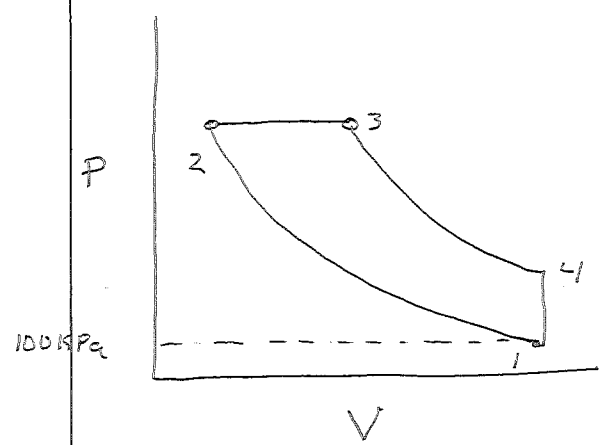


For analysis, assume piston expands during fuel injection such that P stays constant.



Compression ratio =  $\frac{V_1}{V_2}$

Cut-off ratio =  $\frac{V_3}{V_2}$

(Volume ratio between TDC and point where fuel shuts off).

$q_{in}: 2 \rightarrow 3$        $q_{in} = \left\{ \begin{matrix} u_3 - u_2 \\ h_3 - h_2 \end{matrix} \right\} ?$

$q_{out}: 4 \rightarrow 1$

Why?

$Q - W = \underbrace{\sum m_e h_e - \sum m_i h_i}_{=0} + [m_2 u_2 - m_1 u_1]_{cv}$

$m_1 = m_2 = m$

$q - w = u_2 - u_1$ , but  $w \neq 0$ ,       $w = \int_2^3 P dV = P_2 (V_3 - V_2)$

$\Rightarrow q_{in} = P_2 (V_3 - V_2) = u_2 - u_1$        $\Rightarrow q_{in} = \underbrace{u_3 - u_2}_{h_3} + \underbrace{P_3 V_3 - P_2 V_2}_{h_2}$

$\Rightarrow q_{in} = h_3 - h_2$

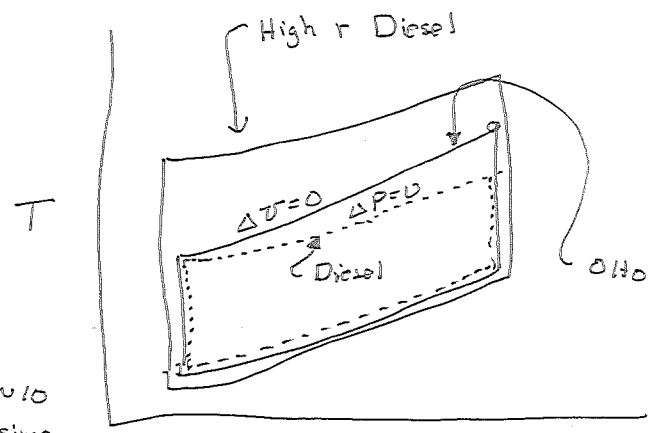
$q_{out} = u_4 - u_1$

Compare Otto and Diesel.

Looks like at same r, Otto  $\eta >$  Diesel  $\eta$  due to higher average T for  $q_{in}$ .

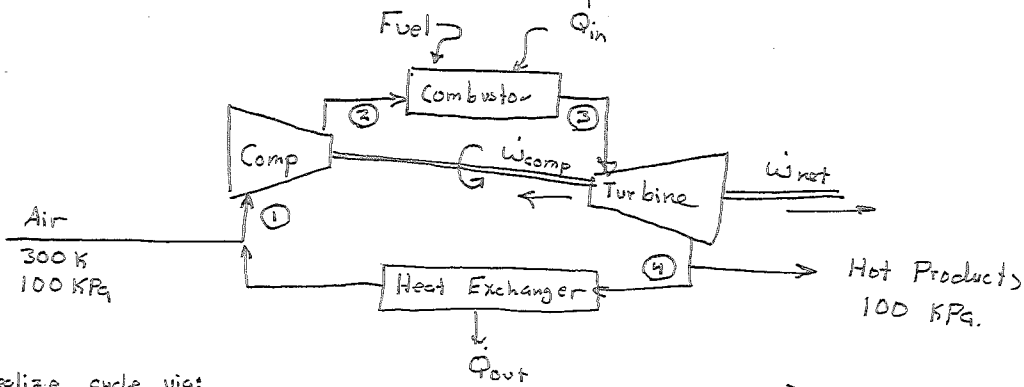
So why use Diesel?

Otto  $r$  limited to  $\sim 10$   
 Diesel can go to  $> 25$  since it just compresses air  $\Rightarrow$  higher  $\eta$



Cycle	$r$	$r_c$	$\eta$
OHD	9.5	—	59
Diesel	25	2.2	67

Now to Gas Turbines - **Brayton Cycle**

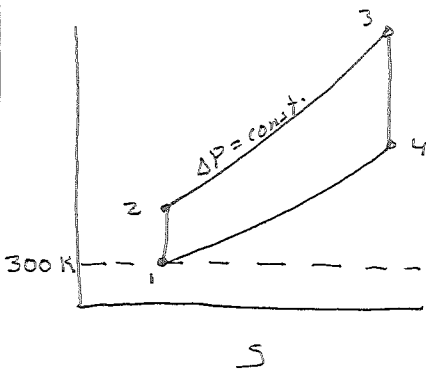


Idealize cycle via:

1. Replace exhaust with virtual heat exchanger to close cycle.
2. Replace fuel with  $\dot{Q}_{in}$

Not as flakey as OHD

- Nuclear - heated He cycles
- Indirectly - Fired gas turbine.



$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$

$$\dot{Q}_{in} = \dot{m} [u_3 - u_2] \Rightarrow \text{Flow system.}$$

Main parameter is  $r_p = \frac{P_2}{P_1}$   
(now a pressure ratio - in OHD it was a volume).

Let  $r_p = 8$ ,  $T_3 = 1650\text{K}$ , air standard assumptions.

①  $\rightarrow$  ② ? How to do it?

$$\left[ \begin{aligned} \frac{P_2}{P_1} &= \frac{P_{r,2}}{P_{r,1}} & \frac{v_2}{v_1} &= \frac{v_{r,2}}{v_{r,1}} \\ \frac{P_1 v_1}{T_1} &= \frac{P_2 v_2}{T_2} \end{aligned} \right. \quad ?$$

$$P_{r,2} = P_{r,1} \left[ \frac{P_2}{P_1} \right] = 1.386 [8] = 11.09$$

Air, 300K

$$T_2 = 540\text{K}$$

$$h_2 = 544.14 \text{ kJ/kg}$$

All work, but only the P-isentropic relation has only one unknown.  
③  $T_3 = 1650\text{K} \rightarrow h_3 = 1818.7 \text{ kJ/kg}$

Get ④ via isentropic:  $P_{r,4} = P_{r,3} \left[ \frac{P_4}{P_3} \right] = 902.25 \left[ \frac{1}{8} \right] = 112.78$  Air, 1650K

$T_4 = 997\text{K}$   
 $h_4 = 1042.9 \text{ kJ/kg}$

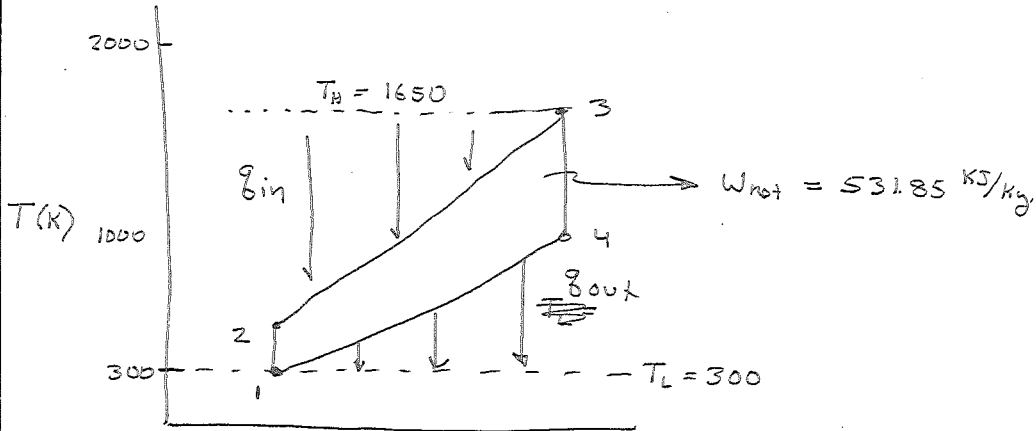
$$W_{net} = \dot{q}_{in} - \dot{q}_{out} = [h_3 - h_2] - [h_4 - h_1] = 531.85 \text{ kJ/kg}$$

$$\eta = 1 - \frac{\dot{q}_{out}}{\dot{q}_{in}} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 41.7\%$$

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Carnot between same two limits:  $\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1650} = 81.8\%$   
 $\Rightarrow$  lots of room for improvement.

Plot to scale and look at irreversibility:



$$i_{2-3} = T_0 \left[ S_3 - S_2 + \frac{q_{R}}{T_R} \right] = T_0 \left[ S_3^0 - S_2^0 - R \ln \frac{P_3}{P_2} + \frac{q_{R}}{T_R} \right]$$

$O_3, P_3 = P_2$

$$q_R = h_3 - h_2 = 1274.56 \text{ kJ/kg}$$

$$i_{2-3} = [300 \text{ K}] \left[ 3.5612 - 2.29906 + \frac{(-1274.56)}{1650} \right] = 146.9 \text{ kJ/kg}$$

$$i_{4-1} = 364.06 \text{ kJ/kg} \quad \Rightarrow \quad \frac{W_{net} + i_{2-3} + i_{4-1}}{q_{in}} = 81.4\% \quad ?$$

(matching Carnot)

Can help hot end via higher  $\Gamma_p$  and  $T_3$ .

$\rightarrow T_3, \text{max} \cong 1700 \text{ K}$  due to turbine blade materials problems.

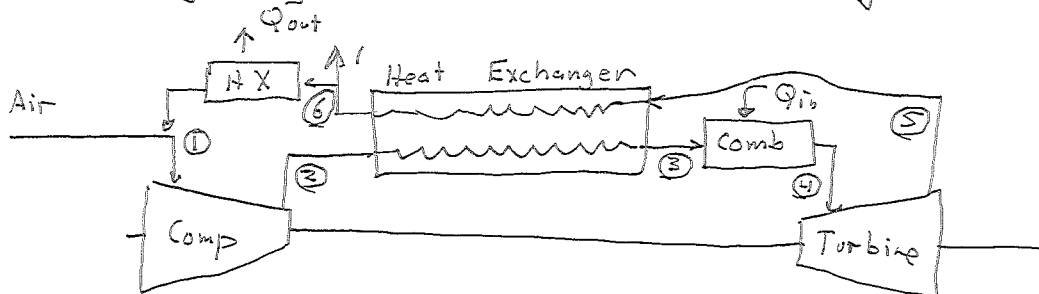
Most  $i$  is in exhaust:  $\rightarrow$  How to address

- One approach is to run a steam cycle off of the exhaust [combined cycle].  $\rightarrow$  Later.

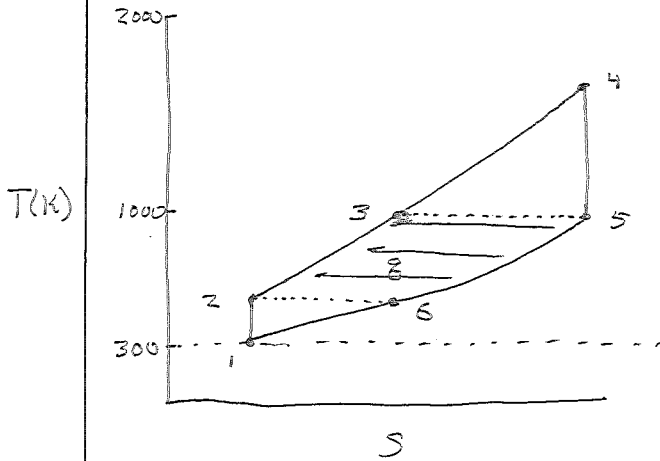
- Another approach.
  - $\rightarrow$  Exhaust leaves at 997 K.
  - $\rightarrow$  Initial  $q_{in}$  is added to gas at 540 K.

} Doesn't make sense?

$\Rightarrow$  Try using hot exhaust to do initial heating instead of fuel.



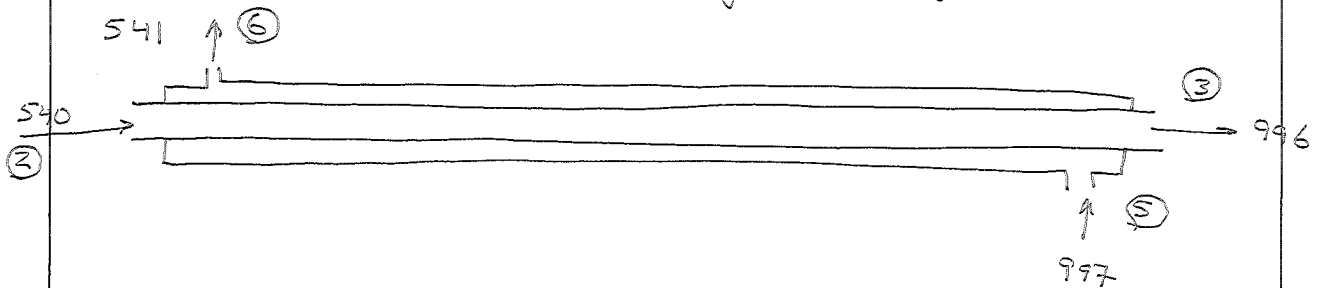
Now take scale system diagram



Point	T(K)	h (KJ/Kg)
1	300	300.19
2	540	544.19
3	→ 997	→ 1042.7
4	1650	1818.7
5	997	1042.7
6	→ 540	→ 544.19

Now locate ③ Can  $T_3 > T_5$ ? No  
 If  $T_3 < T_5$  have  $q$  across  $\Delta T \rightarrow$  irreversibility.  
 Best case is  $T_3 \cong T_5$

Same argument holds for  $T_6 \cong T_2$   
 How can this be? Look at long heat exchanger.



$\Rightarrow$  At each cross-section, stream ⑤  $\rightarrow$  ⑥ is 1 K hotter than stream ②  $\rightarrow$  ③  
 so heat flow occurs, but with very low  $\Delta T$ .  
 $\Rightarrow$  Requires huge exchanger  $\rightarrow$  high capital cost.

$$\Rightarrow h_3 = h_5$$

$$h_6 = h_2$$

So:  $q_{in} \stackrel{?}{=} h_4 - h_3 = 776 \text{ KJ/Kg}$   
 $q_{out} \stackrel{?}{=} h_6 - h_1 = 532.0 \text{ KJ/Kg}$

W/O Regeneration

$$h_4 - h_2 = 1274.6$$

$$h_5 - h_1 = 742.7$$

$$W_{net} = q_{in} - q_{out} = 532.0$$

$$532$$

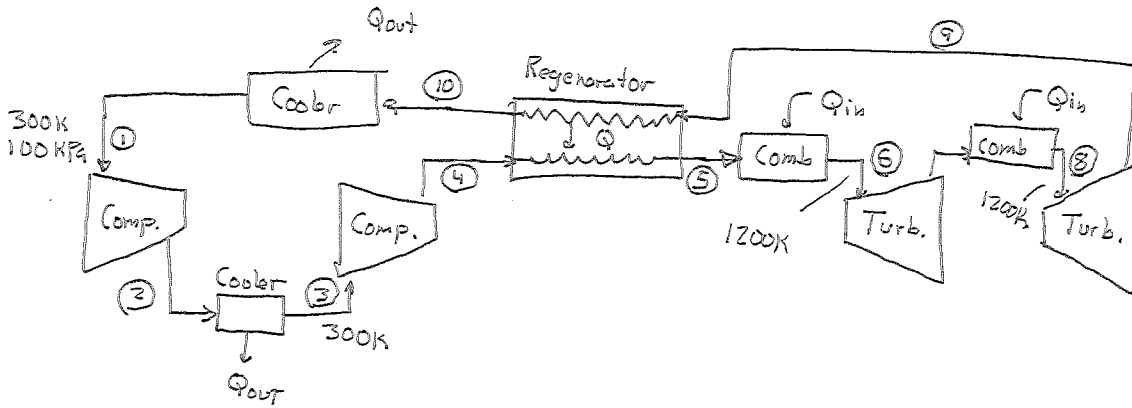
$$\eta = 1 - \frac{q_{out}}{q_{in}} = 68.6\%$$

$$41.7\%$$

This  $\eta$  would ~~be~~ not be possible due to

- ① Compression and turbines not reversible
- ② Heat exchangers require finite  $\Delta T$   
 $\Rightarrow$  Not leaking heat  $q$   
 $\Rightarrow$  Rather  $\Delta T \rightarrow$  lost work potential.

Another approach is to use dual-stage compression with intercooling and " " expansion " reheating.



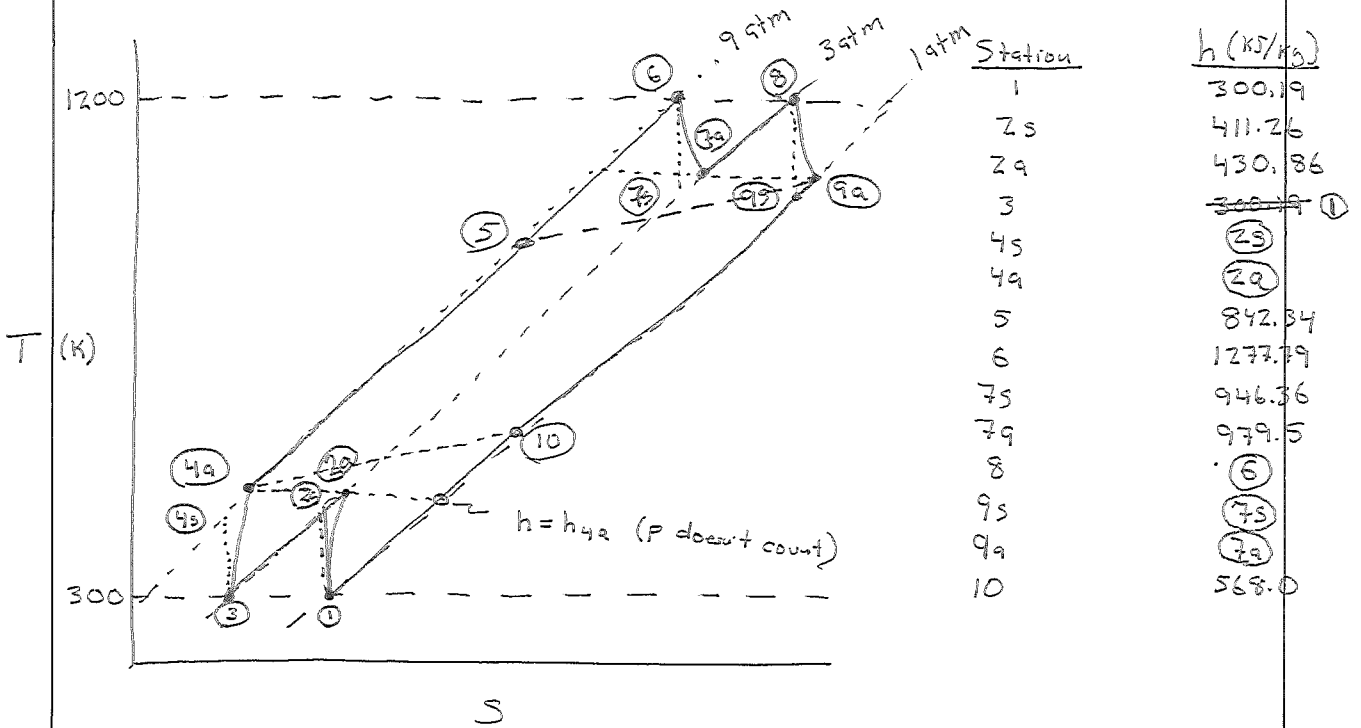
$\Gamma_p = 3$  for each stage

$\gamma_c = 0.85$

$\gamma_T = 0.90$

$\epsilon_{regen} = 0.75$  (will define when we get to it).

Air standard.



① → ② Isentropic compression:  $P_{r,2} = P_{r,1} \left[ \frac{P_2}{P_1} \right] = 1.386 [3] = 4.158 \rightarrow h_{2,s} = 411.26$   
Air, 300K

Compressor  $\eta \Rightarrow \frac{W_s}{W_a} =$  "Real compressors require more w for a given  $\Delta P$  than ideal compressors"

$\Rightarrow \eta_c = \frac{W_s}{W_a} = \frac{h_{2,s} - h_1}{h_{2,a} - h_1} \Rightarrow 0.85 = \frac{411.26 - 300.19}{h_{2,a} - 300.19}$

$\Rightarrow h_{2,a} = 430.86 \text{ kJ/kg}$  [now forget  $h_{2,s}$ ]

Cooler takes us where? Back to 300 K.

$$\left. \begin{array}{l} P_3 = 300 \text{ kPa} \\ T_3 = 300 \text{ K} \end{array} \right\} \begin{array}{l} h_3 = 300.19 \text{ kJ/kg} \\ \gamma_p = 3 \\ \eta_c = 0.85 \end{array} \left. \begin{array}{l} \text{Same as stage 1, except for} \\ P_3 = 300 \text{ kPa} \end{array} \right\}$$

$$P_{r,3} = P_{r,3} \left[ \frac{P_4}{P_3} \right] = \text{Same as stage 1} \quad \eta_c = \frac{h_{4,s} - h_3}{h_{4,a} - h_3} \quad \text{Same as stage 1}$$

300k 3

$$\Rightarrow \begin{array}{l} h_{4,s} = h_{2,s} \\ h_{4,a} = h_{2,a} \end{array}$$

Skip ⑤

$$T_6 = 1200 \text{ K} \rightarrow h_6 = 1277.79 \text{ kJ/kg}$$

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$$T_6 \text{ ⑦} \Rightarrow \text{Isentropic expansion: } P_{r,7} = P_{r,6} \left[ \frac{P_7}{P_6} \right] = 238 \left[ \frac{1}{3} \right] = 79.33 \xrightarrow{\text{Air}} h_{7,s} = 946.36 \text{ kJ/kg}$$

Turbine  $\eta$ : "Real turbines deliver less work for a given  $\Delta P$  than actual turbines"

$$\eta_T = \frac{W_a}{W_s} = \frac{h_6 - h_{7,a}}{h_6 - h_{7,s}} \Rightarrow 0.9 = \frac{1277.79 - h_{7,a}}{1277.79 - 946.36} \Rightarrow h_{7,a} = 979.5 \text{ kJ/kg}$$

Again, 6  $\rightarrow$  7s, 7s  $\rightarrow$  7a  
Same as 8  $\rightarrow$  9s, 9s  $\rightarrow$  9a

Now finish with regenerator:

- In absence of regeneration, where would  $q_{in}$  occur.

$$q_{in} = (h_6 - h_{4,a}) + (h_3 - h_{7a})$$

- Next, add an ideal regenerator ( $\Delta T = 0$ ) How much heat is moved? (add dashed).

$$q_{regen, ideal} = h_{9a} - h_{4,a} \quad (\text{doesn't depend on } P_0)$$

- But a real heat exchanger needs a finite  $\Delta T$  to drive the heat flow. [Add dashed lines and numbers]
- Logical definition of "Regenerator Effectiveness" is

$$\epsilon_{regen} = \frac{\text{actual heat transferred}}{\text{max heat transfer if } \Delta T = 0} = \frac{h_5 - h_{4a}}{h_{9a} - h_{4a}}$$

$$0.75 = \frac{h_5 - 430.86}{979.5 - 430.86} \Rightarrow h_5 = 842.34 \text{ kJ/kg}$$

Last: Get  $h_{10}$  from 1<sup>st</sup> law on exchanger.

$$h_{9a} - h_{10} = h_5 - h_{4a} \Rightarrow 979.5 - h_{10} = 847.34 - 430.86$$

$$\Rightarrow h_{10} = 568.0 \text{ kJ/kg}$$

$$\dot{q}_{in} \stackrel{?}{=} [h_3 - h_{7a}] + [h_6 - h_5] = 733.76 \text{ kJ/kg}$$

$$\dot{q}_{out} \stackrel{?}{=} [h_{10} - h_1] + [h_{2,9} - h_3] = 398.48 \text{ kJ/kg}$$

$$W_{net} = \dot{q}_{in} - \dot{q}_{out} = 335.28 \text{ kJ/kg}$$

$$\eta = 1 - \frac{\dot{q}_{out}}{\dot{q}_{in}} = 45.7\% \quad (55.3\% \text{ if } \eta_c = \eta_T = 1)$$

An aside:  $i$  for turbines  $\neq 0$  now. Lets compare.

$$i_{6 \rightarrow 7a} = T_0 \left[ S_{7a}^0 - S_6^0 - R \ln \frac{P_{7a}}{P_6} + \frac{\dot{q}_{6 \rightarrow 7a}}{T_R} \right]$$

$$= [300 \text{ K}] \left[ 2.89914 - 3.17888 - \frac{8.314}{29} \ln \left[ \frac{1}{3} \right] \right] = 10.67 \text{ kJ/kg}$$

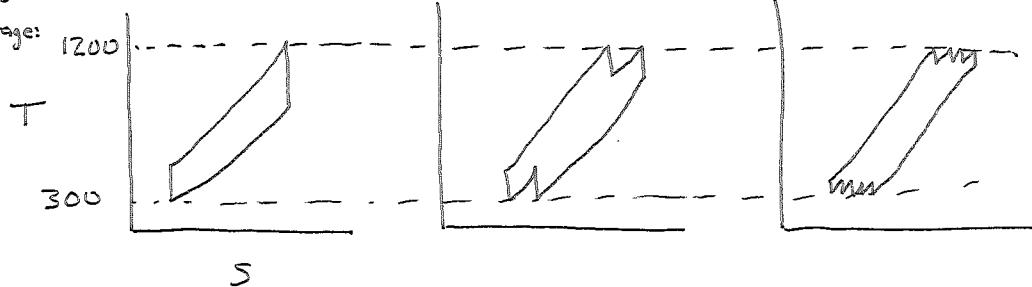
$$i_{10 \rightarrow 1} = \cancel{75.85 \text{ kJ/kg}} [300 \text{ K}] \left[ 1.70203 - 2.3419 + \frac{568 - 300.19}{300} \right]$$

$$i_{10 \rightarrow 1} = 75.85 \text{ kJ/kg}$$

So exhaust heat transfer is still the biggest problem.

Some thoughts:

① Multistage:



• Note that final T is hot

⇒ Need to fix with regenerator.

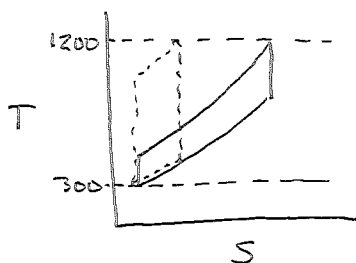
• If you do, what is  $\Delta T$  for  $\dot{q}_{in}$ ,  $\dot{q}_{out}$ ?  $\Delta T \approx 0$ .

⇒ Get Carnot  $\eta$  for  $1200 \rightarrow 300 \text{ K}$ .

⇒ Complexity is up,

⇒ This has lost importance to combined cycles.

② High  $\tau_p$ : Have discussed before

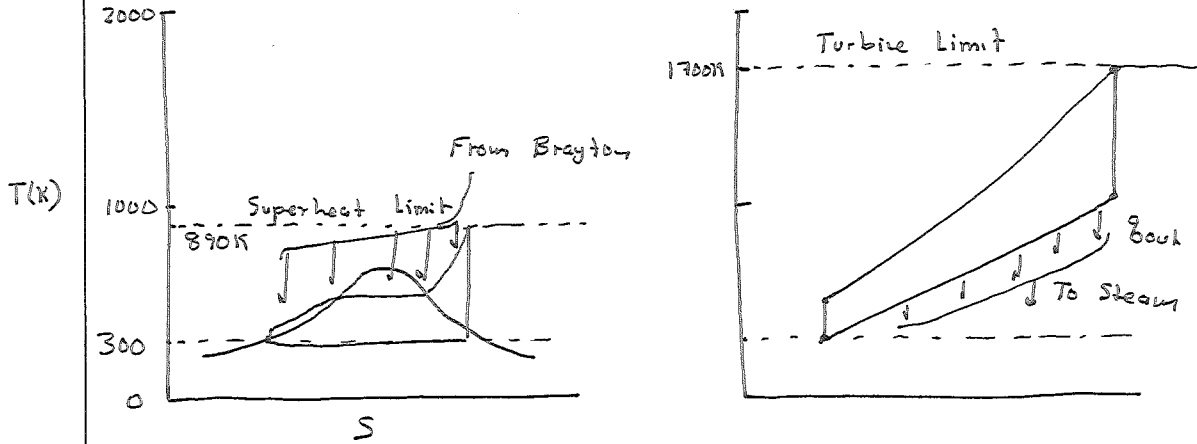


Higher  $\eta$ , but

$\tau_p$  for say 200 MW

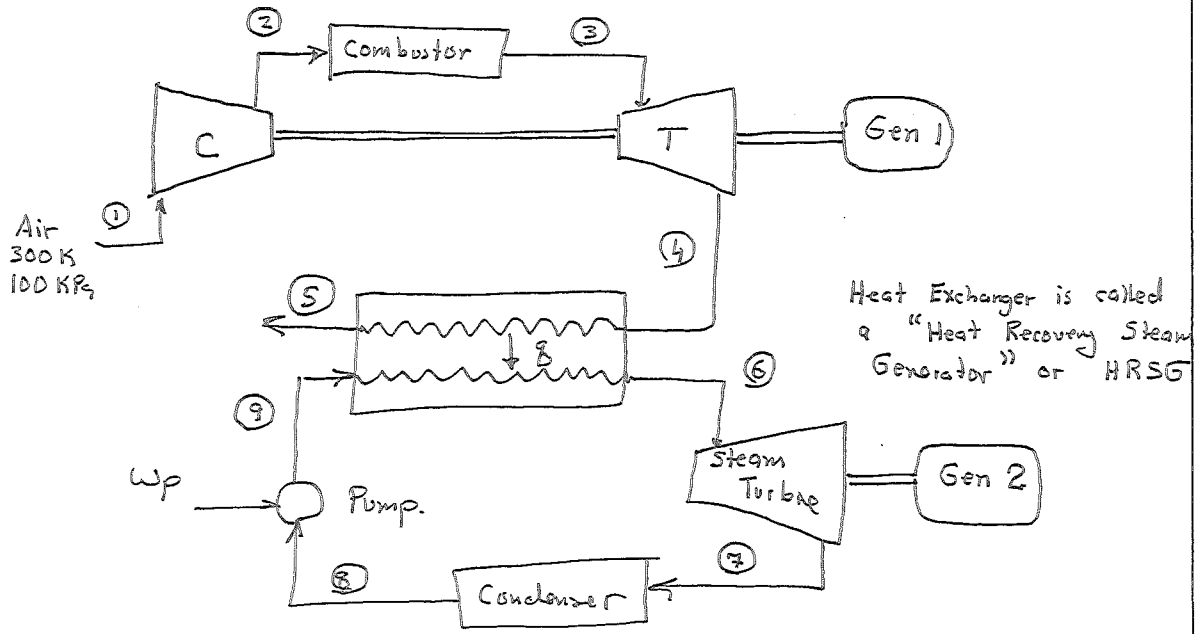
goes up ⇒ Huge plant.

Intro. to Combined Cycle.



- Rankine is weak because  $\Delta T$  between heat source (flame) and steam is large.
- Good because  $Q_{out}$  is across small  $\Delta T$
- Brayton is weak because exhaust  $T$  is high.
- But  $q_{in}$  is at higher average  $\Delta T$ .

⇒ But: Brayton exhaust is hotter (~1000 K) than  $T_{max}$  in Rankine (890 K)  
 ⇒ Use Brayton exhaust to run Rankine.



Heat Exchanger is called a "Heat Recovery Steam Generator" or HRSG

Key points:

- Both cycles are physically separate in each has own  $m_a, m_s$ .
- What is  $\dot{Q}_{in} \stackrel{?}{=} m_a (h_3 - h_2)$
- What is  $\dot{Q}_{out} \stackrel{?}{=} m_a (h_5 - h_1) + m_s (h_7 - h_8)$

$$\dot{W}_{net} = \dot{Q}_{in} - \dot{Q}_{out}$$

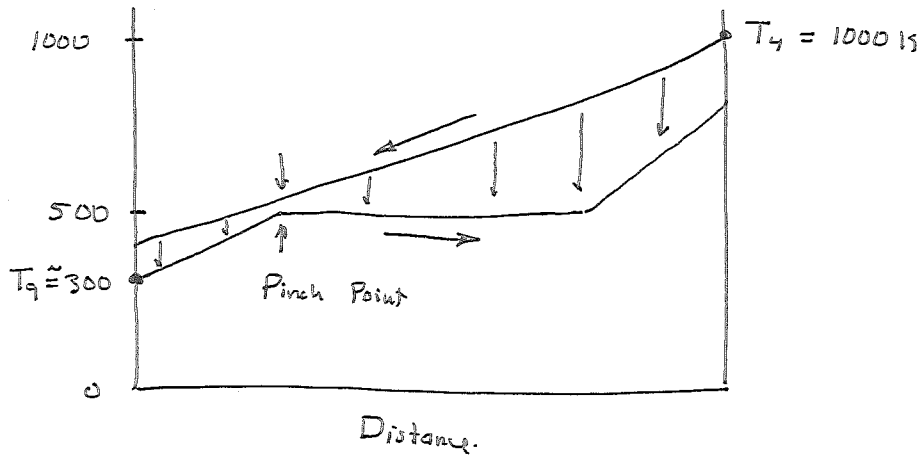
$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$



Think about some design issues:

- High or low  $r_p$ ? Not too high because of low  $T_4$ .
- Regeneration? No - heat not lost, but goes to Rankine.
- HRSG:

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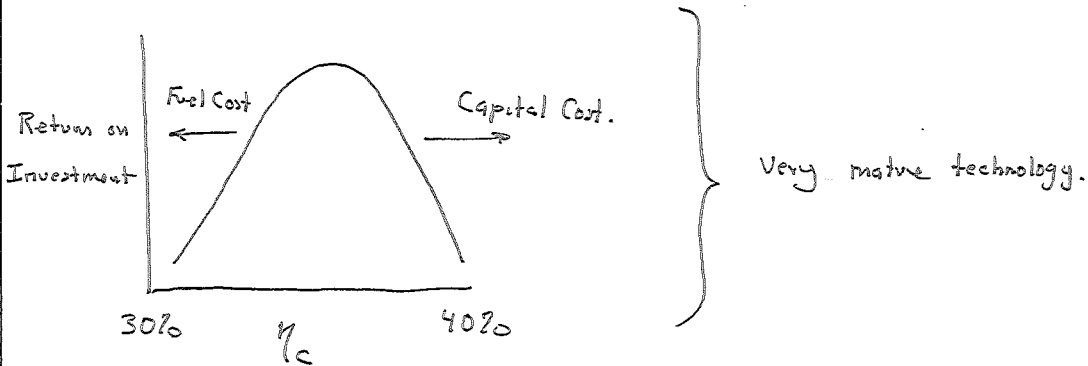


- Hard to get a low  $i$  because HRSG has a big  $\Delta T$  built in.
- "Pinch Point" is a limit.
  - ⇒ Advanced designs make use of complex systems with multiple pressure levels. (ME 430).
  - ⇒ Handout on web shows one example.

Historically, power generation is mostly Rankine; why?

- Can handle dirty fuels (coal).
- Early gas turbines were very inefficient → Fuel Hogs!
  - Low turbine inlet  $T$ : New materials and aerodynamic film cooling
  - Bad  $\eta_c, \eta_T$ : Modern CFD design codes.

Rankine  $\eta \approx 36-37\%$



Opening of Canadian and Rocky Mt. gas fields drove prices down.

Coal - \$1.5/10<sup>6</sup> Btu.  
 Gas - \$2.4/10<sup>6</sup> Btu. } ~~cheap gas~~

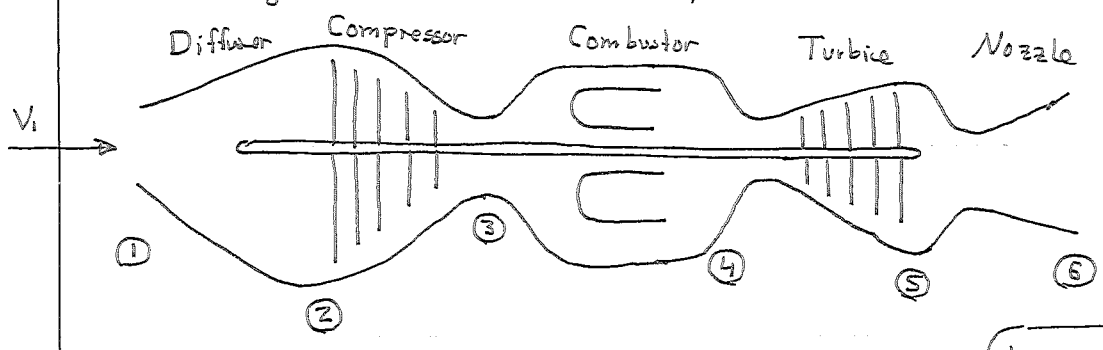
- ⇒ Combined Cycle because
  - 1) Although coal cheaper, gas easier to use and requires no pollutant cleanup (SO<sub>2</sub>, NO<sub>x</sub>, particulates).
  - 2)  $\eta_{cc} \approx 60\%$
  - 3) Fast, modular construction.

Blip last winter → Gas to \$10/10<sup>6</sup> BTU  
 Now down to < 3\$. } FPLW is tying its future to gas.

Jet Propulsion Cycles

A gas turbine that produces thrust instead of net shaft work:

- Let engine sit still and flow air past it.



Key differences from land-based gas turbine.

- Turbine only generates enough power to drive compressor - No extra.
- This means  $P_5 > P_{stm}$ .
- Use this pressure to drive flow through nozzle → thrust  $\vec{P}$

Live: 600 mph, 30,000 ft.  
 $T_1 = 220\text{K} (-64^\circ\text{F})$   
 $P_1 = 40\text{ kPa}$   
 $V_1 = 270\text{ m/s}$

So we need to retain kinetic energy terms:

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

**Diffuser** Converts KE of air to enthalpy by slowing it isentropically (like 1<sup>st</sup> stage of compression). Inlet  $\leftarrow$   
 (like 1<sup>st</sup> stage of compression).  
 negligible, but not = 0.

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \Rightarrow h_2 = h_1 + \frac{V_1^2}{2}$$

$$h_2 = 219.17 \frac{\text{kJ}}{\text{kg}} + \frac{[270\text{ m}]^2}{2} \frac{\text{kJ/kg}}{1000\text{ m}^2/\text{s}^2} = 256.42 \frac{\text{kJ}}{\text{kg}} \quad (256\text{ K} = 0^\circ\text{F})$$

$$\text{Get } P_2 = P_1 \left[ \frac{P_{T,2}}{P_{T,1}} \right] = [40\text{ kPa}] \left[ \frac{0.801}{0.469} \right] = 68.3\text{ kPa}$$

$\uparrow$  220 K

**Compressor** Nothing new here.  $\xrightarrow{1^{\text{st}}}$   $r_p = 25$  (usually high for aero engines).

$$P_{T,3} = P_{T,2} \left[ \frac{P_3}{P_2} \right] = [0.801] [25] = 20.03 \rightarrow T_3 = 632\text{ K}$$

$$h_3 = 641.1 \frac{\text{kJ}}{\text{kg}}$$

$$P_3 = 25P_2 = 1708\text{ kPa}$$

$$w_c = h_3 - h_2 = 384.7 \frac{\text{kJ}}{\text{kg}}$$

**Combustor** Constant P heat input: Add as much as turbine blades will handle  
 $T_4 = 1700\text{ K}; h_4 = 1880.1 \frac{\text{kJ}}{\text{kg}}$

**Turbine** Isentropic expansion, but only to point where  $w_c = w_T$

$$w_c = w_T = h_4 - h_5 \Rightarrow 384.7 = 1880.1 - h_5 \Rightarrow h_5 = 1495.4 \frac{\text{kJ}}{\text{kg}}$$

$$P_5 = P_4 \left[ \frac{P_{r,5}}{P_{r,4}} \right] = [1708 \text{ kPa}] \left[ \frac{428.5}{1025} \right] = 714 \text{ kPa}$$

1380.315  
1700K  
T<sub>5</sub> = 1380.3 K

**Nozzle** Drop pressure to atmospheric isentropically → Get V<sub>6</sub>

$$h_5 - h_6 = h_5 + \frac{V_6^2 - V_5^2}{2} \rightarrow \text{negligible}$$

$$\Rightarrow V_6 = [2(h_5 - h_6)]^{1/2}; \text{ But how to get } h_6$$

⇒ Known ΔP & assume ΔS = 0

$$P_{r,6} = P_{r,5} \left[ \frac{P_6}{P_5} \right] = [428.3] \left[ \frac{40}{714} \right] = 24.01 \Rightarrow h_6 = 667.49 \frac{\text{kJ}}{\text{kg}}$$

T<sub>6</sub> = 666.6 K

$$V_6 = \left[ \frac{2 | 1495.4 - 667.49 \text{ kJ} | 1000 \text{ m}^2/\text{s}^2}{\text{kg} \text{ kJ/kg}} \right]^{1/2} = \underline{1279 \text{ m/s}}$$

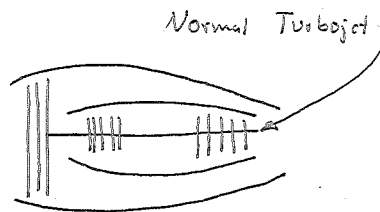
(supersonic, shocks, noisy).

$$\text{Thrust} = \text{Force} = \dot{m} (V_6 - V_1) = [1 \text{ kg/s}] [1279 - 270 \text{ m/s}] = 1009 \frac{\text{N}}{\text{kg/s}}$$

@150 lb, you put 670N on your chair, so you would need 0.66 kg/s to levitate!

Two augmentations:

① Turbofan



• Use extra shaft work to blow air around outside.

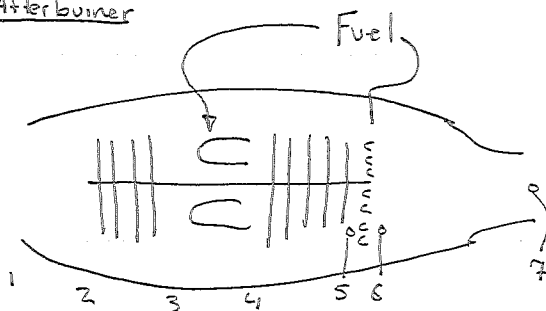
→ Better η

→ Thrust ∼ ṁ V<sub>6</sub>

⇒ ṁ higher, V<sub>6</sub> lower

⇒ No shocks, quieter engine

② Afterburner



• Add more heat after turbine

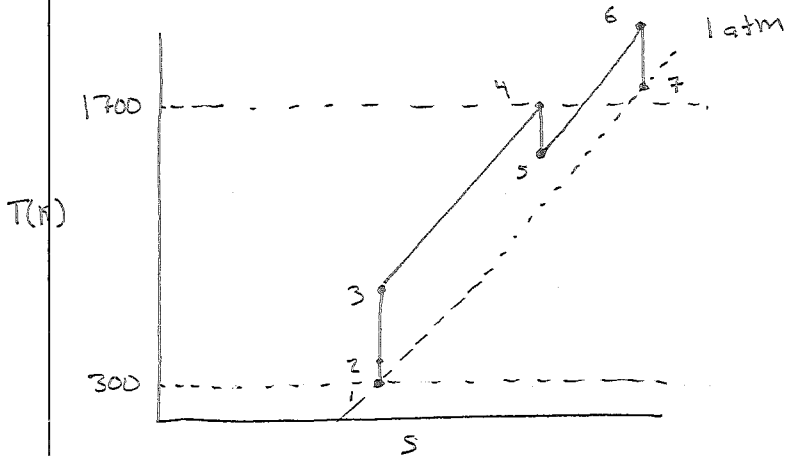
→ Expand gas

→ Higher velocity

→ More thrust.

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Look @ T-S:

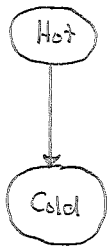


- But: Heat added after partial expansion @ lower T
- Also, exhaust T is hotter.
- ⇒ Very poor  $\eta$ .
- ⇒ Use briefly when you need high thrust, not otherwise.
- Thermodynamically better to add extra fuel to combustor, but this will ruin blades.

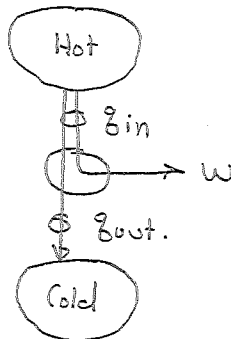
Chapter 9 - Refrigeration Cycles

The Basic Problem.

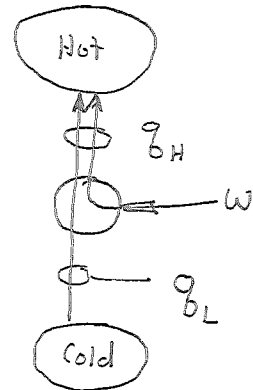
2<sup>nd</sup> Law lets heat go from hot to cold



- Heat Engine grabs some of thermal energy as work.
- Rejects rest to cold.



Reverse the heat engine



• This is irreversible, and entails a loss of work potential.

• Same thermal energy @  $T_H$  is more valuable than that @  $T_L$

$$dS = \frac{dq}{T}_{rev}$$

Higher T, lower S, more order.

In a perfect cycle, the loss in "value" of  $q_{out}$  going from hot  $\rightarrow$  cold exactly balances the gain in value of  $q_{in} - q_{out}$  being converted from thermal energy at  $T_H \rightarrow W$ .

$$\begin{aligned} & \left[ \text{Increase in value of } q_L \text{ going from } T_L \text{ to } T_H \right] \\ & = \left[ \text{Decrease in value of } W \text{ becoming thermal energy at } T_H \right] \end{aligned}$$

	<u>Cold</u>	<u>Hot</u>
Refrigerator:	Icebox	Environment
Heat Pump	Environment	Warm House

Efficiencies:

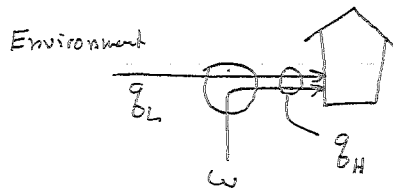
$$\text{Coefficient of performance} = \beta = \frac{\text{What we want}}{\text{What we pay for}} = \frac{Q_L}{W}$$

$$= \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1} \quad \text{Carnot Only} \quad \frac{1}{\frac{T_H}{T_L} - 1}$$

For home refrigerator:  $T_L = 0^\circ\text{F}$  icebox  
 $T_H = 70^\circ\text{F}$  room }  $\beta = 6.57$

- 100 Watts of electricity pulls
- 657 Watts of heat from coldspace, and delivers
- 757 Watts of heat to environment.

Heat Pump:



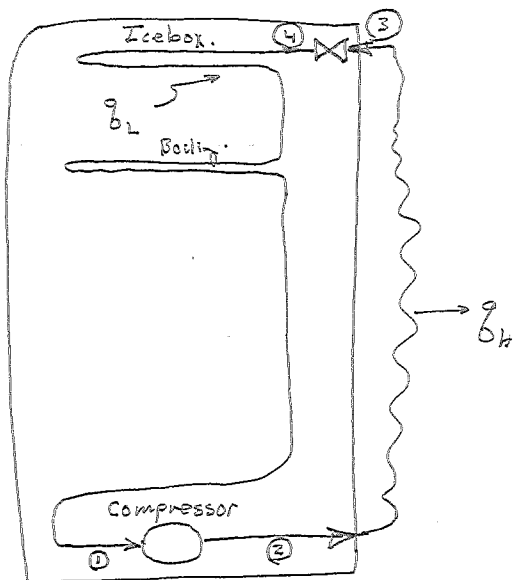
$$\text{Coefficient of Performance} = \gamma = \frac{\text{What we want}}{\text{What we pay for}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{\frac{Q_H - Q_L}{Q_H}}$$

$$= \frac{1}{1 - \frac{Q_L}{Q_H}} \quad \text{Carnot} \quad \frac{1}{1 - \frac{T_L}{T_H}}$$

Environment =  $40^\circ\text{F}$   
 House =  $70^\circ\text{F}$  }  $\gamma = 17.67$

- 100 Watts of power yields
  - 1767 Watts of heating
- } Compare with resistance heaters  
 → 100W power  
 → 100 W of heat.

Basic Vapor Compression Cycle:



- Spray an aerosol can long enough and it gets cold due to boiling inside } Propane.
- Icebox contains low-p boiling refrigerant. ⇒ Product is cond vapor
- Compress the vapor, make it hot.
- Send out back, let hot vapor condense to ~~warm~~ liquid room-T liquid.
- Expand liquid through throttle. to start boiling at low-T.

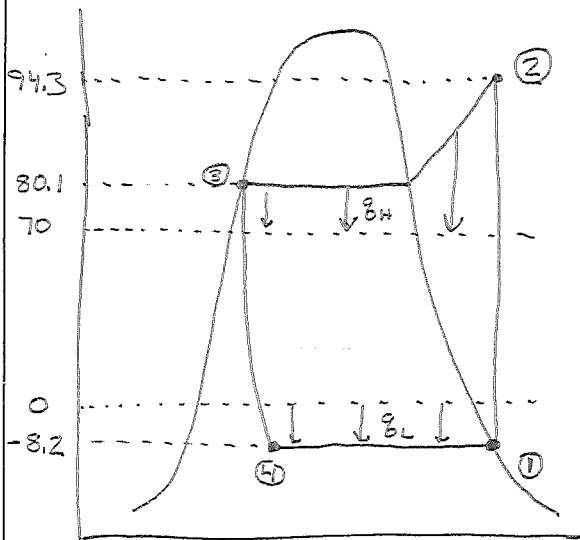


Ideal Cycle: R-134a

$P_H = 0.7 \text{ MPa}$   
 $P_L = 0.12 \text{ MPa}$   
 $\dot{m} = 0.1 \text{ kg/s}$

Get  $\dot{W}$ ,  $\dot{Q}_H$ ,  $\dot{Q}_L$ ,  $\beta$

T  
(°F)



Point	$h$ (kJ/kg)	$S$ (kJ/kg-K)
1	233.86	0.9354
2	270.22	"
3	86.78	0.3242
4	"	0.3489

① Sat. Vapor,  $P = 0.12 \text{ MPa} \Rightarrow h_1 = 233.86 \text{ kJ/kg}$   
 $S_1 = 0.9354 \text{ kJ/kg-K}$   
 $T_1 = -22.36^\circ\text{C} = -8.2^\circ\text{F}$

② Before we used  $W_p = v_f(P_2 - P_1)$ 

- Will this work?
- Why not?

Isentropic compression  $P_2 = 0.7 \text{ MPa}$  }  $134a$   $\rightarrow$   $h_2 = 270.22 \text{ kJ/kg}$   
 $S_2 = S_1$  } Superheat  $T_2 = 34.6^\circ\text{C} = 94.3 \text{ F}$

③ Cool at constant  $P$  to a saturated liquid.  $P = 0.7 \text{ MPa}$ .  
 $\Rightarrow h_3 = 86.78 \text{ kJ/kg}$   
 $S_3 = 0.3242 \text{ kJ/kg-K}$   
 $T_3 = 26.72^\circ\text{C} = 80.1^\circ\text{F}$

④ Expansion valve  $\rightarrow$  Throttle to  $0.12 \text{ MPa}$ .  
 $T_4 = -8.2^\circ\text{F}$

$\dot{Q} - \dot{W} = \sum \dot{m}_e h_e - \sum \dot{m}_i h_i$   
 $\dot{m}_e = \dot{m}_i \Rightarrow h_e = h_i$

Get  $S_4 \Rightarrow x_4 = \frac{h_4 - h_f}{h_{fg}} = \frac{86.78 - 21.32}{212.54} = 0.308$

$S_4 = S_f + x_4 S_{fg} = 0.0879 + [0.308][0.9354 - 0.0879] = 0.3489 \text{ kJ/kg-K}$

Add icebox, ambient  $T$ 's now

Note that cycle is bigger than needed  $\rightarrow$  more work needed to move required  $Q_L$

$\dot{W}_{in} = \dot{m} [h_2 - h_1] = 3.64 \text{ kW} \quad (36.4 \text{ kJ/kg})$

$\dot{Q}_H = \dot{m} [h_2 - h_3] = 18.3 \text{ kW} \quad (183 \text{ kJ/kg})$

$\dot{Q}_L = \dot{m} [h_1 - h_4] = 14.7 \text{ kW} \quad (147 \text{ kJ/kg})$

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$$\beta = \frac{\dot{Q}_L}{\dot{W}} = \frac{14.7}{3.64} = 4.04$$

For Carnot 0°F → 70°F  
 $\beta = 6.57$

So where do irreversibilities occur, and which are biggest?

$$i_{1 \rightarrow 2} = 0$$

$$i_{2 \rightarrow 3} = T_0 \left[ (S_3 - S_2) + \frac{\dot{Q}_R}{T_R} \right] = [294K] \left[ (0.3242 - 0.9354) + \frac{+183}{294} \right] = 3.31 \frac{kJ}{kg} = 0.331 \text{ kW}$$

$$i_{3 \rightarrow 4} = T_0 \left[ (S_4 - S_3) + \frac{\dot{Q}_R}{T_R} \right] = 7.26 \text{ kJ/kg} = 0.726 \text{ kW}$$

$$i_{4 \rightarrow 1} = T_0 \left[ (S_1 - S_4) + \frac{\dot{Q}_R}{T_R} \right] = [294K] \left[ (0.9354 - 0.3489) + \frac{-147}{294} \right] = 2.95 \frac{kJ}{kg} = 0.295 \text{ kW}$$

$$\beta_{max} = \frac{\dot{Q}_L}{\dot{W} - \sum i's} = \frac{14.7}{3.64 - 0.331 - 0.726 - 0.295} = \underline{\underline{6.42}} \text{ (= Carnot?)}$$

How to improve?

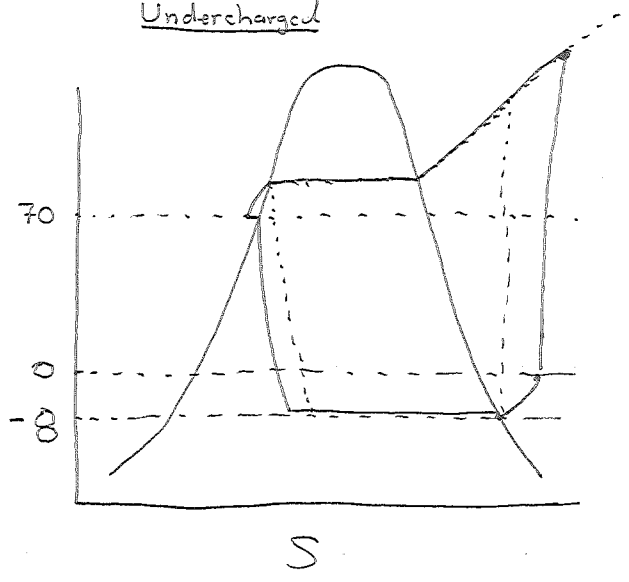
Icebox: Larger surface area.

Condenser: 1) Don't fully evaporate so you stay in dome.  
2) Larger area.

Throttle: Replace with turbine. But 1) Cost of turbine  
2) Hard to expand liquid efficiently  
3) Mechanical energy recovery (→ compressor).

Look at real systems (two versions)

Undercharged

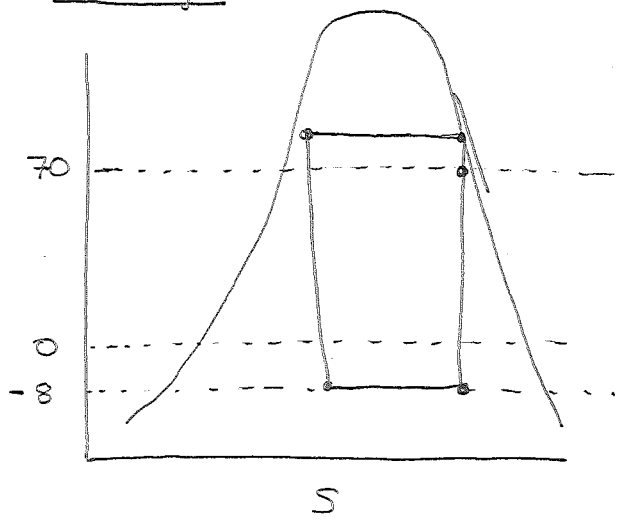


- Evaporator: All refrigerant evaporates and vapor reaches ice box temperature.
- Compressor: Non reversible ( $S \uparrow$ ) and not adiabatic ( $S \downarrow$ ) so cannot tell.
- Condenser: Condenses all liquid and takes T to ambient.
- Throttle starts on compressed liquid ( $\Delta T \approx 0$ ).



Overcharged

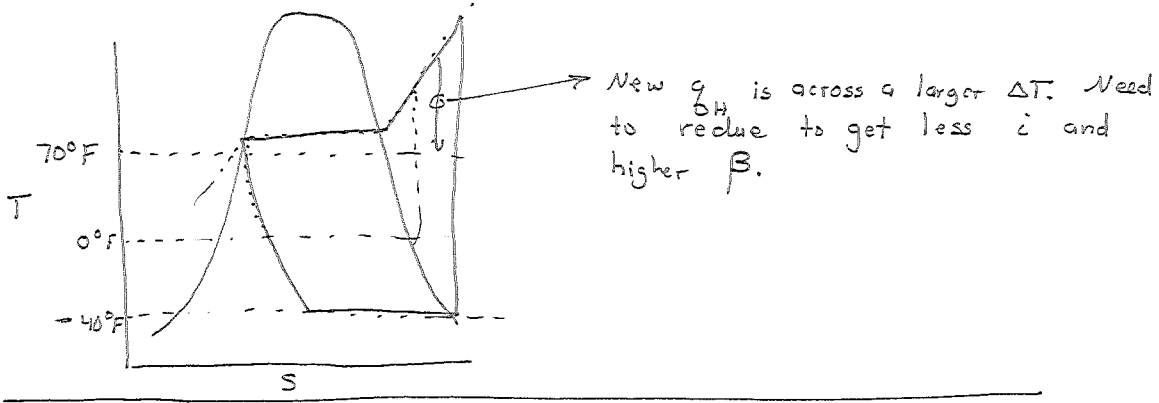
T  
(°F)



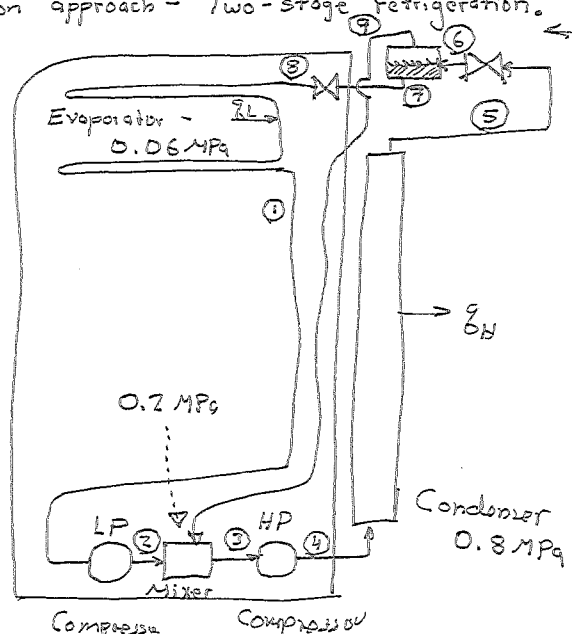
- Evaporator: Not all refrigerant evaporates.
  - Compressor: Same as above
  - Condenser: Not all vapor condenses
- ⇒ Closer to rectangle.  
 ⇒ Like Carnot.  
 ⇒ Higher efficiency.
- ⇒ Note: Recharge car AC until T into evaporator = T<sub>out</sub>.

- Another practical observation: Auto A/C charge gets low and you notice it cycles on and off. Why?
  - ① Low charge reduces P<sub>max</sub>, P<sub>min</sub>.
  - ② Evap < D°C ⇒ Form frost plug
  - ③ P-sensor on low-p side shuts System down to avoid freezeup.

If you want a very low-T (~40°F), you just need to run condenser at low-P.

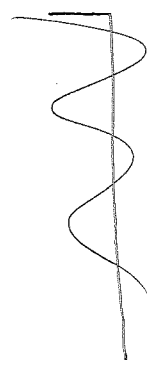


Common approach - Two-stage refrigeration.



Flash Chamber (separates vapor and liquid)

Idea is to partially expand  
 → Pass only liquid to final expansion  
 → Send vapor (which doesn't help icebox) to cool charge between compressors (like an internal intercooler).

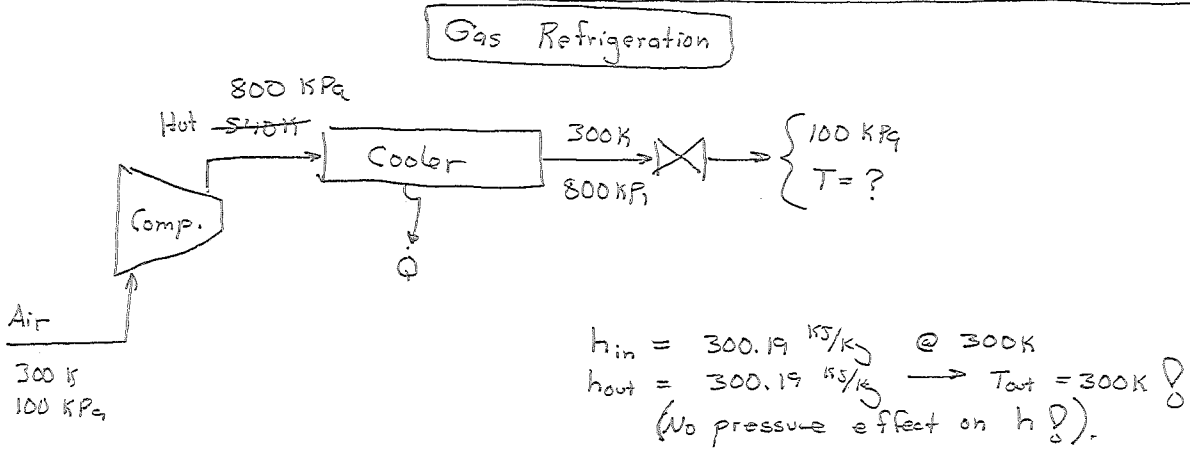


$$\dot{Q}_H = \dot{m}_4 (h_4 - h_5) = (2 \text{ kg/s})(275.98 - 93.42) = 345.1 \text{ kW}$$

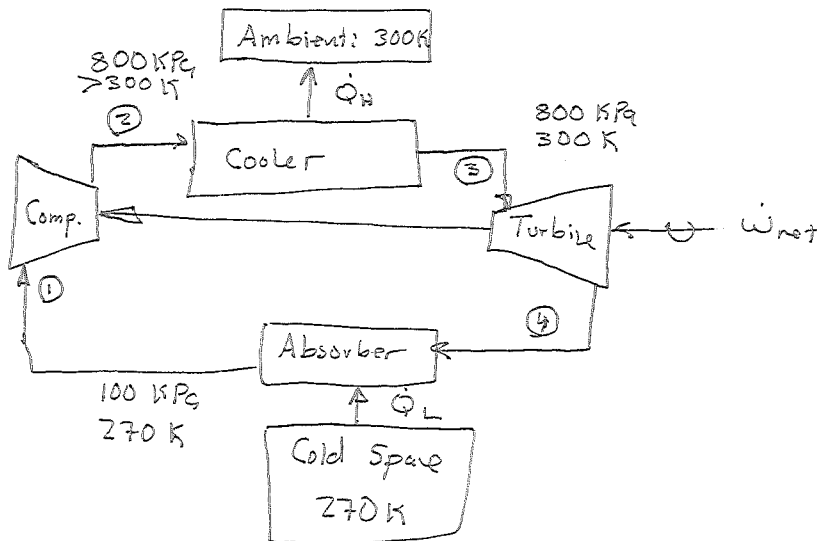
$$\dot{Q}_L = \dot{m}_7 (h_1 - h_6) = (2 \text{ kg/s})(1 - 0.28)(224.72 - 36.84) = 270.6 \text{ kW}$$

$$\dot{W} = \dot{Q}_H - \dot{Q}_L = 94.55 \text{ kW} ; \beta = \frac{\dot{Q}_L}{\dot{W}} = 2.86$$

(Equivalent simple cycle = 2.45)



So you need to use a turbine:



Look at performance with cold air assumptions.

$$T_2 = T_1 \left[ \frac{P_2}{P_1} \right]^{k-1/k} = (270 \text{ K})(8)^{0.4/1.4} = 489 \text{ K}$$

$$T_4 = T_3 \left[ \frac{P_4}{P_3} \right]^{k-1/k} = (300 \text{ K})(1/8)^{0.4/1.4} = 166 \text{ K}$$

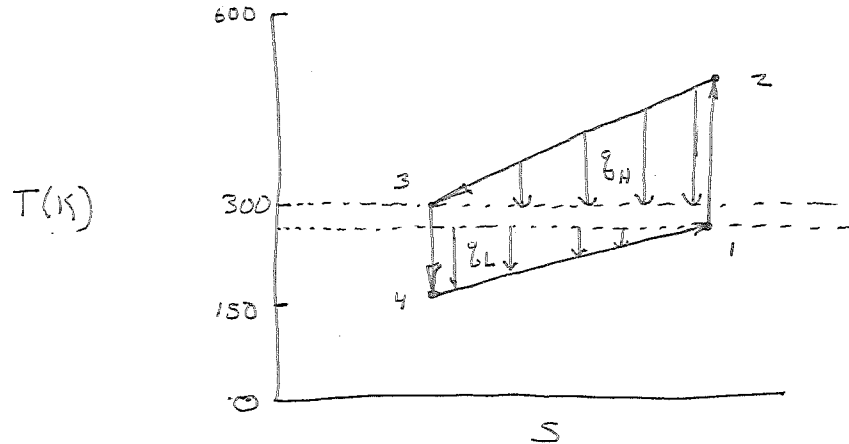
$$q_H = C_p (T_2 - T_3) = (1.005)(489 - 300) = 190 \text{ kJ/kg}$$

$$q_L = C_p (T_1 - T_4) = (1.005)(270 - 166) = 105 \text{ kJ/kg}$$

$$W_{net} = q_H - q_L = 190 - 105 = 85 \text{ kJ/kg (going in)}$$

$$\beta = \frac{q_L}{W_{net}} = \frac{105}{85} = 1.24 \text{ (really bad!)} \quad \beta_{Carnot} = \frac{1}{\frac{300}{270} - 1} = \underline{\underline{9}}$$

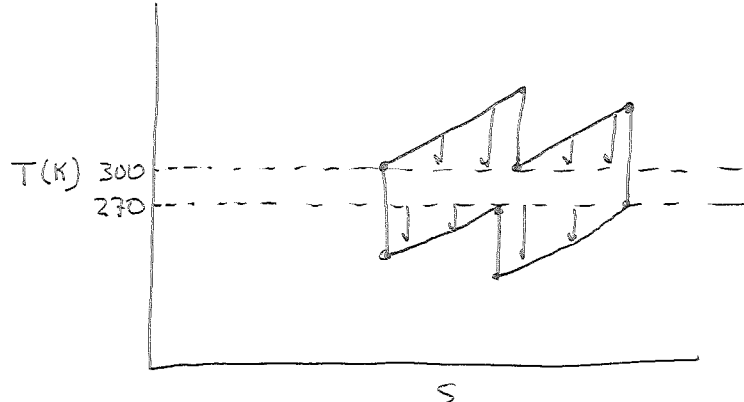
Why? What can we do?



- Big  $\Delta T$  for  $q_H$  and  $q_L$
- Lots of  $i$ .

What to do?

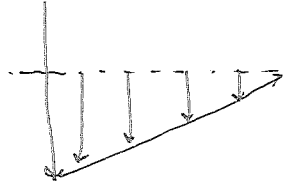
- Compress in two stages with an intercool.
- Use a two-stage absorber.



$\Rightarrow \beta = 2.89$  !

The key difference is:

Air



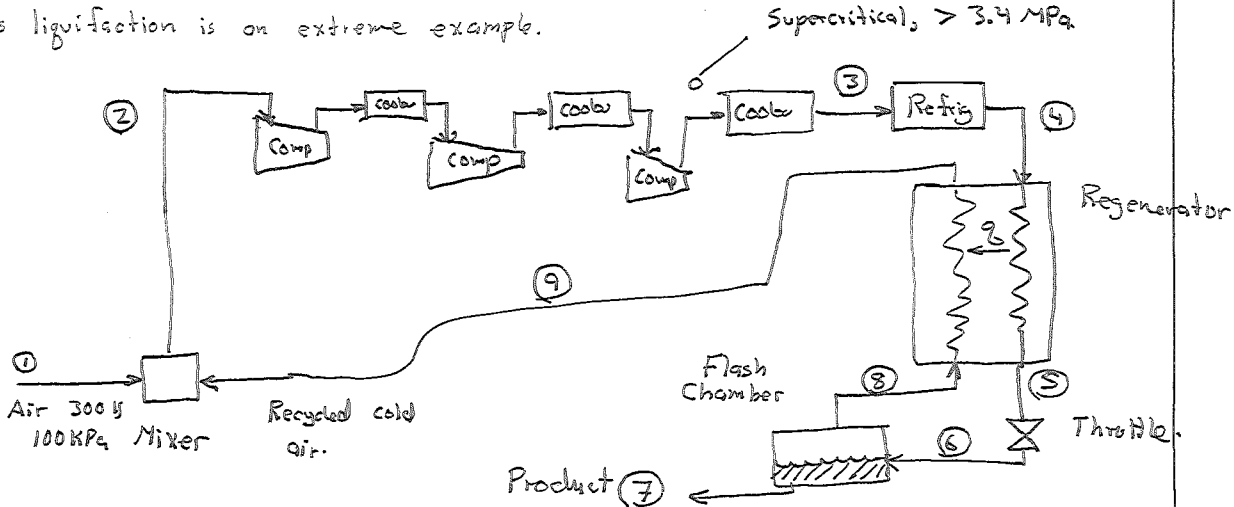
- Absorbs heat by heating up.
- Must start cold  $\Rightarrow i$

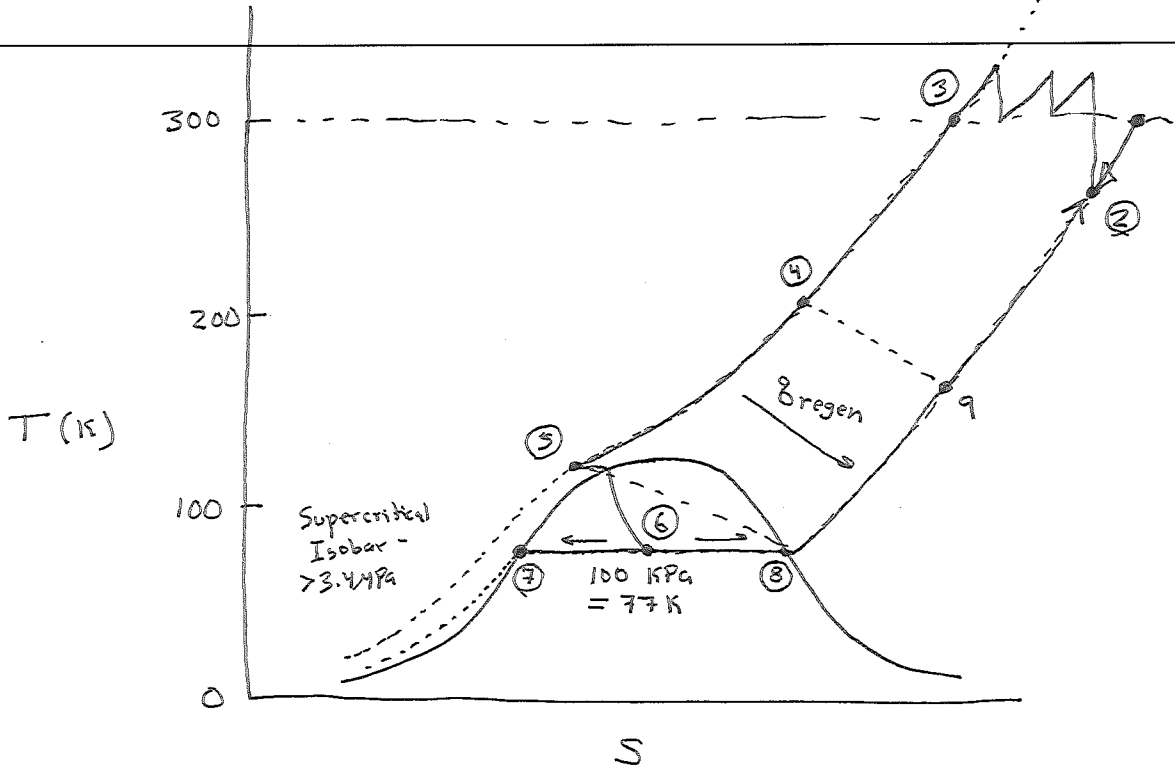
R-134a



- Absorbs heat by boiling
- Can run with small  $\Delta T$ .

Gas liquifaction is an extreme example.

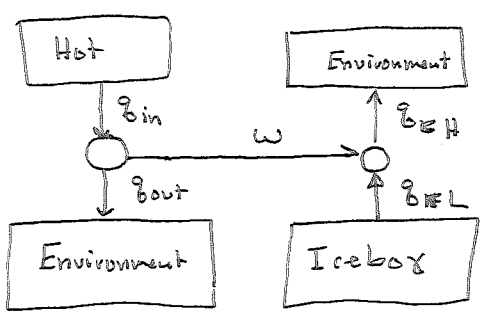




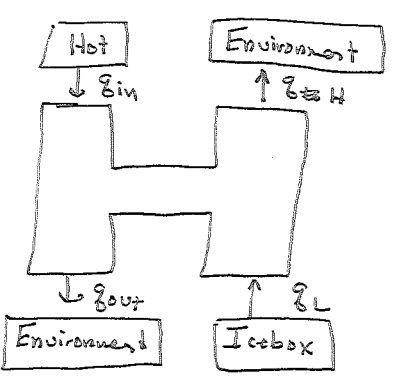
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Absorption Refrigeration: Colman-Type refrigerators that run off a flame.

Normal refrigeration



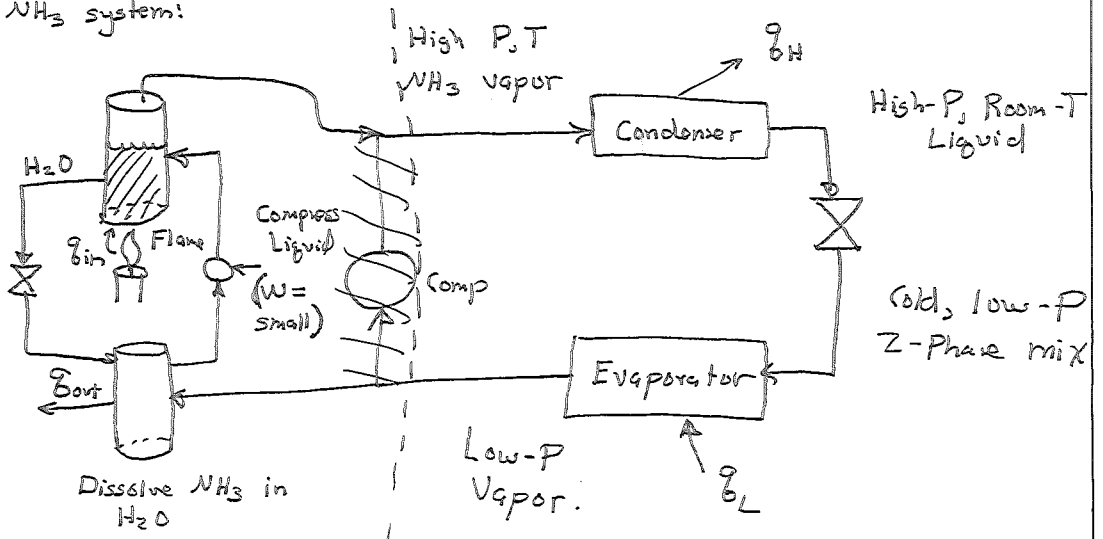
Need  $w$ ; usually get from heat engine.



Put them together!

⇒ Eliminate discrete work step.

Look at  $NH_3$  system!



Goal is to replace compressor. - All else stays the same.