3/(4\epsilon)$
1.31	$0.104\,N \cdot s/m^2$
1.32	$3.27\,m/s$
1.33	(a) 10,016, (b) $77.5 \times 10^{-5}\,lb_f/ft^2$, (c) $0.0413\,lb_f$
1.34	$5.9\,\mu m$, $106\,\mu m$, $78\,km$
1.35	(a) 220,000, (b) 21×10^6, (c) 475×10^6, (d) 74×10^6, (e) 1.9×10^6, (f) 667
1.36	(a) $240\,m/s$, (b) $2360\,m/s$, (c) $11.8\,m/s$
1.37	$Re = 1667$ (laminar)
1.38	$9.8\,mm$
1.39	$0.033\,N/m$
1.40	$104,327\,Pa$
1.41	$584\,Pa$
1.42	$107\,mm$

Chapter 2

2.1 (a) $36.3\,psi$, (b) $62.4\,psi$, (c) $15.2\,psi$, (d) $8.67\,psi$
2.2 (a) $2.06 \times 10^4\,Pa$, (b) $430 \times 10^3\,Pa$, (c) $105 \times 10^3\,Pa$, (d) $78.5 \times 10^3\,Pa$
2.3 (a) $-80.7 \times 10^3\,Pa$, $-11.7\,psig$, (b) $149 \times 10^3\,Pa$, $21.6\,psig$, (c) $3.66 \times 10^3\,Pa$, $0.53\,psig$, (d) $329 \times 10^3\,Pa$, $47.7\,psig$, (e) $-41.5 \times 10^3\,Pa$, $-6.03\,psig$
2.4 $2000\,N$
2.5 $1820\,N$
2.6 $\rho g(a+b)Wc$
2.7 $A_2\left(F_1/A_1 - \rho g H\right)$
2.8 $0.714\,m$
2.9 (a) $2.81\,m$, (b) $3.48\,m$, (c) $0.208\,m$
2.10 $0.072\,m$
2.11 (a) $23.1 \times 10^3\,N$, (b) $> 2.5\%$, (c) $> 0.12\%$
2.12 $\rho_B/\rho_A = 2.8$
2.13 $20\,mm$
2.14 $gA\left(\rho_2 h_2 + \rho_1 h_1\right)$
2.15 $1.06\,m$
2.16 $69.9 \times 10^3\,Pa$
2.17 $-900g\,Pa$
2.18 $p_g = (\rho_0 z + mz^3/3)g$
2.19 $5.8\,atm$
2.20 $3.12\,m$
2.21 $2.34\,m$
2.22 $R_1 = 3496\,lb_f/ft$, $R_2 = 3700\,lb_f/ft$
2.23 (a) $\rho g H^2 w/2$; (b) $2H/3$
2.24 $3.12\,in.$
2.25 $h_2/h_1 = (\rho_1/\rho_2)^{1/3}$
2.26 $h_1/h_2 = \sqrt{\rho_2/\rho_1}$
2.27 (a) $\rho g w H(d + H/2)$, (b) $\rho g w H^2(d/2 + H/3)$
2.28 $4\rho g w b^2$, acting $7b/6$ below top of door
2.29 (a) $\rho_1 g z$, $\rho_1 g D + \rho_2 g(z - D)$, (b) $F/(\rho_1 D g L w) = 1 + (\rho_2/2\rho_1)(L/D)$, (c) $(L/2)\left(1 + (2\rho_2 L)/(3\rho_1 D)\right)\left(1 + (\rho_2 L)/(2\rho_1 D)\right)$ from top of gate
2.30 $p_w = 2M_h/(WB^2) - \rho g h + \rho g B/3$
2.31 $\rho_2/\rho_1 = (3H_1 - b)/(3H_2 - b)$
2.32 (a) $2\rho W g h^2\sqrt{2}/9$; (b) $8\rho W g h^2/81$
2.33 $D^3 - 3HD^2 + 2H^3 = 3(MH/\rho b)\cos\theta\sin\theta$
2.34 Need solution
2.35 $M = \rho W L/\cos\theta[H + (2L/3)\sin\theta]$
2.36 $\rho g b W(3h - b)/(6\cos\theta)$, normal to the gate
2.37 $\rho_2/\rho_1 = (H_1/H_2)^3\cos^2\theta$
2.38 $14d/9$ from surface along wall
2.39 $F = (\rho_1 H_1^2 - \rho_2 H_2^2)Wg/2$, $(\rho_1 H_1^3 - \rho_2 H_2^3)/3(\rho_1 H_1^2 - \rho_2 H_2^2)$ above apex
2.40 $\rho_2/\rho_1 = 2$
2.41 $\rho_2/\rho_1 = 2$
2.42 $\left[3mL\cos\theta\sin^2\theta/\rho w\right]^{1/3}$
2.43 $\rho_2/\rho_1 = \left(\frac{H_1}{H_2}\right)^3$
2.44 $\left(2h_1 - 2L\sin\theta\right)/\left(3h_2 - 2L\sin\theta\right)$
2.45 $\rho g\left(H - (b/3)\sin\theta\right)$
2.46 $5\rho g h/6 + W\sin 2\theta/nw$
2.47 $2c/b$
2.48 $\rho\left(3h - 2L\sin\theta\right)/\left(3\rho_s\cos\theta\right)$
2.49 $7\rho w b^3/\left(3L\sin^2\theta\cos\theta\right)$
2.50 $\rho g w L\left[h - (L/2)\sin\theta\right]$, $3hL - 2L^2\sin\theta/\left(6h - 3L\sin\theta\right)$ from O along the window
2.51 $\left(\sin\theta_2/\sin\theta_1\right)^{2/3}$

2.52 $(5/6)\rho g w L^2 \sin\theta / \left((1/2)\cos\theta + \sin\theta\right)$
2.53 $\rho g \left(h - L\sin\theta/3\right)$
2.54 (a) $p_a/\rho g + h/2$, (b) $\left(wh/\sin\theta\right)\left(p_a + \rho g h/2\right)$, (c) $\left(wh^2/2\sin^2\theta\right)\left(p_a + \rho g h/3\right)$ CW
2.55 (a) $(2/3)\rho g w L\left(H - L\sin\theta/3\right)$, (b) $(4/9t)\left(2H - 5L\sin\theta/9\right)$,
(c) $\rho_t w t L^2 \cos\theta/2 - \rho g w L^2\left(H/2 - L\sin\theta/3\right)/18$
2.56 (a) $\rho g\left(H^3/6 + H^3\cos\theta/81\right)$, (b) normal to the window $H/3$ from the hinge
2.57 (a) $3\rho g w H^3/(2\sin\theta)$, (b) $\rho g w H^2/(2\sin\theta)$, (c) $\rho g w H^2(1/3 + \sin^2\theta)/(2\sin^3\theta)$
2.58 (a) $3\rho g h^2/(8\sin\theta)$, normal to the surface, (b) $z = 2h/9$, (c) $-(3\rho g h^2\cot\theta/4)\mathbf{j}$
2.59 0.8
2.60 $W - \rho V$
2.61 $(6M/\pi\rho)^{1/3}$
2.62 6.4
2.63 8.33 hrs
2.64 (a) 0.08, (b) sinks, (c) no
2.65 volume $= 2.24\,m^3$
2.66 $389\,N$, $87.3\,lb_f$
2.67 1.5
2.68 0.75
2.69 2
2.70 $3b/4$
2.71 $d = \rho_w(H - D)/\rho_o$
2.72 (a) $3\rho g L/4$, (b) $3\pi\rho g L D^2/16$
2.74 D^3/nV
2.75 $2.06\rho_w g A$
2.76 $0.255D$
2.78 (a) 0.85
2.79 $[3M/(4\pi(\rho_w - \rho_b))]^{1/3}$
2.80 $3.22b$
2.81 $\rho w^2 L g/8$
2.82 Need solution
2.83 $2b/3$ below interface
2.84 $3\rho_w g V(1 - \rho_f/\rho_w)$
2.85 $D^2(h - D/3) - 2abL(0.86 - (h - D)/b) = 0$
2.86 $\rho H^3 - 3\rho H L^2 - 3mL^2 = 0$
2.87 $d = 2h^3/(3b^2)$
2.88 $b = 5.2H$
2.89 $B = 2h$
2.90 (a) $16,564\,lb_f$, (b) $3.93\,ft$ along the plate from its top edge, (c) $10.6\,ft$ from surface
2.91 $2MgL\cos\theta/(wd^2) - 2\rho g d\sin\theta/3$
2.92 $M = \left(\rho w D/\cos\theta\right)\left(H/2 + D\sin\theta/3\right)$
2.93 $\rho g(D + 2L\cos\theta/3)$
2.94 $h^3 = 6WL\sin^2\theta/(\rho w)$
2.95 (a) $\frac{1}{2}\rho g w h^2\sqrt{(b/h)^2 + 81}$, (b) $b/h = 9$
2.96 $2\rho g H^2 a/3$ at $3H/4$
2.97 $\rho_2/\rho_1 = 2.5$
2.98 $\rho g a b\left(D - 2a/3\right)$, $D = 3WL/(\rho g b a^2) + a/2$
2.99 (a) $\pi\rho g h R^2/2$, (b) $\pi\rho g R^3/8$
2.100 $80\,lb_f$
2.101 $\rho(a + g)H$
2.102 (a) $0.547\,ft$, (b) $69.1\,mph$
2.103 $0.134g$
2.104 $39.6\,rad/s$

Chapter 3

3.1 $0.382\,lb_m/s$; $10\,ft/s$
3.2 (a) $499\,kg/s$, (b) $63.7\,m/s$, (c) $31,800\,N$

3.3 $V_2 = 1.33\,m/s$, $V_3 = 2.08\,m/s$
3.4 $8.33\,ft/s$; $6.67\,ft/s$
3.5 $119\,m/s$; $215\,m/s$
3.6 $0.8\,m/s$
3.7 $0.0736\,m^3/s$ or $1167\,US\,gpm$; 12 hoses
3.8 $1011\,s$
3.9 Level falls at $0.0026\,m/s$
3.10 $V_2/V_1 = 2$
3.11 $0.00610\,m^3/s$
3.12 (a) $5V_1A_1/4$, $5\rho V_1A_1/4$, (b) $\rho V_1^2 A_1((1 - 25\cos\theta/32)\mathbf{i} + (1/8 - 25\sin\theta/32)\mathbf{j})$
3.13 (a) $\tau = -0.0146y/h$, (b) $0.666\,m/s$, (c) $1.33 \times 10^{-3}\,m^3/s$, (d) $1.61 \times 10^{-3}\,kg/s$, (e) 0.0273
3.14 $\rho V^2 \pi D^2/2$
3.15 $U_2 = U_1 + \dot{q}/2D$, $F/(2\rho U_1^2 DW) = (p_2 - p_1)/(\frac{1}{2}\rho U_1^2) + (\dot{q}/DU_1)\left((\dot{q}/4DU_1) + 1\right)$
3.16 $V_B = 0.6V_1$; $p_B - p_A = \rho V_1^2/25$
3.17 $4275\,lb_f$
3.18 $\rho V^2 \pi R^2 \left(\cos 30^\circ \mathbf{i} - (1 - \sin 30^\circ)\,\mathbf{j}\right)$
3.19 $\theta = \pi/4$
3.20 $\rho h w U_1^2\left((2/27)\cos\alpha - 1\right)\mathbf{i}$
3.21 $U_2 = 8U_1/5$; $F_D = \rho h^2 (g\Delta\rho_m/\rho - 3U_1^2/25)$
3.22 $4\rho b V_m h/3$, $2V_m/3$
3.23 $\dot{m}_3 = 4\rho QWb/9$
3.24 $U_3 = 3(U_1 + U_2)/4$
3.25 $2U_1W(h_2 - h_1)$
3.26 $U_m/U_o = 3b/2h$
3.27 $3(2U_1 + U_m)/4$, $16\rho W b U_m^2/15$
3.28 $U_m = 3aU_1/2b$, $6\rho W a^2 U_1^2 \sin\theta/(5b)$
3.29 $U_m = 2U_1 a/b$, $(4\rho W a^2 U_1^2/(3b))\mathbf{j}$
3.30 $\rho_2 = 3\rho_1/2$; $2\rho_2 W h U_2^2(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})/3$
3.31 $(\rho_1 U_1 - \rho_2 U_m/2)/(2L)$
3.32 (a) $U_2/U_1 = (1 - \delta/2H)^{-1}$, (b) $1 - (U_2/U_1)^2$, (c) $1 - (U_2/U_1)^2/(1 - 5\delta/(3H))$
3.33 (a) $b = 3a/2$, (b) $2\rho W U_{av}^2 a(\mathbf{i} - 0.8\mathbf{j})$
3.34 (a) $2U_1/3$, (b) $\rho U_1^2 aw\left(2p_{1g}/\rho U_1^2 - \cos\theta + 5/9\right)\mathbf{i} - \rho U_1^2 aw\sin\theta\mathbf{j}$
3.35 $U_2 = 2U_1$; $F_x = -\rho U_1^2 Dw/3$
3.36 $T/(\frac{1}{8}\rho U_1^2 \pi D^2) = (p_2 - p_1)/(\frac{1}{2}\rho U_1^2) + (U_m/U_1)^2 - 2$
3.37 (a) $\delta = 0.25D_1$, (b) 1/15
3.38 (a) $V_2 = V_1 h_1/h_2$, (b) $F = (p_1 - p_2)Wh_2 + 8\rho W(V_1^2 h_1 - V_2^2 h_2)/15$
3.39 $U_m/U_{av} = 1.5$; $C_D = C_p - 0.4$
3.40 (a) $U_2 = 3U_1$, $U_m = 9U_1/4$, (b) $p_1 - p_2 = 4\rho U_1^2$, (c) $F/(\rho U_1^2 hW) = -3p_1/(\rho U_1^2) + 12/5$
3.41 (b) $C_p = (U_2/U_1)^2 - 1$
3.42 $(16/15)\rho U_m^2 wb\,(1 - \cos\alpha)$
3.43 (a) $2V$, (b) $-\rho V^2 A\cos\theta\mathbf{i} + \left[\rho V^2 A\,(\sin\theta - 3/4)\right] bfj$
3.44 (a) $5V_1/2$, (b) $C_F = F/(\frac{1}{2}\rho V_i^2 wh) = (160\sin\theta)\mathbf{i} + (12 - (160\cos\theta/9))\mathbf{j}$
3.45 (a) 0.5, (b) $-\left(2wh\rho V_1^2\cos\alpha + 2hwp_{1g}\cos\alpha + (4/5)hw\rho V_1^2\right)\mathbf{i} - \left(2wh\rho V_1^2\sin\alpha + 2hwp_{1g}\sin\alpha\right)\mathbf{j}$
3.46 (a) $3V_1/4$, (b) $\rho V_1^2 \pi D^2/4\,[(7/16 - \sin\theta)\,\mathbf{i} + \cos\theta\mathbf{j}]$
3.47 (a) 2, (b) $-2\rho V_1^2 hw\left(8/15 + 16\cos\theta/15 + p_{1g}/\rho V_1^2 + p_{2g}\cos\theta/2\rho V_1^2\right)\mathbf{i}$
$- 2\rho V_1^2 hw\sin\theta\left(16/15 + p_{2g}/2\rho V_1^2\right)\mathbf{j}$
3.48 $-\rho hwV^2/4$
3.49 (a) $8\rho U_m^2 D/15L$, (b) $8\rho U_m^2 Da/15$
3.50 (a) 3/4, (b) $hw(3/5)\rho V_1^2\mathbf{i} - 2hw\left(p_{ig} + \rho V_1^2/3\right)\mathbf{j}$
3.51 (a) $U_0\delta W/3$; $\rho U_0^2\delta W/3$
3.52 $C_D = 1/3$
3.53 (a) $U_\infty H/2L$, (b) $F = \rho U_\infty^2 HW/6$
3.55 $2gh^2\sin\theta/\pi\nu$
3.56 $4\mu V_s L^2/\left(\pi d^2\rho V_e^2\cos\alpha\right)$
3.57 $\rho\pi D^2 V^2 L/4$ CCW
3.58 $F_y/F_x = \sin\theta/(1 + \cos\theta)$

3.59	(a) $\rho V^2 WD((\cos\theta - 1)\mathbf{i} + \sin\theta\mathbf{j})$, (b) $\rho V^2 WD((\cos\theta - 1)\mathbf{i} + \sin\theta\mathbf{j})/4$
3.60	$\mathbf{F} = 998\mathbf{i} - 1729\mathbf{j}\ N$
3.61	(a) 5, (b) $4.4\,m/s$, (c) $-\rho V_2^2 H_2 w((1 + \cos\theta)\mathbf{i} + \sin\theta\mathbf{j})$, (d) magnitude $\times 1/4$
3.62	$3210\,lb_f$
3.63	(a) $F = 0$, (b) $113\,N$, (c) $1.63\,m$

Chapter 4

4.1	$y - 2 = \ln x$
4.2	Need solution
4.3	Need solution
4.4	$y = 2x$; 0.25 time units
4.6	(d) $p_{2g} = -8\rho V_1^2/9$, (e) $p_{1g} = 11\rho V_1^2/18$
4.8	$p_1 - p_2 = 15\rho V_1^2/2$
4.9	24
4.10	7°, $(A_1/A_2)^2 - 1$
4.11	$42.8\,m/s$
4.12	$15\rho V_0^2/8$
4.13	$-75\,Pa$
4.14	$\sqrt{2\,(p_2 - p_1)\,/\rho}$
4.15	$10.16\,psia$, $8.64\,psia$
4.16	$81.2\,m/s$; $C_p = -0.833$
4.17	$0.517\,psi/ft$
4.18	(a) $p_{1g} = 15\rho V_1^2/2$, (b) 25.2° from horizontal
4.19	$F_x/(\rho U_1^2 A_1) = 4.5$
4.20	$365\,lb_f$
4.21	$-\rho V_2^2 \pi R^2((1 + \cos\theta)\mathbf{i} + \sin\theta\mathbf{j})$
4.22	$-\rho V_2^2 \pi R^2((17/2 + 4\cos\theta)\mathbf{i} + 4\sin\theta\mathbf{j})$
4.23	(a) $\frac{15}{2}\rho U_1^2$, (b) $\rho A_1 V_1^2\,(-11.96\mathbf{i} - 2\mathbf{j})$
4.24	$-9\rho V_1^2 A_1/2$
4.26	$8.58\,ft^3$
4.27	$h = 8\dot{q}^2/(g\pi^2 d^4)$
4.28	$A\sqrt{2gH}$, $\rho g H$
4.29	$h \sin^2\theta$
4.30	$V = \sqrt{gL}$
4.31	$8.21\,ft$
4.32	(b) $A/A_t = \sqrt{H_1/(H_1 - y)}$, (c) $H_3 = H_1 \sin^2\theta$, (d) vol $= 2H_1 A_t \sin\theta$
4.33	(a) $V_e = 4.95\,m/s$, (b) $V_e' = 4.89\,m/s$
4.34	(a) $7.67\,m/s$, (b) $2.67\,m/s$, $25,600\,Pa$, (c) $7.75\,m$
4.35	$H\sin^2\theta$; $-2\rho g H A\cos\theta$
4.36	(a) $\sqrt{2gH}$, (b) $-2\rho g H A\cos\theta$, (c) $H\sin^2\theta$
4.37	$H\sin^2\theta$; $\rho g H\,(A_T + 2A_j \sin\theta)$
4.38	$H_3 = H_1 \sin^2\theta$; vol $= 2H_1 A_2 \sin\theta$
4.39	(a) $255\rho V^2/2$, (b) $289\pi\rho V^2 D^2/8$ (vertical)
4.41	$F = \rho V_1^2 HW/2$ (to right)
4.42	(a) $\rho_2 g D$, (b) $(A_1/A_2)^2 = 2g(\rho_2 D - \rho_1(z_2 - z_1))/\rho_1 V_1^2) + 1$
4.43	$V^2 = (2\rho_m gh/\rho_a)/((D/d)^4 - 1)$
4.44	$409\rho_1 U_1^2/16$
4.45	$\rho_2/\rho_1 = 16/3$; $C_p = 4$
4.46	(a) $A_3/A_2 = \sqrt{h_1/(h_1 + h_2)}$, (b) vol $= 2\sqrt{h_1}A_2\left(\sqrt{h_1 + h_2} - \sqrt{h_1}\right)$
4.47	$(d^3 V_i/8D)\sqrt{(\rho\pi/2Mg)}$
4.49	(a) 0.676, 1.172, (b) $-0.711\rho V_1^2$
4.50	(a) $2V$, (b) $3\rho V^2/2$, (c) $5\rho V^2 A/2$, 51.3°
4.52	(a) $V_1/2$, (b) $-p_{1g}A_1 - \rho V1^2 A_1\,(1 + (3/4)\cos\theta)$ (c) $(1/4)\rho V_1^2 A_1 \sin\theta$, (d) $(3/2)\rho V_1^2$
4.53	(a) $-\rho A_2^2 V_1^2/A_2^2\,(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$, (b) $(1/2)\rho V_1^2\left[(A_1/A_2)^2 - 1\right]$, (c) $-(1/2)\rho V_1^2 A_1\left[(A_1/A_2)^2 - 2\,(A_1/A_2)\cos\alpha + 1\right]$, (d) $\rho V_1^2 A_2\,(A_1/A_2)^2 \sin\alpha$

4.54 (a) $3\rho V_1^2/2$, (b) $+\rho V_1^2(\cos\theta - 5/2)$
4.55 (a) $0.5\rho V_1^2\left[(A_1/A_2)^2/4 - 1\right]$, (b) $-0.5\rho V_1^2\left[(A_1/A_2)^2/4 + 1 + (A_1/A_2)\sin\theta\right]\mathbf{j}$

Chapter 5

5.2 4/3
5.3 45°
5.4 $\nabla\cdot\mathbf{V} = 2 + 5z^2$
5.5 No
5.6 (a) [2], (b) $4xy$, (c) 63.4°
5.7 (a) $-\rho(2yt + y^2x)$, (b) $2xy + 4xy^2t^2 + 2y^3x^2t/3$
5.8 (a) $-z + 4xy + x^4y/3 - ztx^2$
5.9 $x = 3$
5.10 yes, flow is incompressible
5.11 incompressible
5.12 yes, flow is incompressible
5.13 $-3/x$; $-2(6x + 3yx + 2t + yt)/(xt)$
5.14 $2at + 18xy + 16x^2$; $-18(at^2 + xy)$
5.15 (a) 2, (b) $-2t - yx$, (c) $2x(1 + 2t^2)$
5.16 (b) $8(\mathbf{i}+3\mathbf{j})$, (c) 24, (d) L^3/T
5.17 (a) $-4xy$, (b) $(8x^3y^2 + 6x^2)\mathbf{i} + 12\mathbf{k}$
5.18 (c) $D\mathbf{V}/Dt = 18x^3\mathbf{i} + (4zx + 12x^2zt + 16zx^2t^2)\mathbf{k}$, (d) $\nabla\cdot\mathbf{V} = 6x + 4xt \neq 0$
5.19 (d) $a_x = 2xy + 4xy^2t^2 + 2x^2y^3t/3$
5.20 (d) $\theta = \pi/4$, (e) $-2xy^2\mathbf{i} + 18y^3\mathbf{j}$
5.21 (d) $-z^3/3 + z^5t^2/3$
5.22 (c) n0, (d) $a_x = xy^2z^2 + xzt^2 + 3xy$
5.23 (c) $2\mathbf{i} - 2\mathbf{j}$, (d) 0.5
5.24 (d) $4z + 5xy^3t$, (e) $-\rho(4z + 5xy^3t)$
5.25 (b) $a_x = x^2(x^3/3 - 2)$
5.26 [2], steady, $-2xz$
5.27 (a) Yes, (b) x^2z^3
5.28 [2], incompressible, rotational
5.29 [2], unsteady, incompressible, $2x\left(1 + 2t^2\right)\mathbf{i} - 2y\left(1 - 2t^2\right)\mathbf{j}$
5.30 [3], incompressible, $2xy^2z^2$, $-26.6°$, (d) $2U_m/3$
5.31 [3], incompressible, $(9 + 4z)(x\mathbf{i} + y\mathbf{j})$, $-\rho(9 + 4z)(x\mathbf{i} + y\mathbf{j})$
5.32 [3], incompressible, $x\mathbf{i} + 4y\mathbf{j} + z\mathbf{k}$
5.33 [3], steady, incompressible, 0, $4\mu\mathbf{k}$
5.34 [3], [-1,0.5], $\left(ab + 2b^2\right)x^2y^3$
5.35 [2], steady, incompressible, $8x^3\mathbf{i} + 24xz^3\mathbf{j} + 8x^2z\mathbf{k}$, $-\rho\left(8x^3\mathbf{i} + 24xz^3\mathbf{j} + 8x^2z\mathbf{k}\right)$
5.36 (a) $-26.6°$, (b) $1/y = 0.5\ln x + 1$, (c) $-14.85\,Pa$
5.37 (a) [2], (b) compressible, (c) no, (d) $4x\mathbf{i} + 2y^3\mathbf{j}$
5.38 (a) [2], (b) no, (c) incompressible, (d) $-(1/3)\left(z^3 + z^5t^2\right)$
5.39 (a) [2], (b) incompressible, (c) $4x^3y^2$, (d) $-4x\mathbf{i} + ay\mathbf{j} - \left(4y^2 + 2x^2\right)\mathbf{k}$
5.40 incompressible, $-6x\mathbf{i} - 6x^2\mathbf{j}$, -6μ
5.41 [3], incompressible, $3x(y\mathbf{j} + z\mathbf{k})$, $-3\rho x(y\mathbf{j} + z\mathbf{k})$
5.42 [2], unsteady, incompressible, $-6z^2$, $6z$
5.43 $-y^3/3$
5.44 $v = y^2$
5.45 (a) $-2y$, (b) $p_0 - (\rho/2)\left(4x^2 - 12xy + 13y^2\right)$
5.46 $\rho = c/x^2$
5.47 $a + e + j = 0$
5.48 $u = -2yx - 2x + f(y)$
5.49 $v = Ay/x^2$
5.50 $(4/r^2 - 1)\cos\theta$
5.51 (a) $u_r = \overline{v}r/2h$, (b) $a_r = 2.22\times 10^5\,m/s^2$

Chapter 6

6.3 $\psi = -(A/k)e^{ky} \cos kx$
6.4 (a) yes, (b) irrotational, (c) $p = \rho A^2 k^2 \sin^2 kx$
6.5 (a) yes, (b) rotational, (c) $p = -2\rho A^2 k^2 \sin^2 kx$
6.6 (b) $u = V_0$, $v = 0$, (c) V_0
6.7 $V_0(-\sin\alpha\ x + \cos\alpha\ y)$
6.8 (b) $u = x$, $v = -y$, (c) 1
6.9 $\psi = r^3 \sin 3\theta$; $|\mathbf{V}| = 3r^2$
6.10 (a) $V_0 x$, (b) $V_0(\cos\alpha\ x + \sin\alpha\ y)$, (c) $x^2 + y^2$, (d) $r^3 \cos 3\theta$
6.11 $\psi = 2xy$
6.12 $\psi = c\left(x^2 y + a^2 y - y^3/3\right)$, $\phi = c\left(y^2 x - a^2 x - x^3/3\right)$
6.13 $u_r = 0$; $u_\theta = 20\,m/s$
6.14 (a) $ky^2/2$, (b) no, flow is rotational
6.15 $\psi = 1.04y^2 + 1.5y\ m/s$
6.16 $\theta - \log r$
6.17 (b) $u_r = 0$, $u_\theta = A/r$, (c) $-A \log r$
6.18 (b) 0, $\sqrt{2}$ and 1.225, -1.225, (c) $(2r^{3/2} \cos 3\theta/2)/3$
6.19 (a) $\phi V_0 x + \frac{q}{2\pi}(\ln\sqrt{x^2+y^2} - \ln\sqrt{(x-b)^2+y^2}$, (b) $b/2 - \sqrt{(2qb + \pi b^2 V_0)/4\pi V_0}$
6.20 $\theta = \pi/6$
6.21 (a) $r = Q/(2\pi U_\infty)$, $\theta = \pi$, (c) $2\,m$, (d) $(p - p_\infty)_{max} = 2.408\,Pa$, $(p - p_\infty)_{min} = -1.4\,Pa$
6.23 $q = 0.6\,m^2/s$
6.24 $q = 3.75\,m^2/s$
6.25 $\Gamma = -21\,m^2/s$
6.26 $1.592\,m/s$ for $\Gamma = -10\,m/s$, $s = 4\,m$

Chapter 7

7.7 $0.0129 \times 10^{-6}\,m^2/s$
7.8 $12.5\,mm$
7.9 (b) $30 \times 10^8\,lb_f/in^2$, (c) $0.05\,Hz$
7.10 (b) $\omega_m = 4\omega_p$, $T_m = T_p/2$
7.11 (a) 4, (c) $P_p = 4P_m$, $V_m = V_p$
7.12 (b) $1794\,rpm$, $0.776\,N \cdot m$
7.13 same
7.14 $2gL(\rho_f a/\dot{m}_f)^2 = 1 - (\rho_f/\rho_a)(a/A)^2(\dot{m}_a/\dot{m}_f)^2$
7.15 (a) $50\,ft/s$, (b) $3.75\,rps$
7.16 (b) $V_m = V_p/2$, $f_m = 2f_p$
7.17 (b) $d_2 = d_1/\sqrt{2}$, $N_2 = N_1\sqrt{2}$, (c) none
7.18 (b) $V_m = V_p$, $F_m = F_p/100$
7.19 (b) $D_m = D_p/2$, $\omega_m = 4\omega_p$
7.20 (b) $V_{1m} = V_{1p}/2$, $\nu_{1m} = \nu_{1p}/8$
7.21 (b) 0.2
7.22 (b) $15,600\,N$
7.24 (b) $V_m = 2\,m/s$, $\nu_{1m} = \nu_{1p}/125$
7.25 (b) drag increases by 10%
7.26 (b) $V_2 = 0.183V_1$, $\nu_2 = 0.0061\nu_1$
7.27 (b) 10, 1/100
7.28 (b) $0.033\,m/s$, $0.04\,Hz$
7.29 (b) 0.1, 0.1, 0.001
7.30 (b) $\sqrt{3}\,m/s$, $1/\sqrt{3}\,Hz$, $3\sqrt{3}\nu_{water}$
7.31 $1/\sqrt{10}$, 10, or $\sqrt{10}$
7.32 (b) 4, 1/2
7.33 (b) $5V_p$ or V_p
7.34 (b) 0.5, 1/80
7.35 (b) $2V - p$ or V_p
7.36 (b) 5, 5, 25

7.37 (b) 0.5, 0.125
7.38 (b) $2V_p$, $0.5\omega_p$
7.39 Need solution
7.40 (b) V_p, $2\omega_p$
7.41 (b) $0.25\dot{q}_p$, Δp_p, $0.25P_p$
7.42 (b) $\omega_p/100$, $10^5 P_p$
7.43 $0.1\,kg/m^2$, $\sqrt{10}\,m/s$, $1.9\,Hz$
7.44 (b) $V_m = V_p$, $\Delta p_m = \Delta p_p/2$

Chapter 8

8.1 600 (laminar)
8.2 10,000 (turbulent)
8.3 $\overline{V} < 0.065\,ft/s$
8.4 $0.0028\,ft/s$, $0.0141\,ft/s$
8.5 (a) 519 (laminar), (b) 173,000 (turbulent), (c) 5.19×10^6 (turbulent)
8.6 64.2; 0.0787
8.7 22.0
8.8 $4.50\,psi$
8.9 $87\,min$
8.10 15
8.11 $\Delta T = -15^\circ K$
8.12 $26.8\,m/s$, $448\,m/s$
8.13 $325^\circ K$
8.14 (a) $5.25 \times 10^5\,Pa$, (b) $5881\,N \cdot m$
8.15 $p_{Bg} = 81,500\,Pa$
8.16 $5.10\,m/s$
8.17 $\dot{q} = 0.022\,m^3/s$
8.18 $p_{1g} = 24.7\rho V^2$
8.19 $480\,m$
8.20 $H = (V^2/2g)(1 + fL/16D + C_{D1}/16 + 2fL/D + C_{D2})$
8.21 $0.22\,m/s$, $33 \times 10^{-6}\,m^3/s$
8.23 (b) $y/h = u/U_0$
8.24 $6\mu V/D$
8.25 (a) $-(R/2)(dp/dx)$, (b) $-2\mu U_{CL}/R$
8.26 $40.0\,ft^3/s$, $0.057\,psig$
8.27 $0.456\,ft^3/s$, $-6.74\,psig$
8.28 $0.0748\,m^3/s$ $(0\,yrs)$; $0.0738\,m^3/s$ $(5\,yrs)$; $0.0728\,m^3/s$ $(10\,yrs)$; $0.0716\,m^3/s$ $(20\,yrs)$
8.29 13.5°
8.30 6.5°
8.31 $h_1/h_2 = 7.9$
8.32 $2.5\,in.$
8.33 $V_e = \sqrt{80gd}$
8.34 $4.7\,hp$
8.35 $+200\,kW$ (pump)
8.36 $-768\,kW$ (turbine)
8.37 $131\,kW$
8.38 (a) $0.0924\,Pa$, (b) $3.70\,Pa$, (c) $Re = 865$
8.39 $\overline{V} = U_{CL}/2$; $C_f = 16/Re$
8.40 (b) $-8U_0\mu/D^2$, (c) $-8U_0 y/D^2$
8.41 (a) $\tau_w = \rho g D/4$, (b) $\nu = gD^2/(16U_c)$
8.42 (a) $u = (h^2 g \sin\theta/\nu)(y/h - \frac{1}{2}(y/h)^2)$; $\dot{q}$/width $= gh^2 \sin\theta/(3\nu)$, (b) $\dot{q}$/width $= h\sqrt{2gh\sin\theta/f}$
8.44 $227.5\,W$
8.45 $(16 + C_{D1} + 32fL/D + 16C_{D2})(\overline{V}^2/32g)$
8.46 (a) $10.4\,m/s$, (b) $44.3\,m/s$, (c) $482\,kW$
8.47 (a) $1.81\,m/s$, (b) $1/6\,m$, (c) $990\,W$
8.48 (a) $1.89\,m/s$, (b) $-30,322\,Pa$, (c) $109\,W$

8.49 $75.7\,kW$
8.50 (a) turbulent, (b) $5.9\,m$
8.51 $57.2\,kPa$
8.52 (a) $\sqrt{(2gH)}$, (b) $\sqrt{(gH/3)}$, (c) $0.15\pi gH\sqrt{(gH/3)}$

Chapter 9

9.1 $Re_L = 43,000$ (laminar)
9.2 $1/3$
9.3 $\mu U_e/\delta$, $2/Re_\delta$, $\delta/2$, $\delta/6$
9.4 (a) $16/Re_D$, (b) $\pi R/2$, $\pi R/6$
9.5 (c) $\overline{V} = U_m/3$; $C_f = 12/Re$
9.6 $16\mu/(\rho h U_m) = 16/Re_h$
9.7 (a) $\delta^*/x = (K/2)/\sqrt{Re_x}$, $\theta/x = (K/6)/\sqrt{Re_x}$, (b) $9/7$
9.10 (a) 1.5, 0.5, (b) $((3\mu w U_e/c)\sqrt{Re_L}$, (c) 0.375δ
9.11 (a) $\rho g\delta\sin\theta$, $\mu V_s/\delta$, (b) $2\sin\theta/F_s^2$, $2/Re_\delta$
9.12 $\delta/10$, $9\delta/110$, $11/9$
9.13 (a) $Re_x = 1.28\times10^6$ (turbulent), (b) $22\,mm$, (c) 0.00444, (d) $1.05\,N$
9.14 (a) $U_e = 1.153\,m/s$, (b) $\delta = 22\,mm$, (c) 0.00444, (d) $2.94\,N$
9.15 (a) $1.32\,ft$, (c) $0.462\,lb_f$
9.16 1.35
9.17 (a) $(0.037U_e/Re_x^{0.2})(y/\delta)^{8/7}$, (b) 0.0067°, 0.0268°, 0.107°
9.18 (a) $7/72$, (c) $0.296\rho U_e^2 k/Re_x^{0.2}$
9.19 (b) $p_1 - p_2 = \frac{1}{2}\rho U_1^2((U_2/U_1)^2 - 1)$
9.20 (a) $\tau_w = 2\mu U_{max}/h$; (b) $2U_{max}/h$
9.21 $\pi\mu U_e/(2\delta)$, π/Re_δ
9.22 $4/3$
9.24 (a) $3\delta/8$, $39\delta/280$, $35\delta/13$, (b) $F_D = 2\rho w U_e^2\theta$
9.25 $d\theta/dx = (d\delta/dx)/6$; $C_f = 2\nu/U_e\delta$
9.26 $\tau_w = 1.5\mu U_e/\delta$
9.27 (a) $3\mu U_e/2\delta$, (b) $C_f = 3/Re_\delta$, (c) 0.375
9.28 (a) 0.364, 0.137, (b) δ^*/x (half angle), (c) $1.308/\sqrt{Re_L}$
9.29 $193\,hp$
9.31 $1.85(L/D)(1/C_D)Re_L^{-0.2}$
9.34 $1200\,N$
9.35 $83\,lb_f$
9.36 $406\,lb_f$
9.37 $72\,lb_f$
9.38 (a) $1.20\,m/s$, (b) $35.6\,N$
9.39 (a) $138\,lb_f$, (b) $92\,mph$, $62\,mph$
9.40 $3.61\,m/s$
9.41 $\approx 30\,m/s$
9.42 $31.3\,m/s$
9.43 $6.95\,m/s$
9.44 $0.026\,m/s$
9.45 $28.6\,m/s$, $3.16\,m/s$
9.46 (a) $1035\,Hz$, $1922\,Hz$, (b) $690\,Hz$, $1281\,Hz$, (c) $517\,Hz$, $961\,Hz$
9.47 (a) $1.76\,Hz$, $25\,ft$, (b) $422\,Hz$, $2.5\,in.$

Chapter 10

10.1 30°
10.2 $4.64\,m$
10.3 $140\,m/s$
10.4 $U \pm \sqrt{gy}$
10.7 $F_1 < 1$, $F_2 > 1$
10.9 $15.6\,ft^3/s$
10.10 $8.3\,ft$

10.11 $F_1 = 0.386$, $F_2 = 0.932$
10.12 $F_1 = 0.192$; $H/Y_1 = 0.519$
10.13 $F_1 = 0.316$; $F_2 = 2.53$
10.14 $F_1 = \sqrt{2h_1/Y_1}$; $Y_2 = 2(h_1 - h_2)/3$
10.15 0.990, 0.316
10.16 (a) $F_1 \approx \sqrt{2H/h_1}$; (b) $F_1 = 2.53$; (c) $b = 1.41h_1$
10.17 $h/H_1 = 1 - W_1/W_2$
10.18 (a) 0.543, (b) 0.266, (c) $C_D = F_x/\rho g w H_1^2 = 0.218$
10.19 (a) 0.5, (b) 0.77, (c) 0.10
10.20 (a) 2, (b) 0.707, (c) 0.25
10.21 (a) 0.5, (b) 0.77
10.22 (a) 2.4, 0.075, (b) 2.2
10.23 (a) 0.567, (b) 0.44
10.24 (a) supercritical, (b) 1.837, (c) 0.438, (d) 3.541
10.25 (c) 0.5, 0.77, (d) 2/3
10.28 $V_1 = 5.43\,m/s$, $V_2 = 2.71\,m/s$
10.29 $6.07\,ft$
10.30 (d) $H_2/H_1 = (\sqrt{1+8F_2^2} - 1)/4F_2^2$, or $H_1/H_2 = (\sqrt{1+8F_2^2} - 1)$, (e) $2.26\,m$
10.31 $2.67\,ft$
10.32 $1.813\,m$
10.33 $2.65\,m$
10.34 $U_2/U_b = 0.5$
10.35 $2.04\,ft$, $2.84 ft/s$
10.36 $2.96\,m$
10.37 $4.64\,m/s$
10.38 $0.74\,m$
10.39 $2.25\,m/s$, $3.34\,m/s$
10.40 $4.42\,m/s$
10.41 $3.42\,m/s$, $0.71\,m/s$
10.42 (a) $5.5\,m/s$, (b) $2.7\,m/s$
10.43 $6.43\,m/s$
10.45 (a) $Y_2 = 2Y_1/3$, (b) $F_3 = 2$, (c) $Y_4 = 0.791Y_1$, (d) $F_4 = 0.547$, (e) $F_5 = 0.30$
10.46 (a) $Y_2 = 3Y_1$, (b) $Y_3 = 7.12Y_1$, (c) $F_3 = 0.547$, (d) $F_4 = 0.872$, (e) $B_4 = 0.318B_1$
10.47 (a) $Y_2/Y_1 = 2\left(F_1^2/2 + 1 - H/Y_1\right)/3$, (b) $F_3 = 2\sqrt{2}F_1$, (c) $Y_4 = Y_1$
10.48 (a) 0.707, (b) $(h_1/h_3)^3 - 5\,(h_1/h_3) + 4 = 0$, (c) 0.97
10.49 $F_1 = 4.0$, $Y_2 = 12.9\,mm$
10.50 $h_3/h_2 = 1.863$
10.51 $Y_2/Y_1 = 0.63$, $Y_3/Y_1 = 0.19$
10.52 (a) $F_2 = 2.55$, $B_2/B_1 = 1.66$, (b) $Y_3/Y_1 = 1.57$
10.53 (a) $F_3 = 2.83$, (b) $B_3 = 2B_1$, (c) $Y_4 = 0.883Y_1$
10.54 (a) 0.35, (b) 0.31, (c) 0.25, (d) 0.74
10.55 (a) $1/\sqrt{3}$, (b) $\sqrt{8/3}$, (c) 2.5, (d) 0.642
10.56 (a) 0.75, (b) 0.77, (c) 2, (d) 0.55
10.57 $F_1 = \sqrt{2(H - Y_1)/Y_1}$
10.58 (a) $F_2 = 1.51$, (b) $3.38Y_1$
10.59 0.39
10.62 (a) $F_2 = 1.98$, (b) $F = -0.115\rho g w Y_1^2$, (c) $F = 0$ (ignoring friction), (d) $F_3 = 0.552$
10.63 (a) $F_2 = 1$, (b) $Y_2 = 0.63Y_1$, $h = 0.18Y_1$, (c) $C_D = 0.069$
10.64 (a) 0.152, (b) 0.529
10.65 (a) $15\rho g h^2/32 - 3\rho V^2 h$, (b) $Y_d = (h/8)(\sqrt{1 + 512F^2} - 1)$, $F_d = (h/Y_d)^{1.5}$
10.66 (a) $F_1 = 0.193$, (b) $H = 0.519Y_1$, (c) $0.298\rho g w Y_1^2$

Chapter 11

11.1 $347.2\,m/s$, $1017\,m/s$
11.2 1.823, 1.856, 1.929, 2.010
11.3 Subsonic

11.4 $14.78\,psi$, $60.8^\circ F$ ($520.5^\circ R$)
11.5 $122\,kPa$, $29.9^\circ C$ ($303.1^\circ K$
11.6 $128\,kPa$, $34.9^\circ C$; $102\,kPa$, $21.9^\circ C$; $35.7\,kPa$, $-30.0^\circ C$
11.7 0.437, 2.033
11.8 $3444\,ft/s$ ($2348\,mph$)
11.9 2.13, 93×10^6; 2.03, 113×10^6
11.11 $266\,kPa$, $3.74\,kg/m^3$, $-25.6^\circ C$ ($247.6^\circ K$)
11.12 (a) $255\,m/s$, (b) $551\,kPa$, compared with $881\,kPa$
11.13 $6100\,Pa$, $0.236\,kg/m^3$, $104\,kPa$, $475\,m/s$
11.14 $247\,m/s$, $670^\circ K$, $320\,J/kg \cdot {}^\circ K$
11.15 a factor of 6
11.16 $68.6^\circ K$, $668\,Pa$, $0.91\,kg/s$
11.17 3.40, $0.0247\,m^3$, $0.0040\,m^3$
11.18 (a) $1.10 \times 10^6\,Pa$, (b) $196^\circ K$, (c) $19.6\,kg/m^3$, (d) $30\,kg/s$
11.19 (a) $0.120\,kg/s$, (b) $3.73\,kPa < p_b < 33.5\,kPa$
11.20 (a) $4.05\,kg/s$, (b) 0.147, 2.94, (c) $670\,kPa$, $20.3\,kPa$
11.21 14.74°
11.22 43.79°
11.23 (a) 2.68, (b) 2.24, (c) 1.93
11.24 (a) 2.00, $183\,kPa$, $XX^\circ K$, (b) $6.45\,J/kg \cdot {}^\circ K$
11.25 (a) 1.64, $312\,kPa$, $379^\circ K$, (b) $4.07\,J/kg \cdot {}^\circ K$, (c) 15.5°
11.26 1.67; $p_2/p_1 = 0.779$
11.27 23.7°, 20.6°
11.28 5.92°, $41,400\,N$
11.29 $L = 779\,N$, $D = 99.5\,N$

Chapter 12

12.1 $N'_{sd} = 53$, Francis
12.2 $303\,rpm$
12.3 $\dot{q} = 50.5\,ft^3/s$, $r = 9.25\,ft$
12.4 $\dot{q} = 3.34\,m^3/s$, $31.8\,MW$
12.5 $2r = 2.2\,m$, $N'_{sd} = 2.4$
12.6 $D = 0.82\,m$, $P = 5.4\,MW$, $N = 18\,rad/s$
12.7 $N_s = 1650$
12.8 $N_s = 6552$
12.9 centrifugal, mixed (respectively)
12.10 $1280\,rpm$
12.11 $1.85 \times 10^6\,gpm$
12.12 4 in series
12.13 (b) 1370
12.14 $6.5\,ft$
12.15 $347\,rpm$, $73,000\,gpm$
12.16 $1.86\,ft$, $172\,hp$, $121\,ft$
12.17 $0.076\,hp$
12.18 92%
12.19 $38,800\,ft \cdot lb_f$, $1480\,hp$
12.20 $393\,rpm$
12.21 $25.1\,kW$
12.22 $1580\,rpm$, $30.4\,hp$
12.23 (a) 61.6°, (b) $1.05\,MW$
12.24 0.71, $39\,hp$
12.25 $154\,kW$, $25\,rpm$, $5.6\,m/s$
12.26 0.74, 85%
12.27 82%, $170\,mph$, $1180\,hp$, $2609\,lb_f$, $0.36\,psi$
12.28 $420\,km/hr$, $10,220\,N$, $1.02\,MW$, $851\,kW$, 83%, $4020\,Pa$
12.29 4 shafts, $244,000\,lb_f \cdot ft$

12.30 $314\,kW$, $11\,m/s$, yes
12.31 $1.94\,ft^3/s$, $1.40\,ft^3/s$

Chapter 13

13.4 (a) very stable, (b) stable, (c) unstable
13.5 (a) neutral, (b) unstable, (c) stable
13.6 (a) $2.83\,m/s$, $3.33\,m/s$, (b) $5.02\,m/s$, $7.80\,m/s$
13.7 $13.6\,m/s$
13.8 (a) $6952\,ft$, (b) $4861\,ft$
13.9 (a) $2176\,m$, (b) $948\,m$
13.10 $378\,m$
13.11 (a) $3.0 \times 10^{-3}\,m/s$, $0.3\,m/s$, $4.3\,m/s$, (b) $6.0 \times 10^{-3}\,m/s$, $0.6\,m/s$, $6.0\,m/s$
13.12 (a) $5.4 \times 10^{-5}\,m/s$, $5.4 \times 10^{-3}\,m/s$, $0.13\,m/s$,
(b) $1.1 \times 10^{-4}\,m/s$, $1.1 \times 10^{-2}\,m/s$, $0.23\,m/s$
13.13 $0.37\,m/s$
13.14 $77\,ft$
13.15 $44\,\mu m$