**Mass Defect & Binding Energy**

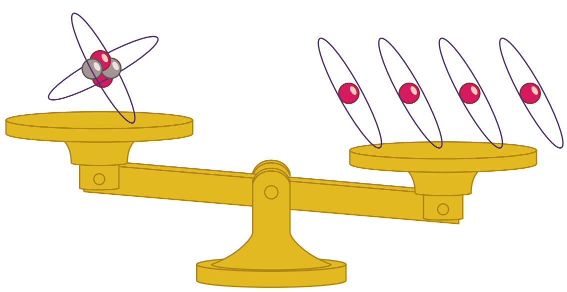
The nuclear reaction used by stars to produce energy (for most of their lives anyway) is the proton proton cycle:

$$^1H + {^1H} \rightarrow {^2H} + e^- + \nu$$
$$^1H + {^2H} \rightarrow {^3He} + \gamma$$
$$^1H + {^3He} \rightarrow {^4He} + e^- + \nu \quad \text{or} \quad {^3He} + {^3He} \rightarrow {^4He} + {^1H} + {^1H}$$

In the overall reaction, 4 hydrogen-1 atoms combine to form 1 He-4 atom plus some other particles and energy. But examining the atomic masses of hydrogen and helium on the periodic table, the mass of four hydrogen atoms is greater than the mass of one helium atom. Subtracting the difference gives:

$$4 \times 1.007825 - 4.002603 = 0.028697 \text{ amu}$$

This difference is called the mass defect and measures the amount of "binding energy" stored in an atom's nucleus or the amount of energy required to break up the nucleus back into the individual protons and neutrons.



In general, the mass defect is calculated by summing the mass of protons, neutrons, and electrons in an atom, and subtracting the atom's actual atomic mass. The general formula is:

$$M_d = Z\, m_p + N\, m_n - M_a$$

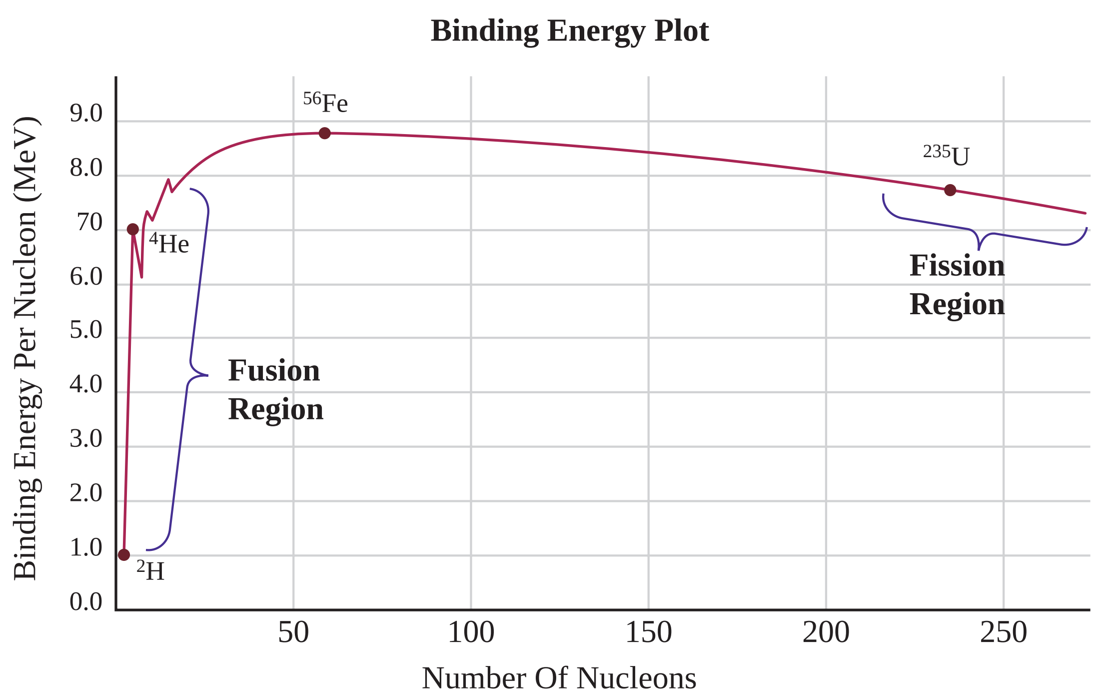
where $Z$ is the atomic number, $N$ is the number of neutrons in the atom, and $M_a$ is the actual measured mass of the atom. Placing $M_d$ into Einstein's equation for relating mass and energy gives the energy release from forming the atom from its constituent particles:

$$E = M_d\, c^2$$

where $c$ is the speed of light ($\approx 3.00 \times 10^8$ m/s). For example, using the information from the reaction above, one can deduce that to form 1 kg of $^4He$ from requires 1.0073 kg of $^1H$. The mass defect per kg is 0.0073 kg/ per kg $^4He$ (this difference can be measured using a precision balance without difficulty). Using Einstein's equation gives $6.601 \times 10^{14}$ J/kg $^4He$.

Do chemical compounds have a mass defect associated with their bonds? Yes, but their mass defects are much smaller. For example, the enthalpy of formation for carbon dioxide is –393 520 kJ/kmol and the individual atoms summed together is 498 380 kJ/kmol. The energy difference is 891 900 kJ/kmol. Converting the energy to per kg of $CO_2$, the energy becomes $20.27 \times 10^6$ J/kg. Substituting this into Einstein's equation, the mass defect is $2.255 \times 10^{-10}$ kg per kg $CO_2$ formed, a very small difference that is difficult to measure.

The binding of elements varies according to their atomic number. In order to compare different atoms easier, the binding energy is often given per nucleon (protons plus neutrons). From the chart below, the most tightly bound nucleus is $^{56}$Fe.




The plot shows how nuclear reactions liberate energy. Fusion reactors combine light nuclei like hydrogen to form heavier ones (like the example above) while fission reactors split heavy nuclei like $^{235}$U and $^{239}$Pu to release energy.