So: for ① → ② along path Ⓐ $\Delta S + \left[\int_1^2 \left(\frac{\delta Q}{T}\right)_{\cancel{\text{sys}}\ \text{Surround}}\right] > 0$ Irr.

$$\Delta S + \left[\int_1^2 \left(\frac{\delta Q}{T}\right)\right]_{\text{surr}} = 0 \quad \text{Reversible}$$

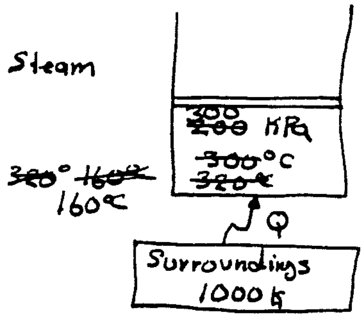
This principle is frequently written as ⇒

$$S_{\text{generated}} = \Delta S_{\text{system}} + \left[\int_1^2 \left(\frac{\delta Q}{T}\right)\right]_{\text{surroundings}}$$

Where $S_{gen} \geq 0$

The implication here is that S_{universe} always increases for any real operation. ΔS_{system} may actually decrease, but increase in entropy of surroundings more than makes up for it, always.

Example:



Expand at constant P until double volume

⇒ Find S_{gen}.

① System: 300 ~~200~~ KPa, 160°C ~~320°C~~ $\xrightarrow[\text{A-4}]{\text{Superheat}}$

$S_1 = 7.1276$ ~~7.7722~~ KJ/Kg-K

$v_1 = 0.651$ m³/Kg

~~$u_1 = 2587.1$~~ KJ/Kg

$h_1 = 2782.3$ KJ/Kg.

② System: 300 KPa, $v_2 = 2v_1 = 1.302$ m³/Kg $\xrightarrow[\text{A-4}]{\text{Superheat}}$

$S_2 = 8.522$ KJ/Kg-K

$h_2 = 3648.2$ KJ/Kg

$$\Delta S_{sys} = S_2 - S_1 = 1.394 \text{ KJ/Kg-K}$$

Surroundings: $\int_1^2 \left(\frac{\delta Q}{T}\right)_{surr} \overset{?}{=} \frac{Q_{surr}}{T_{surr}} \Rightarrow$ $Q = h_2 - h_1$ (const P) $= 865.9$ KJ/Kg

But in this case, heat flow is referenced to surroundings, not system.

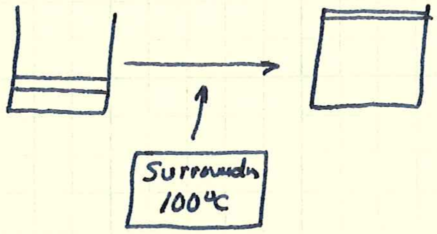
$$\frac{Q_{surr}}{T_{surr}} = \frac{-865.9 \text{ KJ/Kg}}{1000 \text{ K}} = -0.8659 \text{ KJ/Kg-K}$$

$$S_{gen} = 1.394 - 0.8659 = 0.5281 \text{ KJ/Kg-K}$$

⇒ Some comments with respect to this —

So we made entropy... process cannot be reversed... why?

Suppose Sat Liquid
100°C
1 Kg.



Sat. Vapor - 100°C.

Reversible? Yes! $S_{gen} \underset{\uparrow\ \text{probably}}{=} 0$

Check quickly $\Delta S_{system} = S_g - S_f = 6.0480\ \text{kJ/kg-K}.$

$\underset{\uparrow\ p=\text{const}}{Q} = h_2 - h_1 = h_g - h_f = 2257\ \text{kJ/kg}$

Here, "-" because Q is ref'd to surroundings.

$$S_{gen} = \Delta S_{sys} + \left[\int_1^2 \frac{\delta Q}{T}\right]_{surroundings} = 6.0480 + \frac{(-2257\ \text{kJ/kg})}{373.15\ \text{K}} = 0$$

~~May 21~~

Got through increase in entropy principle: Three forms (for surroundings at constant temperature).

σ in book

Closed System: $S_{gen} = m(s_2 - s_1) + \frac{Q_{surr}}{T_{surr}} \geq 0$

Steady Flow: $\dot{S}_{gen} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i + \frac{\dot{Q}_{surr}}{T_{surr}} \geq 0$

System

$- \int_1^2 \left(\frac{\delta Q}{T}\right)_{sys\ bdy}$

$- \frac{\dot{Q}_{sys}}{T_{sys}}$

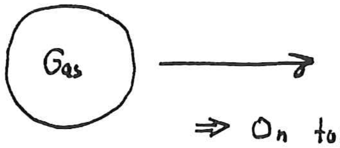
Unsteady (Fixed Endstates) $S_{gen} = \sum m_e s_e - \sum m_i s_i + (m_2 s_2 - m_1 s_1)_{cv} \geq 0$

$+ \frac{Q_{sur}}{T_{sur}}$ $- \int_1^2$

⇒ Key point: Q_{surr} is referenced to <u>surroundings</u> so heat flow <u>out</u> of system and <u>into</u> surroundings is <u>positive</u>.

One way to think of entropy is as a measure of the <u>disorganization</u> of energy.

Case 1:

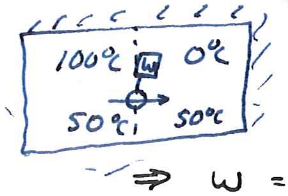


Moving: Has kinetic energy.

⇒ On top of random motion, we have an organized motion. Could bring the gas to a rest and recover work … organized energy that we can use elsewhere.

Case 2:

KE from case 1 is converted to internal energy



⇒ Now energy is random and disorganized. Cannot directly convert it to organized energy ⇒ Work.

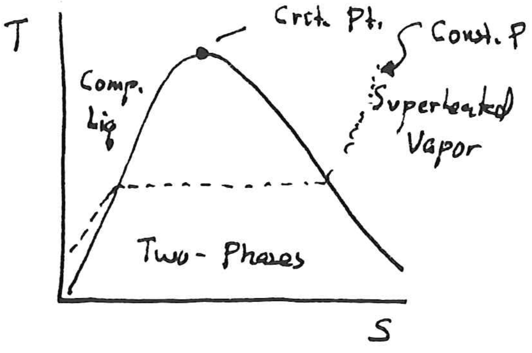
⇒ W = flow of organized energy $\xrightarrow{100\%}$ Heat … Disorganized Energy

Q = flow of disorganized energy $\xrightarrow{<100\%}$ Work.

⇒ Its a one way street, and the increase in s principle expresses this.

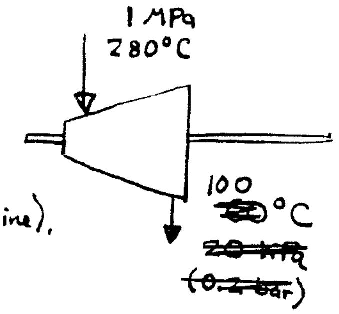
Entropy Data: Steam, R-12, R-134a, etc. Read it off the chart.

The T-s diagram:



Advantage: Constant entropy processes (<u>isentropic</u> processes) are represented as straight lines.

Simple example: Steam Turbine



$\dot{W} = ?\ \frac{kJ}{kg}$

You've gone to the catalog, and ordered a <u>reversible</u>, <u>adiabatic</u> turbine. (Top of the line).

How do I solve this? $-\frac{\dot{W}}{\dot{m}} = h_e - h_i$

→ Know state 1. But only know T_2 ∴ could be any pressure.

→ Go back to increase in entropy principle for flow system (one inlet & outlet).

$$S_{gen} = \dot{m}\left(\overset{s_e - s_i}{\cancel{s_2} - \cancel{s_1}}\right) + \frac{\dot{Q}_{surr}}{T_{surr}} \geq 0$$

$$\left.\begin{array}{ll} \text{① What does reversible mean?} & S_{gen} = 0 \,! \\ \text{② How about adiabatic?} & \dot{Q}_{surr} = 0 \end{array}\right\} \Rightarrow s_e - s_i$$

So for a reversible, adiabatic process, $s_i = s_e$.

$$\text{ⓘ}\ \left.\begin{array}{l} 1\ \text{MPa} \\ 280°C \end{array}\right\} \xrightarrow{A-4} \left\{\begin{array}{l} h_i = 3008.2\ \text{kJ/kg} \\ s_i = 7.0465\ \text{kJ/kg-K} \end{array}\right.$$

		s_f	s_g
A-2	100°C	1.3069	7.3549

$$\text{ⓔ}\ \left.\begin{array}{l} T_e = 100°C \\ s_e = s_i = 7.0465\ \text{kJ/kg-K} \end{array}\right\} \xrightarrow[\text{superheated ?}]{\text{Two-Phase or}}$$

⇒ Since we are between, we are in two-phase.

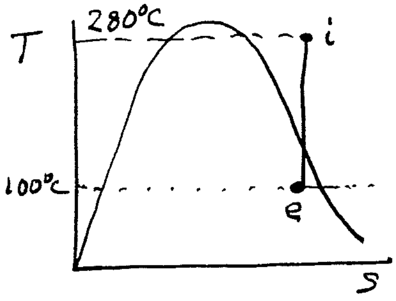
⇒ If $s_e > s_g$ → superheat.

$$\text{Get } x_e = \frac{s_e - s_f}{s_g - s_f} = \frac{7.0465 - 1.3069}{7.3549 - 1.3069} = 0.949$$

$$\text{Get } h_e = h_f + x_e h_{fg} = 419.04 + (0.949)(2257) = 2561\ \text{kJ/kg}$$

$$-\frac{\dot{W}}{\dot{m}} = h_e - h_i = 2561 - 3008.2\ \text{kJ/kg} \Rightarrow \boxed{\frac{\dot{W}}{\dot{m}} = 447.25\ \text{kJ/kg}}$$

On the T-s diagram, what would this look like? Where would we start & end?

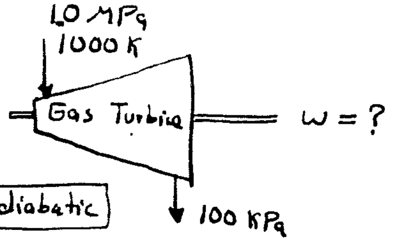


May 13

BM

Gases are a little bit trickier. Need to go back to fundamentals.

For gases, the basic problem we wish to solve is:



Put on right Side - Save ?

If we know T_e, then we could solve ⇒ $-\frac{\dot{w}}{\dot{m}} = \underbrace{h_e - h_i}_{\rightarrow \text{ Air Tables.}}$

⇒ Don't have tabular entropy data for air like we do for steam (do have S°, but this isn't directly entropy).

– – – – – – –

Start with definition of entropy: $\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev.}$ or $dS = \left(\frac{\delta Q}{T}\right)_{rev.}$

So: $(\delta Q)_{rev} = Tds$

Now first law, written specifically for case of reversible process:

$$(\delta Q)_{rev} - (\delta W)_{rev} = dU$$

But for a reversible path: $(\delta W)_{rev} = PdV$ ... so substituting for δQ, δW.

$TdS - PdV = dU$ $\boxed{\cancel{dU = TdS - PdV}}$ $\cancel{= dU}$ One of the Gibbs Relations.

or $\boxed{TdS = dU + PdV}$

Can get the other from the definition of enthalpy. $H = U + PV$

⇒ Can you figure out what the correct substitution into this equation would lead to? $Tds = \cancel{dh + Pd}\ dh$?

Chain rule: $dH = dU + PdV + VdP$

$$\Rightarrow dH - VdP = \underbrace{dU + PdV}_{\rightarrow \text{ Subst.}}$$

$\boxed{TdS = dH - VdP}$ ⇒ Second Gibbs Relation.

~~So we know~~ A major utility of these equations is that they allow s data to be developed from properties that are easier to measure.

Work with the 2nd equation in specif formal: $\cancel{dS = \frac{dH}{T} - \frac{V}{T}dP}$ $ds = \frac{dh}{T} - \frac{v}{T}dP$

So far, we have made no assumptions in model.

Assume ideal gas: $Pv = RT$, $\frac{v}{T} = \frac{R}{P}$

$$ds = \frac{dh}{T} - \frac{R}{P}dP \qquad \text{Integrate:} \quad \int_i^e ds = \int_i^e \frac{dh}{T} - \int_i^e \frac{R}{P}dP$$

Ask about each term →

$$s_e - s_i = \underbrace{\int_i^e \frac{dh}{T}}_{} - R\ln\left[\frac{P_e}{P_i}\right]$$

→ Cannot directly integrate because T varies with h.

but $dh = C_p dT$

$$s_e - s_i = \int_i^e \frac{C_p}{T}dT - R\ln\left[\frac{P_e}{P_i}\right]$$

Key point. While u, h. depend only on T, not on P for ideal gases, $s = s(T, P)$.

For constant C_p (inert gases, or real gases over small ΔT) can integrate this:

$$\boxed{s_e - s_i = C_p\ln\left[\frac{T_e}{T_i}\right] - R\ln\left[\frac{P_e}{P_i}\right]}$$ - Ideal gas... const C_p.

Conceptually, can you see how you would solve our original turbine problem?

$$\Rightarrow \text{Know} \left.\begin{array}{l} \text{① Pressure Ratio} \\ \text{② } \Delta s = 0 \end{array}\right\} \Rightarrow T_e$$

→ First Law

But if C_p varies with T, then we need to do the integral from C_p vs T data.

⇒ The integral depends <u>only</u> on T, so can define:

$$s^o(T) = \int_0^T \frac{C_p}{T}dT \qquad \text{performed at 1 atm.}$$

⇒ This is tabulated in Table A-16.

⇒ So, For <u>variable</u> C_p ⇒

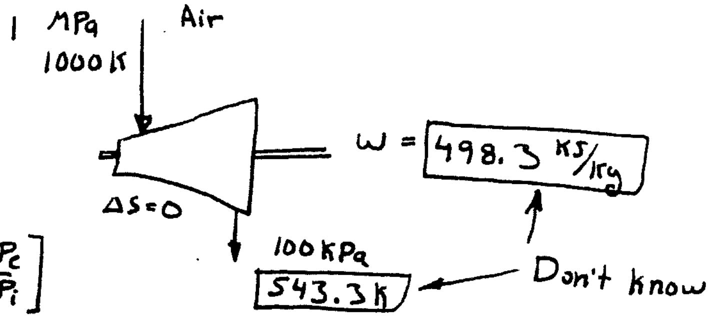
$$\boxed{s_e - s_i = \underbrace{s_e^o - s_i^o}_{\text{Expresses T part}} - \underbrace{R\ln\left[P_e/P_i\right]}_{\text{Expresses P part}}}$$

$$s_e - s_i = \int_i^e \frac{C_p}{T}dT - R\ln\left[\frac{P_e}{P_i}\right]$$

$$\overbrace{\int_0^e \frac{C_p}{T}dT - \int_0^i \frac{C_p}{T}dT} - \overbrace{\left[R\ln\frac{P_e}{1\text{atm}} - R\ln\frac{P_i}{1\text{atm}}\right]}$$

s_e s_i

Now one more trick:



Solved by $\Delta S = 0 = \underbrace{S_e^\circ}_{\to \text{A-16} \to T_e,\, h_e} - S_i^\circ - R\ln\left[\frac{P_e}{P_i}\right]$

For isentropic: $$\underbrace{\exp\left[\frac{S_e^\circ - S_i^\circ}{R}\right]} = \frac{P_e}{P_i} = \underbrace{\frac{\exp\left[\frac{S_e^\circ}{R}\right]}{\exp\left[\frac{S_i^\circ}{R}\right]}}_{\text{Depends only on } T,}$$

Define $P_r = \exp\left[\frac{S_e^\circ}{R}\right] \longrightarrow$ In table A-16.
(relative pressure) not reduced pressure $\boxed{P_r = P/P_c}$

⇒ For isentropic processes, $(\Delta S = 0)$ $$\boxed{\frac{P_e}{P_i} = \frac{P_{r,e}}{P_{r,i}}}$$ Depends only on T!!! in Table A-16.

For our problem: What do we know? All but $P_{r,e}$ ⇒

$$P_{r,e} = P_{r,i}\left[\frac{P_e}{P_i}\right] = \underset{\text{A-16, 1000 K}}{114}\left[\frac{0.1}{1.5}\right] = 11.4 \xrightarrow{\text{A-16}} \begin{array}{l} T_e = 543.3\ \text{K} \\ h_e = 547.75\ \text{kJ/kg} \end{array}$$

$$-\frac{\dot{W}}{\dot{m}} = h_e - h_i \Rightarrow W = 498.3\ \text{kJ/kg}$$

⇒ Same answer as before:

There is a reduced volume function that serves the same purpose.

May 4

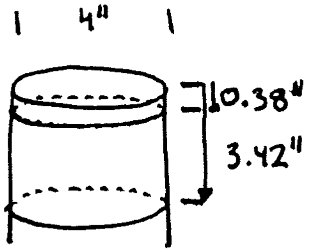
$$\boxed{\frac{V_2}{V_1} = \frac{V_{r,2}}{V_{r,1}}}$$

- For $\Delta S = 0$
- V_r's are a function only of T!!! can be found in Table A-16.

Example: Automobile Engine: 5.0 liter displacement V-8. Compression Ratio = 9 →

$V_{max} - V_{min} = \frac{0.005\ m^3}{8}$ $\qquad \frac{V_{max}}{V_{min}} = 9$ $\qquad 8V_{min} = \frac{0.005}{8}$ ⇒ $V_{min} = 7.81\times10^{-5}\ m^3$

$V_{max} = 7.03\times10^{-4}\ m^3$



$$\left.\begin{array}{l} V_1 = 7.81\times10^{-5}\ m^3 \\ V_2 = 7.03\times10^{-4}\ m^3 \end{array}\right\} \begin{array}{l} P_1 = 7.0\ \text{MPa} \\ T_1 = 2000\ \text{K}. \end{array}$$

⇒ Find how much work you get out of one expansion.

- Assume its reversible,
- Time fast enough so Q=0 $\Big\} \Rightarrow \quad S_2 = S_1$
- Pretend gas has same props as air.

Preliminaries. Mass in cylinder:

$$m = \frac{PV}{RT} = \frac{7000\text{ KPa}\;\big|\;7.81\times10^{-5}\text{ m}^3\;\big|\;\text{Kmole-K}\;\big|\;28.9\text{ Kg}}{2000\text{ K}\;\big|\;\;\big|\;8.314\text{ KPa-m}^3\;\big|\;\text{Kmole}}$$

$$m = 9.50\times10^{-4}\text{ Kg}$$

What is first law?

$$Q - W = m(u_2 - u_1) \Rightarrow W = m(u_1 - u_2) \qquad ①\ 2000\text{K} \rightarrow u_1 = 1678.7\ \text{KJ/Kg}.$$

Since $\Delta S = 0$, Have available:

$$\left.\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}} \qquad \frac{V_2}{V_1} = \frac{V_{r,2}}{V_{r,1}}\right\}$$

⇒ Both are valid.

⇒ Which has the greater utility here?

Know volume ratio

Can look up @ 2000K, Table A-16.

$$V_{r,2} = V_{r,1}\left[\frac{V_2}{V_1}\right] = 2.776\left[\frac{7.03\times10^{-4}}{7.81\times10^{-5}}\right] = 24.99 \xrightarrow{A\text{-}16} T_2 = 1002.5\text{K}$$

$$u_2 = 761.1\ \text{KJ/Kg}$$

$$W = m(u_1 - u_2) = \left[9.50\times10^{-4}\text{ Kg}\right]\left[1678.7 - 761.1\ \text{KJ/Kg}\right] = \underline{\underline{0.87\text{ KJ}}}$$

How would you find the final pressure?

One Way

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \qquad P_2 = P_1\left[\frac{V_1}{V_2}\right]\left[\frac{T_2}{T_1}\right] = (7000\text{ KPa})\left[\frac{7.81\times10^{-5}}{7.03\times10^{-4}}\right]\left[\frac{1002.5}{2000}\right] = 390\text{ KPa}.$$

Alternate way:

$$\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}} \Rightarrow P_2 = P_1\left[\frac{P_{r,2}}{P_{r,1}}\right] = (7000\text{ KPa})\left[\frac{115.2}{2068}\right] = 390\text{ KPa}.$$

~~Next topic:~~

This describes how we handle <u>variable</u> C_p, $\Delta S = 0$ problems. Now we want to get into constant C_p.

$$S_2 - S_1 = C_p \ln\left[\frac{T_2}{T_1}\right] - R\ln\left[\frac{P_2}{P_1}\right] \qquad \text{so for } \Delta S = 0$$

$$\ln\left[\frac{T_2}{T_1}\right] = \frac{R}{C_p}\ln\left[\frac{P_2}{P_1}\right] = \ln\left[\left(\frac{P_2}{P_1}\right)^{R/C_p}\right]$$

Take exp of both sides ⇒ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{R/C_p}$

$$\boxed{\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{K} \qquad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{K-1}}$$

↑ Similar Derivation

$R = C_p - C_v$ for ideal gas. but $K = \frac{C_p}{C_v}$

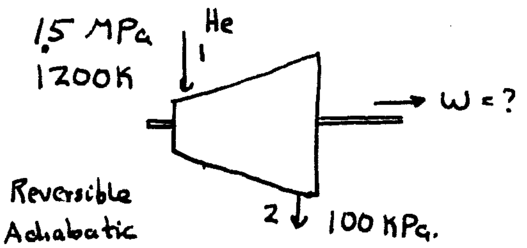
$$\frac{R}{C_p} = \frac{C_p - C_v}{C_p} = 1 - \frac{C_v}{C_p} = 1 - \frac{1}{K} = \frac{K-1}{K} \Rightarrow \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{K-1/K}}$$

Note that the P-V relationship says that $P_1 V_1^{K} = P_2 V_2^{K} = PV^{K}$
For all points between.

So this is where the Polytropic Processes we discussed way back come in:

For isentropic processes, ideal gas, const. $C_p \Rightarrow PV^{K} = \text{const}$
Air, $K = 1.4$ at room T.

Example: Nuclear power plant operating on helium cycle.



- Reversible
- Adiabatic

First Law $-\frac{\dot{W}}{\dot{m}} = h_2 - h_1$

No helium tables... why?

$$\Rightarrow \frac{\dot{W}}{\dot{m}} = C_p (T_1 - T_2)$$

Isentropic relations say: $\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{K-1}{K}}$

$$\overline{C_p} = \frac{5}{2} R$$

$$\overline{C_v} = \overline{C_p} - R = \frac{3}{2} R$$

$$T_2 = T_1 \left[\frac{P_2}{P_1}\right]^{\frac{K-1}{K}} = [1200\text{K}]\left[\frac{0.1\text{ MPa}}{1.5\text{ MPa}}\right]^{\frac{0.667}{1.667}} = 406.07\text{ K}$$

$$\Rightarrow \frac{\overline{C_p}}{\overline{C_v}} = \frac{5/2\, R}{3/2\, R} = \underset{\uparrow}{1.667}$$

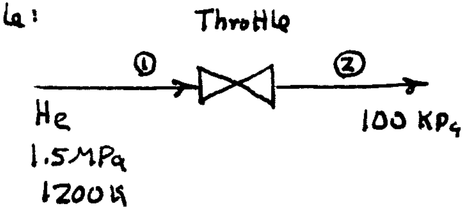
All inert gases.

$$\frac{\dot{W}}{\dot{m}} = \frac{5 \,\big|\, 8.314\text{ KJ} \,\big|\, \text{Kmole} \,\big|\, 1200 - 406.07\text{ K}}{2 \,\big|\, \text{Kmole-K} \,\big|\, 4\text{ Kg} \,\big|} = 4{,}125.46\ \text{KJ/Kg}$$

~~Now, same example, but its an internally reversible turbine, with a heat loss of 1000 KJ/kg.~~

~~First Law: $\frac{\dot{W}}{\dot{m}} = \frac{\dot{Q}}{\dot{m}} + C_p(T_1 - T_2)$~~ ~~Same problem, but how to find T_2?~~

Now another example:



What is T_2?
$T_2 = 1200\text{K}$,
How about non-ideal? $\Rightarrow h_1 = h_2$, but $T_1 \neq T_2$.

Find entropy generation for this process two ways ① Normal Way
② from definition.

Ideal gas: Constant $C_p, C_v \Rightarrow$ ~~S_2 S_1~~

$$(\dot{S}_{gen})_{sys} = \dot{m}(S_2 - S_1) - \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_{sys}}^{\,0}$$

$$S_2 - S_1 = \cancel{C_p \ln\left[\frac{T_2}{T_1}\right]}^{\,0} - R\ln\left[\frac{P_2}{P_1}\right] = -\frac{8.314\text{ KJ} \,\big|\, \text{Kmole}}{\text{Kmole-K} \,\big|\, 4\text{ Kg}} \ln\left[\frac{0.1}{1.5}\right] = \boxed{5.63\ \frac{\text{KJ}}{\text{Kmole-K}}}$$

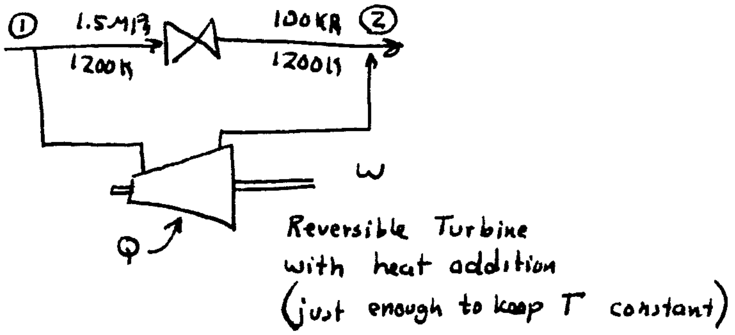
May 6
DTP on
η_c, η_T

From definition:

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev.}$$

May 9

Need to construct a reversible path between ① and ②, which the throttle isn't!
This will be a little bit different than last time:



Reversible Turbine
with heat addition
(just enough to keep T constant)

First Law: $\dot{Q} = \dot{W}$ ($\Delta T = 0$) [Note that this ~~isn't~~ isn't a cycle... in a cycle $Q_{in} > W_{net}$]

How to get the work... only thing we have is P_1, P_2.

Within the turbine, for each parcel of gas $\frac{PV}{T} = \text{const}$... but since $T = \text{const}$

$$\Rightarrow PV = \text{const} = P_1V_1 \qquad w = \int P dV = \int \frac{P_1V_1}{V} dV = P_1 v_1 \ln\left[\frac{v_2}{v_1}\right]$$

but since $Pv = RT \Rightarrow \frac{v_2}{v_1} = \frac{P_1}{P_2}$.

$$w = RT_1 \ln\left[\frac{P_1}{P_2}\right] = \frac{8.314\ \text{KJ}}{\text{Kmole-K}} \bigg| \frac{1200\,\text{K}}{} \bigg| \frac{\text{Kmole}}{4\,\text{Kg}} \ln\left[\frac{1.5}{0.1}\right]$$

$$w = 6754\ \text{KJ/Kg} = Q$$

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{Rev} = \frac{Q}{T} = \frac{6754\ \text{KJ/Kg}}{1200\ \text{K}} = \boxed{5.63\ \text{KJ/Kg-K}}$$

Same as before

Review of Major Relations:

Definition of entropy: $\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev.}$

→ Can be used directly if path is internally reversible

→ If process between 1 and 2 is not reversible - must construct alternate reversible path to evaluate.

~~From definition~~

$$(S_{gen})_{system} = (S_2 - S_1)_{system} - \int_1^2 \left(\frac{\delta Q}{T}\right)_{system.}$$

$$(S_{gen})_{sys + surroundings} = (S_2 - S_1)_{system} + \underbrace{\frac{Q_{surr}}{T_{surr}}}$$

→ Assumes surroundings are at constant T_0

① Note that you have to use correct sign on Q

② Other forms include steady flow & unsteady process.

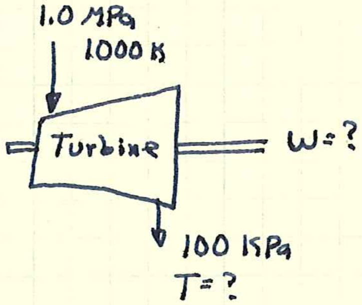
For reversible ($S_{gen} = 0$), adiabatic ($Q = 0$) processes - ⇒ This converts to $S_2 = S_1$

PCM gave May 13.

Exam on May 15.

May 17

Brief Summary:



First Law: $\delta Q - \delta W = dU$

Assume reversible paths $(\delta Q)_{rev} - (\delta W)_{rev} = dU$

$(\delta W)_{rev} = PdV$ $\qquad dS = \frac{(\delta Q)_{rev}}{T}$

$\Rightarrow TdS - PdV = dU$ or $TdS = \underbrace{dU + PdV}$ $\Rightarrow$ Gibbs. 1

$H \equiv U + PV$ so $dH = dU + PdV + VdP$ or $dH - VdP = \overbrace{dU + PdV}$

$\Rightarrow TdS = dH - VdP$ Gibbs 2.

$dS = \frac{dh}{T} - \frac{V}{T}dP$ Get S from simple props.

Integrate: ~~$\Delta S =$~~ $dh = C_p dT$, $\frac{v}{T} = \frac{R}{P}$

$$\Delta S = \int_{T_1}^{T_2} \frac{C_p}{T} dT - \int_{P_1}^{P_2} \frac{R}{P} dP = \int_{T_1}^{T_2} \frac{C_p}{T} dT - R \ln \frac{P_2}{P_1}$$

Now can go a couple of ways:

① C_p = constant (He, Air when ΔT isn't too big ... I will make it clear)

$$\Delta S = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

If turbine is reversible, adiabatic $\left[S_{gen} = (\Delta S)_{sys} + \frac{Q_{surr}}{T_{surr}} \right]$

$\Rightarrow \Delta S = 0$

$\ln\left[\frac{T_2}{T_1}\right] = \frac{R}{C_p} \ln\left[\frac{P_2}{P_1}\right]$ or $\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{R/C_p}$ $\qquad R = C_p - C_v$

$\boxed{\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{K-1/K}}$ where $K = C_p/C_v$ $\cong$ 1.4 for air
= 1.67 for inerts.
$\cong$ 1.3 for hot air.

Similar ~~Relation~~ Derivations. $\boxed{\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{K}}$ $\qquad \boxed{\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{K-1/K}}$

② C_p = variable:

Get from Table A-17

$$\boxed{\Delta S = \overbrace{\underbrace{S_2^0 - S_1^0}_{\text{T-only}}} - \underbrace{R\ln\frac{P_2}{P_1}}_{\text{P-only}}}$$

$$\overbrace{\int_0^{T_2}\frac{C_p}{T}dT - \int_0^{T_1}\frac{C_p}{T}dT}$$

Note that relation says that for
- Ideal gas.
- Const C_p, C_v
- $\Delta S = 0$ (most work).

$\Rightarrow$ $P_1 v_1^K = P_2 v_2^K = \text{const}$

$\Rightarrow$ Polytropic Process, where $K = C_p/C_v$

For turbine where $\Delta S = 0$; $\exp\left[\frac{S_2^0 - S_1^0}{R}\right] = \frac{P_2}{P_1}$

$$\Rightarrow \underbrace{\frac{\exp\left[S_2^0/R\right]}{\exp\left[S_1^0/R\right]}}_{\text{Depends only on T}} = \frac{P_2}{P_1}$$

Define $\exp\left[S_2^0/R\right] = P_{r,2}$ $\Rightarrow$ Table A-17

$\Rightarrow$ For $\Delta S = 0$ $\boxed{\frac{P_{r,2}}{P_{r,1}} = \frac{P_2}{P_1}}$ $\Rightarrow$ Gives you way to solve turbine problem.

- Know P_2/P_1, $T_1 \Rightarrow P_{r,1}$

$\Rightarrow$ Equation $P_{r,2} \Rightarrow T_2$, $\Rightarrow h_2$ $\}$ $\boxed{w = h_1 - h_2}$

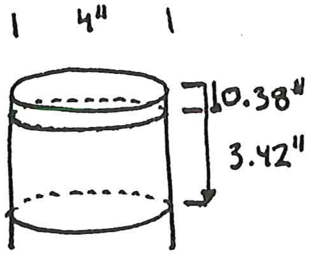
Similar idea for volume: $\boxed{\frac{v_{r,2}}{v_{r,1}} = \frac{v_2}{v_1}}$ For $\Delta S = 0$

$v_r = f(T)$ only - Table A-17

Example: Automobile Engine: 5.0 liter displacement V-8. Compression Ratio = 9 $\}\rightarrow$

$V_{max} - V_{min} = \frac{0.005\,m^3}{8}$ $\frac{V_{max}}{V_{min}} = 9$ $8V_{min} = \frac{0.005}{8} \Rightarrow$ $V_{min} = 7.81\times10^{-5}\,m^3$

$V_{max} = 7.03\times10^{-4}\,m^3$



$V_1 = 7.81\times10^{-5}\,m^3$
$V_2 = 7.03\times10^{-4}\,m^3$ $\}$ $P_1 = 7.0\,MPa$, $T_1 = 2000\,K.$

$\Rightarrow$ Find how much work you get out of one expansion.

- Assume its reversible,
- Time fast enough so $Q=0$

$\Rightarrow S_2 = S_1$

- Pretend gas has same props as air.

Preliminaries. Mass in cylinder:

$$m = \frac{PV}{RT} = \frac{7000\,\text{KPa}}{2000\,\text{K}}\Bigg|\,\frac{7.81\times10^{-5}\,\text{m}^3}{}\Bigg|\,\frac{\text{Kmole-K}}{8.314\,\text{KPa-m}^3}\Bigg|\,\frac{28.9\,\text{Kg}}{\text{Kmole}}$$

$$m = 9.50\times10^{-4}\ \text{Kg}$$

What is first law?

$$Q - W = m(u_2 - u_1) \Rightarrow W = m(u_1 - u_2)$$

① 2000K → $u_1 = 1678.7\ \text{KJ/Kg}$.

Since $\Delta S = 0$, Have available:

$$\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}} \qquad \frac{V_2}{V_1} = \frac{V_{r,2}}{V_{r,1}}$$

⇒ Both are valid.
⇒ Which has the greater utility here?

Know volume ratio

Can look up @ 2000K, Table A-16.

$$V_{r,2} = V_{r,1}\left[\frac{V_2}{V_1}\right] = 2.776\left[\frac{7.03\times10^{-4}}{7.81\times10^{-5}}\right] = 24.99 \xrightarrow{A\text{-}16} T_2 = 1002.5\,\text{K}$$

$$u_2 = 761.1\ \text{KJ/Kg}$$

$$W = m(u_1 - u_2) = \left[9.50\times10^{-4}\,\text{Kg}\right]\left[1678.7 - 761.1\ \text{KJ/Kg}\right] = \underline{\underline{0.87\ \text{KJ}}}$$

May 20

How would you find the final pressure?

One Way

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad P_2 = P_1\left[\frac{V_1}{V_2}\right]\left[\frac{T_2}{T_1}\right] = (7000\,\text{KPa})\left[\frac{7.81\times10^{-5}}{7.03\times10^{-4}}\right]\left[\frac{1002.5}{2000}\right] = 390\ \text{KPa}.$$

Alternate way:

$$\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}} \Rightarrow P_2 = P_1\left[\frac{P_{r,2}}{P_{r,1}}\right] = (7000\,\text{KPa})\left[\frac{115.2}{2068}\right] = 390\,\text{KPa}.$$

~~Next topic:~~

This describes how we handle variable C_p, $\Delta S=0$ problems. Now we want to get into constant C_p.

$$S_2 - S_1 = C_p \ln\left[\frac{T_2}{T_1}\right] - R\ln\left[\frac{P_2}{P_1}\right] \quad \text{so for } \Delta S = 0$$

$$\ln\left[\frac{T_2}{T_1}\right] = \frac{R}{C_p}\ln\left[\frac{P_2}{P_1}\right] = \ln\left[\left(\frac{P_2}{P_1}\right)^{R/C_p}\right]$$

Take exp of both sides $\Rightarrow \dfrac{T_2}{T_1} = \left(\dfrac{P_2}{P_1}\right)^{R/C_p}$

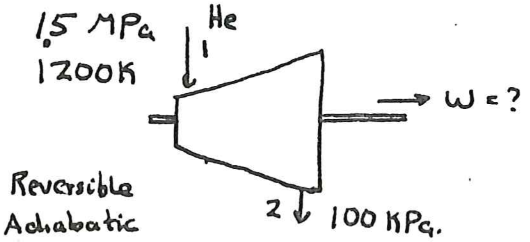
$R = C_p - C_v$ for ideal gas. but $k = \dfrac{C_p}{C_v}$

$$\frac{R}{C_p} = \frac{C_p - C_v}{C_p} = 1 - \frac{C_v}{C_p} = 1 - \frac{1}{k} = \frac{k-1}{k} \Rightarrow \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{k-1/k}}$$

$$\boxed{\begin{aligned}\frac{P_2}{P_1} &= \left(\frac{v_1}{v_2}\right)^{k} \\ \frac{T_2}{T_1} &= \left(\frac{v_1}{v_2}\right)^{k-1}\end{aligned}}$$

↑ Similar Derivation

Example: Nuclear power plant operating on helium cycle.



- Reversible
- Adiabatic

First Law $-\frac{\dot{w}}{\dot{m}} = h_2 - h_1$

No helium tables... why ?

$\Rightarrow \frac{\dot{w}}{\dot{m}} = C_p(T_1 - T_2)$

Isentropic relations say: $\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{\frac{K-1}{K}}$

$\bar{C}_p = \frac{5}{2}R$

$\bar{C}_v = \bar{C}_p - R = \frac{3}{2}R$

$$T_2 = T_1\left[\frac{P_2}{P_1}\right]^{\frac{K-1}{K}} = \left[1200\,K\right]\left[\frac{0.1\text{ MPa}}{1.5\text{ MPa}}\right]^{\frac{0.667}{1.667}} = 406.07\,K$$

$\Rightarrow \frac{\bar{C}_p}{\bar{C}_v} = \frac{5/2\,R}{3/2\,R} = 1.667$

All inert gases.

$$\frac{\dot{w}}{\dot{m}} = \frac{5}{2}\left|\frac{\overset{2.08}{\cancel{8.314}}\ KJ}{\cancel{Kmole}\cdot K\ \ Kg}\right|\frac{\cancel{Kmole}}{4\cancel{Kg}}\left|1200 - 406.07\,K\right| = 4{,}125.46\ KJ/Kg$$

May 3

Now we can solve the turbine problem:

$$S_e - S_i = 0 = S^\circ_e - S^\circ_i - R\ln\frac{P_e}{P_i}$$

$$S^\circ_e = S^\circ_i + R\ln\frac{P_e}{P_i} = 2.96770\,\frac{KJ}{Kg\text{-}K} + \frac{8.314\ KJ}{Kmole\text{-}K}\left|\frac{Kmole}{28.9\ Kg}\right.\ln\left[\frac{0.1\ MPa}{1.0\ MPa}\right]$$

$$S^\circ_e = 2.305\,\frac{KJ}{Kg\text{-}K} \xrightarrow{A\text{-}16} \begin{array}{l} T_e = 543.3\ K \\ h_e = 547.75\ KJ/kg \end{array}$$

$$1000\,K \longrightarrow h_i = 1046.04\ KJ/kg$$

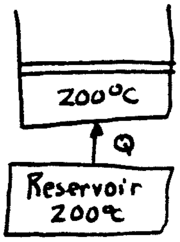
$$-\frac{\dot{W}}{\dot{m}} = h_e - h_i = 547.75 - 1046.04 \Rightarrow \boxed{\frac{\dot{W}}{\dot{m}} = 498.3\ KJ/Kg}$$

~~A slightly different derivation leads you to another useful expression:~~

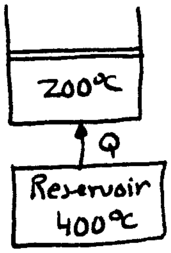
$$\cancel{S_e - S_i =}$$

~~A~~ Another concept: Internal Reversibility: Everything inside boundaries is reversible.

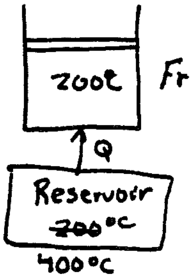
Isothermal Expansion



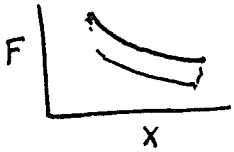
Totally Reversible



Internally Reversible
(Q across ΔT is outside body)



Friction.

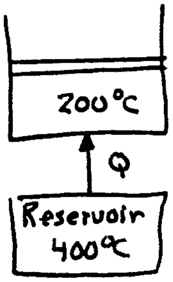


Less W out
Beginning and end states same.
⇒ less Q in.

Also works

$$\boxed{\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\substack{\text{Internally}\\\text{Reversible}}}}$$

⇐ Lets you find Q.



~~Find~~

- Q Added until Volume doubles.
- T stays constant.
- Internally Reversible.

$T_1 = 200°C$ Sat Vapor $\longrightarrow$

$P_1 = 1.554\ MPa$

$u_1 = 2595.3\ \cancel{m^3}\ KJ/Kg$

$v_1 = 0.1274\ m^3/Kg$

$S_1 = 6.4323\ KJ/Kg\text{-}K$

Find Q, W, entropy gen. for sys.
entropy gen for universe.

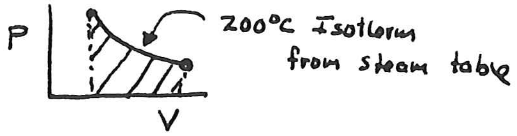
$P_2 = 0.8441\ MPa$

$u_2 = 2628.6\ KJ/Kg.$

$S_2 = 6.794\ KJ/Kg\text{-}K$

For Q, W, ⇒ First Law looks like $Q - W = m(u_2 - u_1)$ $W = \int P\,dV$, but P varies.

One approach is to use brute force ~~to~~



⇒ Get W... then Q from first law.

Alternate. Since process is <u>internally</u> <u>reversible</u>: $\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)$

Since T = constant: $T(s_2 - s_1) = Q = [200 + 273\,\text{K}][6.794 - 6.4323\ \text{KJ/Kg-K}]$

$= \boxed{+171.1\ \text{KJ/Kg.}}$

Now use first law to get $W = Q - m(u_2 - u_1)$

$= 171.1\ \text{KJ/Kg} - [2628.6 - 2595.3\ \text{KJ/Kg}] = \boxed{137.78\ \text{KJ/Kg}}$

⇒ This gives us an additional tool in our arsenal ⇒ ~~If T is~~

⇒ If T is constant & process is internally reversible, can quickly get Q.

⇒ If P is constant, can quickly get W.

⇒ Whichever you get, use first law to find the other.

$$(S_{gen})_{system} = m(s_2 - s_1) - \int_1^2 \left(\frac{\delta Q}{T}\right)_{sys}$$

$$= m(s_2 - s_1) - \frac{Q_{sys}}{T_{sys}} = 6.794 - 6.4323\ \frac{\text{KJ}}{\text{Kg-K}} - \frac{171.1\ \text{KJ/Kg}}{473\ \text{K}}$$

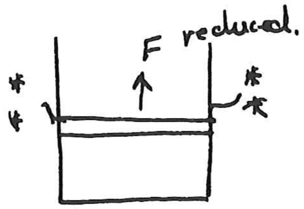
$$(S_{gen})_{sys} = \boxed{0}$$

$$(S_{gen})_{\substack{sys+\\surr}} = m(s_2 - s_1) + \frac{Q_{surr}}{T_{surr}} = 6.792 - 6.4323 + \frac{(-171.1\ \text{KJ/Kg})}{673.\ \text{K}}$$

$$\boxed{(S_{gen})_{\substack{sys\\+surr}} = 0.105\ \frac{\text{KJ}}{\text{Kg-K}}}$$

Since S was generated, what was the source of irreversibility?
Q across ΔT: 400 → 200°C.

Now think about case where there is friction so that $W < \int P\,dV$.
Begining & End states the same:



$F = 0.75 F_{\text{No friction}}$

$W = 0.75\, W_{\text{Reversible}} = 0.75\left[137.78\ \frac{\text{KJ}}{\text{Kg}}\right]$

$= 103.34\ \text{KJ/Kg.}$

First Law: $Q = m(u_2 - u_1) + W = [2628.6 - 2595.3\ \text{KJ/Kg}] + 103.34\ \text{KJ/Kg}$

$Q = 136.64\ \text{KJ/Kg.}$

$$(S_{gen})_{system} = m(s_2 - s_1) - \frac{Q_{sys}}{T_{sys}} = 6.794 - 6.4323\ \frac{\text{KJ}}{\text{Kg-K}} - \frac{136.64}{473\text{K}} = 0.073\ \text{KJ/Kg-K}$$

From definition of S, and first law for reversible systems ⇒ Gibbs relations.

$$\left.\begin{aligned} Tds &= du + Pdv \\ Tds &= dh - v\,dP \end{aligned}\right\}$$

Gives us a way to find ΔS from measurable props.

① Develop s values in steam tables via numerical integration.

② For ideal gas - can ~~numerically~~ analytically integrate.

2nd Equation:

I. $$(s_2 - s_1)_{sys} = \left[\int_1^2 \frac{dh}{T} - \int_1^2 \frac{v\,dP}{T}\right]$$

$$(s_2 - s_1)_{sys} = \int_1^2 \frac{c_p}{T}dT - R\ln\left[\frac{P_2}{P_1}\right]$$

⇒ For ideal gas - s depends on both P, T.

Variable C_p.

$$(s_2 - s_1)_{sys} = \underline{s_2^\circ - s_1^\circ} - R\ln\left[\frac{P_2}{P_1}\right]$$

- From gas tables
- Function of T only.

For isentropic processes: $(\Delta S)_{sys} = 0$

$$\underbrace{\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}} \qquad \frac{v_2}{v_1} = \frac{v_{r,2}}{v_{r,1}}}$$

- Function of T only
- Found in Table A-16.

⇒ For other gases that do not list P_r, v_r values (e.g. O_2, N_2), need to use s°

Constant C_p (inerts & small ΔT)

⇒ Integrate directly

$$(s_2 - s_1)_{sys} = c_p \ln\left[\frac{T_2}{T_1}\right] - R\ln\left[\frac{P_2}{P_1}\right]$$

for convienence in some probems, a slight transformation yields.

$$(s_2 - s_1)_{sys} = c_v \ln\left[\frac{T_2}{T_1}\right] + R\ln\left[\frac{v_2}{v_1}\right]$$

For isentropic processes $(\Delta S)_{sys} = 0$

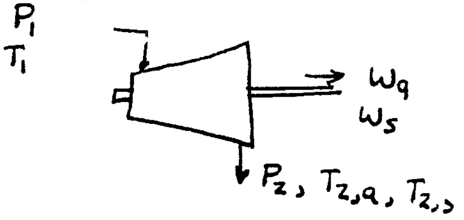
$$\frac{T_2}{T_1} = \left[\frac{P_2}{P_1}\right]^{k-1/k}$$

$$\frac{T_2}{T_1} = \left[\frac{v_1}{v_2}\right]^{k-1}$$

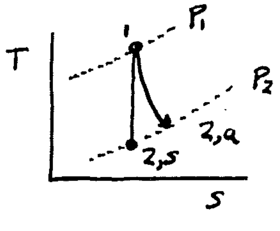
$$\frac{P_2}{P_1} = \left[\frac{v_1}{v_2}\right]^{k}$$

$k = c_p/c_v$

Compressor and Turbine efficiency:



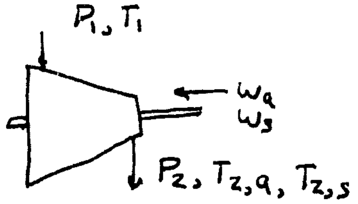
$$\eta_T = \frac{\text{actual } w}{\text{isentropic } w} = \frac{h_1 - h_{2,a}}{h_1 - h_{2,s}} < 1$$

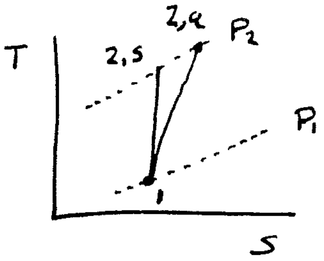


Real η ⇒ Think of some useful work being lost to friction

⇒ Less work comes out because exiting fluid is hotter.

Compressor:





Defined as inverse

$$\eta_c = \frac{\text{isentropic work needed to decomp}}{\text{actual work needed.}}$$

$$= \cancel{h_2}\ \frac{h_{2,s} - h_1}{h_{2,a} - h_1} < 1$$

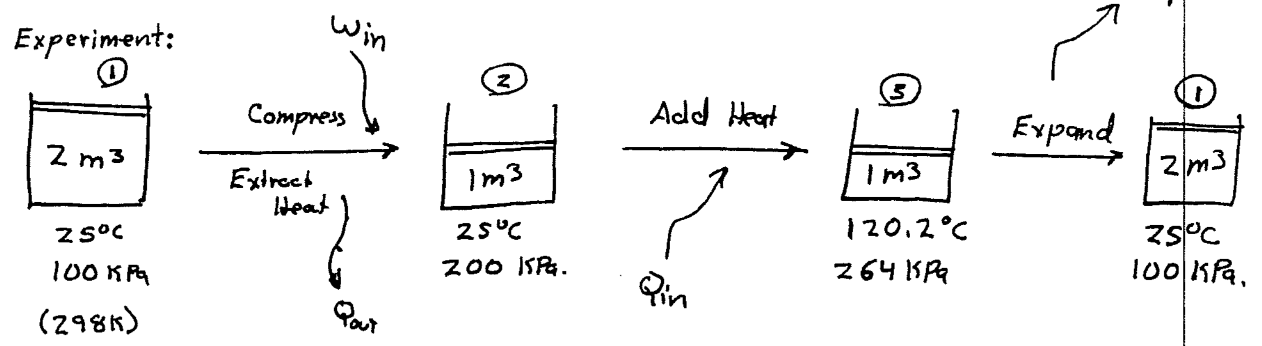
Think of some w_{in} going to increase P,
and some going to friction
➡ Exhaust gas is hotter ⇒ need <u>more</u> work

Announcements: ① HW Notes
② Quiz: Ch. 2.

Cycle Analysis - An Introduction:

$\frac{P_2}{P_1} = \frac{V_1}{\;}$

Thought Experiment:



First Law: Neglect KE, PE.

$Q - W = \Delta U$ ⇒ What is it for entire process?
For entire process, start and end at same place.
⇒ $\Delta U = 0$.

$$\Rightarrow \Sigma Q_{cycle} = \Sigma W_{cycle}.$$

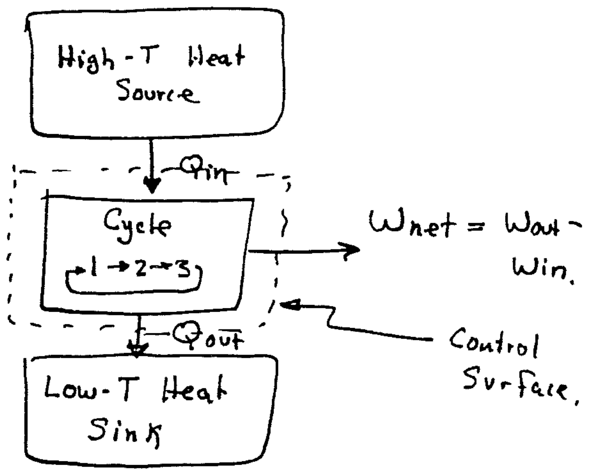
$$\cancel{M}\,\underset{-}{Q_{1-2}} + \underset{+}{Q_{2-3}} = \cancel{M}\,\underset{-}{W_{1-2}} + \underset{+}{W_{2-3}}$$

Last question: ~~Is~~ Which is numerically larger: Win (W_{1-2}) or Wout (W_{2-3})?

$W = \int P dV$ for Win, $P = 100 \rightarrow 200$
for Wout, $P = 264 \rightarrow 100$

⇒ Average pressure is higher for Wout: ⇒ Net work for cycle is positive.

⇒ ~~This~~ Simply process to this:



- Heat Engine: Turns heat into work.
- First Law: (Convention for cycles).

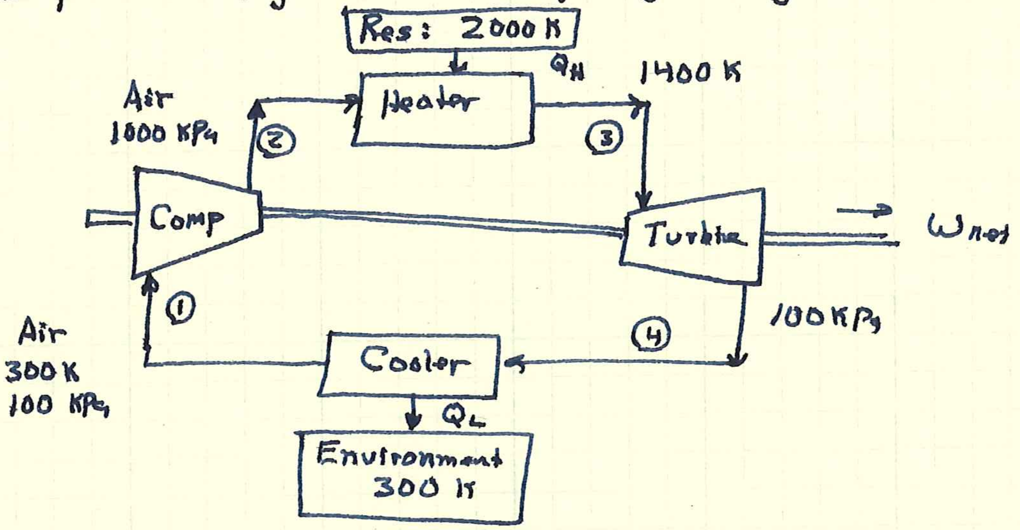
$$\boxed{Q_{in} = Q_{out} + W_{net}}$$

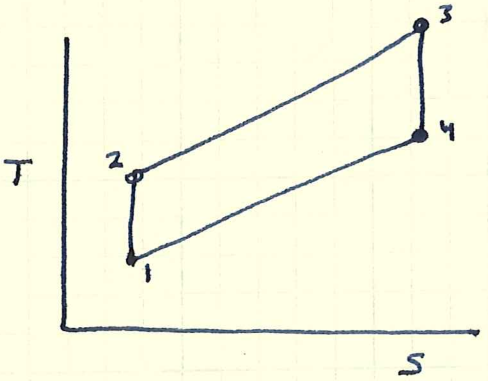
⇒ Drawing the cycle boxes is the one place where we ignore usual first law sign convention. Pos. in direction of arrow.

Think of your automobile as a power cycle:

Q_{in} - Heating value associated with the fuel.
W_{net} = shaft work.
Q_{out} = hot exhaust + radiator.

$\boxed{\text{May 22}}$ Lets put these together into a simple cycle analysis: Gas turbine cycle.





~~Add~~ 1 → 2 Compress: $\Delta S = 0$

2-3 Add heat, $\Delta P = 0$

~~3-4~~ $\Delta S = \int \frac{\delta Q}{T}$ $Q > 0$

3-4 Expand, $\Delta S = 0$

4-1 Lose heat, $\Delta P = 0$

$\boxed{1\to 2}$ Isentropic Compression: $T_1 = 300\text{ K} \Rightarrow h_1 = 300.19\ \text{KJ/Kg}$

$$\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}} \Rightarrow P_{r,2} = P_{r,1}\left[\frac{P_2}{P_1}\right] = \underset{\boxed{\text{A-17, 300 K}}}{1.386}\left[\frac{1000}{100}\right] = 13.86 \xrightarrow{\text{A-17}} T_2 = 574.1\text{ K},\ h_2 = 579.9\ \text{KJ/Kg}$$

$$W_{comp} = h_2 - h_1 = 579.9 - 300.19 = 279.7\ \text{KJ/Kg}$$

$\boxed{2\to 3}$ $q = h_3 - h_2 = 1515.42 - 579.9 = 935.5\ \text{KJ/Kg}$

$\boxed{3\to 4}$

$$\frac{P_4}{P_3} = \frac{P_{r,4}}{P_{r,3}} \Rightarrow P_{r,4} = P_{r,3}\left[\frac{P_4}{P_3}\right] = \overset{1400\text{ K}}{450.5}\left[\frac{100}{1000}\right] = 45.05 \xrightarrow{\text{A-17}} T_4 = 787.7\text{ K},\ h_4 = 808.5\ \text{KJ/Kg}$$

$$W_{Turb} = h_3 - h_4 = 1515.42 - 808.5 = 706.9\cancel{3}\ \text{KJ/Kg}$$

$\boxed{4\to 1}$ $q = h_4 - h_1 = 808.5 - 300.19 = 508.3\ \text{KJ/Kg}$

$$\text{Net Work} = W_{Turb} - W_{comp} = 706.9 - 279.7 = 427.2\ \text{KJ/Kg}$$

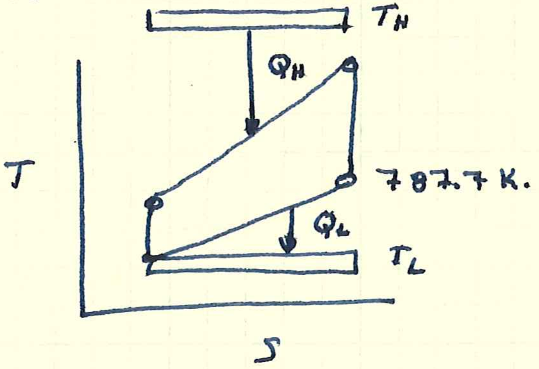
$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{508.3}{935.5} = 45.7\%$$

How about a Carnot between tle same two reservoirs?

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{2000} = 85\%$$ ⇒ Best any engine could do.

~~So where is the s~~

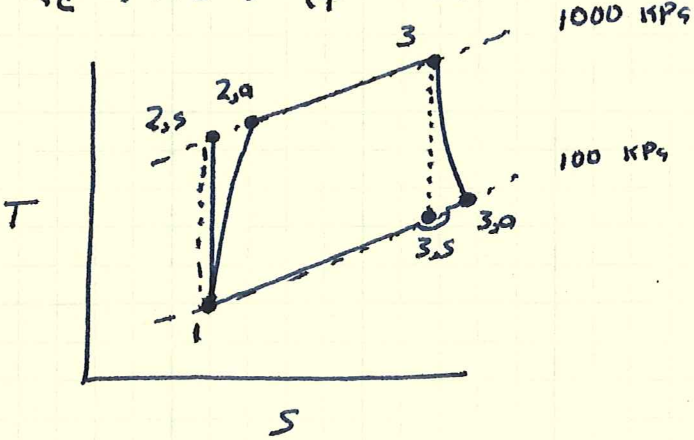
So where is tle source of inefficiency?



- It is the transfar of heat across finite ΔT ⇒ Lost work, like water over dam.
- Which is bigger problem... Q_H or Q_L?
  - ⇒ Don't answer.
  - ⇒ It's Q_L. (Chapter 7).
- What to do? Invert cycle... use this energy.

Now look at finite ~~cycle efficienc~~ compressor & turbine efficiencies.

$\eta_c = 0.8$; $\eta_T = 0.85$



① 100 KPa, 300 K ⇒ $h_1 = 300.19$ KJ/Kg

$\boxed{1 \to 2s}$ same as before $\frac{P_2}{P_1} = \frac{P_{r,2}}{P_{r,1}}$ ⇒ $h_{2,s} = 579.9$ KJ/Kg

$$\eta_c = \frac{W_{ideal}}{W_{actual}} = \frac{h_{2,s} - h_1}{h_{2,a} - h_1} \Rightarrow h_{2,a} = h_1 + \frac{h_{2,s} - h_1}{\eta_c}$$

$$h_{2,a} = 300.19 + \frac{579.9 - 300.19}{0.8} = 649.83 \text{ KJ/Kg}.$$

$$W_{1\to2} = h_{2,a} - h_1 = 649.83 - 300.19 = 349.64 \text{ KJ/Kg} \quad [279.7 \text{ KJ/Kg}]$$

Think about this for a second: { Both ideal and real compressors go from 100→1000 KPa, ideal needs less work, Real needs more, this shows up as extra h (higher T) -- Think of as internal frictio

$\boxed{2a \to 3}$ $q_{in} = h_3 - h_{2a} = 1515.42 - 649.83 = 865.59 \text{ KJ/Kg}.$

$\boxed{3 \to 4a}$ $3 \to 4s$; same as before; $\underbrace{\frac{P_4}{P_3} = \frac{P_{r,4}}{P_{r,3}}}_{\Delta S = 0} \Rightarrow h_{4,s} = 808.5 \text{ KJ/Kg}$

$$\eta_T = \frac{W_{actual}}{W_{ideal}} = \frac{h_3 - h_{4,a}}{h_3 - h_{4,s}} \Rightarrow h_{4,a} = h_3 - \eta_T (h_3 - h_{4,s})$$

$$h_{4,a} = 1515.42 - 0.85 [1515.42 - 808.5] = 914.54 \text{ KJ/Kg}.$$

$$\Rightarrow W_{T,a} = h_3 - h_{4,a} = 1515.42 - 914.54 = 600.88 \text{ KJ/Kg}$$

[706.9 KJ/Kg]

$\boxed{4a \to 1}$ $q = h_{4,a} - h_1 = 914.54 - 300.19 = 614.35 \text{ KJ/Kg}.$

$$\Rightarrow \text{Net Work} = W_T - W_c = \overset{600.88}{\cancel{614.35}} - 349.64 = \overset{251.24}{\cancel{264.71}} \text{ KJ/Kg}$$

[427.2 KJ/Kg]

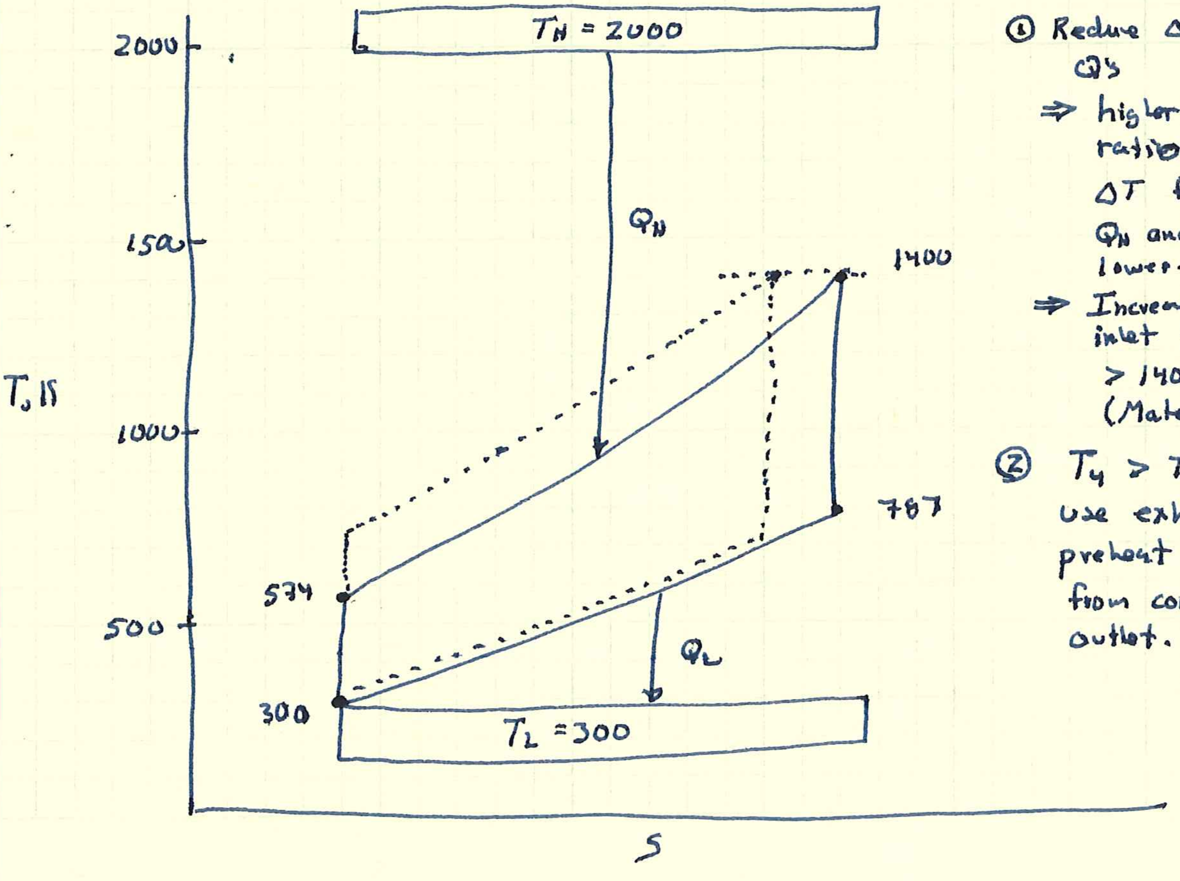
$$\eta = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{614.35}{865.59} = 29\%$$

[45.7%]

May 24

Lets think qualitatively about ways to improve cycle efficiency, say $\eta_T = \eta_c = 1$

Draw to scale



Reservoirs Fixed

① Reduce ΔT for Q's

⇒ higher compression ratio.

ΔT for both Q_H and Q_L are lower.

⇒ Increase turbine inlet temperature > 1400K.

(Materials Limits)

② $T_4 > T_2$, use exhaust to preheat gas from compressor outlet.