International Journal of Fatigue 21 (1999) S15–S34

International Journal of Fatigue

www.elsevier.com/locate/ijfatigue

Near-threshold fatigue: a review

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Abstract

Interest in extending the high cycle fatigue life of structures has increased in the last decade of the 20th century. Since a long fatigue life depends on keeping cyclic loads near or below the fatigue threshold, understanding near-threshold behavior is now all the more important. This paper reviews the status of the 'threshold of fatigue' and to a lesser extent its relation to the 'fatigue limit'. The fatigue threshold for crack propagation is not a single number. And, when it appears expressed in crack propagation equations as ΔK_{th}, it is usually involved with crack closure. To explore what it is, the published information on near-threshold fatigue crack growth is reviewed here with emphasis on current trends. Linear elastic fracture mechanics (LEFM) is the formalism used, but effects such as overload are difficult to explain using a purely elastic model. A single threshold stress intensity range is inadequate to explain the effects of the stress ratio. Concepts of crack closure, shielding and closure-induced shielding have been introduced, leading to the emergence of dual-parameter threshold models. Interpretation of these models has revived debate about the physical significance of crack closure because contributions to fatigue crack growth have been observed from portions of the loading cycle taking place while the crack is closed. In seeking alternatives to closure, fatigue mechanisms based on dislocation generation, residual stresses and microstructural stress singularities have recently been elucidated to supplement work advancing the understanding of what is termed partial closure.

Keywords: Fatigue crack propagation; Fatigue threshold; Crack closure; Load ratio; Fatigue limit; Effective stress intensity

1. Introduction

Damage tolerant design has been the most highly regarded design philosophy in the last 10 years. Under its paradigm, structures are designed to allow limited crack propagation so that growing cracks can be located and remedied. This approach, principally applied to aircraft, replaced a former 'safe life' design paradigm [1] in which inspections were not essential. The 'safe life' approach led to overly-conservative designs and short lives. Damage-tolerant design extended life, but recent economic considerations have led to further increases in the expected lifespan of existing aircraft and other infrastructure, giving rise to increased concern about reliability. Since damage tolerance is based on the likelihood of finding cracks and their estimated propagation rates, delaying the retirement of aircraft has resulted in increased concern about crack propagation rates, initiation probabilities and thresholds in general. There are two types of thresholds currently in use for fatigue. One is the *fatigue crack propagation threshold* which defines a loading criterion under which cracks will not grow significantly. (Henceforth in this paper, the phrase 'fatigue threshold' will refer only to the fatigue crack propagation threshold.) The other is the *fatigue limit* which defines a loading criterion under which significant cracks will not form. The former threshold presumes the existence of the crack and is used in damage tolerant design. The latter is traditional and was associated with the 'safe life' approach; it emphasizes initiation rather than propagation. Both types of thresholds are functions of the loading cycle parameters and environment. Both tend to decrease with increasing stress ratio.

Damage tolerant design has fostered the selection of materials having slow crack growth rates and high thresholds for fatigue crack propagation. But such materials tend to be soft, relatively weak and, as will be discussed, have low fatigue limits, implying cracks are more easily initiated. A major contributor to keeping crack propa-

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0142-1123/99/$ - see front matter
PII: S0142-1123(99)00053-5

gation thresholds high has been thought to be crack closure, a mechanism highlighted and studied by Elber [2,3]. Crack closure is the premature closure of a crack by the presence of an obstacle within it such as might result from plastic deformation, facets, oxide or metal particles. Considerable emphasis has been placed on the role of plastic deformation in the wake of the crack, although its relevance to near-threshold fatigue is questionable. By limiting the range of relative motion of the two fracture surfaces of a crack, crack closure reduces the apparent stress intensity range as seen at the crack tip. Closure has been a key element in Newman's numerical spectrum-loading fatigue model, FASTRAN [4] which remains the standard for aircraft life prediction models [5,6]. Perhaps the most salient event in the last decade has been the questioning of the significance and even the existence of crack closure [7]. This comes at a time of frustration with the limitations of conventional damage-tolerant materials which could be perceived as timid in their performance and yet not unequivocally safe considering the number of aircraft accidents which have occurred [1]. General reviews of the literature related to fatigue thresholds have been written [1,4–10] and additional specialized reviews are listed in the references.

Near-threshold fatigue crack propagation takes place at growth rates generally less than 10^{-9} m/cycle. Its phenomena differ somewhat from the more-studied fatigue modality, low cycle fatigue (failure in less than 40,000 cycles). There are four characteristic periods in fatigue life and three stages of crack propagation. The four periods are: initiation, stage I growth in which cracks presumably grow in shear (mode II) followed by stage II growth and finally by post-critical or uncontrolled propagation. Although the cracks in the stage I period of fatigue are always short (while in terms of propagation, stage I cracks are usually long), stage I and II periods in fatigue are loosely identified with stage I and II of long crack propagation. Crack propagation stages are defined in terms of the second derivative of the logarithm of the rate versus the logarithm of the stress intensity range. In stage I, the threshold region, this is negative; it is zero in stage II and positive in stage III. Fig. 1 illustrates all three stages of fatigue crack propagation. Near-threshold fatigue crack propagation takes place on and below the border between propagation stages I and II. When cracks initiate in high cycle fatigue, their propagation rates usually fall in this same range. Since the majority of civil structures are designed to withstand many stress cycles, high cycle fatigue is economically important. It is less well studied than low cycle fatigue probably because the tests take longer to perform.

High cycle fatigue phenomenology exhibits thresholds: the fatigue limit and the fatigue threshold for crack propagation. Practical structural designs try to exploit

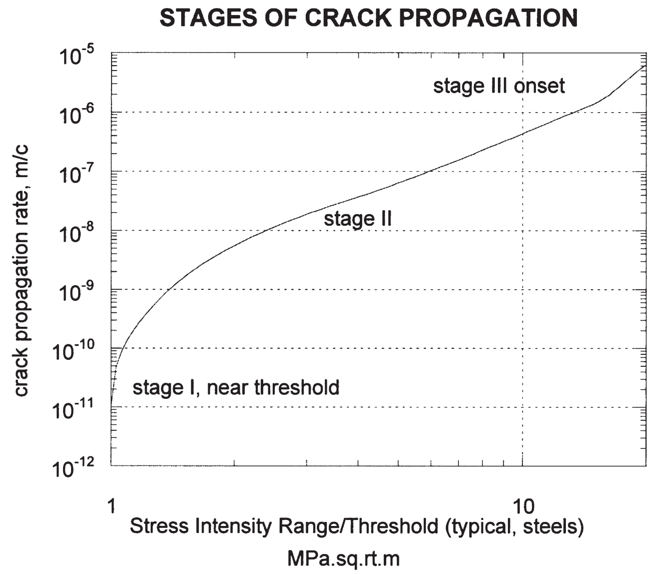

Fig. 1. The three stages of crack propagation.

these thresholds. The propagation threshold is usually stated as an alternating stress intensity, ΔK_0, under a specified stress ratio and other circumstances. For many materials, these thresholds are not absolute in the sense that the initiation or crack growth rates drop to zero. If a crack fails to grow perceptibly in say 10,000,000 cycles, the alternating stress intensity is often taken as an operational fatigue threshold. This threshold condition can also be defined as a crack growth rate of 10^{-11} m/cycle [8, pp. 222–313]. This rate is less than one crystal lattice spacing of growth in ten cycles!

Because, in the absence of overloads, plasticity is very limited in near-threshold fatigue; fracture mechanics (LEFM) is the tool of choice for describing near-threshold crack propagation. LEFM is not the only tool (elastic–plastic fracture mechanics has been used with short cracks near the fatigue threshold) but, by consensus, remains the most effective tool [8, pp.222–313, 9–11] for characterizing the rate of growth of long fatigue cracks in terms of da/dN, the growth increment per cycle. In LEFM, the fatigue crack growth rate is a function of the applied *stress intensity range*, $\Delta K = K_{max} - K_{min}$. K_{max} and K_{min} are respectively the maximum and minimum stress intensities in the loading cycle. In the lower end of the Paris regime, the rate of crack propagation is frequently approximated by the 'thresholded Paris relation', a form of which was introduced by Elber [3]

$$\frac{da}{dN} = C(\Delta K - \Delta K_{th})^n \quad (1)$$

where a is crack length, N is the number of elapsed stress cycles, ΔK_{th} is the threshold stress intensity range, and C and n are constants. In the near-threshold stage, crack growth rates range from 10^{-10} to 10^{-8} m/cycle. The threshold in Eq. (1) is not a vertical asymptote to the low rate end of the propagation curve in Fig. 1, but rather is a correlation constant originally associated with crack closure. Load ratio, $R = K_{min}/K_{max}$, along with microstructure and environment influence the rate of crack growth and ΔK_{th}.

In contrast to long cracks near the fatigue threshold, short crack growth shows considerable scatter and the use of LEFM is often debated [8, pp. 222–313, 11]. The application of LEFM to short cracks raises an issue called ‘similitude’, the notion that all cracks in the same type material should behave alike so long as their crack-tip driving force (K) history is the same. It is not clear whether it is yet possible to formulate crack growth in such a way that similitude is preserved. The existence of similitude would considerably simplify the near-threshold crack propagation problem. In the Paris region of growth, similitude is justified by the Paris relation because the Paris relation is a power law in length and therefore independent of scale. Successful use of the Paris relation for short cracks reinforces the concept that similitude exists [12], but frequently thresholds differ between long cracks and short cracks in the same material. While this seemingly negates similitude, there is usually enough doubt as to the exact nature of crack tip conditions to keep the concept of similitude plausible. Differences in crack closure, microstructural stresses and relative plastic zone size are the reasons most often given for differences between small and large crack behavior. As a result of these the crack tip driving forces are altered under otherwise similar conditions. Many conflicting rules have been put forward to correlate fatigue crack propagation data. One reference alone lists 54 equations for crack growth [13].

2. Thresholds for initiation

The earliest studies of fatigue were concerned with failure of a workpiece after a number of loading cycles. The early workers, most significantly Woehler (see Ref. [1]) did not isolate crack growth as a separate phenomenon. Rather they counted the number of cycles to failure as a function of the loading conditions. Early on, it was recognized that the workpiece would appear to be crack free for a large fraction of its life. This fraction was termed the ‘initiation’ period. The portion of life occupied by propagation was relatively short and could be ignored. Gerber, later Goodman and finally Soderberg, studied the effects of mean stress on fatigue life and developed a type of diagram which has sometimes been called a ‘Haigh–Soderberg diagram’ (Fig. 2) for the fatigue threshold or ‘infinite fatigue life’. Such a diagram plots a curve in coordinates of mean stress vs. cyclic stress amplitude. Such a diagram can also be constructed for say ‘50% of fatigue life being 10,000 cycles’. In that case the space between the curve and the origin would be occupied by points representing combinations of mean and cyclic loading for which failure would occur at a minimum of 20,000 cycles. Steels appear to show a true fatigue limit and the Gerber parabola for steels is such that the space between the curve and the origin is occupied by points representing loading conditions for which failure would *never* occur. Redefining the threshold conditions as ‘not having failed at say 1,000,000 cycles’, such a diagram, as shown, can be constructed for ductile materials besides steels. Fig. 2 is often called a *fatigue limit* diagram and the point on the curve for zero mean stress is the conventionally-defined *fatigue limit* (the stress amplitude at a stress ratio $=-1$).

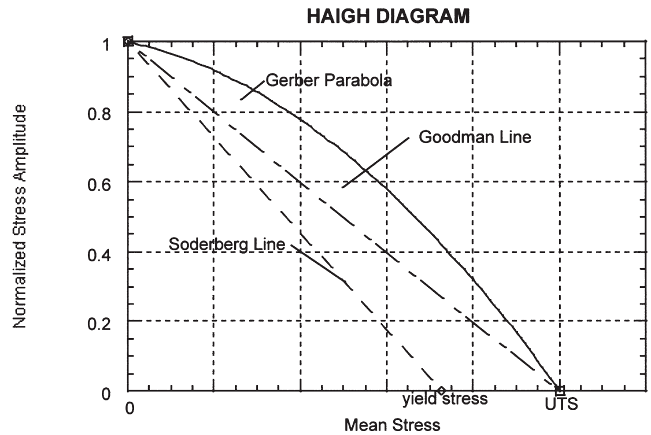

Fig. 2. Haigh diagram for the *fatigue limit* plotting stress amplitude vs. stress ratio. Points under the appropriate curve represent conditions where fatigue failure will not take place, viz. significant cracks will not initiate. The Gerber curve is used for steels. The Goodman and Soderberg curves are used for non-ferrous alloys. The *Y*-axis scale is normalized by dividing by the fatigue limit for fully reversed loading.

Fig. 2 was cast in terms of mean stress. The mean stress increases with the *stress ratio*, R, of the minimum to the maximum applied cyclic stress. The condition ($R=-1$) defines fully-reversed loading in which the mean stress is zero. Stress ratio is one of several variables which can be chosen to introduce mean-stress effects. Replotting the Haigh diagram in terms of stress ratio does not change it much in qualitative appearance except for a change in scale of the abscissa; there is an abrupt drop to zero in the curve when the mean stress approaches the ultimate tensile strength which occurs at a value of R less than unity. For steels the fatigue limit is usually 35% or more of the ultimate tensile strength. If the applied stress amplitude equals the fatigue limit (at $R=-1$), the allowable stress amplitude given by the Haigh diagram goes to zero at less than $R=0.5$. In the framework of fracture mechanics, R is a more popular variable than mean stress. Goodman–Gerber type relationships are empirical in origin. They are founded on the assumption that some sort of a smooth curve should connect the limits for alternating stress and static loading. At the present time fatigue crack initiation is not well understood. It has been best studied in terms of persistent slip bands and the various models are reviewed in standard texts, e.g. [8, pp. 222–313]. In all cases, a microstructural process converts a remotely applied stress into a local singular stress through some process of progressive strain localization. Once the crack is initiated, the strain singularity at the crack tip re-initiates the crack after each advance. The fatigue limit is thus related to the ability of the material to resist strain localization and ultimately its resistance to plasticity.

3. Effect of crack length

Fracture mechanics (LEFM) is mostly concerned with how cracks propagate. It bypasses the unknown details of crack tip atomistic processes. The approach is simplest for long cracks. For these it is possible to draw a curve of crack propagation rate vs. the range of the alternating stress intensity (rather than the stress). The curve itself (Fig. 1) is a function of R and is usually drawn on a log-log scale. The curve is a rotated mirror of the letter s in shape. The spine of the s forms a linear region called the stage II region. At lower values of the stress intensity range, this curve turns downward, forming the stage I or threshold region. The other end of the curve turns sharply upward, forming the stage III or fast crack stage. In the stage II region the crack growth rate is a power law function of the alternating stress intensity, usually represented as the stress intensity range. The domain of this power law is termed the *Paris regime*. *Stress intensity* is proportional to the product of stress and the square root of crack length. This is a statement of its units of measure and says nothing of how either stress or crack length are to be measured in a given situation. Its use as a correlating variable stems from its being the only factor of the singular terms in the stress field for a crack (in an elastic material) which contains the remote applied stress. Although there are non-singular terms as well, these are customarily neglected in the long crack problem.

Since plasticity is the sine qua non of fatigue, the success of LEFM in describing fatigue has an element of mystery. Cracked materials are only superficially elastic. Plasticity explicitly appears in elastic–plastic formulations such as the J-integral or the crack tip opening displacement (CTOD). The variables K and J are, however, closely related under small scale yielding conditions; the additional work involved in using J has not appeared to yield a commensurate improvement in predictive ability under near-threshold conditions except in special cases. The relative success of LEFM is illustrated by Fig. 3 which shows the correlation of data for 2024-T3 obtained by Paris and Erdogan [14,15] from various sources. Correlation is made with the Paris–Erdogan relation, Eq. (1) with the threshold set to zero.

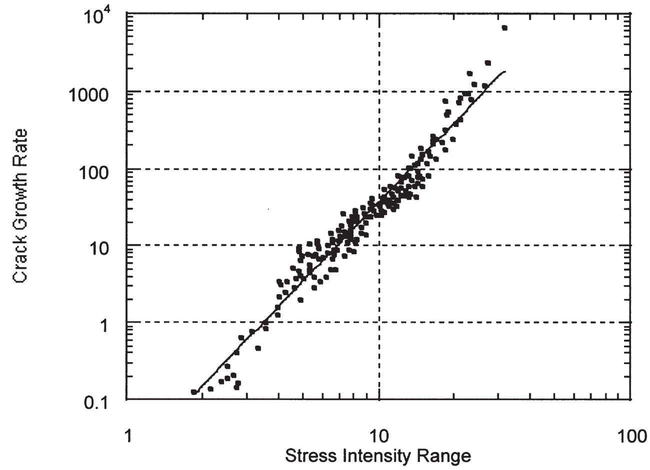

Fig. 3. Data for 2024-T3 aluminum from various sources used by Paris and Erdogan [15]. The stress intensity range is in units of lb/in$^{1.5}$×10^3. The crack growth rate is in units of in/cycle×10^{-6} (microinches per cycle). The straight line represents the Paris–Erdogan relation.

Fig. 3 suggests that the threshold for crack propagation may not be an intrinsic part of the growth (e.g. Paris) relation. For any given material, thresholds are apt to vary more with changing test conditions than do the Paris constants. But, within the threshold region, other effects seem to dominate any *intrinsic* threshold itself. These *extrinsic* effects have been subsumed under the heading of 'crack closure'. As defined by Elber, when closure is present, the effective stress intensity range is not the applied maximum stress intensity minus the applied minimum stress intensity, but rather the applied maximum stress intensity minus a closure stress intensity.

The growth of small cracks is even more complicated. It is not unusual in the case of small cracks for crack propagation within the threshold region to be fitted, as with a spline of local small-crack Paris relations defining the mean value of the growth rate for the given conditions [12,15,16]. Thresholds can thereby be neglected, minimizing the number of constants (to two) needed at one time. This typically results in a variable exponent as cracks become smaller. (The exponent increases for cracks which 'feel' the threshold; the exponent tends to zero for the average *growing* crack due to the Smith radius effect.) Stress intensity is the principal LEFM driving force and has length embedded in it. For this reason, the conventional LEFM approach breaks down as the crack approaches zero length. Stress intensity would not seem to be a meaningful variable for cracks of zero length. The subject of short cracks is a complicated one and has been reviewed by many, including Suresh and Ritchie [17]. It is still not fully resolved. Short cracks can be viewed from a long crack perspective through the Kitagawa diagram (Fig. 4) [18]. This plots observed threshold stress range (at say R=0) vs.

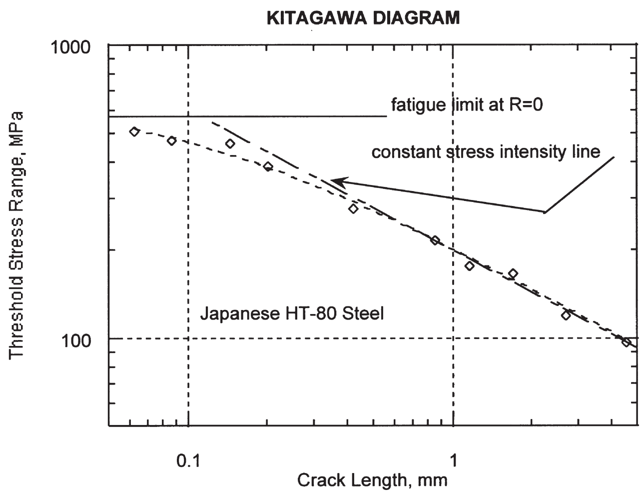

Fig. 4. Kitagawa diagram for HT-80 steel based on Kitagawa and Takahashi's data [18]. The dashed line is a least squares fit to Eq. (2).

crack length on a log-log scale. On the right side of the diagram the line slopes with a rise over run of -0.5. This corresponds to a constant stress intensity independent of length. At the left-hand-side of the diagram, the curve rolls over, becoming parallel to the abscissa as it reaches zero length. The stress range at zero length is that corresponding to the fatigue limit.

Several workers have described the fatigue limit portion of the curve by means of a fictitious length, a_0, often termed the Smith radius [19,20]. For a given stress ratio, it can relate the stress intensity threshold to the fatigue limit simply by making the crack length the sum of the observed length and a_0.

$$\Delta K_{th} = \Delta\sigma Y\sqrt{\pi(a+a_0)} \quad (2)$$

where a is the crack length and Y is a geometric factor. As seen in Fig. 4, the fictitious (Smith) length drops out asymptotically as the crack becomes long. The Smith radius has been related to the slip band length [21–23]. Whether or not the fatigue threshold, cf. Eq. (1), is intrinsic, there is an apparent reduction in its magnitude with decreasing crack length. If there is a threshold at near-zero length, it is concealed by the stress intensity associated with this fictitious length. This disappearance has been ascribed to a larger apparent plastic zone size (see [8, pp. 222–313]) and the absence of closure for small cracks [24]. However, small cracks have not always been observed to be closure-free [25]. Microstructural stress singularities, discussed later, provide another explanation for the Smith radius.

4. Effect of crack geometry

The near-threshold region in metals is generally associated with a reversed-shear mode of growth which at least implies a mode II component. At the same time, plasticity is largely confined to select crystallographic planes, e.g. {111} in Fe–Ni alloys [26]. This gives rise to a faceted fracture surface. Since growth is taking place by a shear mechanism on planes inclined to the mode I stress plane, a certain amount of mode II displacement is expected. If this were unreversed, as might happen in a tensile overload, registry of the peaks and troughs between the upper and lower crack faces could be lost and the peaks would contact each other before the crack fully closed. Ritchie [9] pointed out that in steels the transgranular faceting reaches a maximum in the threshold region at which nearly 80% of the fracture surface may be faceted. Close to the threshold, where the crack growth rate is near 10^{-10} m/cycle, this can drop to near 1%. At stress intensity ranges near the threshold, a large oxide buildup is likely. Lists of papers on this subject are given in [9,28]. The presence of this oxide, believed to be due to fretting, has been thought to prop the crack open [8, pp. 222–313, 10,27,28]. At very low fatigue loads, oxide, and at higher loads, misaligned facets act as wedges reducing the effective stress intensity range by preventing the crack from closing. These effects are called, respectively, oxide- and roughness-induced *crack closure*. Near the fatigue threshold, the stress ratio exerts a strong effect. Although stress ratio affects the fatigue limit in a roughly similar way, its ability to alter the fatigue threshold (that for propagation) has been linked more to crack closure than any cause which could also be related to initiation.

Environmental effects similarly reach a maximum at stress intensity ranges near threshold but then diminish [8, pp. 222–313, 9]. Environment has been thought to act more through alteration of crack geometry than through any direct chemical effect and is seen to vanish in some materials at growth rates less than 10^{-8} m/cycle [29,30] where the crack surface becomes very planar. Cracks have even been observed to slow down in their growth in aggressive environments [31]. Under conditions intended to 'avoid crack closure', Matsuoka et al. [32] found that steels immersed in saline show slightly reduced growth rates in the near-threshold region; but under freely corroding conditions, threshold stress intensities were significantly lowered and unaffected by cathodic bias. Others have also seen a variable (but sometimes negative) increase in the crack growth rate of freely-corroding steel in saline vs. air [33]. Other investigators have shown that similar steels immersed in saline under cathodic bias demonstrate substantially increased propagation thresholds. This has been explained in terms of calcareous deposits causing closure [34] and closure due to crystallographic asperities [35]. But, closure is not the only cause of crack retardation in corrosive environments. Corrosion blunting has long been observed in both steel and aluminum alloys to reduce the stresses at the crack tip [36].

In some materials, notably Al–Li alloys, planar slip resulting from shearing of ordered precipitates results in exaggerated surface roughness and crack tortuosity [37,38]. Tortuosity, illustrated in Fig. 5, has been referred to as having a 'shielding' effect by reducing the effective stress intensity range. Tests have been conducted with planar-slip materials by applying a large maximum stress intensity to keep the cracks open and then gradually reducing the cyclic component to find threshold stress intensity range independent of closure. These tests suggest that closure may not be a major factor in explaining the low propagation rates seen [37,39].

Fretting fatigue has geometries where there is a significant cyclic abrasion. For this reason, it has a different phenomenology. The concept of a threshold for fretting fatigue is less well established and tends to follow different lines from those discussed here [40,41].

5. Effect of strength

While it is probably impracticable to list all the parameters which affect fatigue thresholds apart from the load ratio, R, the following are perhaps the most important factors (in no particular order):

1. strength or yield stress;
2. microstructure;
3. load history;
4. environment;
5. elastic modulus [6].

Ritchie [9,42,43] collected data for steels and plotted threshold stress intensity vs. yield stress. Fig. 6 illustrates his data in a pooled form. He noted that there was a negative slope; higher yield stresses led to lower thresholds. (This trend is not true for all materials [44], and a reverse trend has been noted for non-ferrous alloys [45].) Ritchie [9] attempted to explain the effect in steels through hydrogen embrittlement. His explanation introduces a concept, Neuber's radius [46], which has been used by several authors in modeling material behavior. If we imagine a crack tip as having an intrinsic radius, this leads to a formula relating applied stress intensity to a finite stress at the crack tip. (In the absence of such a radius and in a purely elastic material — infinitely sharp crack — the stress would be singular and infinitely large over some infinitesimal circle in the crack tip plane.) Ritchie treated the effect of the resulting stress field on the chemical potential of hydrogen in a small volume at the tip of the crack [9]. The tensile stress field attracted hydrogen to the tip, thus weakening the metal. Since yielding limited the stress, the amount of hydrogen scaled with the yield stress in agreement with the generally observed lower propagation thresholds at higher yield stresses. On the other hand, higher yield stresses usually result in higher fatigue limits for all metals because the plastic strain per load cycle is reduced. Fig. 7, a Kitagawa diagram based on Ritchie's [9] result, illustrates the simultaneous reduction in fatigue threshold and increase in fatigue limit.

Other explanations for the reduction of the threshold stress intensity range in steels with increasing strength have been given. All mentioned are based on the idea of a sharper crack tip. In the Dugdale plastic strip model [8, pp. 222–313] the CTOD scales directly with the square of the stress intensity and inversely with the yield stress. It follows that the crack tip is sharper when the

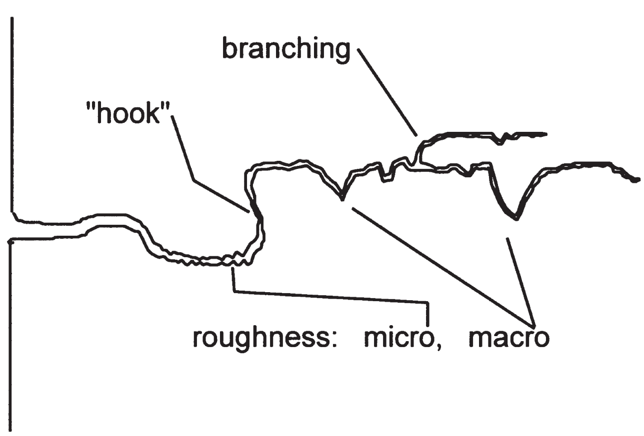

Fig. 5. Crack tortuosity. Tortuosity tends to reduce the effective stress intensity range below that of the applied range and involves several mechanisms. Roughness-induced closure limits the minimum stress intensity. Hook or 'lock-up' mechanisms limit the maximum stress intensity. Branching produces true elastic shielding. Tortuosity also increases the ratio of the true length to that projected on the plane of the stress axis reducing the energy release rate.

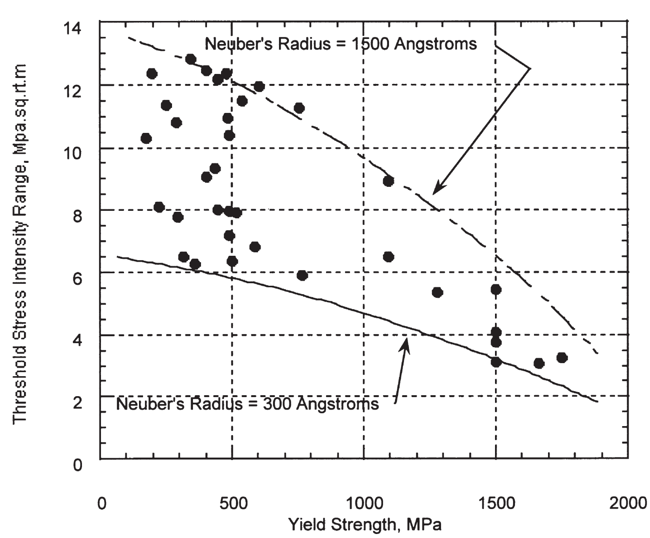

Fig. 6. General trend of yield stress versus threshold stress intensity range at R=0 as gathered by Ritchie [9]. The bounding curves represent Ritchie's hydrogen model.

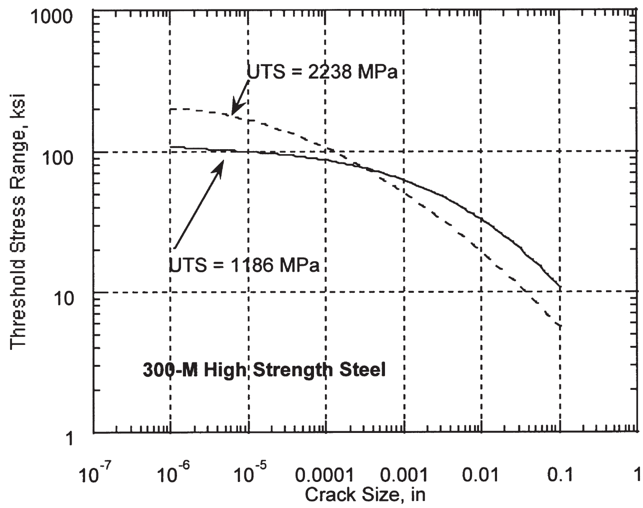

Fig. 7. Model behavior of 300-M steel as a function of heat treatment [9] displayed on a Kitagawa diagram. Note that the softer material has a lower fatigue limit but a higher fatigue propagation threshold.

yield stress is higher although the CTOD range would be expected to be smaller. In a purely elastic material, a sharper crack tip will place higher stresses in the material [36]. Similarly incorporating plasticity through the Neuber's radius concept, Weiss and Lal [47] equated the stress at the crack tip to a fracture stress for the material ahead of the crack tip. Sadananda and Shahinian [48] equated this same stress to that for dislocation emission (needed for plasticity) and thus created a series of dislocation models of the fatigue threshold. More recently, Lal and co-workers [49–52] extended their earlier fracture stress idea by considering two competing processes in the same characteristic volume at the crack tip. One process is a stress-dependent cleavage process while the other is a strain-dependent reversed shear process. Having two processes in the model increases its flexibility. In general, raising the yield stress has a variable effect on mode I crack propagation rates depending on the specific metal. Its effect on crack propagation rates above the threshold is generally small [53].

6. The effect of grain size

Microstructure has a strong effect on fatigue thresholds and the explanations are as varied as the effects. Among the best studied effects are those of grain size. Grain size effects are not entirely separable from yield stress effects due to the Hall–Petch relation which sets yield stress as inversely proportional to the square root of grain size. Grain refinement has long been used to raise fatigue limits. This is reviewed by Ritchie [9]. The observation of small cracks arresting at grain boundaries has been made by many observers [54]. Taira et al. [55] examined the growth rates of small cracks and noted that minima occurred when the size of the plastic zone was approximately equal to that of the grain as illustrated in Fig. 8. This led to the blocked slip band concept of Tanaka et al. [22]. In the resulting type of model, transference of the slip band from one grain into the next controls the rate of propagation [22,56]. It argues that smaller grains allow the slip band to reach the grain boundaries at lower stress intensities, thus reducing threshold and increasing the speed of propagation. This is discussed further in [8, pp. 212–333]. Consistent with the blocked slip band model, grain refining in titanium alloys and steels has been observed to increase mode I propagation rates [57–61].

The blocked slip band model has been extended by Li [62,63] into a Stroh-type dislocation based model in which edge dislocation pileups are blocked by grain boundaries. This model mimics features of crack closure without closure actually present. Lal also modeled the effect of grain size using his Neuber's radius approach [49]. Which of his two competing processes, cleavage and reversed shear, dominates depends on the defect concentration in the stressed volume at the crack tip. Grain boundaries are modeled as planar defects. Sufficiently high defect concentration results in cleavage when matched by a sufficiently high peak stress intensity. Failure by reversed shear is slower and (discounting shearable precipitates and other mild obstacles to slip) depends on the stress intensity range rather than the peak stress. Thus, whether a crack leaps across a grain boundary by cleavage or stalls there waiting for sufficient accumulated plastic strain is governed by the maximum local stress intensity present which depends on the stress ratio. For crack propagation to take place, minimum values of two parameters which may be taken to be K_{max} and ΔK must be simultaneously exceeded. This model

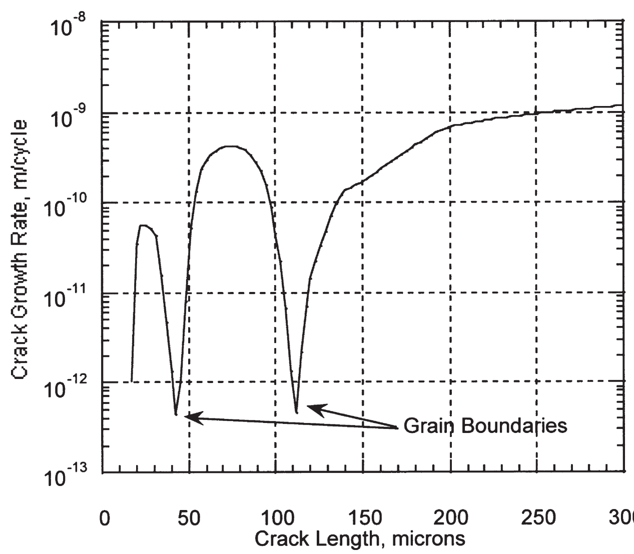

Fig. 8. Typical effect of grain boundaries on retarding the crack propagation rate, adapted from data quoted in Ref. [54].

also mimics the effects of closure without closure being present.

7. The effect of stress ratio on propagation

Stress ratio has already been mentioned in the context of mean stress and the fatigue limit. A plot can be constructed where the threshold stress intensity is plotted vs. the stress ratio. Such plots have been compiled for steels by Bulloch [64]. Using Bulloch's data, Fig. 9 shows that there is considerable scatter even when the data are normalized. Nevertheless, there is an overall negative slope (reminiscent of that of the Haigh diagram for the fatigue limit) showing a trend toward lower thresholds with higher stress ratio. Analogous to the limit set by the ultimate tensile stress on the Haigh diagram, the fracture toughness in this case sets a limit at high stress ratios. Blacktop and Brook noted that the fatigue threshold must touch zero at R=1 since at that point the maximum stress intensity is infinite for any finite stress intensity range [65]. On the other hand, Masuva and Radon had noticed that some fatigue threshold data show a leveling off at high stress intensities rather than the expected drop to zero [66]. An evolving series of semi-empirical approaches to understanding R-effects have been put forth which do not assume crack closure. These include two-parameter fittings derived from the Barsom linear fit [67] or the Klesnil and Lucas [68] power fit to the fatigue threshold which has a resemblance the Gerber [69] line for the fatigue limit. These two fits take the following forms respectively,

$$\Delta K_{th}=A-B\cdot R \quad (3a)$$

$$\Delta K_{th}=A\cdot(1-R)^{\gamma} \quad (3b)$$

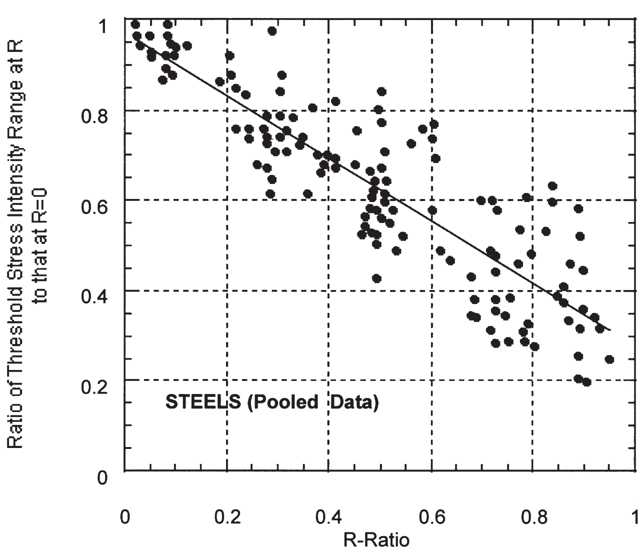

Fig. 9. Normalized threshold stress intensity range as a function of stress ratio showing the general trend for steels. The data are taken from Ref. [64]. The line illustrates a Barsom fit.

where A, B and γ are constants. Voisikovsky [33,44] modified Barsom's fit by tying one parameter, A, to the observed value of threshold stress intensity at R=0. The Barsom type relationships tended to provide a better fit to ferrite/ pearlite steels than did the Klesnil and Lucas type expressions. Fig. 10 illustrates some Klesnil–Lucas fits to pooled data for tempered bainitic and martensitic steels adapted from [64]. Mazumdar and Conrad [70] found that they could tie the exponent of a Klesnil and Lucas expression to the strain hardening exponent. But the results were not uniformly successful [64]. McEvily and Groeger [71] developed a modification of the Klesnil and Lucas expression in which an extra term was added but the exponent was set to 0.5, leaving only one adjustable parameter. The idea of the model was that the threshold stress intensity range was in fact a constant CTOD range. Although theoretically laudable and quite simple, the model did not fit the data well, especially for austinitic steels [64].

To provide a physical explanation for R-effects, crack closure has been cited. Because closure is an extrinsic or test-dependent variable it could give at least a qualitative explanation for the large scatter observed and illustrated in Figs. 9 and 10. Crack closure was first studied by Christensen [72] who matched photoelastic models to fatigue experiments in which fretting-generated debris accumulated, limiting the opening displacement range of cracks. The propagation rates were strongly affected by whether or not the debris was allowed to work itself out of the crack. Nearly a decade later, crack closure was reintroduced and formalized by Elber [2,3]. Elber's closure was that due to the plastically-deformed wake of the crack making contact with itself while some tensile loading was still applied. Some other causes of crack closure

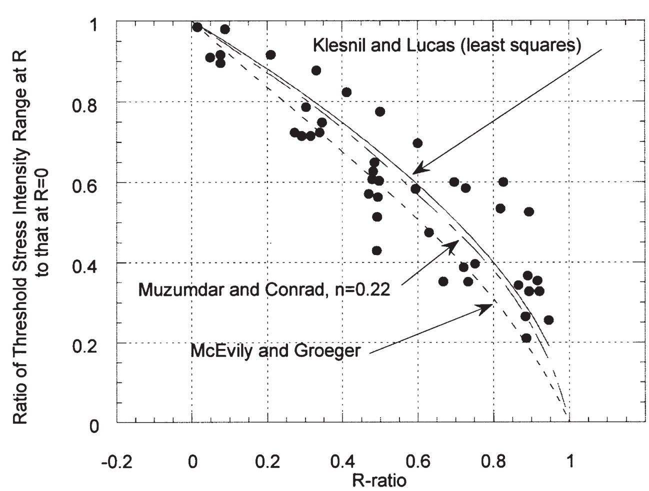

Fig. 10. Klesnil–Lucas-type fittings to data for martensitic and tempered bainitic steels from Ref. [64]. Mazumdar and Conrad's fit is based on assuming a strain hardening exponent, n, which becomes a quasi-adjustable parameter. Sweeping n over the entire domain of its possible values makes a crescent locus which only covers only a small fraction of the data points.

have been mentioned and several may be enumerated [9]:

1. plastic deformation of the wake;
2. roughness and disregistry;
3. oxidation or corrosion products;
4. viscous liquids;
5. martensitic transformations;
6. metal particles from fretting [72];
7. hydrogen-induced deformation [73].

As mentioned in connection with tortuosity, some authors consider closure to be a form of crack tip shielding. In such cases, it is wise to remember the distinction between elastic shielding which affects K and closure which affects ΔK [74,75].

Not all data for stress ratio effects take the form illustrated in Fig. 10. Fig. 11 shows some outliers which hint at a different functional relationship. A key line of reasoning began when Schmidt and Paris [76] plotted the threshold stress intensity of 2124-T3 aluminum vs. R (Fig. 12). By overprinting lines corresponding to a constant K_{max} and a constant ΔK on their diagram they were able to show that the sloping portion of the curve related to a constant K_{max}, while the flat portion of the curve related to a constant ΔK. It was a short step to argue that the part of the curve controlled by K_{max} was governed by the need to open the crack while the constant ΔK portion represented the true threshold hidden under closure. This follows from the simple relation:

$$K_{max} \geq \Delta K_{th,int} + K_{closure} \tag{4}$$

where $\Delta K_{th,int}$ is an intrinsic threshold stress intensity. This relation must be satisfied in order to satisfy the closure condition and the intrinsic threshold simultaneously. Application of this relation to explain Fig. 12 carries the tacit assumption that $K_{closure}$, a suitably chosen effective stress intensity at which the crack opens, does not vary with R. Later Döker [77] plotted threshold stress intensity range vs. maximum stress intensity and obtained a squarish plot as did Marci [78] using Schmidt and Paris's data. Fig. 13 shows such a plot.

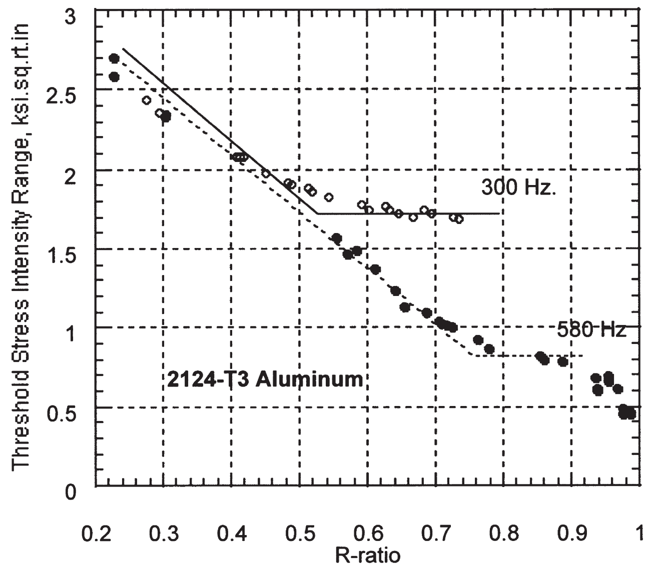

Fig. 12. Threshold stress intensity versus stress ratio for 2124-Al at two different test frequencies adapted from Schmidt and Paris [76]. Negative-slope lines represent constant peak stress intensity. Horizontal lines represent constant stress intensity range. The effect of frequency is probably due to heating of the crack tip.

The next step was made by Vasudevan, Sadananda and Louat [79] when they postulated that such a squarish

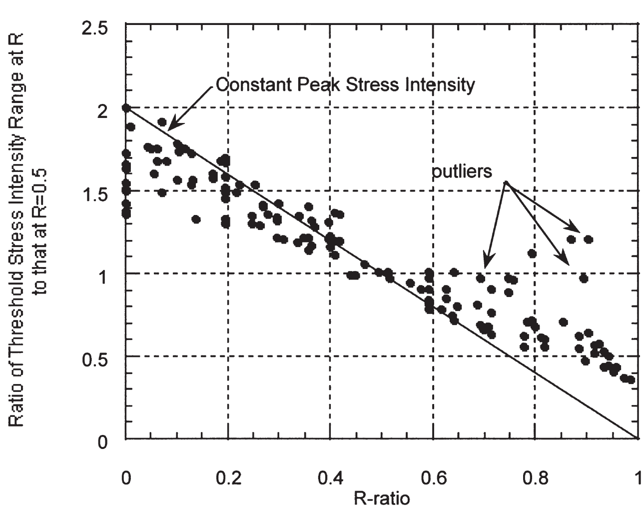

Fig. 11. Normalized threshold stress intensity range versus stress ratio for a variety of metal alloys from Ref. [54]. Most of the data show the expected nearly linear decrease with increasing R. Outliers suggest the possibility of a different behavior pattern. Below R=0.6, the data tend to crowd a line of constant maximum stress intensity.

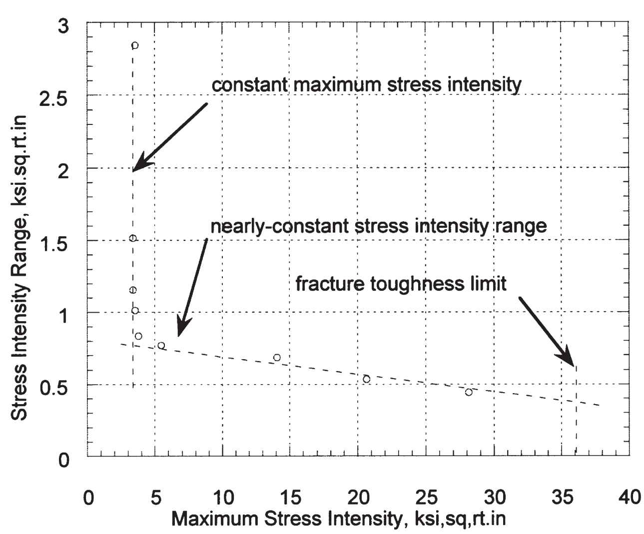

Fig. 13. Döker plot of Schmidt and Paris's data adapted from Ref. [78]. Values of R increase as one moves counter-clockwise from the upper left corner of the plot. At the limit of fracture toughness, the metal fails in one-half cycle.

plot could be reduced to two nearly constant thresholds which they contended were intrinsic properties of the material rather than effects of closure (they actually considered a family of four variations on the Döker plot) [80]. These two thresholds are K^*_{max} and ΔK^*_{th} corresponding to the intercepts with the axes when the two nearly orthogonal lines in the Döker plot (Fig. 13) are extended. Up until this moment, K^*_{max} was considered to be due to crack closure (as given by Eq. (4) when taken as an equality). Closure is an extrinsic variable in that it is a function of crack geometry. But, since closure data were often tacitly treated as if they were a property of the material, crack closure had already been treated with some success as if it were intrinsic. The interpretation of phenomena previously attributed to closure as being the result of two intrinsic thresholds spurred interest in finding new explanations for the effects of stress ratio.

8. Measurements of crack closure

Crack closure explained several important effects, including those of stress ratio and overload. The first actual measurements of closure using a near-tip strain gauge were made by Elber [3] who used a crack opening extensometer. But closure is not easily observed. As an example, Fig. 14 adapts data from Schmidt and Paris [76]. Were closure distinct, this plot would show an abrupt shift between two constant compliance values: a low one when the crack was closed and a high one when it was open. The indistinct slope transition suggests that the crack does not close completely until zero load or

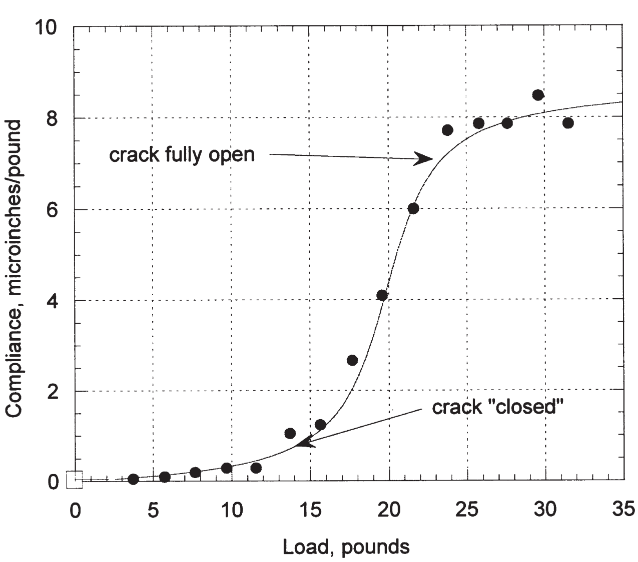

Fig. 14. Graph similar to that used by Schmidt and Paris [76] to obtain the closure load based on their data. This graph is the derivative of the load displacement graph obtained from a crack opening sensor placed near the crack tip. Schmidt and Paris, using the same data, calculated the closure load to be approximately 25 lb. The data are shown fitted to an arctangent function.

less. Customarily, the load at which the crack is completely closed is called the 'opening' load while that at which the compliance begins to change due to contact is the 'closure' load. Schmidt and Paris recorded simultaneous load-displacement curves at different points along the crack and noted an unzipping behavior sometimes expected for plastic closure. Since closure may involve contact of the two sides of the crack at a point beneath the observed surface plane, the only truly direct methods for observing closure are those capable of seeing beneath the surface. These are electrical potential [73] and ultrasonic transmission/diffraction [81,82] methods. Both can identify contact at points away from the crack tip but lose sensitivity rapidly as the tip is approached. Lankford and Davidson [83] used stereoimaging in the SEM to map displacements associated with crack opening. The presence of closure can be inferred from such measurements but contact itself is not observed directly. Similarly, closure loads have also been 'measured' indirectly through fractography [84–86]. Dawicke's three-dimensional fractography [84] has suggested that the interior portion of the crack tip may often remain open while the exterior portion closes. The method of Sunder and Dash [85] has been used to measure both closing and opening loads when discernible striations are formed. In this procedure, K_{max} is held constant while decreasing K_{min} until striations are formed. Then K_{max} is reduced in a step followed by increasing K_{min} until striations disappear. By measuring the spacing of the striations it is possible to estimate closing and opening stress intensities under both load decreasing and load increasing conditions. Closure stress intensities need not remain constant. Most observations of closure are made through observing variations in specimen compliance.

Compliance measurements of the specimen as a whole could in principle diagnose the presence of closure. In reality, the variations in compliance are small and the measurements must be made near to the crack in order to show distinct changes. Crack mouth opening gauges are perhaps the simplest approach and still in use [87], but these have been criticized as giving values of opening and closing stress intensities which are far different from any observed at the crack tip [88]. Unzipping of the crack has been noted but is not always in the same direction. In one test of 2024-T3 aluminum using multiple strain gauges the crack apparently opened at the tip first and then progressively toward the mouth [76]. Davidson [89] however, saw unzipping in the opposite direction progressing from the crack mouth toward the tip in 7091 aluminum alloy.

The use of single strain gauges near the crack tip has been more common than the use of multiple gauges [90–92]. Single gauges are placed in the path of the crack and closure is usually estimated from slope discontinuities in the load-displacement curve as the tip of the crack approaches the gauge. An improvement on strain gauges is to measure point-to-point across the crack from one surface to the other. This can be done using laser interferometry by placing a strong scattering point for laser illumination on each side of the crack tip. This gives two-point diffraction and is in principle quite sensitive so long as the scatterers are not excessively deformed by plasticity [86]. Whatever method is used, there is still a requirement to determine the location of the slope discontinuity which is often gradual. An offset method is commonly used which subtracts an extension of the upper portion of the curve from the lower [8, pp. 222–313]. It is illustrated in Fig. 15. This method increases the apparent deviation at the expense of increasing the noise. As a result it is not surprising that a round robin test of crack opening measurement reproducibility among different laboratories did not show a high degree of reproducibility [93].

Recognizing that a significant contribution to fatigue damage occurs in the load range below the opening load as measured by compliance techniques, Donald has proposed an alternate method for measuring the effective stress intensity range [94]. This method, the compliance ratio, corrects the applied stress intensity range with the ratio of the observed average compliance to what that compliance would have been in the absence of closure. These compliances are obtained by measuring displacements near the crack. A systematic treatment of how such *partial closure,* where fatigue damage takes place

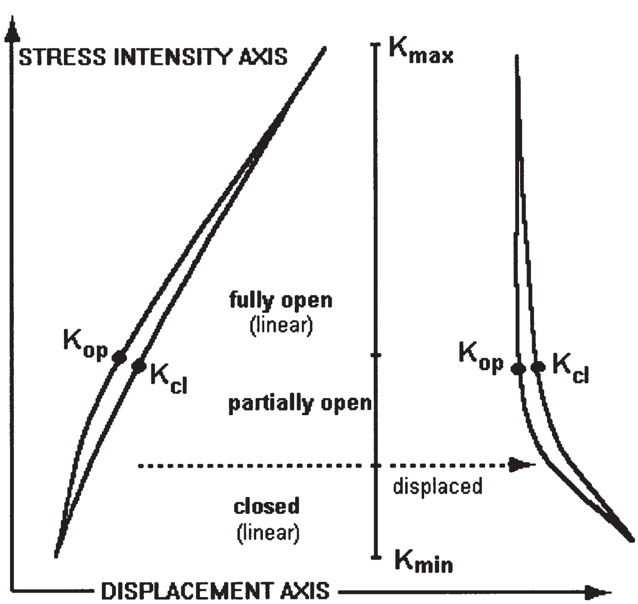

Fig. 15. Offset method of closure stress intensity determination. The actual load displacement curve is shown at the left and takes the form of a hysteresis loop. By subtracting the mean slope in the linear region above the closure points, the closed curve on the right is obtained. The opening stress intensity is located at the point (on the right hand curve) where on ascending the non-linear region joins the linear one. The closing stress intensity is at the corresponding point on the descending side.

below the closing load, would affect displacement measurements in the vicinity of the crack tip does not appear to exist, especially for the elastic–plastic case. Such a treatment would require solving for the displacement field of a mode I elastic–plastic crack — a problem fraught with difficulty [95].

The intended goal of the crack closure measurements was originally to express the effective stress intensity range to a simple form like:

$$\Delta K_{\mathrm{eff}}=\Delta K_{\mathrm{applied}}-(K_{\mathrm{op}}-K_{\mathrm{min}})=K_{\mathrm{max}}-K_{\mathrm{op}} \quad (5)$$

provided $K_{\mathrm{min}}<K_{\mathrm{op}}$ where K_{op} was the opening stress intensity and K_{min} was the applied minimum stress intensity. In most cases, this approach was able to ‘collapse’ data taken at different R-ratios into a single line on the da/dN curve. But the success of the approach may have been largely due to the skill with which K_{op} was chosen. Questions about this subjectivity were partially answered through artificially-induced closure experiments [96]. When closure was induced by placing shims into the mouth of the crack, a compliance change was noted but the effect on crack propagation and threshold were much less than would have been predicted using the stress intensity range of Eq. (5). This indicated that contributions were being made to the advancement of the crack from portions of the loading cycle in which the crack was supposedly closed. Therefore, the closure caused by an asperity was partial [74].

Variations on Eq. (5) have been proposed by several workers to achieve a more realistic estimate of the effective stress intensity in the presence of crack closure. Donald’s previously mentioned method involves multiplying the applied stress intensity by a factor called the ‘Adjusted Compliance Ratio’, ACR, which is obtained from measurements made without determining an opening stress intensity [94]. The product is the effective stress intensity. Paris [6] showed that the effective opening load is only about $2/\pi$ times its value as customarily measured when closure is modeled as being due to an elastic wedge.

9. Plastic closure

Closure due to plastic deformation of the crack wake was one of the earliest forms of closure and its recognition is usually attributed to Elber, although he mentions that others had considered it at the time, notably Rice who he said had already discounted its existence [2]. Once discovered, plastic closure was modeled by Budianski and Hutchinson [97] as a wedge of new material in the crack wake. Since volume must be conserved, most scenarios require out-of-plane plastic displacements in order to provide the wedge’s material. This would prevent plastic closure from occurring in purely plane strain. The shape and significance of this wedge depends to some extent on whether the stress intensities grew as the crack lengthened. Since out-of-plane displacements would be required, plastic crack closure is generally regarded as a plane stress rather than a plane strain effect [8, pp. 222–313, 97,98]. However, specimens sized for nominally plane strain tests may be in plane stress near the surface [84]. Some authors have suggested that plastic closure does not exist at all [7,99]. In support of the absence of plastic closure, Louat [100] developed a (plane strain) dislocation-based calculation to show its absence. The calculation was based on adding a slip-step and dislocation of opposite sign. The authors superimposed the stress field due to the dislocation onto the crack tip and then used a Hilbert transformation integral after the fashion of Muskhelishvili (see Weertman [101]) to assure a traction-free crack surface. The net result largely canceled the displacement of the crack surface due to the slip step. Then choosing a specific geometry, they showed that closure would have to be insignificant. The absence of an *R*-effect in reported studies of steels and aluminum in vacuum was offered as experimental justification since oxide closure was ruled out.

Opposition appeared to the idea that there was no plastic closure. In 1997 Riemelmoser and Pippan [102] published an article describing a dislocation configuration which did result in closure according to a calculation similar to that used by Louat, and in 1998 the same authors [103] questioned the validity of articles gainsaying the existence plastic closure. Several authors have also argued in favor of the continued existence of plastic closure [86,104–106]. Following a line of reasoning begun in a diagram by Riemelmoser and Pippan [107], the third author of this paper, Meshii, points out that, even in plane strain, whether or not a plastic displacement effects closure or opening of the crack depends on when in the cycle the crack advances. If the tensile portion of the cycle creates a plastic stretch at the crack tip and then following that event the crack grows through that stretched zone in the same half cycle, premature closure will occur on unloading because the opening of the new-grown extension will be only that allowed by the remaining tensile displacement in that half cycle.

Finite element analyses of plastic closure have been reviewed by various authors [7,105,106,108]. McClung et al. [108] noted that the differences between the various published treatments of plastic closure were small but noticeable. Ashbaugh et al. [106] arrived at a similar conclusion. Since crack propagation is simulated by releasing a node in the mesh, the question exists of when in the loading cycle to release that node. It is usually released proximate to the maximum load although the exact moment of release varies with different models, resulting in small variations. Another factor seen was the choice of a linear versus a power law hardening model. The latter tended to result in larger closure stresses. Vasudevan et al. [7] contended that the boundary conditions used in finite element models could fail to conserve Burgers vector and were therefore suspect.

Ashbaugh et al. [86,106] conducted an extensive correlation study between fatigue crack closure measurements of 2014-T651 aluminum and a finite element model. Using a combination of laser interferometry and the method of Sunder and Dash [85] to monitor closure, they obtained good agreement between model and the experiment under nominally *plane strain* test conditions. Crack closure nearest to the tip proved to have the strongest effect on thresholds and propagation. Crack propagation behavior correlated well with displacements measured near the crack tip. Closure and opening stresses were found to apparently increase with *R* when plotted with respect to the maximum stress. Fig. 16 illustrates their results. Another and a more extreme example of closure stresses increasing with stress ratio was reported by Marci [109] for a nickel-base alloy undergoing a Sunder and Dash style constant K_{max} load sequence. Here the opening stress stayed near the 40% point in the stress intensity range independent of the mean stress. This sort of behavior would tend to confound any assumptions that tests conducted at high *R* are automatically closure-free.

As mentioned, the non-existence of plastic closure has been linked to the absence of a strong *R*-effect for metals in vacuum [7,100]. This absence has been observed by several authors [27,110,111]. In vacuum the threshold value for stress intensity varies little with *R* but the line lies above that for air. Fig. 17 shows typical results, those obtained by Stewart [27] for low-alloy steel. Pippan [112] conducted environmental fatigue studies of ARMCO iron (a reference material). He saw hysteresis

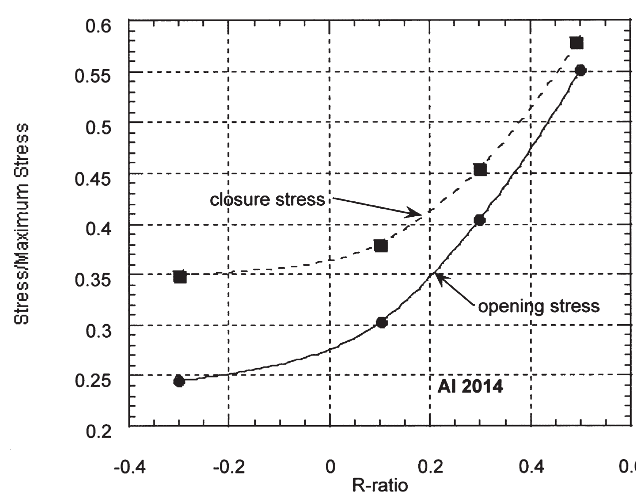

Fig. 16. Plastic closure in an aluminum alloy [106] as determined by correlating load displacement measurements with a finite element model. Note that the closure stresses are defined in terms of the maximum stress which is not constant with *R*.

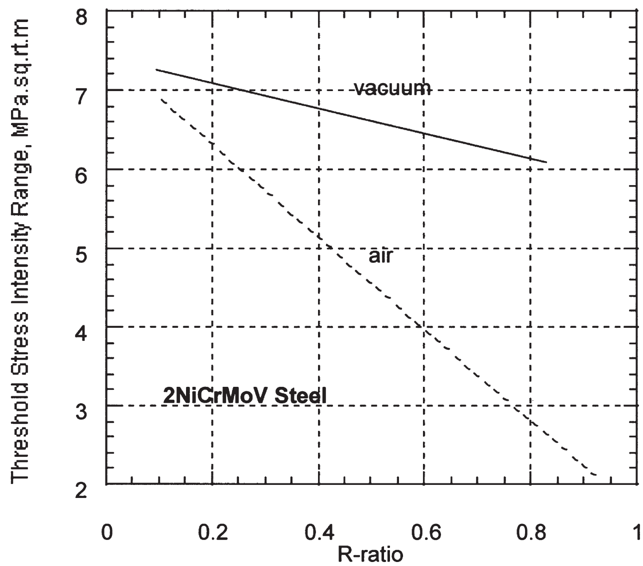

Fig. 17. Threshold stress intensity range versus stress ratio for a low alloy steel in vacuum and air [27]. The flatness of the line for vacuum suggests that closure is absent, but its large values of threshold suggests that closure may be obscured by some other process.

and crack arrest behavior near threshold in air and especially in vacuum. In his interpretation of his data, little effect of R was noted. Lenets [113] re-analyzed Pippan's data and choosing the threshold on the basis of propagation rather than arrest, saw a small but significant R-effect that allowed him to reinterpret the data in terms of crack closure. By subtracting off the closure values, he found an effective threshold value which collapsed the various experimental results into a single broad line, suggesting that closure in vacuum was present. The high values of threshold stress intensity range in vacuum appear to mask the closure effect, since they are larger than any closure stress seen in air. Soboyejo and Knott [114] did studies comparing plane strain and plane stress cracks in vacuum and air. In plane stress the effect of closure appeared to be much higher than in air. While the R-effect in the plane strain specimens in vacuum was small, this was not so for the plane stress specimens which exhibited much larger thresholds. These larger thresholds could be interpreted as the result of a large value of K^*_{max} possibly due to accentuated crack closure in plane stress.

Excluding crack deflection and branching, the effect of overloads on crack propagation have been attributed to plastic closure. Overloading involves plasticity whether through blunting, plastic closure or the generation of a zone of compressive residual stress ahead of the crack tip. For example, Ward-Close and Ritchie [115], studying overload effects in near-threshold crack propagation in titanium alloys, reported detecting a change in specimen compliance, corresponding to near-tip plastic closure, after the application of a tensile overload. However, they still attributed retardation of the crack growth rate following an overload to the compressive residual stresses formed ahead of the crack tip and did not see plastic closure as significant in comparison with roughness-induced closure. They saw overload blunting as the cause of an initial acceleration, rather than as retarding crack growth, through a reduction in roughness-induced closure. A crack growing out of a blunted precursor crack is analogous to one growing from a notch. Its stress intensity is initially relatively small, but after having grown the distance of several notch radii regains the stress intensity it would have had in the absence of blunting. However, blunt tip radii are small in comparison with the size of the zone of plasticity associated with their creation. Yet, cracks usually must grow an additional distance of two to four plastic zone sizes before retardation vanishes. Consequently, as in the example, the retarding effect of plastic blunting is small in comparison with residual stress formation [8, pp. 222–313]. The post-overload pattern of brief acceleration followed by retardation is common. In 1971, Elber referenced a 1960 article by Schijve attributing this pattern to residual compressive stress rather than crack closure [3].

10. Non-plastic closure

Under strictly near-threshold growth conditions, plastic closure would not be expected to be significant even if its existence had not been questioned. The preponderance of the evidence shows that crack closure due to oxidation and surface roughness is significant. The major caveat is that in many cases Eq. (5) will greatly underestimate the effective stress intensity because the closure resulting from contact only at isolated points is partial. Ohta [116], Chen [92] and Donald [94] have studied partial closure experimentally and deserve credit for focusing attention on it, although its possibility was discussed by Elber [2]. Following Hertzberg's experiments, elasticity analyses of asperity-induced crack closure were performed [100,102,117,118]. Chen [117], Louat [100] and Shan [118] analyzed the effects of single asperities. Louat's analysis was for the case where the width of the asperity could be ignored. Chen's emphasized the compliance of the asperity itself. Shan's analysis was directed toward asperities of arbitrary thickness. He showed that the Eq. (5) effective stress intensity required an adjustment similar to Paris's $2/\pi$ correction. However, under realistic assumptions, the correction was not $2/\pi$ but ranged from 0.3 to 0.85 for a single asperity and is a function of R. Even with a rigid, wide asperity the behavior under cyclic loading of the crack tip strain singularity of the 'closed' crack is near to that of an open crack shortened to the tip-to-asperity distance. Riemelmoser and Pippan [102] considered contact by multiple asperities and showed that under some realistic scenarios Eq. (5) could be considered valid using an uncorrected opening stress intensity obtained by the compliance method.

Not all closure must be treated as partial. In iron, for example, compliance measurements of roughness-induced closure have been effective in explaining differences in threshold and near-threshold growth rates effected by heat treating and cold rolling, e.g. Lin and Fine [119]. Ritchie [120] and Ravichandran [121] created and reviewed models of roughness-induced crack closure which include the processes by which asperities are generated. At very low crack growth rates, oxidation is a factor which can enhance roughness by adding to the thickness of the asperities. As the Paris region is approached, plastic closure may become a justifiable consideration in that plasticity may combine to accentuate roughness [90].

Obtaining threshold values which are valid for all loading situations may not be always possible in the presence of crack closure. Wu et al. [37], while studying Al–Li alloys, noted that closure parameters are functions of the method used to obtain the thresholds, viz. the type of load shedding or other procedure. Different procedures gave differing amounts of closure. In general, apparent closure loads increased as the load shedding ramp became steeper and were minimal for a load-increasing procedure in which most closure was eliminated through removal of the crack wake by machining. But, under these circumstances, accounting for closure was still not able to unify the data, as shown in Fig. 18. Nakai et al. [122] observed that even in a simple ferrite pearlite low-carbon steel, anomalous behavior could be found: the threshold stress intensity range at $R=-1$ was less than that for $R=0$. This would conflict with existing threshold models including those having two parameters, since these would have predicted the higher threshold to be at $R=-1$.

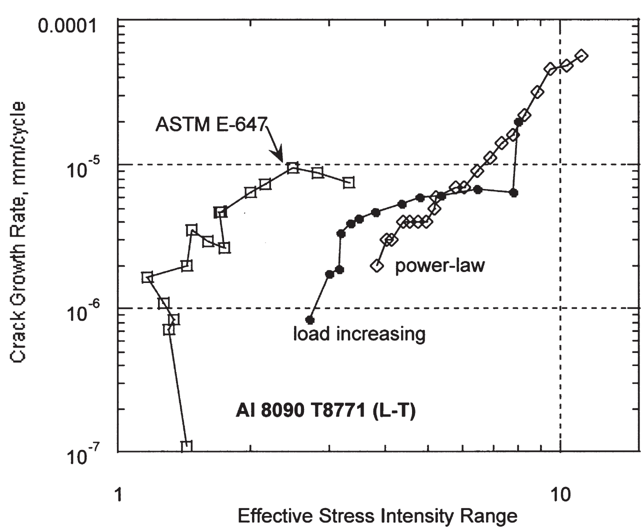

Fig. 18. Fatigue crack growth rate vs. $K_{max}-K_{op}$, for an aluminum–lithium alloy in air at $R=0.1$ [37]. Although most of the data are not taken in the near threshold region, the observations warrant a general caution. Three different load varying procedures were used to determine threshold. In the case of the load increasing procedure, K_{op}, the closure-opening stress intensity, was approximately K_{min}, the minimum stress intensity.

In view of these problems it is not surprising that one group of authors recently abandoned the usual assumptions in order to summarize their data in the form of equations. Dubey et al. [87] used a greatest-likelihood estimation routine in the software, Statistical Package for the Social Sciences, to obtain a multivariate linear regression in a log-log space. For crack growth in Ti–6Al–4V, they obtained, in lieu of the Paris relation, this expression:

$$\frac{da}{dN}=4.9\times10^{-11}\frac{(\Delta K)^{10.33}}{(K_{cl})^{4.97}}R^{0.35} \tag{6}$$

Here the closure stress intensity does not appear as part of a difference giving ΔK_{eff} but rather, the applied stress intensity range is divided by essentially the square root of the closure stress intensity. Another variant form of growth relation has been reintroduced by McEvily and Ritchie [123] and Donald and Paris [124] to take into account residual effects of K_{max} variations which are not otherwise accounted for in the effective stress intensity. The two pairs of authors used different formulas for obtaining the effective stress intensity and consequently might be expected to have residual K_{max} effects from different causes.

$$\frac{da}{dN}\propto(\Delta K_{eff}^{(1-n)}K_{max}^{n})^{m} \tag{7}$$

This form of expression does not negate the concept of a dual threshold for crack propagation but is consistent with the belief that the K_{max} contribution to threshold is merely the result of crack closure and not a driving force for crack propagation per se [123]. The quantity within the brackets is termed ΔK_{norm}. The K_{max} sensitivity exponent is n.

11. Two-parameter threshold models

Two-parameter threshold models are defined here as those which try to circumvent crack closure. Most have already been mentioned. Before Vasudevan and co-workers introduced the hypothesis of two *intrinsic* [79] thresholds, K^{*}_{max} and ΔK^{*}_{th}, Lal [49–52] had introduced models containing a similar concept, a critical R value of approximately 0.6. At R less than the critical value, the threshold behavior is controlled by the need to exceed a critical value of K_{max}. Above the critical R value, the necessary value of K_{max} is always exceeded and threshold is controlled by the need to exceed a critical stress intensity range. Lal equates the two different thresholds to two largely independent mechanisms at work. Lal's model is related to earlier models, such as the BCS model, in which the reversed shear mechanism functions primarily to pump up the defect concentration in the characteristic volume [125,126] until fracture occurs. There is also the previously mentioned blocked slip band model which in part replicates the other two-parameter models [63].

Vasudevan et al. [7] introduced a single process dislocation model in which ΔK^*_{th} is the minimum stress intensity required to generate dislocation dipoles. Vacancy-type dislocation dipoles are a constituent of persistent slip bands and have figured in many models of fatigue crack propagation because of their potential for generating a local material weakness. In the Vasudevan model, K^*_{max} is that minimum required for crack-tip bond breaking; it may be compared with Weertman's BCS model [126]. A related approach begins with a different historical branch of the Neuber's radius theory, that of Sadananda and Shahinian [48] mentioned earlier in connection with yield stress effects. In this model, crack tip stress is required to exceed that needed for dislocation emission. Many versions of this concept followed. One of the most recent is that of Wilkinson and Roberts [127,128].

The first Wilkinson and Roberts model [127] consists of a Frank–Read source on a slip plane connecting with the crack tip. K^*_{max} is the stress intensity at the crack tip required to operate the Frank–Read source. The other parameter, ΔK^*_{th}, is related to the necessary strain to transport the dislocations from the source to the crack tip. This model was later extended to include dislocation dipoles [128]. In the expanded model, the crack is presumed to emit interstitial dipoles, leaving vacancy dipoles behind. Threshold is the condition where one dipole is absorbed per cycle. There is, in the revised model, a dislocation-free zone which results from the location of the source near a bubble of maximum shear stress predicted by blunt crack theory. In this model there is no specific assignment of K^*_{max}. Rather, when K_{max} is plotted against ΔK_{th}, a squarish plot results, showing ΔK_{th} to be insensitive to K_{max} when K_{max} is high.

Crack closure is not essential to threshold or R-effects since there are several alternative explanations. While these non-closure two-parameter models do explain the R-effect, they do not as yet explain the effects of overloads or underloads very well. But, whether or not closure is actually present, a pseudo-closure stress intensity is present in all these models and the effective stress intensity for insertion into the Paris relation can still be given by:

$$\Delta K_{eff} = K_{max} - \sup(\zeta K_c, K_{min}) - \Delta K_{th,int} \quad (8)$$

where ζ is a stress-ratio dependent adjustment to K_c (note that some authors do not explicitly subtract $\Delta K_{th,int}$ but try to incorporate it into the closure component). For convenience in plotting, Eq. (9) is an analytical approximation to Eq. (8), when partial or varying closure can be expressed in terms of its parameters such as K^*_{max}. It plots as a family of rectangular hyperbolas with the threshold curve being similar to the Döker plot, Fig. 13. The constant, c, is a number which defines the sharpness of the vertex.

$$c = (\Delta K - \Delta K^*_{th} - \Delta K_{eff})(K_{max} - K^*_{max} - \Delta K_{eff}) \quad (9)$$

K^*_{max} is related to the parameters in Eq. (8) by Relation (4) taken as an equality.

The effective closure stress intensity is not usually constant. Fig. 19 plots a version of Eq. (9) representing the ΔK_{norm} portion of Eq. (7), the equivalent of ΔK_{eff} in the presence of power-law K_{max} dependence. The K_{max} dependence in this plot is seen through the slight downward sloping of the curves at high K_{max}.

Even when the closure geometry is unvarying, the ζ factor (Eq. (8)) in the case of partial crack closure can be viewed as dependent both on R and on the applied stress intensity range. It is therefore not constant nor as separable as Eq. (8) might suggest. According to both Lauat [100] and Shan [118], partial closure leads to a two-term expression for the effective closure stress intensity linear in K_{min} and K_c, the apparent closure stress intensity from a compliance-type measurement. Allowing a function in Louat's expression, designated C in Eq. (10), to take a wider range of values than strictly

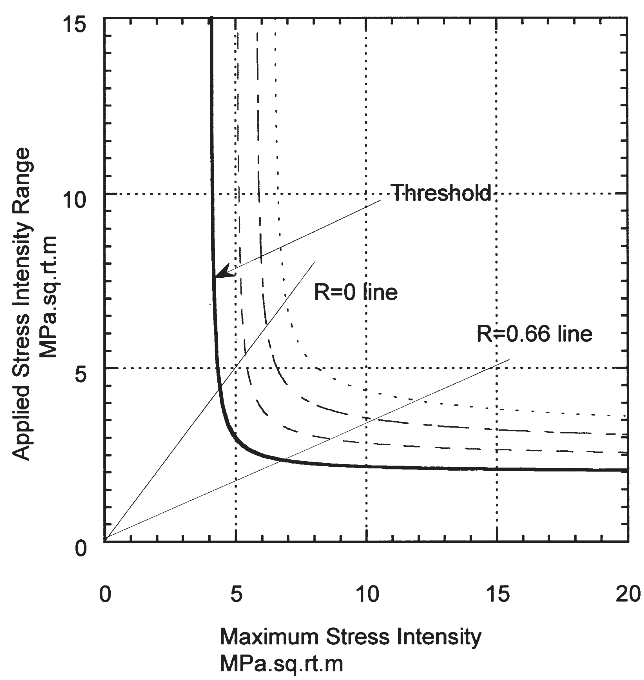

Fig. 19. A family of equal-ΔK_{norm} curves (see Eq. (7)) for an hypothetical material having an effective closure stress intensity of 2 MPa√m, a true fatigue threshold of 2 MPa√m and a K_{max} sensitivity exponent of 0.25. The constant, c, has been set to unity giving a blunt elbow at the vertex. Above the threshold, these curves would represent lines of constant crack growth rate.

possible in its original formulation, gives a simple approximation to Shan's expression (eq. (9) in Ref. 118). In terms of R,

$$\Delta K_{\mathrm{eff}}=\frac{\Delta K}{1-R}-CK_{\mathrm{c}}-(1-C)\Delta K\frac{R}{1-R} \qquad (10)$$

where C is a geometric constant between zero and 1. When C is unity, closure is total. The first term is $K_{\max}$ and $K_{\min}$ should be easily recognized in the third term. Unlike Eqs. (8) and (9), Eq. (10) applies only at low values of $K_{\max}$ where closure effects are present. Krenn and Morris discuss how through incorporating $K_{\max}$ sensitivity, Eq. (7) may compensate for residual R-ratio dependence of closure stress intensities [129].

12. Residual stress effects

Another class of mechanism which alters near-threshold behavior is residual stress. In the near-tip region, distinguishing what happens in front of the crack tip from what happens behind it may be difficult. Closure is conventionally defined as taking place behind the crack tip while residual stress affects the material in front of it. Perhaps one of the most illuminating discoveries in the area of residual stress was that of fatigue cracks initiating ahead of notches in cyclic compression. The phenomenon has been studied extensively and reviewed by Suresh [8, pp. 222–313]. The phenomenon is related to the formation of residual tensile stresses as a result of underloading in principally the first cycle. Similarly, overloads can result in zones of compression ahead of the crack tip, producing an effect similar to crack closure and blunting as well as augmenting already existing closure. Vasudevan and co-workers have emphasized these mechanisms as an alternative to closure and as an explanation for short crack behavior [11]. Because of the difficulty of measuring residual stresses over small distances, most of the published work is theoretical. One group of workers have actually suggested growing cracks as a way of measuring residual stress [130]. The literature on the role of residual stresses in near-threshold crack propagation is not extensive and very scattered.

Some attention has been directed toward understanding the short crack problem in terms of residual stress singularities. A body of theoretical mechanics literature has evolved which deals with the formation of stress singularities apart from those associated with crack tips, a few examples of which are given in the bibliography [131–134]. Weertman [133] considered the dislocation density function for a crack growing from a singular pileup of dislocations at a microstructural obstacle. Picu [134] was among those who looked at sliding of grains and saw that stress singularities sufficiently strong to nucleate and propagate cracks could form through shear displacement of sharp-cornered elastic bodies. Lawson [135] estimated the maximum stress intensity at the tip of a crack growing outward from such singularities and applied it to a stochastic model of the length distribution of microcracks in an austinitic steel. Currently, a consensus of opinion appears to favor such singularity models as useful in interpreting crack initiation behavior and the Smith radius. It is possible that such mechanisms could be related to the hysteresis occasionally thought to be observed in the thresholds of both short and long cracks when these thresholds are deliberately traversed many times [113].

13. Conclusion

After a period of quiescence, the subject of near-threshold crack growth is now in a state of flux. Near-threshold fatigue crack growth behavior remains a phenomenological subject while linear elastic fracture mechanics, which has successively weathered nearly four decades, remains its best tool. Predicting the effects of changes in material strength under cyclic loading conditions can be accomplished most of the time using various concepts such a crack closure, blocked slip bands, Smith's and Neuber's radii as guides. Since about 1980, crack closure has been in fashion as near-threshold fatigue's major mechanism. Recent reexamination of the subject shows that the original concept of crack closure needs modification to emphasize partial closure which is expected to be more common than total closure. The perceived importance of plastic closure, at least in near-threshold fatigue, has lessened. Also, theoretical modeling has shown that mechanical closure in the crack wake need not be the sole cause of closure-like behavior, since a wide variety of other processes may result in pseudo-closure stress intensity thresholds unrelated to the wake of the crack. These new models have been put forth with great vigor and a correct sense that the significance of crack closure needs reinterpretation. However, the astute reader may notice inconsistencies within these models. Apart from closure, one potential new area for the growth of LEFM is the study of stress singularities of microstructural origin not at the tips of cracks. In addition to causing fracture and initiating fatigue cracks, these singularities may also provide a short range driving force for their growth.

Note on references: In many cases where the number of possible references is very large, e.g. to investigators using electric potential to monitor crack closure, only one or two examples are listed. Whenever possible, these examples are drawn from papers already cited for some less general purpose.

Acknowledgements

This paper was partly prepared with the financial support of the US Department of Energy, under Cooperative Agreement No. DE-FC21-95MC31176, the GE Company and Northwestern University. Any opinions, conclusions or recommendations expressed herein are those of the authors and do not necessarily reflect the views of the DOE, the GE Company or Northwestern University. The authors wish thank Mary Patterson and the Cornell University Engineering Library for her valuable help.

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