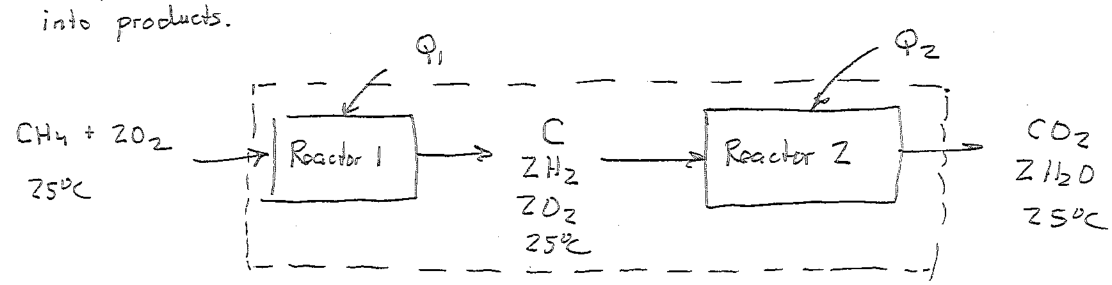
The concept is to first turn the reactants into elements, and then recombine these into products.



$$Q_1 = -\left[\bar{h}^\circ_{f,CH_4} + \cancel{2\bar{h}^\circ_{f,O_2}}^{\,0}\right]$$

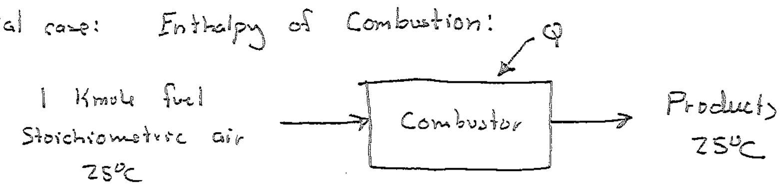
$$Q_2 = +\left[\bar{h}^\circ_{f,CO_2} + 2\bar{h}^\circ_{f,H_2O(\ell)}\right]$$

$$Q_1 + Q_2 \equiv \boxed{\Delta\bar{h}_{rxn} = \bar{h}^\circ_{f,CO_2} + 2\bar{h}^\circ_{f,H_2O(\ell)} - \bar{h}^\circ_{f,CH_4}}$$

$$\text{or} \quad \Delta\bar{h}_{rxn} = (-393{,}520) + 2(-285{,}830) - (-74{,}850) = -890{,}330 \ \frac{kJ}{kmole\ CH_4}$$

So in general for any reaction
$$\boxed{\Delta\bar{h}_{rxn} = \left[\sum n_i \bar{h}^\circ_{f,i}\right]_P - \left[\sum n_i \bar{h}^\circ_{f,i}\right]_R}$$

A special case: Enthalpy of Combustion:



$$\Delta\bar{h}_c = Q$$

$\Rightarrow$ This is a measure of the heating value of a fuel.

Example: Δh_c for propane:

$$C_3H_8 + \underset{a_{th}}{\overset{5}{}}\left[O_2 + 3.76N_2\right] \rightarrow 3CO_2 + 4H_2O + \overset{18.8}{3.76a_{th}}N_2$$

$$\text{O-atom} \qquad 2a_{th} = 6 + 4 \Rightarrow a_{th} = 5$$

$$\Delta\bar{h}_c = \left[\sum n_i \bar{h}^\circ_{f,i}\right]_P - \left[\sum n_i \bar{h}^\circ_{f,i}\right]_R$$

$$= 3\bar{h}^\circ_{f,CO_2} + 4\bar{h}^\circ_{f,H_2O} + \cancel{18.8\bar{h}^\circ_{f,N_2}}^{\,0}$$

$$- \bar{h}^\circ_{f,C_3H_8} - \cancel{5\bar{h}^\circ_{f,O_2}}^{\,0} - \cancel{18.8\bar{h}^\circ_{f,N_2}}^{\,0}$$

(A-23 has both).
Lower heating value (LHV)
If H_2O is vapor

$$= 3(-393{,}520) + 4(-241{,}820) - (-103{,}850) = -2{,}043{,}990 \ \frac{kJ}{kmole\ C_3H_8}$$

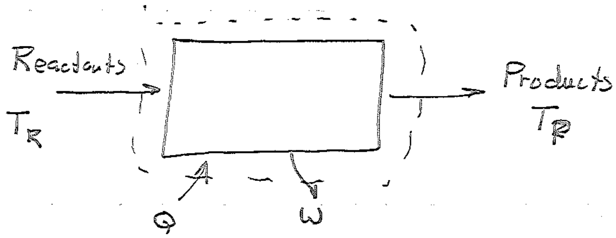
$$\text{or} \quad = \quad '' \quad + 4(-285{,}830) - \quad '' \quad = -2{,}220{,}030 \ \frac{kJ}{kmole\ C_3H_8}$$

If H_2O is liquid

Higher heating value (HHV)

Nov 30

Now we're ready to do 1^{st} law around any chemically reacting system:



$$Q - W = \sum_P n_i \bar{h}_i - \sum_R n_i \bar{h}_i$$

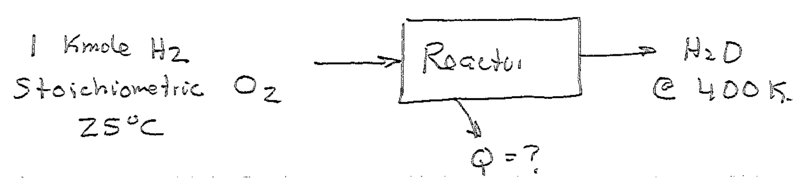
$$= \cancel{\sum_P n_i \left[\bar{h}^o_{f,i} + \bar{h}_{T_P} - \bar{h}_{298}\right]}$$

$$= \sum_P \left[\bar{h}^o_{f,i} + \left(\bar{h}_{i,T_P} - \bar{h}_{i,298}\right)\right] - \sum_R \left[\underset{(a)}{\bar{h}^o_{f,i}} + \underbrace{\left(\bar{h}_{i,T_R} - \bar{h}_{i,298}\right)}_{(b)}\right]$$

A-25 — A-23 @ T_P — A-23 @ 298 K = 25°C

reference point: $h = 0$ for elements @ 298 K
(a) = Δh for making compound from elements
(b) = Δh for taking compound from 298 K $\rightarrow T_R$.

A simple example, then a tougher one —

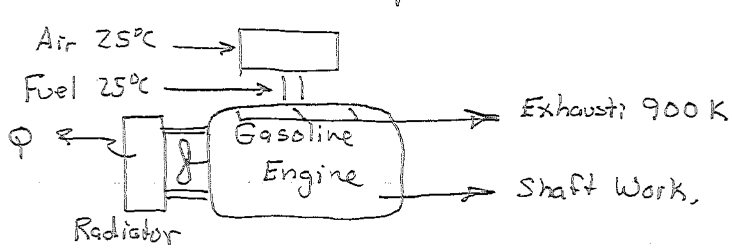


Reaction: $H_2 + 0.5 O_2 \rightarrow H_2O$

	Species	Kmoles	$\bar{h}^o_f$ (KJ/Kmole)	$\bar{h} - \bar{h}_{298}$	$n_i[\bar{h}^o_{f,i} + (\bar{h} - \bar{h}_{298})]$	
R	H_2	1	0	8468 − 8468	0	$H_R = 0$
	O_2	0.5	0	8682 − 8682	0	
P	H_2O	1	−241,820	13,356 − 9902	−238,368	$H_P = -238,368$

$$\boxed{Q = H_P - H_R = -238,368 \text{ KJ/Kmole } H_2 \text{ reacted}}$$

Now a more detailed example:



Info:
- 30 mi/gal @ 65 mph.
- Fuel injector gives 10% excess air
- Exhaust gas C
  - → 90% CO_2
  - → 10% CO
- Measure ΔT & $\dot{m}$ Coolant

$$\dot{Q}_{Rad} = \dot{m}_{cool} C_{P,cool} \Delta T_{cool} = 15 \text{ KW}$$

→ Get horsepower.

First, pick a basis for calculation (1 Kmole C_8H_{18}), then convert later.

① Reaction Equation

If completely burned

$$C_8H_{18} + \underbrace{\frac{12.5}{a_{th}}}\ \frac{1.1}{\Phi}\left[O_2 + 3.76N_2\right] \rightarrow \underbrace{8CO_2} + 9H_2O + 51.7N_2 + 1.25O_2$$

90% CO_2, 10% CO

$7.2\,CO_2 + 0.8\,CO$
$+\ 0.4\,O_2$

$\Rightarrow$ Final Products = $7.2CO_2 + 0.8CO + 9H_2O + 51.7N_2 + 1.65O_2$

② First Law: $Q - W = H_P - H_R$

	Species	Kmoles	$\bar{h}_f^\circ$	$\bar{h} - \bar{h}_{298}$	$n_i\left[\bar{h}_f^\circ + (\bar{h} - \bar{h}_{298})\right]$	
R	C_8H_{18} (f)	1	−249,950	0	−249,950	$H_R =$ −249,950
	O_2	13.75	0	0	0	
	N_2	51.7	0	0	0	
P	CO_2	7.2	−393,520	37,405 − 9364	−2,631,500	$H_P =$ −3,710,424
	CO	0.8	−110,530	27,066 − 8669	−73,706	
	H_2O (g)	9	−241,820	31,829 − 9904	−1,979,000	
	N_2	51.7	0	26,890 − 8669	942,026	
	O_2	1.65	0	27,928 − 8682	31,756	

$$H_P - H_R = -3{,}710{,}424 - (-249{,}950) = -3{,}460{,}474\ \frac{KJ}{Kmole\ C_8H_8}$$

Dec 3

So: $\dot{Q} - \dot{W} = \dot{n}_{C_8H_{18}}\left[H_P - H_R\right]$

Get fuel flow rate:

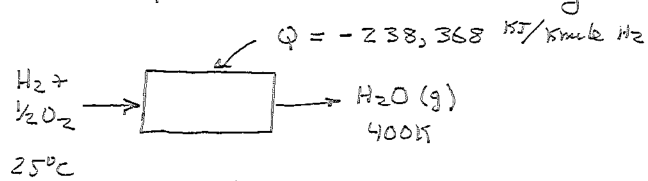
$$\frac{65\ mi}{hr}\left|\frac{gal}{30\ mi}\right|\frac{5.6\ lb}{gal}\left|\frac{0.454\ Kg}{lb}\right|\frac{hr}{3600\ s}\left|\frac{Kmole}{114\ Kg}\right. = 1.34E{-5}\ \frac{Kmoles}{s} \quad \left[\sim 1.5\ g/s\right]$$

$$-15\ \frac{KJ}{s} - \dot{W} = \frac{1.34E{-5}\ Kmoles}{s}\left|\frac{-3{,}460{,}474\ KJ}{Kmole\ C_8H_8}\right. \Rightarrow \dot{W} = 31.37\ KW.$$

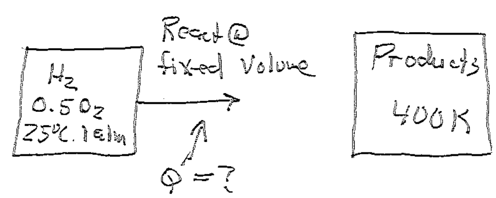
Converting: $\dot{W} = \frac{31.37\ KJ}{s}\left|\frac{737.6\ ft\text{-}lbf}{KJ}\right|\frac{HP}{550\ ft\text{-}lbs/s} = 42.1\ HP \Rightarrow$ @ VW !

Of course, this is for a car at cruise, not wide-open throttle.

To this point, we have been dealing with flow systems. How about fixed volume?



How about:



Will heat released be more or less? $Q - W = \Delta U$

~~Intuition says less because some internal energy will go to W as gases expand.~~

$$Q - W = \cancel{[\Sigma n_i \bar{h}_i]_P - [\Sigma n_i \bar{h}_i]_R} \quad [\Sigma n_i \bar{u}_i]_P - [\Sigma n_i \bar{u}_i]_R$$

but $\bar{h} \equiv \bar{u} + P\bar{v}$ or $\bar{u} = \bar{h} - P\bar{v}$ but $P\bar{v} = \bar{R}T$

$\rightarrow \bar{u} = \bar{h} - \underbrace{\bar{R}T}$ → correction to existing approach.

$$Q - W = \sum_P n_i \left[\bar{h}_f^\circ + (\bar{h} - \bar{h}_{298}) - \bar{R}T\right] - \sum_R n_i \left[\bar{h}_f^\circ + (\bar{h} - \bar{h}_{298}) - \bar{R}T\right] \; !$$

For our problem:

	Species	Kmols	$\bar{h}_f^\circ$	$\bar{h} - \bar{h}_{298}$	$-\bar{R}T$	$n_i[\bar{h}_f^\circ + (\bar{h}-\bar{h}_{298}) - \bar{R}T]$	
R	H_2	1	0	0	−2478	−2478	$U_R = -3717$
R	O_2	0.5	0	0	−2478	−1239	
P	H_2O	1	−241,820	13,356 − 9904	−3325	−241,693	$U_P = -241,693$

$$Q = U_P - U_R = [-241,693] - [-3717] = -237,976 \ \text{KJ/Kmole } H_2$$

(Was −238,368, so heat release did go down a little)

Get P_2: $V_1 = \dfrac{n_1 \bar{R} T_1}{P_1} = \dfrac{1.5 \,|\, 8.314 \,|\, 298}{101} = 36.80 \ m^3 = V_2$

$$P_2 = \frac{n_2 \bar{R} T_2}{V_2} = \frac{1 \,|\, 8.314 \,|\, 400}{36.8} = \underline{\underline{90.37 \ \text{KPa}}}$$

$Q - W = \underset{\text{same}}{\underline{\Delta U}}$ $\quad Q_p - W_p = Q_v \;\Rightarrow\; Q_p = Q_v + W$ ← Volume shrinks $W < 0$

└ Q $\quad \Rightarrow Q_p < Q_v$

Adiabatic Flame Temperature

Fuel + air 25°C → Combustor → Products T = ?

Q = 0

For a flow system: $\cancel{Q}^{0} - \cancel{W}^{0} = H_P - H_R$

For fixed volume $\quad U_P = U_R$ (as set up above)

Flow: $$0 = \sum_P n_i \left[\bar{h}_f^\circ + \bar{h} - \bar{h}_{298}\right] - \sum_R n_i \left[\bar{h}_f^\circ + \bar{h} - \bar{h}_{298}\right]$$

Approach:
1. Know all except $\bar{h}$ for products $(= f(T))$
2. Guess T_P, look up $\bar{h}$
3. Plug in and see if it works! (RHS = 0)

- One of the problems with gas turbine combustion is NO, made by high-T.
- Current approach is "lean, premixed" combustion to keep T-low.

CH_4, 200% Theoretical Air, 25°C $\longrightarrow$ $\boxed{\text{Combustor}}$ $\longrightarrow$ $T = ?$

$Q = 0$

$$CH_4 + \overset{2}{a_{th}}\,\overset{2}{\Phi_{AF}}\left[O_2 + 3.76N_2\right] \rightarrow CO_2 + 2H_2O + 15.04N_2 + 2O_2$$

	Species	Kmols	$\overline{h}_f^\circ$	$\overline{h} - \overline{h}_{298}$	$N_i\left[\overline{h}_f^\circ + (\overline{h} - \overline{h}_{298})\right]$	
R	CH_4	1	−74,850	0	−74,850	$H_R = -74,850$
	O_2	4	0	0	0	
	N_2	15.04	0	0	0	
P	CO_2	1	−393,520	$\overline{h}_{CO_2} - 9364$	$-393,520 + \overline{h}_{CO_2} - 9364$	
	$H_2O(g)$	2	−241,820	$\overline{h}_{H_2O} - 9904$	$2\left[-241,820 + \overline{h}_{H_2O} - 9904\right]$	
	N_2	15.04	0	$\overline{h}_{N_2} - 8669$	$15.04\left[\overline{h}_{N_2} - 8669\right]$	
	O_2	2	0	$\overline{h}_{O_2} - 8682$	$2\left[\overline{h}_{O_2} - 8682\right]$	

$$0 = H_P - H_R$$
$$= \left[-393,520 + \overline{h}_{CO_2} - 9364\right] + 2\left[-241,820 + \overline{h}_{H_2O} - 9904\right] + 15.04\left[\overline{h}_{N_2} - 8669\right]$$
$$+ 2\left[\overline{h}_{O_2} - 8682\right] - \left[-74,850\right]$$

$$\boxed{0 = \overline{h}_{CO_2} + 2\overline{h}_{H_2O} + 15.04\,\overline{h}_{N_2} + 2\overline{h}_{O_2} - 979,228}$$

Now ~~look up~~ Guess T_P, look up h's until RHS = 0

Tricks to speed the process:

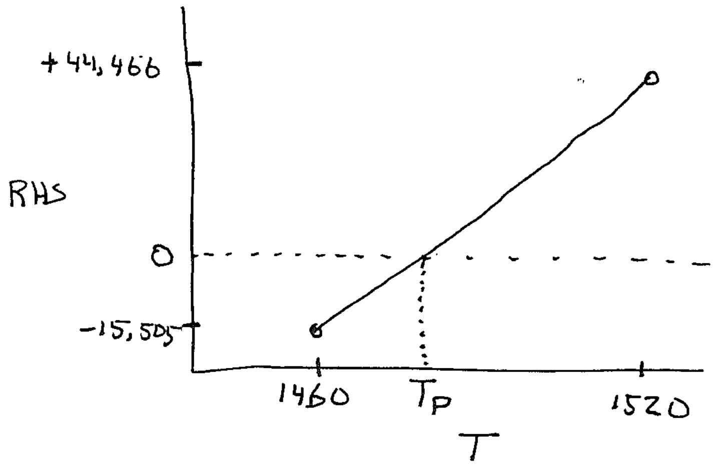
① Temporarily pretend all products are N_2 to get an initial guess.

$$0 = 20.04\,\overline{h}_{N_2} - 979,228 \Rightarrow \overline{h}_{N_2} = 48,864 \ \frac{\text{KJ}}{\text{Kmole}\ N_2}$$

$$\longrightarrow T_P \cong 1540\,\text{K}.$$

Now Guess $T_P = 1540K$, read $\bar{h}$'s and go to equation.

T_P	$\bar{h}_{CO_2}$	$\bar{h}_{H_2O}$	$\bar{h}_{N_2}$	$\bar{h}_{O_2}$	RHS	
1540	73,417	59,888	48,470	50,756	+44,466	$\Rightarrow$ T_P too hot.
1460	68,748	56,128	45,682	47,831	−15,505	$\Rightarrow$ T_P too low.



$$\frac{0-(-15{,}505)}{44{,}466-(-15{,}505)} = \frac{T_P - 1460}{1520-1460} \Rightarrow \boxed{T_P = 1481\ K}$$

Chapter Summary.

1. Balancing equations.
   (a) Get a_{th} from O-atom balance @ stoichiometric.
   (b) Then use Φ_{AF} for off-stoichiometric.
   (c) $y_i = N_i/N_m$.

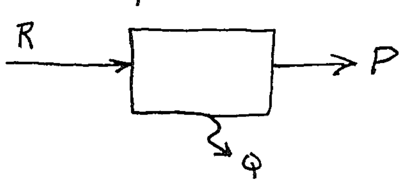
Two kinds of problems:
- Given ϕ, find y_i
- Given y_i, find ϕ.

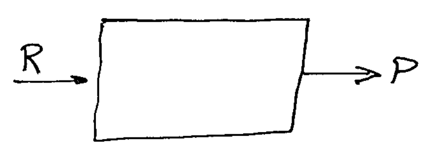
2. First Law: $\bar{Q}-\bar{W} = \sum_P N_i\left[\bar{h}_f^\circ + (\bar{h}-\bar{h}_{298})\right] - \sum_R N_i\left[\bar{h}_f^\circ + (\bar{h}-\bar{h}_{298})\right]$

Baseline: Elements @ 298K
$\Rightarrow h \equiv 0$

$$\underset{25^\circ C}{H_2, O_2} \xrightarrow{\bar{h}_f^\circ} \underset{25^\circ C}{H_2O} \xrightarrow{\bar{h}-\bar{h}_{298}} \underset{400\,K}{H_2O}$$

Two kinds of problems:





Know in & out, find Q

Know in & Q (=0), find T_P

There are others!

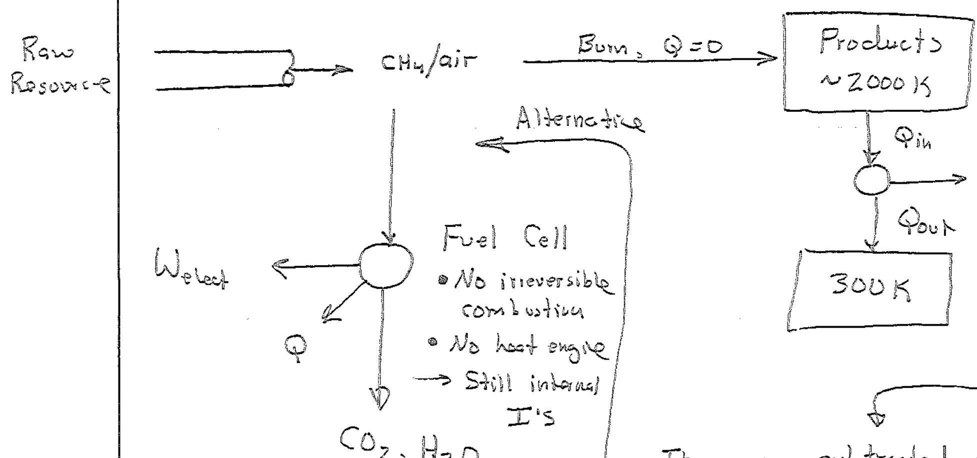
Dec. 5

~~Webster~~

First law for fixed volume: $Q - W = \sum_P n_i \bar{u}_i - \sum_R n_i \bar{u}_i$

$$\bar{u}_i = \bar{h}_i - P\bar{v} = \bar{h}_i - \bar{R}_u T = \bar{h}_f^\circ + (\bar{h} - \bar{h}_{298}) - \bar{R}T$$

Promised no 2^{nd} law stuff, <u>but</u>:



Look at irreversibilities

- Combustion: $= T_0 [S_P - S_R]$
- ΔT for Q_{in}
- ΔT for Q_{out}
- Internal cycle i.

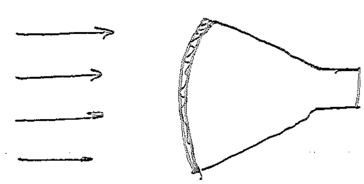
These are subtracted off the original work potential of the CH_4, the original raw resource

$$W_{max,\,CH_4} = h_R - h_P - T_0 [S_R - S_P]$$

$\boxed{\text{High-Speed Flow}}$ → Last part of Ch. 9.

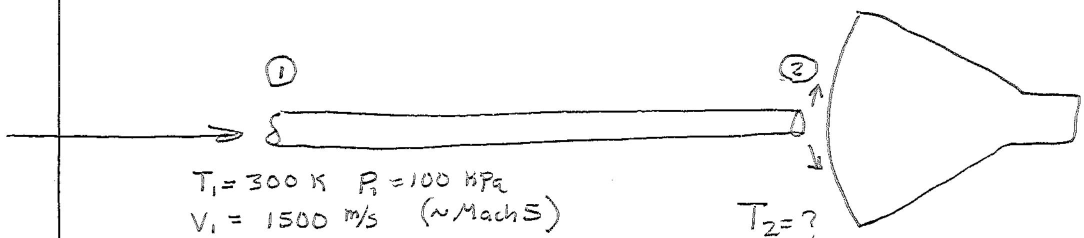
Three Topics: ① 1^{st} Law: Exchange KE for h
→ Slow a flow, it gets hot
→ Speed a flow, it gets cool.
② Isentropic Flows: How to get supersonic and back.
③ Shock Waves: Non-isentropic compression. (blast waves).

~~Space~~ When ~~[illegible]~~ Apollo returned from moon, tiles for re-entry were on bluff side.



Why did tiles heat? Not friction, at least not as we know it.

Shift coordinates so capsule is stationary and fluid approaches:



$T_1 = 300\ K \quad P_1 = 100\ kPa$

$V_1 = 1500\ m/s$ (~Mach 5)

~~$T_2 = ?$~~

negligible energy contribution.

Apply 1st law on CV $\quad \cancel{q}^{\,0} - \cancel{w}^{\,0} = \underbrace{h_2 - h_1}_{C_p(T_2 - T_1)} + \cancel{\frac{V_2^2}{2}} - \frac{V_1^2}{2}$

$$\Rightarrow T_2 = T_1 + \frac{V_1^2}{2C_p} = 300\,K + \left[\frac{1500\,m}{s}\right]^2 \left|\frac{1}{2}\right| \frac{kg\text{-}K}{1.005\,kJ} \left| \frac{kJ/kg}{1000\,m^2/s^2} \right. = 1419\,K \cong 2100\,°F$$

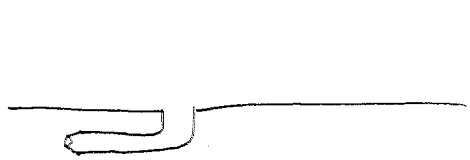
(for car @ 60 mph, $\Delta T \cong 0.6\,°F$, so you don't need heat tiles on your hand).

What is P_2? This is tricky:

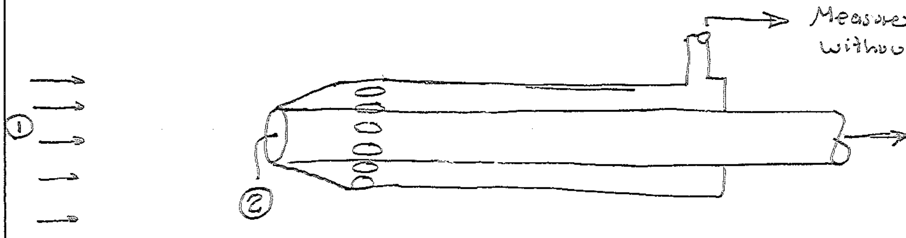
1. If approaching flow is subsonic, slow-down will be ~ isentropic
   $\Rightarrow$ Can calculate, e.g., $\frac{P_2}{P_1} = \left[\frac{T_2}{T_1}\right]^{k/k-1}$

2. If approaching flow is supersonic, will have shock wave (sonic boom).
   $\Rightarrow$ later.

Example: Pitot measurement:



When you are stuck getting onto an airplane, look left out of door and you will see the pitot probe used as an airspeed indicator.



240 K
40 kPa

If aircraft is moving at 545 mph = 243.5 m/s, find the ΔP that would be read by instrument.

(a) Get T_2
(b) Use isentropic $\rightarrow P_2$.

$$0 = C_p(T_2 - T_1) + \cancel{\frac{V_2^2}{2}}^{\,0} - \frac{V_1^2}{2}$$

So again:

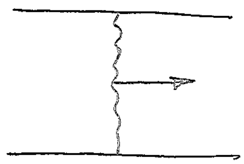
$$T_2 = T_1 + \frac{V_1^2}{2C_p} = 240\,K + \left[\frac{243.5\,m}{s}\right]^2 \Big| \frac{1}{2} \Big| \frac{1}{1.005\,KJ/kg\text{-}K} \Big| \frac{KJ/kg}{1000\,m^2/s^2} = 269.5\,K$$

$\Rightarrow \sim 30°C$ temp increase

Remember that isentropic means that a given T increase corresponds to only one P increase. Anything else is not $\Delta S = 0$.

$$\frac{P_2}{P_1} = \left[\frac{T_2}{T_1}\right]^{K/K-1} \qquad P_2 = [40\,KPa]\left[\frac{269.5}{240}\right]^{\frac{1.4}{0.4}} = \underline{\underline{60\,KPa}}$$

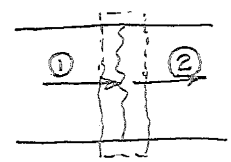
In what follows, the <u>speed of sound</u> is a key property of a fluid.



Speed with which a vanishingly small disturbance moves to right.

Apply conservation laws to get:

① Draw CV around disturbance and make it stationary



② Apply conservation of mass:

$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$, but $A_1 = A_2$

for differential: $d[\rho V] = 0$ or $\rho dV + V d\rho = 0$

or $\boxed{\dfrac{dV}{d\rho} = -\dfrac{V}{\rho}}$

③ Apply conservation of energy:

$h_1 + \dfrac{V_1^2}{2} = h_2 + \dfrac{V_2^2}{2}$ or $d\left[h + \dfrac{V^2}{2}\right] = 0$ for differential

$\Rightarrow \boxed{dh + VdV = 0}$

④ Apply $\Delta S = 0$ 2nd Gibbs: $dh = \cancel{Tds}^{\,0} + v dP$

$\Rightarrow \boxed{dh = \dfrac{dP}{\rho}}$

Start with ③ $dh + VdV = 0$

Subst in ④ $\dfrac{dP}{\rho} + VdV = 0$

mult by $\dfrac{\rho}{d\rho}$ $\Rightarrow$ $\dfrac{dP}{d\rho} + \rho V\underbrace{\dfrac{dV}{d\rho}}_{\text{Subst from ②}} = 0$

$\dfrac{dP}{d\rho} + \rho V\left(-\dfrac{V}{\rho}\right) = 0$ or $V^2 = \dfrac{dP}{d\rho}$ at const s $\Rightarrow$ $\boxed{V^2 = \left(\dfrac{\partial P}{\partial \rho}\right)_s}$

Use symbol $\underline{\underline{C}}$ for sonic velocity:

$$C = \left[\left(\frac{\partial P}{\partial \rho}\right)_s\right]^{1/2} = \left[k\left(\frac{\partial P}{\partial \rho}\right)_T\right]^{1/2} \longleftarrow \text{any material.}$$

see p. 581

For $\underline{\text{ideal gas}}$ only: $P = \rho R T$

$$\left(\frac{\partial P}{\partial \rho}\right)_T = RT \quad \Rightarrow \quad C = (kRT)^{1/2}$$

For air:
$$C = \left[\frac{1.4 \mid 8.314\ \text{kJ} \mid \text{kmole} \mid 298\ \text{K} \mid 1000\ \text{m}^2/\text{s}^2}{\mid \text{kmole-K} \mid 29\ \text{kg} \mid \mid \text{kJ/kg}}\right]^{1/2} = 346\ \text{m/s}$$
$$= 774\ \text{mph.}$$
or 4.6 s/mile.

For water?: need to use $C^2 = k \underbrace{\left(\frac{\partial P}{\partial \rho}\right)_T}$ → From compressed liquid tables

$C \cong 1500$ m/s $\cong$ 1 miles/s.

⇒ Note: If water were truly incompressible, $C = \infty$.

Dec 6

<u>One Dimensional Isentropic Flow.</u>

Pick ↓

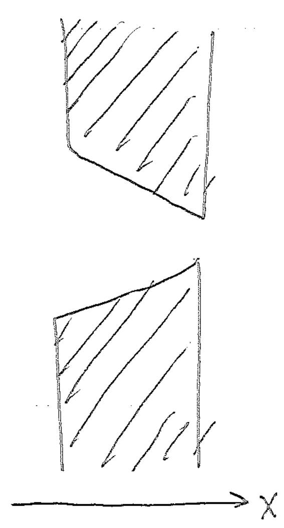
$q = w = 0$
S = constant
P, T, V, A } Vary.
ρ

Note that $q = w = 0$ means V and T are locked via 1st law
$\Delta S = 0$ means P and T are locked.
Ideal gas means P & $T \Rightarrow \rho$
Mass conservation means $\rho_1 V_1 A_1 = \cancel{\rho_2 V_2 A_2}$ constant
$\Rightarrow A$ is locked.

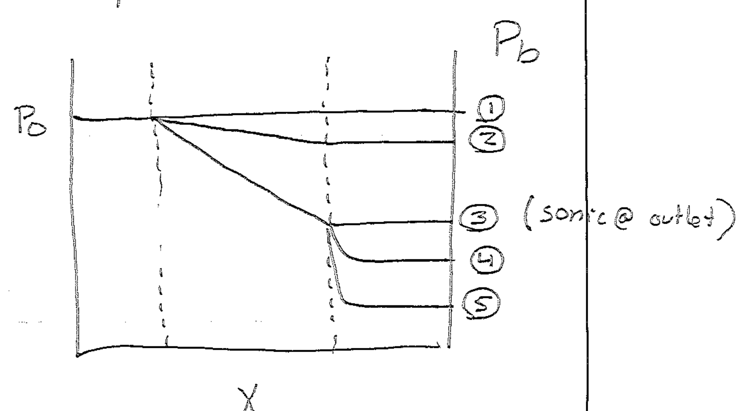
⇒ Means that only one free variable exists! e.g. pick $A(x)$, all else follows.

<u>Thought experiment:</u>

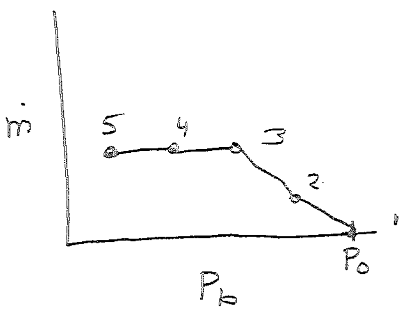
$P_0 = 100$ kPa
$T_0 = 300$ K



P_b = back pressure = variable.

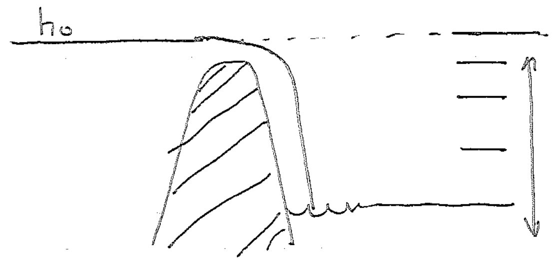


So flow looks like:



⇒ Flow has become "choked"
⇒ When you reach sonic velocity @ throat, pressure info cannot move upstream
⇒ Lower P_b has no effect.

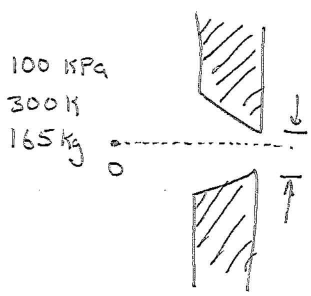
Analogy, dam:



- Above a certain h_b, the downstream height influences ṁ.
- Below a critical height, the flow is in free fall and is choked
  ⇒ ṁ doesn't depend on h.

Key point: If sonic velocity occurs, it is always at smallest area.

An example:
- Your first job is with NASA
- Assigned to shuttle
- Just when things couldn't look better, you get hit by a meteroir in an inaccessible region
  ⇒ Leaves converging hole in skin.



- Since you are fresh out of school, you get the job of finding the leak rate.
- <u>But</u>: No book, so you have to go from first principles

---

① $\dot{m} = \rho V A$
② @ throat, $M = 1 \Rightarrow V = c = \sqrt{kRT}$

} need ρ, T @ throat.

At "o" $V \cong 0$ ⇒ Stagnation properties
⇒ Write 1st law between stagnation point and any point in flow:

$$\cancel{q}^{\,0} - \cancel{w}^{\,0} = \underbrace{h - h_o}_{C_p(T-T_o)} + \frac{V^2}{2} - \cancel{\frac{V_o^2}{2}}^{\,0}$$

$$0 = C_pT - C_pT_o + \frac{V^2}{2} \qquad \text{Divide by } C_pT$$

$$0 = 1 - \frac{T_o}{T} + \frac{V^2}{2C_pT} \quad \Rightarrow \quad \frac{T_o}{T} = 1 + \frac{V^2}{2C_pT}$$

but: $C_p = \frac{kR}{k-1}$ for ideal gas

but: $C = [kRT]^{1/2}$

$$\Rightarrow \frac{T_o}{T} = 1 + \frac{k-1}{kR}\frac{V^2}{2T} = 1 + \left(\frac{k-1}{2}\right)\frac{V^2}{\underbrace{kRT}_{= C^2 !}}$$

$$\boxed{\frac{T_o}{T} = 1 + \left(\frac{k-1}{2}\right)M^2}$$

$\Rightarrow$ key point: $\frac{T_o}{T} = f(M)$ only

$M = 0 \quad T = T_o$

Higher $M \rightarrow$ Faster flow $\rightarrow$ colder $T \rightarrow$ intuitive! 1st law!

At throat $M = 1 \Rightarrow \frac{300}{T} = 1 + \frac{k-1}{2}$ (1.4) $\Rightarrow T = 230\text{ K} = T^*$

Props @ $M = 1$.

Now get pressure: Flow is isentropic, so:

$$\frac{P_o}{P} = \left[\frac{T_o}{T}\right]^{k/k-1} = \left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{k/k-1} \underset{M=1}{\Longrightarrow} P^* = 52.8\text{ KPa}.$$

Need ρ to solve problem:

$$\rho = \frac{P}{RT} \Rightarrow \frac{\rho_o}{\rho} = \cancel{\frac{R}{R}}\left(\frac{P_o}{P}\right)\left(\frac{T}{T_o}\right) = \cancel{\frac{R}{R}}\left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{k/k-1}\left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{-1}$$

$$= \left[1 + \frac{k-1}{2}M^2\right]^{1/k-1}$$

But we can get ρ^* directly $= \frac{P^*}{RT^*} = \frac{52.8\text{ KPa}}{250\text{ K}}\Big|\frac{\text{Kmole - K}}{8.314\text{ KPa-m}^3}\Big|\frac{29\text{ Kg}}{\text{Kmole}} = 0.736\ \frac{\text{Kg}}{\text{m}^3}$

$$V = C = [kRT] = \left[\frac{(1.4)(8.314\ \cancel{\text{KPa-m}}\ \frac{\text{KJ}}{\text{Kmole-K}})(250\text{ K})(1000\ \frac{m^2/s^2}{KJ/Kg})}{29\ \text{Kg/Kmole}}\right]^{1/2} = 317\text{ m/s}$$

$A = \frac{\pi D^2}{4} \longrightarrow 1.96\text{E-}5\text{ m}^2 \qquad \dot m = \rho V A = \frac{0.736\text{ Kg}}{m^3}\Big|\frac{317\text{ m}}{s}\Big|1.96\text{E-}5\text{ m}^2 = 4.57\text{E-}3\ \frac{\text{Kg}}{s}$

~~$\frac{m}{\dot m}$ 1.05 Kg 4.57~~

How about area? $\dot m = \rho V A = \rho^* V^* A^*$

$$\frac{A}{A^*} = \frac{\rho^* V^*}{\rho V} = \frac{1}{M}\frac{\rho^*}{\rho} = \frac{1}{M}\left[\frac{\rho^*}{\rho_o}\right]\left[\frac{\rho^o}{\rho}\right]$$

$$= \frac{1}{M}\left[1 + \frac{k-1}{2}\right]^{1/1-k}\left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{1/k-1}$$

OK, so P/P_o, T/T_o, ρ/ρ_o, $A/A^* = f(M)$ only. I feel a Table coming on!

Table 9-1, p 493.

M	P/P_o	T/T_o	A/A^*
0	1	1	∞
0.5	~~0.952~~ 0.843	0.952	1.34
1.0	0.528	0.833	1
2	0.128	0.556	1.69
∞	0	0	∞

P and T decrease, but A goes through a minimum @ A^*

$\Rightarrow$ $M=1$ occurs @ smallest area

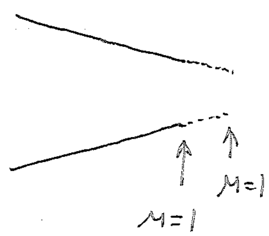
So if we have our tables, just ~~go to M~~ → get P_o, T_o
→ go to $M=1$ ⇒ P^* T^*
→ ideal gas ⇒ ρ^*
→ $V = c = [kRT]^{1/2}$
⇒ $\dot{m}$

With the table in mind, can establish some rules of the road.

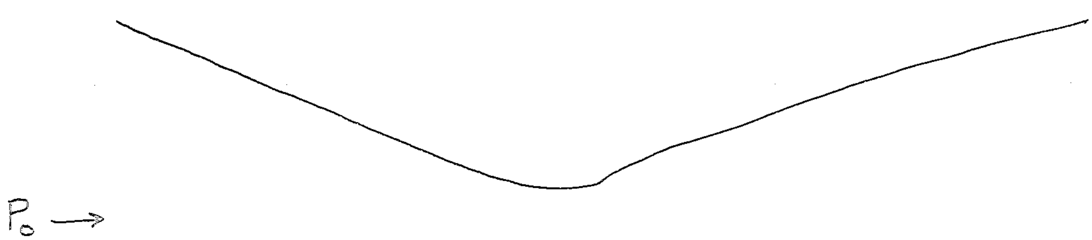
Nozzle:
Subsonic in $A\downarrow \Rightarrow P\downarrow\ T\downarrow\ M\uparrow$ Subsonic $A\uparrow\ P\uparrow\ T\uparrow\ M\downarrow$

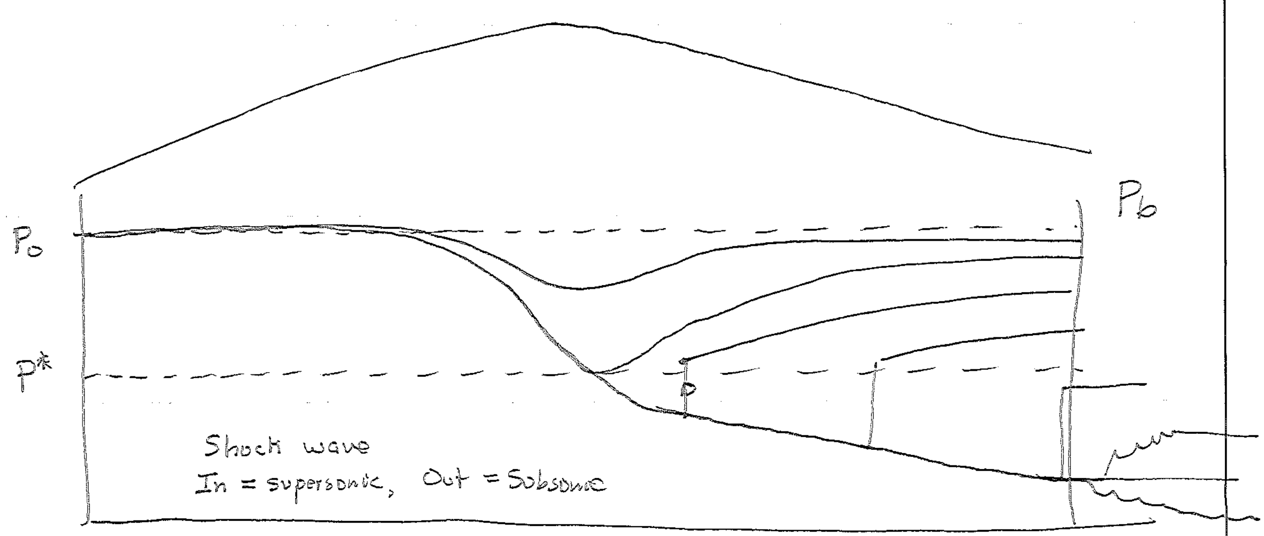
Supersonic in $A\downarrow \Rightarrow P\uparrow\ T\uparrow\ M\downarrow$ Supersonic in $A\uparrow\ P\downarrow\ T\downarrow\ M\uparrow$

Assume you've got sonic flow. How to get supersonic?
→ Add more nozzle?
→ Just moves the point for $M=1$ and reduces area.



Answer is to couple nozzle and diffuser together.





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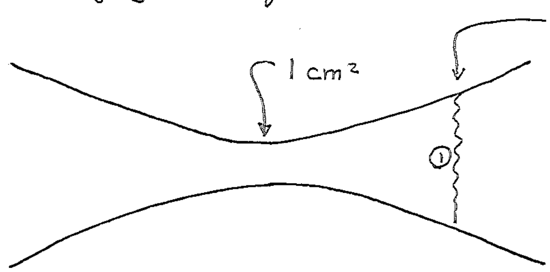
Example: Converging/diverging rocket nozzle with throat of 1 cm².

$P_0 = 10$ MPa

$T_0 = 2000$ K



Find $\dot{m}$ and conditions upstream of shock.

Since ~~inlet o condi~~ shock is supersonic in, subsonic out, $M=1$ @ throat $\Rightarrow A^* = 1$ cm²

Table 9-1

M	P/P_0	T/T_0	A/A^*
0	1	1	∞
0.31	0.98	0.98	2
1	0.524	0.833	1
2.2	0.09352	0.50813	2
∞	0	0	∞

So we are faced with two solutions $\Rightarrow$ must pick supersonic.

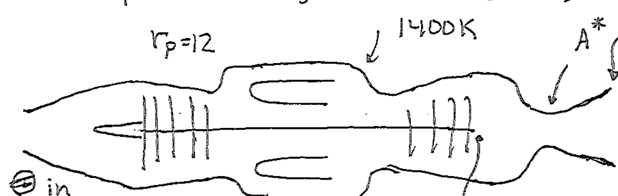
$\Rightarrow T_1 = 0.50813\, T_0 = 1016.3$ K

$\Rightarrow P_1 = 0.09352\, P_0 = 935$ KPa

$$C_1 = [kRT_1]^{1/2} = 639 \text{ m/s}$$
$$V_1 = M_1 C_1 = 2.2 C_1 = 1406 \text{ m/s}$$
$$\rho_1 = \frac{P_1}{RT_1} = 3.21 \text{ kg/m}^3$$
$$\dot{m} = \rho_1 V_1 A_1 = \left[3.21 \tfrac{\text{kg}}{\text{m}^3}\right]\left\{1406 \text{ m/s}\right\}\left[2E\text{-}4 \text{ m}^2\right] = 0.90 \text{ kg/s}$$

$\Rightarrow$ Could have based $\dot{m}$ on throat properties also $\dot{m} = \rho^* C A^*$

Another example: PS 5, #1 (9-76, text) was a turbojet engine:



→ Design the nozzle on the back end.

~~[illegible] A_1 so $P = 22$ KPa~~

in: 22 KPa, 220 K, 250 m/s

$P_0 = 158.52$ KPa

$T_0 = 1140$ K

$\dot{m} = 100$ kg/s

Throat: $\quad \dot{m} = \rho^* C A^* \quad \rho^* = \dfrac{P^*}{RT^*} \quad C = (kRT^*)^{1/2}$

$\qquad\qquad\qquad\quad ?$

Table 9-1 $\quad M=1 \Rightarrow T^* = 0.833\, T_0 = 950$ K

$$C = \left[(1.4)\frac{(8.314)}{(29)}(950)(1000)\right]^{1/2} = 617.5 \text{ m/s}$$

$$P^* = 0.524\, P_0 = 83.1 \text{ KPa}$$

$$\rho^* = \frac{P^*}{RT^*} = \frac{83.1\ \text{kPa}\ \big|\ 29}{950\ \big|\ 8.314} = 0.305\ \text{kg/m}^3$$

~~100~~ $\dot{m} = \rho^* c A^*$

$$\left\{100\ \frac{\text{kg}}{\text{s}}\right\} = \left[0.305\ \frac{\text{kg}}{\text{m}^3}\right]\left[617.5\ \frac{\text{m}}{\text{s}}\right] A^* \Rightarrow \quad A^* = 0.531\ \text{m}^2$$

$$\underline{D = 0.822\ \text{m}}$$

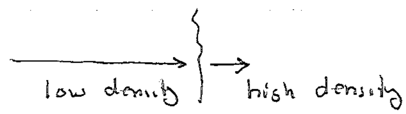
<u>Outlet:</u> How to select nozzle outlet area? ~~Pick~~ Match P's.

$$\frac{P_1}{P_0} = \frac{22}{158.52} = 0.139 \xrightarrow{9\text{-}1} \frac{A_1}{A^*} = 1.620 \qquad \underline{M = 1.95}$$

$$A_1 = 1.62 A^* = 0.860\ \text{m}^2 \Rightarrow \quad D = 1.046\ \text{m} \quad \Rightarrow$$ So it doesn't diverge that much.

<u>Shock Waves</u>

<u>What can we say?</u>

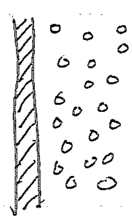


~~Sub~~ Supersonic → Subsonic

P, T increase

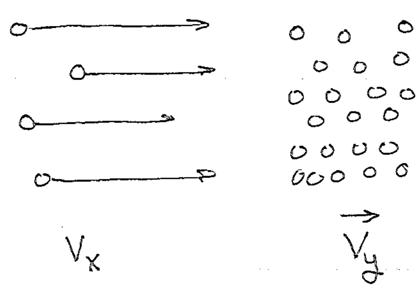
S increases <u>Cannot have reverse shock!</u>

<u>What is it on microscale?</u>



- Dense gas
- Collisions with wall restrains gas.

Replace barrier with high-speed approaching flow.

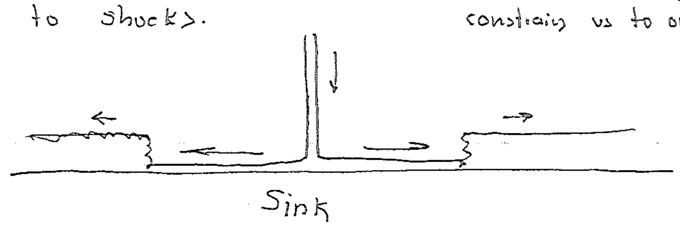


- High-speed molecules pile into dense phase and collect
- Move to right.
- If everything is balanced, discontinuity is stationary.

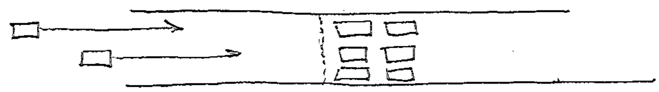
V_x, V_y. ⇒ Just like isentropic flow, will see balancing all this constrains us to only one free variable.

Two other phenomena are similar to shocks.

① Hydraulic jump:

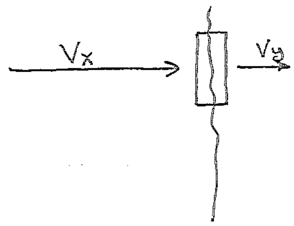


② Freeway slowdown



High-speed, low density   Low-Speed, high density

Go back to basics to figure out how things change across shock.



~~ⓐ Conservation of mass:~~ $\cancel{\dot m = \rho VA = \text{constant} \quad A_x = A_y}$

$$\cancel{\Rightarrow \rho_x V_x = \rho_y V_y}$$

ⓐ 1st Law:

$$\cancel{q}^{\,0} - \cancel{w}^{\,0} = h_y - h_x + \frac{V_y^2}{2} - \frac{V_x^2}{2} \quad \Rightarrow \quad \underbrace{h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2}}$$

This says that h_0 = stagnation enthalpy is same on both sides
T_0 = constant across shock.

On left side: $\frac{T_0}{T_x} = 1 + \left(\frac{K-1}{2}\right) M_x^2$ On right side: $\frac{T_0}{T_y} = 1 + \left(\frac{K-1}{2}\right) M_y^2$

Ratio left over right:
$$\boxed{\frac{T_y}{T_x} = \frac{1 + \left(\frac{K-1}{2}\right) M_x^2}{1 + \frac{K-1}{2} M_y^2}}$$

ⓑ Conservation of Momentum:

$$P_x + \rho_x V_x^2 = P_y + \rho_y V_y^2$$

Substitute $\rho = \frac{P}{RT} \Rightarrow \quad P_x + \frac{P_x}{RT_y} V_x^2 = P_y + \frac{P_y}{RT_y} V_y^2$

$$P_x \left[1 + \underbrace{\frac{V_x^2}{RT_x}} \right] = P_y \left[1 + \frac{V_y^2}{RT_y} \right]$$ Almost a C^2 in denominator!

$$= \left. \frac{K \, V_x^2}{KRT_y} \right\} K M_x^2$$
$KRT_y - C^2$

$$\Rightarrow \boxed{\frac{P_y}{P_x} = \frac{1 + K M_x^2}{1 + K M_y^2}}$$

ⓒ Conservation of Mass: $\dot m_x = \dot m_y$

$\rho_x V_x A_x = \rho_y V_y A_y$ but $A_x = A_y$

$$\frac{P_x}{RT_x} V_x = \frac{P_y}{RT_y} V_y \quad \Rightarrow \quad \frac{V_x}{(KRT_x)^{1/2}} \cdot \frac{(KRT_x)^{1/2} P_x}{RT_x} = \frac{V_y}{(KRT_y)^{1/2}} \cdot \frac{(KRT_y)^{1/2} P_y}{RT_y}$$

$$\frac{M_x P_x}{T_x^{1/2}} = \frac{M_y P_y}{T_y^{1/2}} \Rightarrow \frac{P_y}{P_x} = \left[\frac{T_y}{T_x}\right]^{1/2} \frac{M_x}{M_y}$$

Now substitute for P & T ratios from above:

$$\frac{1 + kM_x^2}{1 + kM_y^2} = \left[\frac{1 + \frac{k-1}{2} M_x^2}{1 + \frac{k-1}{2} M_y^2}\right]^{1/2} \frac{M_x}{M_y}$$

Only M_x, M_y left!

Solve for $\boxed{M_y^2 = \dfrac{M_x^2 + \frac{2}{k-1}}{\frac{2k}{k-1} M_x^2 - 1}}$

OK, so you know $M_x \to M_y$, $\frac{P_y}{P_x}$, $\frac{T_y}{T_x}$ $\Longrightarrow$ suggests another table:

See 9-1

M_x	M_y	P_y/P_x	T_y/T_x	$P_{o,y}/P_{o,x}$
1	1	1	1	1
2	0.58	4.5	1.69	0.72

Another useful idea:

Shock

T_o, P_o $\xrightarrow[\text{isentropically}]{\text{Accelerate}}$ T_x, P_x, M_x | T_y, P_y, M_y $\xrightarrow[\text{Isentropically}]{\text{Decelerate}}$ T_o, P_o

First law says T_o is same, but will find $P_{o,y} < P_{o,x}$. This is where the $\Delta S \neq 0$ loss comes in.

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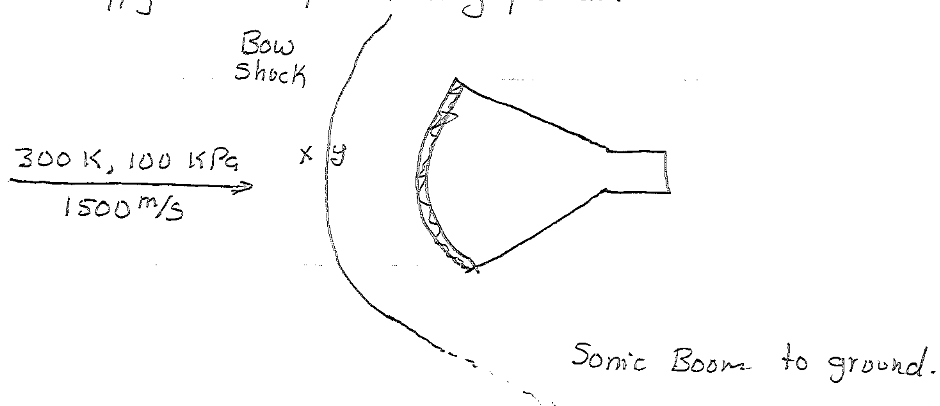
$$\frac{P_{o,y}}{P_{o,x}} = f(M_x)$$

Add.

Now apply to our Apollo reentry problem.



Before shock, no disturbance:

$$C_x = \left[KRT_x\right]^{1/2} = \left[(1.4)\frac{(8.314)}{29}(300)(1000)\right]^{1/2} = 347 \text{ m/s}$$

$$M_x = \frac{1500}{347} = 4.32$$

T 9-1 $\rightarrow$ $M_y = 0.428$
$P_y/P_x = 21.4 \longrightarrow P_y = 2140$ KPa
$T_y/T_x = 4.53 \longrightarrow T_y = 1359$ K.

Now that we are on the y-side, can stagnate isentropically.

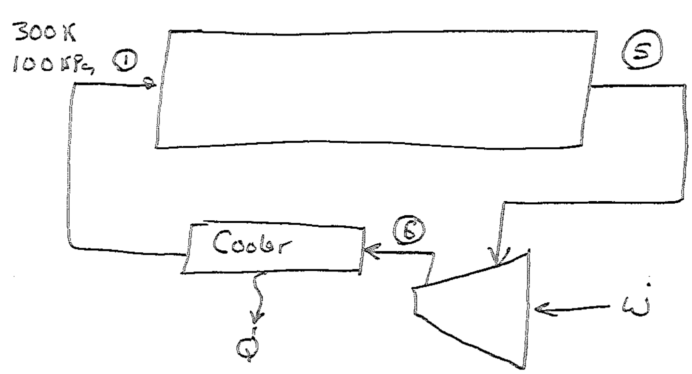
T 9-1 $\quad M = 0.428 \Rightarrow \frac{P}{P_o} = 0.881 \Rightarrow P_o = 2429$ KPa

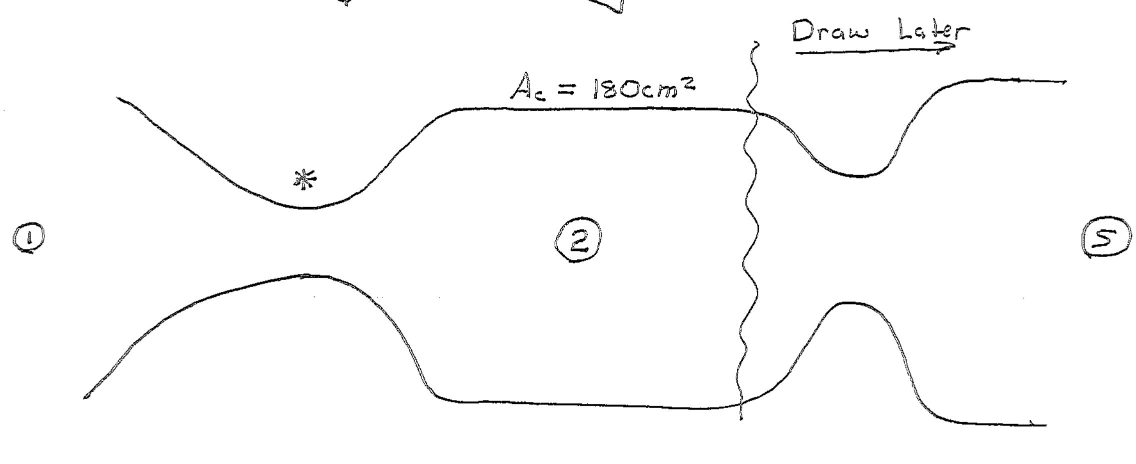
$\frac{T}{T_o} = 0.964 \Rightarrow T_o = 1410$ K

- Same T as from simple first law.
- $P_o \cong 25$ atm, so structural design a problem!

Now lets put the whole thing together into a closed circuit wind tunnel design:

Want: $M=2$ in test section
6" diameter $= 180$ cm^2
$\longrightarrow$ $\dot{m}$
$\longrightarrow$ Dimensions
$\longrightarrow$ $\dot{W}$





This first part is straightforward: Accelerate flow to $M = 2$ isentropically.

$\boxed{9\text{-}1}$

M	P/P_0	T/T_0	A/A^*
0	1	1	∞
			1.25 Down
1	0.524	0.833	1
2 $\rightarrow$			1.25, 1.6875 up
∞	0	0	∞

$M_2 = 2 \Rightarrow P/P_0 = 0.1278 \Rightarrow P_2 = 12.8$ KPa
$T/T_0 = 0.5556 \Rightarrow T_2 = \cancel{1.66}$ 166.7 K
$A_2/A^* = 1.6875 \Rightarrow \dfrac{180\ cm^2}{A^*} = 1.6875 \Rightarrow A^* = 106.7\ cm^2$
[D = 4.6 in]

$$\rho_2 = \frac{P_2}{RT_2} = 0.268\ kg/m^3$$

Get $\dot{m} = \rho_2 V_2 A_2 \Rightarrow C_2 = [kRT_2]^{1/2} = 259\ m/s$
$V_2 = M_2 C_2 = 518\ m/s$

$$\dot{m} = (0.268)(518)(0.018) = 2.49\ kg/s$$

Now the issue is what to do next.

- $\longrightarrow$ Want to stagnate flow to recover T, P before compressor.
- $\longrightarrow$ Isentropic would recover original P_0, but this isn't possible
- $\longrightarrow$ What is initial shape?
  Slowing a supersonic flow, narrow.
- $\longrightarrow$ Shock at some point.
- $\longrightarrow$ Continue to slow ~~flow~~ subsonic flow $\longrightarrow$ expand
- $\longrightarrow$ Let $A_s = 1.25 A^*$

Conditions @ 3 Two places for $A_3/A^* = 1.25$
$\Rightarrow M_3 = 1.6$
$P_3/P_0 = 0.235 \Rightarrow P_3 = 23.5$ KPa
$T_3/T_0 = \cancel{\ }$ 0.661 $T_3 = 198.3$ K.

Now step across shock:

M_x	M_y	P_y/P_x	T_y/T_x
1.6	0.688	2.82	1.39

$T_4 = 1.39 T_3 = [1.39][198.3] = 275.6$ K
$P_4 = 2.82 P_3 = [2.82][23.5] = 66.3$ KPa

Now back to isentropic to stagnate flow:

	P_5/P_0	T_5/T_0
$M = 0.688 \longrightarrow$	0.742	0.919

$\cancel{P_4/P_5}$ $P_5 = P_4/0.742 = \dfrac{66.3}{0.742} = 89.3$ KPa $\longleftarrow$ Less due to shock.

$T_5 = T_4/0.919 = \dfrac{275.6}{0.919} = 300$ K $\longleftarrow$ Same $\Rightarrow$ No energy loss

Need to pump to 100 KPa

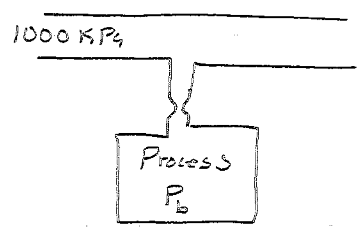
$$T_6 = T_5 \left[\frac{P_6}{P_5}\right]^{k-1/k} = [300]\left[\frac{100}{89.3}\right]^{0.4/1.4} = 309.6\ \text{K}.$$

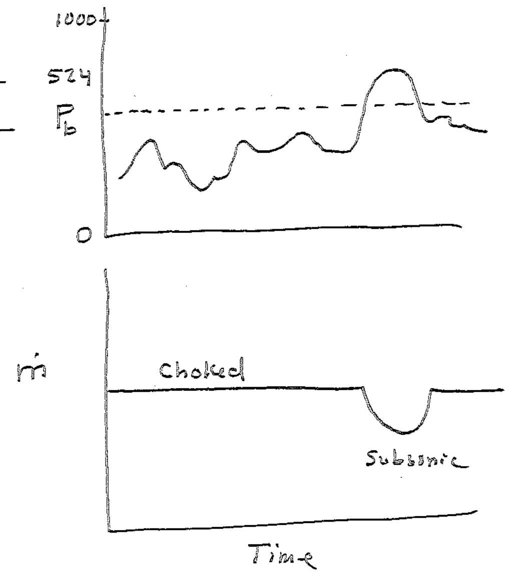
$$\dot{w} = \dot{m} C_p (T_6 - T_5) = \frac{2.49\ \text{Kg}}{\text{s}} \left| \frac{1.005\ \text{KJ}}{\text{Kg}-\text{K}} \right| 309.6 - 300\ \text{K} = \underline{\underline{24.12\ \text{KW}}}$$

$$\boxed{\dot{Q} = \dot{w}}$$

<u>Some Applications:</u>

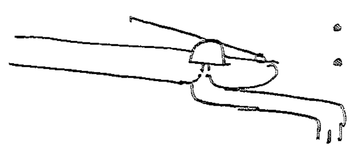
Flow control:





Anytime you throttle an air flow with $P_0 \gtrsim 2 P_b$, you will go sonic at narrowest point

→ e.g. gas stations air line

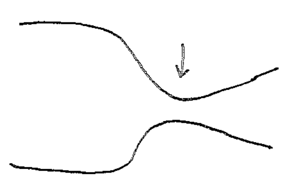


- Sonic @ Narrow point
- Shock wave downstream.

Brain Teaser:

Said throttling ideal gas leads to $\Delta T = 0$ in refrigeration chapter.

But: Conv/Div. nozzle does get cool?



The throttle has negligible KE on either end, so $T_{in} = T_{out} = T_0$

⇒ Same as nozzle.

But could have heat absorption @ throat ⇒ supersonic refrigerator?