ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

MGJ/08 Feb 1999

PURPOSE

The purpose of this exercise is to obtain a number of experimental results important for the characterization of materials. In particular, the results from the torsion test will be compared to the results of the engineering tensile test for a particular alloy using the effective stress-effective strain concept.

EQUIPMENT

- Constant-diameter gage section torsion specimen of 6061-T6 aluminum
- Torsion test machine with grips, troptometer, and force sensor.

PROCEDURE

- Measure the diameter (D=2R) of the gage section for each test specimen to 0.02 mm.
- Install the bottom end of the torsion test specimen in the lower grip of the test machine. Rotate the lever arm as far to the right as possible. (Note: unscrew the horizontal threaded drive rod as much as possible).
- Rotate the top grip as far as possible in the direction necessary to remove the 'slack' from the reaction cables and install the top end of the torsion test specimen in the top grip of the test machine.
- Zero the output of the force sensor.
- Use the threaded drive rod to apply torque to the base of the test specimen and record the applied torque, T, versus angular rotation, θ, at 2° increments until 30° of rotation.
- Remove the horizontal threaded drive rod and find the torque after 90° and 360° of rotation, being careful not to allow elastic unloading.
- After 360° of rotation, unload and remove the specimen. Measure the gage length L (grip to grip length) of the installed specimen to 0.1 mm.

RESULTS

- Plot measured torque, T, versus angular displacement per unit length, $\theta' = \frac{d\theta}{d\ell}$. Using linear regression, fit the curve to 30° of relative rotation. (It is assumed that T is proportional to θ' from θ=0° to θ = 30°). (Note that θ must be in radians, i.e. π radians = 180°).
- Calculate the shear modulus, G, from the linear portion of the T-θ' using linear regression to find dT/dθ' from θ=0° to θ = 30°. Compare this value of G to the shear modulus determined from the tensile test results (i.e. $G = \frac{E}{2(1+\nu)}$) using ν=0.345 for aluminum.
- Using the strength coefficient coefficient, K (or H), and the strain hardening exponent, n, determined from the tensile test for the approximate constitutive relation $\sigma = K\varepsilon^n = H\varepsilon^n$, integrate the predicted shear stress, τ, versus radial distance, r, to obtain the predicted torque, T, after 90° and after 360° of rotation. Compare these values of T to those measured experimentally. (Note that θ must be in radians for the calculations, i.e. π radians = 180°). Use the attached "cook book" method to facilitate your work.
- On the same graph, plot shear stress, τ, and engineering shear strain, γ as functions of radial distance, r, at 30° of rotation. Construct similar plots τ and γ vesus r for 90° and 360° of rotation. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

LABORATORY REPORT

1. As a minimum include the following information in the laboratory report.

a. Raw data (typed in tabular form)

b. Two values for the shear modulus, G (tension and torsion)

c. Two values of the torsional yield stress, τ_o (tension and torsion)

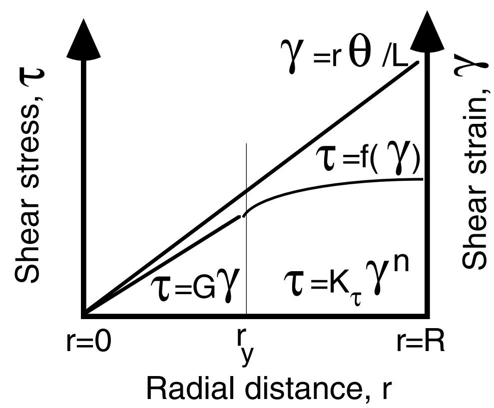
d. "n" and "K" from the tension test (use these in the calculations)

e. Total torque as required in the table:

Angle of Rotation	90°	360°
Predicted Torque ( )		
Measured Torque ( ) .		
% Difference		

f. Plot of Torque vs. Angular displacement per unit length (T vs. θ')

g. One graph each of τ and γ as functions of radial distance, r, for θ = 30°, 90°, and 360° (2 plots on each graph for a total of 3 graphs)



h. Discuss comparisons of basic mechanical properties as determined from tension and torsion tests. Compare results of these tests for each alloys to 'book' values from such sources as the ASM Metals Handbook. Comment on any differences. Compare the shapes of the stress vs. radial distance curves and the magnitudes of the plastic and elastic torques.

2. Include the following information in the appendix of the laboratory report. THIS MAY NOT BE ALL THAT IS NECESSARY (i.e., don't limit yourself to this list.)

a. Original data sheets and/or printouts

b. All supporting calculations. Include sample calculations if using a spread sheet program.

c. All "cookbook" calculations from the Torsion Test Solution Path.

* REFERENCES

Annual Book or ASTM Standards, American Society for Testing and Materials, Vol. 3.01 E143 Standard Test Methods for Shear Modulus at Room Temperature.

ME354 NOTES on Torsion Testing

ME 354, MECHANICS OF MATERIALS LABORATORY

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DATA SHEET

NAME_______________________________________**DATE**____________

LABORATORY PARTNER NAMES____

____

EQUIPMENT IDENTIFICATION______________________________________

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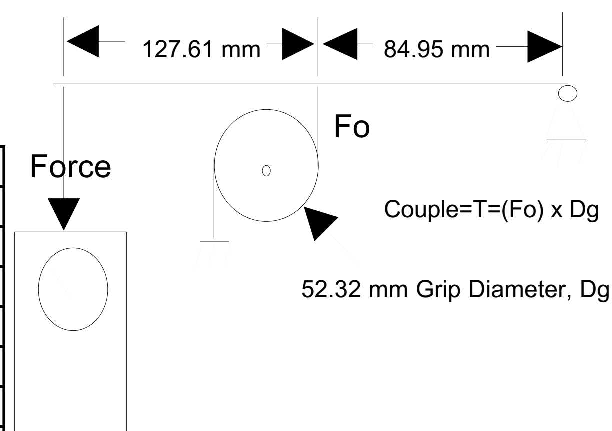
Note: Be sure to record units for each quantity.

Specimen measurments

D ( )	
L ( )	
J ( )	

Angle(degrees)	Force ( )
0	
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	
22	
24	
26	
28	
30	
90	
360	



ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

Torsion Test Solution Path

TORSION TEST

The initial set of calculations has input parameters obtained only from the torsion test. The results of these calculations will later be compared to results calculated with information obtained from the tension test.

1. Record the torsion specimen diameter (D=2R) and the length of the gripped section of the torsion specimen, L. Calculate the polar moment of inertia for a solid rod, $J = \frac{\pi D^4}{32}$.

   D = mm

   L = mm

   J = mm^4

2. From the measured torque, T, versus angular rotation, θ, data points, plot T versus relative angular deflection, $\theta' = \frac{d\theta}{d\ell}$ between two cross sections (i.e. $\frac{\theta}{L}$). Obtain the "best fit" of the linear portion of the T versus θ' data using linear regression. (It is assumed that T is proportional to θ' from θ=0° to θ = 30°). Determine the slope, dT/dθ' from θ=0° to θ = 30°. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

   dT/dθ' = (N-mm) / (rad/mm)

3. The shear modulus, G, from the torsion test can now be calculated from the relation:

   $G = \frac{dT_{(0-30°)}}{d\theta'_{(0-30°)}} \frac{1}{J} =$ (N/mm^2=MPa)

4. Finally, record the measured torques and calculate $\theta' = \frac{d\theta}{d\ell}$ for θ = 90° and 360°. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

   $T_{90°}$ = N-mm

   $\theta'_{90°}$ = /mm

   $T_{360°}$ = N-mm

   $\theta'_{360°}$ = /mm

TENSION TEST

This set of calculations has input parameters obtained only from the tension test. The results of these calculations will later be compared to results calculated with information obtained from the torsion test.

1. Record the uniaxial elastic modulus, E, uniaxial yield stress, σ_o, the strain hardening coefficient, K, and the strain hardening exponent, n, determined from the tension test.

   E = N/mm^2

   σ_o = N/mm^2

   K = N/mm^2

   n =

2. Calculate the value of the shear modulus from the results of the tension test:

   $G = \frac{E}{2(1+\nu)}$ using ν=0.345 for aluminum.

3. Using the effective stress concept, calculate the shear strength indicated by the tension test data such that:

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{xz}{}^2\right)\right]^{\frac{1}{2}}$$

   and setting $\bar{\sigma} = \sigma_o$ and solving for $\tau_{xy} = \tau_y$ (yield stress in shear) with all other stress equal to zero.

   τ_y = N/mm^2

EVALUATION OF TORSION TEST RESULTS FOR YIELDING

This set of calculations has input parameters obtained only from the torsion and tension tests. The results of these calculations are used to evaluate the shear stresses and shear strains across the radius of the torsion specimen as it yields.

1. Find the radius of the torsion specimen at yielding, r_y for $\theta = 90°$ (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

$$r_y = \frac{\tau_y}{G}\frac{R}{\gamma_{max}} \text{ where } \gamma_{max} = (\theta'_{90°})R$$

$$r_y^{90°} = \qquad\qquad\qquad\qquad \text{mm}$$

2. Within the elastic domain, the shear stress is a linear function of radial distance, r, such that

$$\tau(r) = \frac{\tau_y}{r_y} r$$

3. The shear stress as a function of radial distance, r, can now be multiplied by differential area element, $2\pi\,\mathrm{r\,dr}$ and a moment arm, r, and integrated to find the torque over the elastic domain. (i.e. $\sum \mathrm{M} = 0$).

$$T_e = \int_o^{r_y} \left(\frac{\tau_y}{r_y} r\right)(r) 2\pi\,\mathrm{r\,dr}$$

4. In the plastic domain, only the shear strain remains a linear function of radial distance, r. Therefore, it is advantageous to change the integration variable to γ. In order to accomplish this variable change, the shear stress, τ, moment arm, r, and differential area of integration, $2\pi\,\mathrm{r\,dr}$, must be expressed as function of γ.

   In the tension test the uniaxial stress, σ, was expressed as a function of uniaxial strain, ε , through the strenght coefficient, K (or H), strain hardening exponent, n, such that:

$$\sigma = \mathrm{K}\varepsilon^{\mathrm{n}} = \mathrm{H}\varepsilon^{\mathrm{n}}$$

   Since the uniaxial stress is identical to the effective stress, and the uniaxial strain is identical to the effective strain, the equation relating effective stress to effective strain would be exactly the same.

$$\overline{\sigma} = \mathrm{K}\overline{\varepsilon}^{\mathrm{n}} = \mathrm{H}\overline{\varepsilon}^{\mathrm{n}}$$

   When the effective stress and effective strain are evaluated for the case of pure torsion, the shear stress can be found as a function of the shear strain.

$$\tau(\gamma) = \left(\frac{1}{\sqrt{3}} \mathrm{K} \left(\frac{\gamma}{\sqrt{3}} \right)^{\mathrm{n}} \right)$$

Since $\gamma = \theta' r$ it is also true that $r = \left(\frac{\gamma}{\theta'} \right)$ and, since θ' is a constant .

Substituting these relations into the basic torsion integral yields:

$$T_e = \int_o^{r_y} \left(\frac{\tau_y}{r_y} r \right) (r) 2\pi \, \mathrm{r} \, \mathrm{dr}$$ for the elastic torque

$$T_p = \int_{\gamma_y}^{\gamma_{\max}} \left(\frac{1}{\sqrt{3}} \mathrm{K} \left(\frac{\gamma}{\sqrt{3}} \right)^{\mathrm{n}} \right) \left(\frac{\gamma}{\theta'} \right) 2\pi \, \frac{\gamma}{\theta'} \frac{\mathrm{d}\gamma}{\theta'}$$ for the plastic torque.

Note that the limits of integration are $\gamma_y = \frac{\tau_y}{G}$ and $\gamma_{\max} = \theta' R$. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

The total torque, T, is found as the sum of the elastic and plastic torques such that:

$T = T_e + T_p$ This torque value is then compared to the value measured in the torsion test.

For θ = 90°, calculated torques are:

T_e = N-mm

T_p = N-mm

T = N-mm

For θ = 90°, measured torque is:

$T_{90°}$ = N-mm

5. Steps 1 to 4 are repeated for θ = 360° (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

For θ = 360°, calculated torques are:

T_e = N-mm

T_p = N-mm

T = N-mm

For θ = 360°, measured torque is:

$T_{360°°}$= N-mm

6. Finally, plot τ and γ as functions of r after θ = 30° for relative rotations of θ = 90° and θ = 360°. (Note that θ must be in radians for the calculations, i.e. π radians = 180°).

ME 354, MECHANICS OF MATERIALS LABORATORY

MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS: TORSION TESTING*

NOTES on Torsion Testing

STRESSES IN THE ELASTIC RANGE

In the elastic range, stresses in the shaft will remain less than the proportional limit and less than the elastic limit as well. For this case Hooke's law will apply and there will be no permanent deformation. Hooke's Law for shear stress is as follows:

$$\tau = G\gamma$$

G = Modulus of rigidity (shear modulus)
τ = Shear Stress
γ = Shear Strain

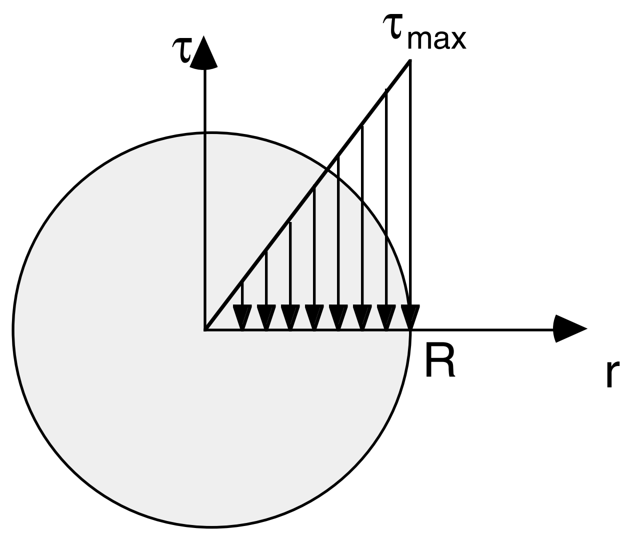


Figure 1. Distribution of Stress in the Elastic Range.

The elementary forces exerted on any cross section of the shaft must be equal to the magnitude T of the torque exerted on the shaft:

$$\int_o^R r(\tau dA) = \text{T} \quad \text{where} \quad \text{dA} = 2\pi \text{ r dr}$$

$$\gamma = \frac{r}{R}\gamma_{\text{max}}$$

$$G\gamma = \frac{r}{R}G\gamma_{\text{max}}$$

$$\tau = \frac{r}{R}\tau_{\text{max}}$$

$$T = \int r\,\tau dA = \frac{\tau_{max}}{R}\int r^2 dA$$

$$\int r^2 dA = J = \frac{1}{2}\pi R^4$$

$$T = \frac{\tau_{max}}{R} J$$

$$\boxed{\tau_{max} = \frac{TR}{J}}$$

$$\boxed{\tau = \frac{Tr}{J}}$$

The last two equations are known as the *elastic torsion formulas*.

ANGLE OF TWIST IN THE ELASTIC RANGE

For this section the entire shaft will again be assumed to be in the elastic range. Therefore Hooke's Law applies.

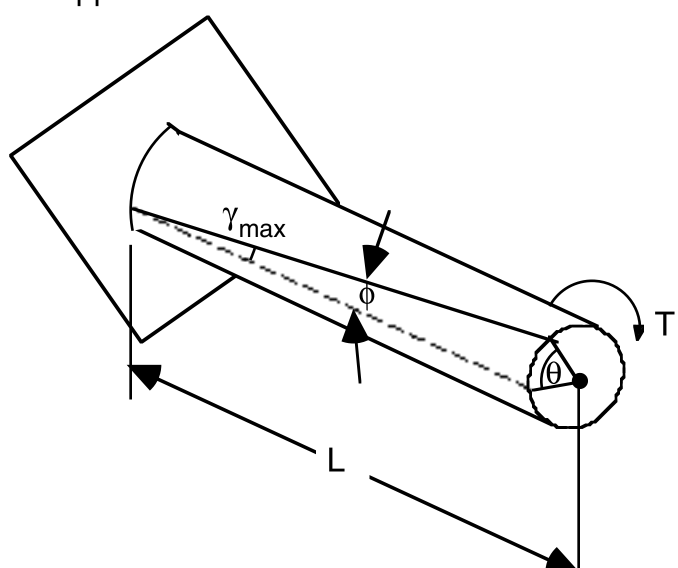


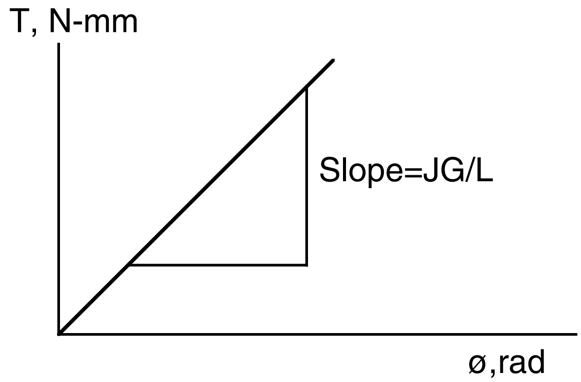
Figure 2. Demonstration of the Angle of Stress and the Shearing Strain.

$$\gamma_{max} = \frac{R\text{ø}}{L}$$

$$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{TR}{JG}$$

$$\boxed{\text{ø} = \frac{TL}{JG}}$$

The angle of twist, Ø, is expressed in radians. The angle of twist is proportional to the torque T applied to the shaft. The above equation provides a convenient method for determining the modulus of rigidity, G. Torques of increasing magnitude T are applied to the specimen, and the corresponding values of the angle of twist in a length L of the specimen are recorded. As long as the yield stress of the material is not exceeded, the points obtained by plotting Ø against T will fall on a straight line. The slope of the line represents the quantity JG/L from which the modulus of rigidity, G, may be computed.



PLASTIC DEFORMATIONS IN CIRCULAR SHAFTS

If the yield strength is exceeded in some portion of the shaft the relations discussed in the earlier sections cease to be valid. The purpose of this section is to develop a more general method for determining the distribution of stresses in the solid circular shaft, and for computing the torque required to produce a given force.

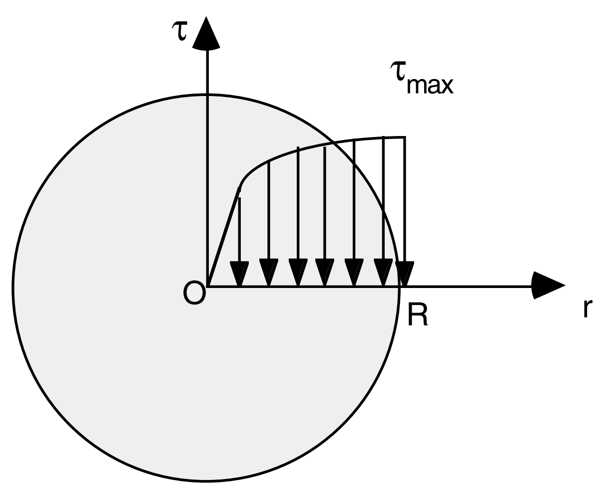


Figure 3. Stress Distribution in a Shaft for Plastic Deformation.

As the torque increases, τ_{max} eventually reaches the shearing yield stress, τ_y, of the material. Solving for the corresponding value of T, we obtain the value of T_y at the onset of yield:

$$T_Y = \frac{J}{R}\tau_Y$$

T_y is referred to as the maximum elastic torque, since it is the greatest torque for which deformation remains fully elastic. Recalling that, for a solid circular shaft, $J/R=1/2\ (\pi R^3)$ we have:

$$T_Y = \frac{1}{2}\pi R^3 \tau_Y$$

The τ_Y can be found using the data from the tension test and the idea of *effective stress*. Using the Distoritonal Energy (von Mises) criterion and the yield stress from the tensile test laboratory τ_Y can be determined.

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}{}^2 + \tau_{yz}{}^2 + \tau_{xz}{}^2\right)\right]^{\frac{1}{2}}$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[6\tau_{xy}{}^2\right]^{\frac{1}{2}} = \sqrt{3}\tau_{xy}$$

$$\bar{\sigma} = \sigma_0 = \text{yield stress from tension test}$$

$$\tau_{xy} = \tau_Y = \frac{\bar{\sigma}}{\sqrt{3}} = 0.577\,\bar{\sigma}.$$

$$\gamma_y = \frac{\tau_y}{G}, \text{measured } G \text{ from the elastic part of the torsion test.}$$

$$r_y = \frac{\gamma_y}{d\theta/d\ell}, \text{ where } d\theta/d\ell = \frac{\pi/2}{L} \text{ (at } 90^\circ\text{) and } \frac{2\pi}{L} \text{ (at } 360^\circ\text{) for this lab.}$$

The total torque is a function of the torque in the elastic range and the torque in the plastic range.

$$T_{total} = T_{elastic} + T_{plastic}$$

$$T_{total} = \int_0^{r_y} r\tau(r)2\pi r dr + \int_{r_y}^{R} r\tau(r)2\pi r dr$$

for the elastic range

$$\tau(r) = \frac{\tau_y}{r_y} r \quad \therefore \quad T_{elas} = \int_0^{r_y} \frac{\tau_y}{r_y} 2\pi r^3 dr$$

for the plastic range

$$\sigma = K\varepsilon^n = \sqrt{3}\tau, \quad \varepsilon = \frac{\gamma}{\sqrt{3}}, \; \sqrt{3}\tau = K\left(\frac{\gamma}{\sqrt{3}}\right)^n$$

$$\tau = \frac{K}{\sqrt{3}}\left(\frac{\gamma}{\sqrt{3}}\right)^n = 0.577K\left(\frac{\gamma}{\sqrt{3}}\right)^n$$

$$\therefore \quad T_{plastic} = \int_{r_y}^{R} \frac{K}{\sqrt{3}}\left(\frac{\gamma}{\sqrt{3}}\right)^n 2\pi r^2 dr$$

$$r = \frac{\gamma}{\frac{d\theta}{d\ell}}, \quad dr = \frac{d\gamma}{\frac{d\theta}{d\ell}}$$

$$T_{plastic} = \int (const)(\gamma)^{n+2} d\gamma$$

$$T_{plastic} = \int_{\tau_y/G}^{\gamma_{\max}} \frac{K}{\sqrt{3}}\left(\frac{\gamma}{\sqrt{3}}\right)^n 2\pi \frac{1}{\left(d\theta/d\ell\right)^3} \gamma^2 d\gamma$$

OR

$$T_{plastic} = \int_{r_y}^{R} \frac{K}{\sqrt{3}}\left(\frac{r\frac{d\theta}{d\ell}}{\sqrt{3}}\right)^n 2\pi r^2 dr, \quad \frac{d\theta}{d\ell} = \frac{\theta}{L}$$