Lecture Notes on Composite Fracture Models

by[1]
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The WEK Model

The Waddoups, Eisenmann, and Kaminski (WEK) fracture model was developed in 1971 [2]. These authors attempted to predict composite fracture using Linear Elastic Fracture Mechanics (LEFM) methods applied to model fracture in isotropic metals. They suggested that a region of intense energy of characteristic length *a* originating from the edge of a hole in a composite specimen can be thought of as a pseudo "crack" (Figure 2.1). This suggests that an analysis based on the stress-intensity factor might be appropriate. However, since self-similar crack growth does not occur in composites, it was implied by Waddoups et al that an energy-based analysis similar to that proposed by Irwin [3] would hold a greater intellectual appeal.

From standard LEFM for isotropic materials in plane strain conditions the mode I strain energy release rate, G_I, is related to the mode I stress intensity factor, K_I, as follows[2]:

$$G_I = \frac{K_I^2}{E}(1-\nu^2) \qquad (1)$$

Also in accordance with standard LEFM, the stress intensity factor is:

$$K_I = \sigma_N^\infty \sqrt{\pi a}\;\; f(a/R) \qquad (2)$$

Where σ_N^∞ is the remotely-applied stress. The function $f(a/R)$ is often called the "geometry factor", and accounts for the finite dimensions of a test specimen or structure. Expressions for $f(a/R)$ for various *isotropic* test specimen geometries are tabulated in many sources, for example in Paris and Sih [5]. However, Waddoups et al assumed that the geometry factor

[1] A few minor edits have been inserted by M. E. Tuttle

[2] The original paper by Waddoups et al contains an erroneous expression for G_I in which an extraneous "π" term appeared in their Eq (1). A related error then appeared in subsequent expressions in the original paper. These errors have been corrected in this summary.

$f(a/R)$ for isotropic materials would not be applicable to composite laminates (a very good assumption, especially since stacking sequences and consequently stress fields vary from laminate to laminate). One of the objectives of their paper was to suggest a method of determining $f(a/R)$ for composite laminates.

Although Waddoups et al imply that an energy-based analysis is appropriate for use with composites, the bulk of their analysis is based on use of the stress intensity factor, Eq (2).

At the moment of fracture the stress intensity factor has reached the critical level, i.e., $K_I\big|_{fracture} = K_{Ic}$. K_{Ic} is called the "fracture toughness" or "critical stress intensity factor". At fracture Eq (2) becomes:

$$K_{Ic} = \sigma_c \sqrt{\pi a}\ \ f(a/R) \tag{3}$$

where σ_c represents the remotely-applied stress at fracture

For an unnotched specimen with no hole, $f(a/R) = 1$. In this case the remote stress at fracture is equals the unnotched fracture strength of the laminate, σ_o, and Eq (2) becomes:

$$K_{Ic} = \sigma_o \sqrt{\pi a} \tag{4}$$

Dividing Eq(3) by Eq (4):

$$\frac{\sigma_o}{\sigma_c} = f(a/R) \tag{5}$$

Waddoups et al used Eq (5) to measure the geometry factor, and hence the fracture toughness K_{Ic} using Eq (3), for laminates with open holes of varying diameters[3]. Diameters less than 0.50 in were termed "small" holes, whereas holes with diameters greater than 0.50 in were termed "large" holes. A reasonably consistent K_{Ic} was measured for specimens with small holes, and a consistent *but different* value for K_{Ic} was measured for specimens with large holes. Since K_{Ic} is thought to be a material property, at least for isotropic metals, these results illustrated a shortcoming of standard LEFM, when applied to composite laminates. For composites, K_{Ic} is not a material property but rather varies with geometry.

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[3] A crticial strain energy release rate can also be calculated using Eq (1), although this calculation was not performed by Waddoups et al

The applications of standard LEFM models to predict composite fracture led to inconsistent results, as evidenced by the MEK approach, leading to refined models described in the following.

The WN Models

J. M. Whitney and R. J. Nuismer proposed two models to predict the fracture strength of notched composites [7,8]. The two models are the “Point Stress Criteria” and the “Average Stress Criteria” models. They are based upon a characteristic dimension that is assumed to be a material property independent of laminate geometry or stress distribution. The theory of the models assumes that fracture occurs when the stress distribution or average stress reaches the unnotched strength at a distance equal to the characteristic dimension away from the edge of the discontinuity. The models were proposed in a effort to explain the hole size effect without resorting to the use of LEFM.

The theory explaining the hole size effect is based upon the difference in the stress distribution ahead of the hole for different sized holes. The normal stress, σ_y, along the x -axis for an isotropic plate of infinite size containing a circular hole is given by [8]:

$$\frac{\sigma_y}{\sigma^\infty} = 1 + \frac{1}{2}\left(\frac{R}{x}\right)^2 + \frac{3}{2}\left(\frac{R}{x}\right)^4 \qquad (13)$$

where σ^∞ is the uniform tensile stress applied parallel to the y -axis at infinity. Plotting the normalized stress as a function of the distance ahead of the hole reveals that, while the stress concentration at the hole edge is 3 for both hole sizes, it is much more localized for the smaller hole, Figure 2.2. It can then be argued that the plate containing the larger hole will have a lower residual strength. This is because there is a larger volume of material subjected to a higher stress and the probability of having a larger flaw in the highly stressed region is greater, thus resulting in a lower strength. Also, the plate with the smaller hole has a greater capability for redistributing the stress, leading to a higher strength.

The first model is the "Point Stress" model. This model assumes that failure occurs when the stress, σ_y, over some distance, d_o, away from the discontinuity is equal to or greater than the strength of the unnotched laminate [7]:

$$\sigma_y(x,0)\big|_{x=R+d_o} = \sigma_o \tag{14}$$

This criterion is represented schematically in Figure 2.3.

The normal stress, σ_y, along the *x*-axis in front of the hole for an infinite orthotropic plate containing a circular hole of radius R subjected to a uniform stress, σ^∞, can be approximated by [10]:

$$\sigma_y(x,0) = \frac{\sigma^\infty}{2}\left\{2+\left(\frac{R}{x}\right)^2+3\left(\frac{R}{x}\right)^4-\left(K_T^\infty-3\right)\left[5\left(\frac{R}{x}\right)^6-7\left(\frac{R}{x}\right)^8\right]\right\};x>R \tag{15}$$

where K_T^∞ is the orthotropic stress concentration factor for a circular hole in an infinite plate as expressed by [11]:

$$K_T^\infty = 1+\left(\frac{2}{A_{11}}\left(\sqrt{A_{11}A_{22}}-A_{12}+\frac{A_{11}A_{22}-A_{12}^2}{2A_{66}}\right)\right) \tag{16}$$

where A_{ij} are the orthotropic in-plane stiffnesses of the laminate as determined from lamination theory [12]. Applying the point stress criteria, Equation 14, with Equation 15 yields the notched to unnotched strength ratio:

$$\frac{\sigma_N^\infty}{\sigma_o} = \frac{2}{\left[2+\xi_1^2+3\xi_1^4-\left(K_T^\infty-3\right)\left(5\xi_1^6-7\xi_1^8\right)\right]} \tag{17}$$

where

$$\xi_1 = \frac{R}{R+d_o} \tag{18}$$

Examining the limits of Equation 17 reveals that for very large holes, $\xi_1 \to 1$, the classical stress concentration factor, $\sigma_N^\infty/\sigma_o = 1/K_T^\infty$, is obtained. Also, for vanishing holes (R$\to$0), $\xi_1 \to 0$, and , $\sigma_N^\infty/\sigma_o = 1$, as expected.

The second model is the "Average Stress" model. This model assumes that failure occurs when the average stress, σ_y, over some distance, a_o, equals the unnotched laminate strength [7]:

$$\sigma_o = \frac{1}{a_o}\int_R^{R+a_o} \sigma_y(x,0)dx \tag{19}$$

This criterion is represented schematically in Figure 2.4. Applying this criterion, Equation 19, with Equation 15 yields the notched to unnotched strength ratio:

$$\frac{\sigma_N^\infty}{\sigma_o} = \frac{2(1-\xi_2)}{\left[2-\xi_2^2-\xi_2^4+(K_T^\infty-3)(\xi_2^6-\xi_2^8)\right]} \tag{20}$$

where

$$\xi_2 = \frac{R}{R+a_o} \tag{21}$$

The Mar-Lin Model

It has been widely accepted that the applicability of LEFM to composites is not appropriate. This is because the fracture toughness of a composite generally increases with the crack length and asymptotically approaches a constant value. In linear elastic fracture mechanics for monolithic brittle materials, the failure stress is related to the crack length by

$$\sigma_N^\infty = K_{I_c}(\pi L)^{-1/2} \tag{32}$$

the exponent -1/2 is the mathematical stress singularity at the crack tip. A direct application of equation (32) to composite materials has been found to be inadequate. The K_{IC} value, which is supposed to be a material constant, generally increases with the crack length and asymptotically approaches a constant value. To account for this Mar and Lin [14] proposed an equation for the fracture of composite materials, but with a stress singularity corresponding to that for a bimaterial interface:

$$\sigma_N^\infty = H_c(2L)^{-m} \tag{33}$$

where H_C is the composite fracture toughness having units of stress x (length)^m and is the property of the laminate material and lay-up. The exponent, m, is related to the stress singularity at the crack tip of the bimaterial interface. The order of singularity is dependent upon the ratio of the shear moduli of the matrix and the fiber, μ_1/μ_2, and the Poisson's ratio, υ_1 and υ_2 [15,16,17]. The order of singularity can be predicted from $m = 1-\lambda$, where λ is calculated from the characteristic equation:

$$\lambda^2\left(-4\alpha^2 + 4\alpha\beta\right) + 2\alpha^2 - 2\alpha\beta + 2\alpha - \beta + 1 + \left(2\alpha^2 + 2\alpha\beta - 2\alpha + 2\beta\right)\cos\lambda\pi \qquad (34)$$

with $\alpha = \dfrac{\left(\dfrac{\mu_1}{\mu_2} - 1\right)}{(1+\eta_1)}$ and $\beta = \dfrac{\mu_1(1+\eta_2)}{\mu_2(1+\eta_1)}$. In the case of plane stress the factor $\eta_i = \left(\dfrac{3-\upsilon_i}{1+\upsilon_i}\right)$, i=1,2, where υ_1 and υ_2 are the Poisson's ratio of the matrix and fiber materials, respectively.

It is important to note that the Mar-Lin model assumes that it is the length not, the shape of discontinuity that is the controlling parameter on fracture. Thus, there is no distinction made between laminates with circular holes and laminates with cracks since both types of defects are merely a geometric discontinuity at the microscopic level.

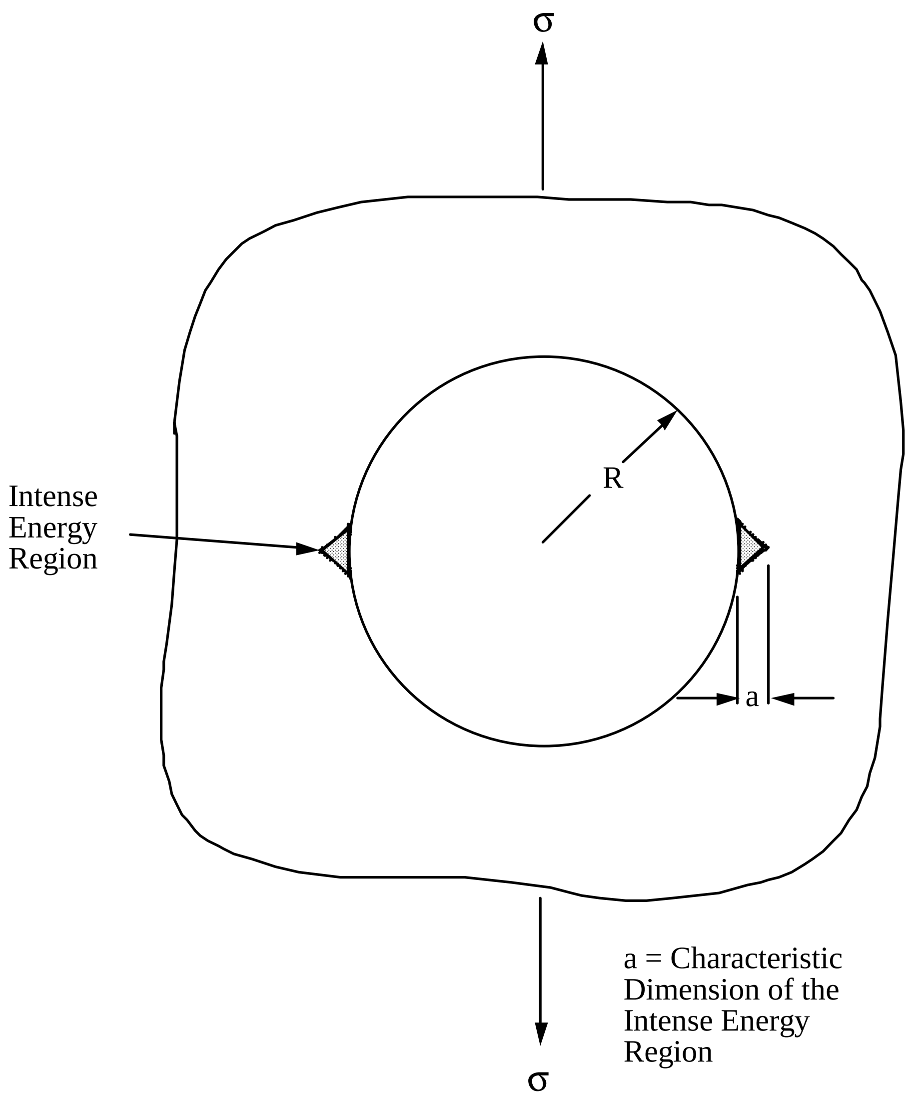

Figure 2.1 WEK Fracture Model [2]

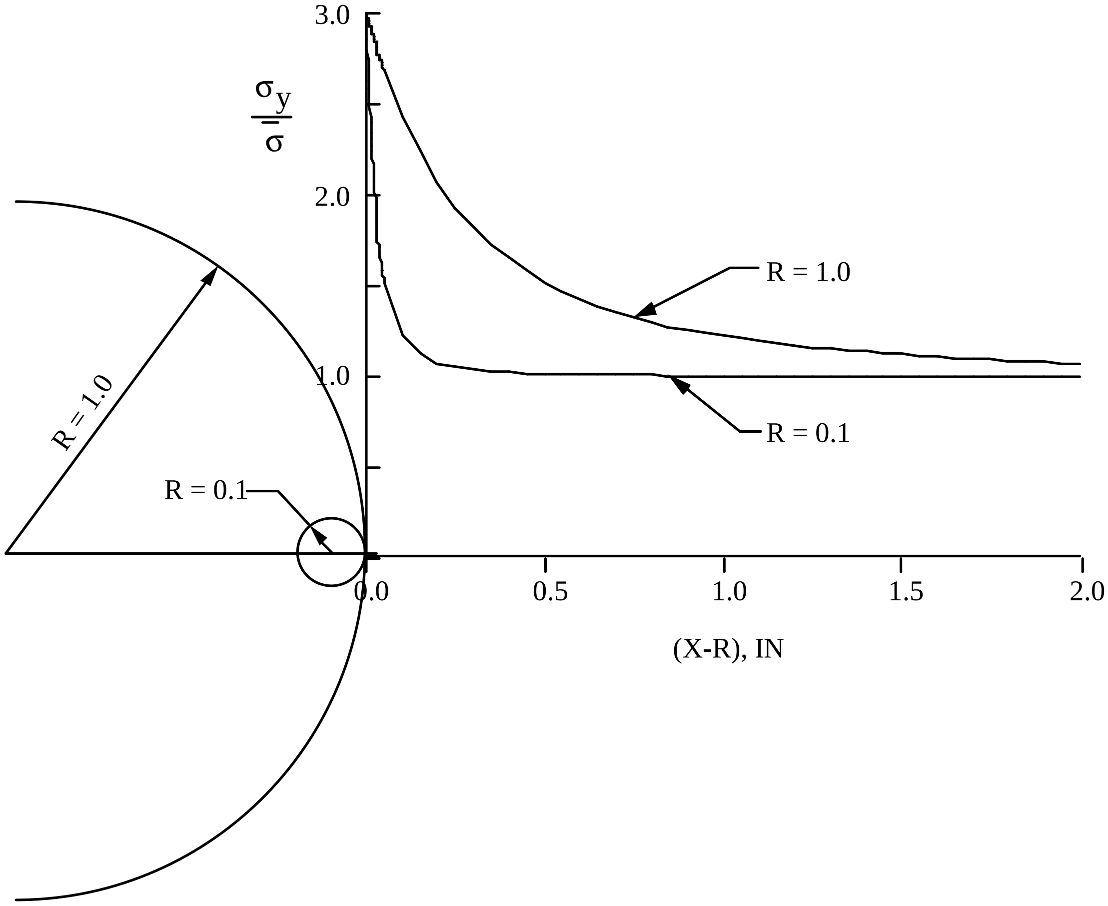

Figure 2.2 Normal Stress Distribution for a Circular Hole
in an Infinite Isotropic Plate.[7]

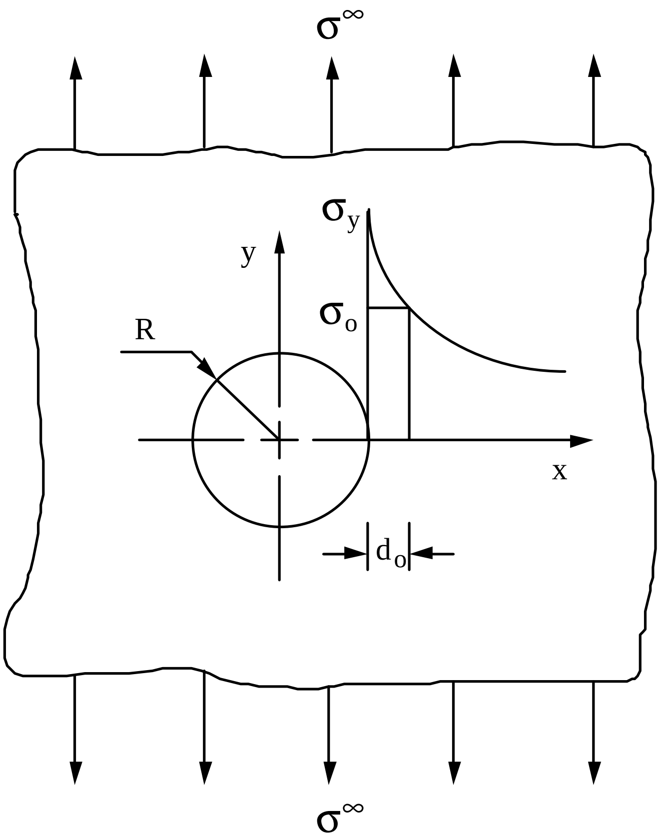

Figure 2.3 WN Point Stress Criteria for a Laminate Containing a Circular Hole.[7]

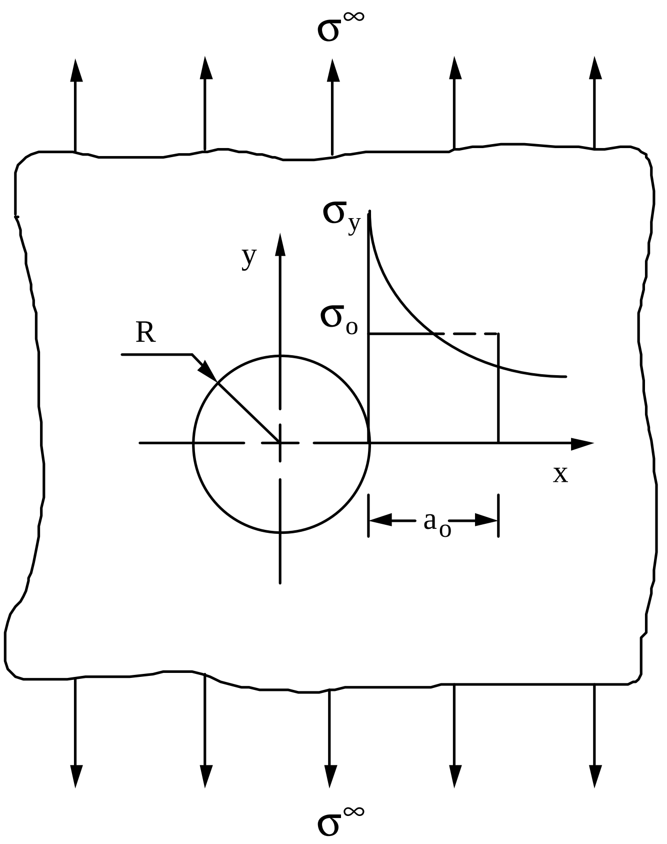

Figure 2.4 WN Average Stress Criteria for a Laminate Containing a Circular Hole.[7]

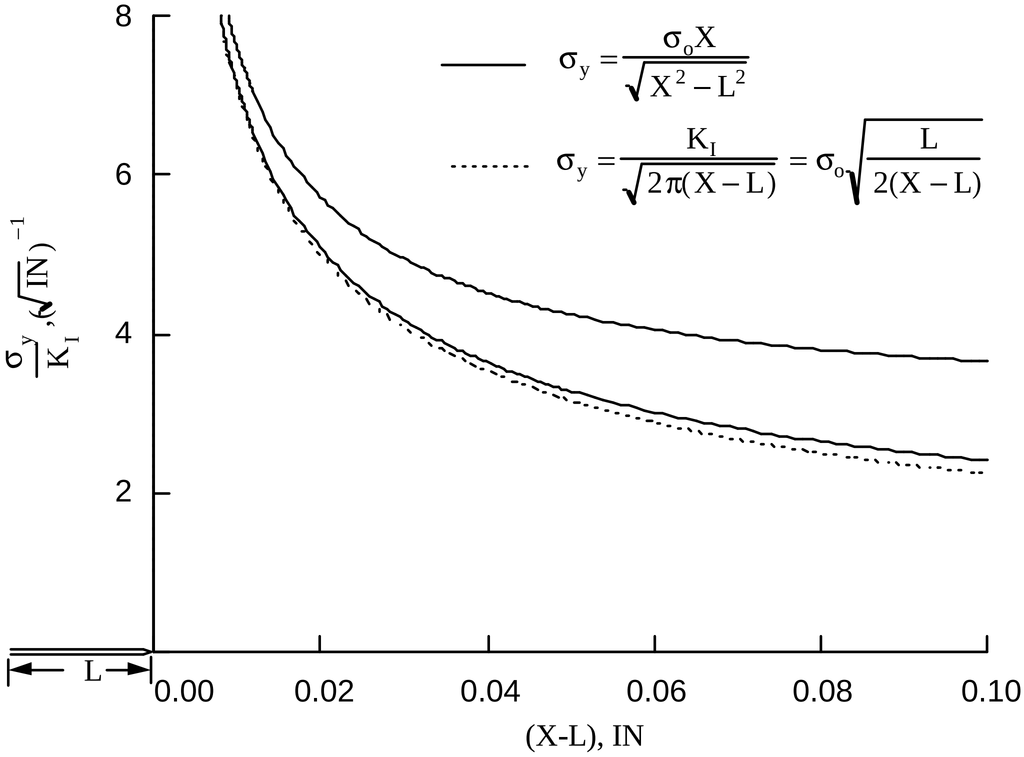

Figure 2.5 Normal Stress Distribution for a Center Crack
in an Infinite Anisotropic Plate.[7]

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