ME 354, MECHANICS OF MATERIALS LABORATORY

PRESSURE VESSELS AND COMPOUND STRESSES/STRAINS

17 november 2002 / mgj

PURPOSE

The purposes of this exercise are to evaluate the compound stress/strain states in a thin walled closed end cylinder and to relate analytically and experimentally determined stresses and strains. The thin walled closed end cylinder is loaded by both an axially-applied load and an internal pressure. Strain gages arranged longitudinally and tangentially are used to experimentally determine axial and hoop strains. Thin wall pressure vessel stress equations and uniaxial loading stress equations along with constitutive relations are used to analytically determine the strains.

EQUIPMENT

- Strain-gaged, closed-end thin-wall pressure vessel
- Bridge completion and strain gage conditioning equipment
- "Dead-weight," lever arm axial-loading device
- Various "dead-weight" masses of 5.0 and 10.0 kg.
- Device for producing air pressure.

PROCEDURE

- Examine the test setup and take note of the following
  - a) Strain gage orientation on the pressure vessel
  - b) Strain gage connections and circuits
  - c) Dead weight lever arm axial loading device and the mechanical advantage

- Ensure that the loading system is disconnected from the freely-hanging pressure vessel
- Zero all the strain gage circuits

Longitudinal Loading Only

- Connect the loading system to the freely hanging pressure vessel• Apply total "dead weight" masses of m_A=5, 10, 15, and 20 kg to the pan of the dead weight lever arm. Record both the applied mass, m_A, the mass, m_P, of the pan and the mass of the lever arm in kg.
- Record the applied mass and the resulting strains of all the strain gages for each applied mass.
- Determine the elastic modulus, E, and the Poisson's ratio, ν, from the recorded strains and stresses calculated from the applied longitudinal forces and the dimensions of the pressure vessel

Pressure Loading Only

- Disconnect the loading systems from the freely-hanging pressure vessel
- Ensure that the strain gage circuits are zeroed
- Introduce a pressure of 500 to 700 kPa to the pressure vessel and record pressure and the resulting strains

Pressure Loading and Longitudinal Loading

- While the pressure vessel is still pressurized, reconnect the loading system and apply a total dead weight mass of 15 kg to the pan of the deadweight lever arm and record the applied mass, the pressure and the resulting strains
- Remove the mass, disconnect the deadweight loading system and release the pressure

BACKGROUND AND ANALYSIS OF RESULTS

Cylindrical or spherical pressure vessels are commonly used in engineering to serve as boilers or tanks. When under pressure, the material from which these vessels are made is subjected to stresses from all directions. Although this is the case, the vessel can be analyzed in a simple manner, provided that it has a “thin wall.”

“Thin wall” refers to a vessel having a ratio of inner radius to wall thickness of 10 or more (i.e., r/t≥10). Specifically, when r/t=10, the equations resulting from the thin-wall pressure vessel analysis will predict a stress that is ~4% less than the actual maximum stress in the vessel. As r/t increases, this difference decreases.

When the vessel wall is “thin”, the stress distribution throughout the thickness will not vary significantly and is therefore assumed constant and uniform. In analyzing stresses in thin wall pressure vessels, the pressure is assumed to be gauge since it measures the pressure greater than atmpospheric pressure which is assumed to exist both inside and outside the vessel’s wall.

An element in the wall of a cylindrical thin wall pressure vessel is subjected to three normal stresses when the vessel is internally pressurized: axial, hoop and radial (see Figure 1):

Axial stress: $\sigma_a = \frac{pR_i}{2t}$ at r=R_i and r=R_o (1)

Hoop stress: $\sigma_h = \frac{pR_i}{t}$ at r=R_i and r=R_o (2)

Radial stress: $\sigma_r = -p$ at r=R_i and $\sigma_r = 0$ at r=R_o (3)

where r is the variable for radius,R_i is the inner radius, R_o is the outer radius, p is the internal gauge pressure, t is the wall thickness,

The three stresses are mutually orthogonal and represent the principal normal stresses in the wall of a thinwall cylindrical pressure vessel as shown in Fig. 1.

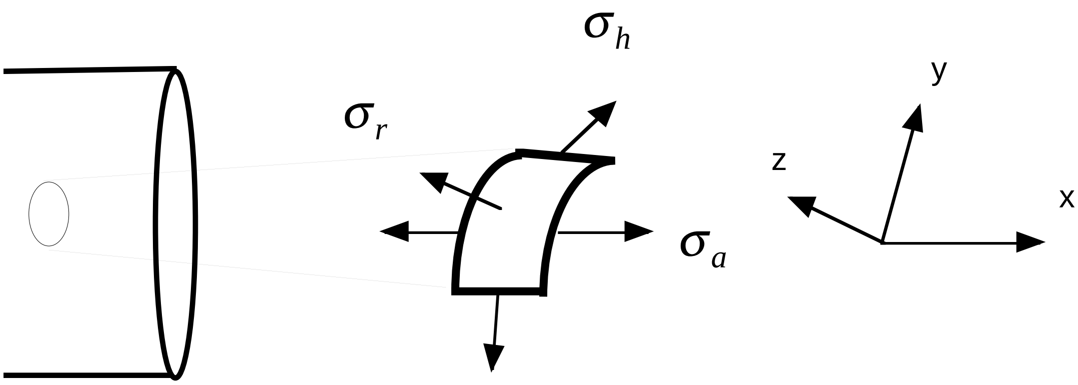

Figure 1 Illustration of three normal stresses acting on the wall of a thinwall pressure vessel

Compound stress states exist in many engineering components. Because stress is a tensor, having both type (normal or shear), direction (e,g, x, y, z or 1, 2, 3) and magnitude, stresses calculated from engineering formulae (e.g., σ=P/A or Eqs 1-3) cannot be added or subtracted indiscriminately. Instead only stresses of the same type and in the same direction can be “superposed” to find the resulting stress.

For example if a longitudinal force, F, is applied to an internally pressurized cylindrical thin wall pressure vessel, the resulting longitudinal normal stress, $\sigma_L = \dfrac{F}{\pi(R_o^2 - R_i^2)}$ can only be added to the axial stress, $\sigma_a = \dfrac{pR_i}{2t}$, resulting from the closed end pressure vessel. The total stress state for a longitudinally loaded, internally-pressurized thin-wall pressure vessel is the radial stress, the hoop stress, and superposed longitudinal and axial stresses. If a Cartesian coordinate system is applied as shown in Fig 1, the resulting coordinate stresses at the outer surface (r=R_O) of the pressure vessel can be expressed as:

x- stress: $$\sigma_x = \sigma_a + \sigma_\ell = \frac{pR_i}{2t} + \frac{F}{\pi(R_o^2 - R_i^2)} \qquad (4)$$

y- stress: $$\sigma_y = \sigma_h = \frac{pR_i}{t} \qquad (5)$$

z- stress: $$\sigma_z = \sigma_r = 0 \qquad (6)$$

Note that if the constitutive relations are applied to the coordinate stresses of Equations 4-6, the resulting strains in the x, y and z directions can be calculated and compared to measured strains such that:

x- strain from coordinate stresses: $$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad (7)$$

y- strain from coordinate stresses: $$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \qquad (8)$$

z- strain from coordinate stresses: $$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \qquad (9)$$

Alternatively, the constitutive relations can be applied to measured coordinate strains to calculate resulting stresses in the x, y and z directions such that:

x- stress from coordinate strains: $$\sigma_x = \frac{E}{(1+\nu)}\varepsilon_x + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \qquad (7)$$

y- stress from coordinate strains: $$\sigma_y = \frac{E}{(1+\nu)}\varepsilon_y + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \qquad (8)$$

z- stress from coordinate strains: $$\sigma_z = \frac{E}{(1+\nu)}\varepsilon_z + \frac{\nu E}{(1+\nu)(1-2\nu)}\left(\varepsilon_x + \varepsilon_y + \varepsilon_z\right) \qquad (9)$$

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DATA SHEET

NAME____DATE______________

LABORATORY PARTNER NAMES____

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EQUIPMENT IDENTIFICATION_______________________________________

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Note: Be sure to note units of each quantity.

Mass of Pan, m_P=___________________________________kg

Mass of Bar, m_B= ___________________________________kg

Half length of lever arm, L_1______________________mm

Half length of lever arm, L_2______________________mm

Longitudinal Loading Only

Added Mass, m_A (kg)	Strain Gage 1, SG1 (µ m/m)	Strain Gage 2, SG2 (µ m/m)	Strain Gage 3, SG3 (µ m/m)	Strain Gage 4, SG4 (µ m/m)	Strain Gage 5, SG5 (µ m/m)	Strain Gage 6, SG6 (µ m/m)

Pressure Loading Only

Applied Pressure, p (kPa)	Strain Gage 1, SG1 (µ m/m)	Strain Gage 2, SG2 (µ m/m)	Strain Gage 3, SG3 (µ m/m)	Strain Gage 4, SG4 (µ m/m)	Strain Gage 5, SG5 (µ m/m)	Strain Gage 6, SG6 (µ m/m)

Pressure Loading and Longitudinal Loading

Applied Pressure, p (kPa)	Added Mass, m_A (kg)	Strain Gage 1, SG1 (µ m/m)	Strain Gage 2, SG2 (µ m/m)	Strain Gage 3, SG3 (µ m/m)	Strain Gage 4, SG4 (µ m/m)	Strain Gage 5, SG5 (µ m/m)	Strain Gage 6, SG6 (µ m/m)

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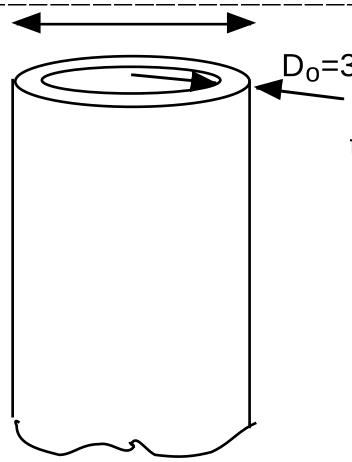
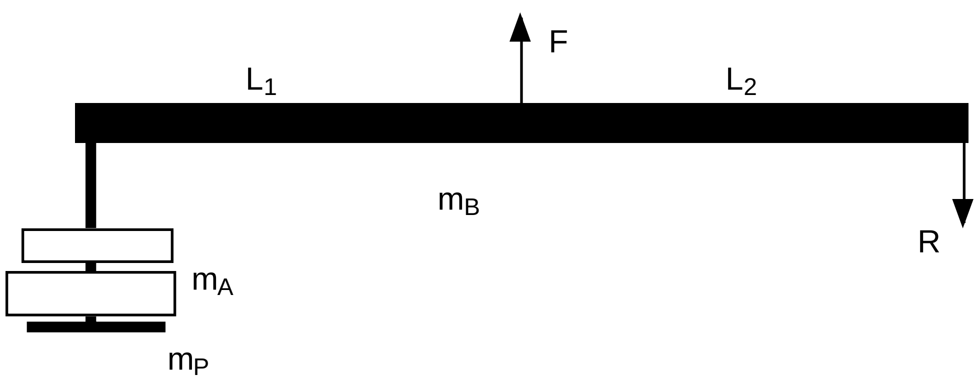
WORKSHEET

NAME____**DATE**_____________



1) **Longitudinal Loading Only** The inner diameter of the pressure vessel is D_o-2t=D_i=_________mm.

2) The cross sectional area of the pressure vessel is $\frac{\pi\left(D_o^2 - D_i^2\right)}{4}$ =A=__________mm^2.



3) The longitudinal force, F, imposed on the pressure vessel is calculated from the mechanical advantage of the lever arm system by summing moments about the fixed reaction point such that the force in Newtons is $F = g\frac{(L_1 + L_2)}{L_2}\left(\frac{m_B}{2} + m_A + m_P\right)$ where g=9.816 m/s^2.

The uniaxial normal stress applied longitudinally to the pressure vessel can be calculated as $\sigma_L = \frac{F}{A}$ in MPa when F is in Newtons and A is in mm^2.

The average longitudinal and transverse strains are calculated as $\varepsilon_L = \frac{\varepsilon_L^1 + \varepsilon_L^2}{2}$ and $\varepsilon_T = \frac{\varepsilon_T^1 + \varepsilon_T^2}{2}$, respectively, the 1 and 2 superscripts refer to rosette locations 1 and 2.

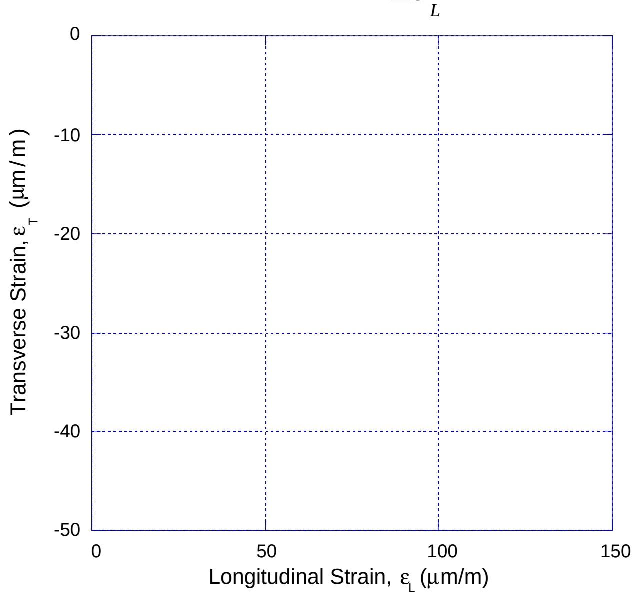
Results are tabulated as follows:

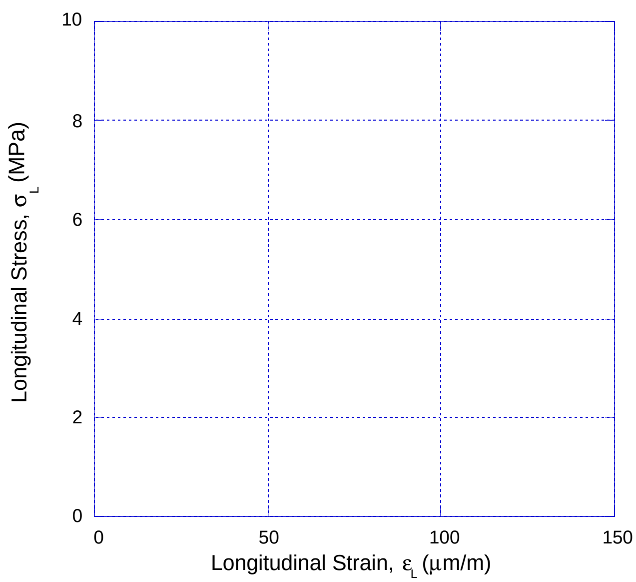
Added Mass, m_A (kg)	Calculated Longitudinal Stress, $\sigma_L = \frac{F}{A}$ (MPa)	Average Londitudinal Strain, $\varepsilon_L = \frac{\varepsilon_L^1 + \varepsilon_L^2}{2}$ (µ m/m)	Average Transverse Strain, $\varepsilon_T = \frac{\varepsilon_T^1 + \varepsilon_T^2}{2}$ (µ m/m)

Two elastic constants can be extracted from these test results...

The slope of the plot of longitudinal stress vs longitudinal strain is the elastic modulus, E such that $\sigma_L = E\varepsilon_L$ where $E = \left(\frac{\Delta\sigma_L}{\Delta\varepsilon_L}\right)(1{,}000{,}000)$ MPa for σ in MPa and ε in µm/m.

The slope of the plot of transverse strain vs longitudinal strain is Poisson's ratio, ν such that $\varepsilon_T = -\nu\varepsilon_L$ where $\nu = -\frac{\Delta\varepsilon_T}{\Delta\varepsilon_L}$.





Measured Elastic Modulus, E (MPa)	Expected Elastic Modulus, E (MPa)	Measured Poisson's Ratio, ν	Expected Poisson's Ratio, ν

4) **Pressure Loading Only:** For a thinwall pressure vessel, the stress in the axial direction due to internal pressure is: $\sigma_a = \frac{pR_i}{2t}$

For a thinwall pressure vessel, the stress in the hoop direction due to the internal pressure is: $\sigma_h = \frac{pR_i}{t}$

Using constitutive relations for plane stress state at the free surface of the pressure vessel, the strain in the axial direction due to internal pressure only is: $\varepsilon_a = \frac{1}{E}\left[\sigma_a - \nu(\sigma_h)\right]$

Using constitutive relations for plane stress state at the free surface of the pressure vessel, the strain in the hoop direction due to internal pressure only is: $\varepsilon_h = \frac{1}{E}\left[\sigma_h - \nu(\sigma_a)\right]$

Applied Presssure, p (kPa)	Calculated Axial Stress, σ_a (MPa)	Calculated Hoop Stress, σ_h (MPa)

Calculated Axial Strain, ε_a (μ m/m)	Measured Axial Strain $\left[\frac{\varepsilon_L^1 + \varepsilon_L^2}{2}\right]$(μ m/m)	Calculated Hoop Strain, ε_h (μ m/m)	Measured Hoop Strain $\left[\frac{\varepsilon_T^1 + \varepsilon_T^2}{2}\right]$(μ m/m)

5) **Pressure Loading + Longitudinal Loading:** For a thinwall pressure vessel, the stress in the axial direction due to internal pressure is: $\sigma_a = \frac{pR_i}{2t}$. The uniaxial normal stress applied longitudinally to the pressure vessel can be calculated as $\sigma_L = \frac{F}{A}$.

For at thinwall pressure vessel, the stress in the hoop direction due to the internal pressure is: $\sigma_h = \frac{pR_i}{t}$

Using constitutive relations for plane stress state at the free surface of the pressure vessel, the total strain in the axial direction due to internal pressure only is:

$$\varepsilon_a = \frac{1}{E}\left[(\sigma_a + \sigma_L) - \nu(\sigma_h)\right]$$

Using constitutive relations for plane stress state at the free surface of the pressure vessel, the strain in the hoop direction due to internal pressure only is:

$$\varepsilon_h = \frac{1}{E}\left[\sigma_h - \nu\left(\sigma_a + \sigma_L\right)\right]$$

Applied Presssure, p (kPa)	Applied Mass, m_a (kg)	Calculated Axial Stess, $\sigma_a + \sigma_L$ (MPa)	Calculated Hoop Stress, σ_h (MPa)

Calculated Axial Strain, ε_a (µ m/m)	Measured Axial Strain $\left[\frac{\varepsilon_L^1 + \varepsilon_L^2}{2}\right]$(µ m/m)	Calculated Hoop Strain, ε_h (µ m/m)	Measured Hoop Strain $\left[\frac{\varepsilon_T^1 + \varepsilon_T^2}{2}\right]$(µ m/m)

6) Discuss the agreement or lack of agreement between the various measured and calculated results. Were the results what you expected? Do you think it is possible to superpose stress solutions and still predict actual stress or strain states?