12.4 Standard, RNG, and Realizable k-ϵ Models Theory

This section presents the standard, RNG, and realizable k-ϵ models. All three models have similar forms, with transport equations for k and ϵ. The major differences in the models are as follows:

- the method of calculating turbulent viscosity
- the turbulent Prandtl numbers governing the turbulent diffusion of k and ϵ
- the generation and destruction terms in the ϵ equation

The transport equations, methods of calculating turbulent viscosity, and model constants are presented separately for each model. The features that are essentially common to all models follow, including turbulent production, generation due to buoyancy, accounting for the effects of compressibility, and modeling heat and mass transfer.

12.4.1 Standard k-ϵ Model

Overview

The simplest "complete models" of turbulence are two-equation models in which the solution of two separate transport equations allows the turbulent velocity and length scales to be independently determined. The standard k-ϵ model in FLUENT falls within this class of turbulence model and has become the workhorse of practical engineering flow calculations in the time since it was proposed by Launder and Spalding [196]. Robustness, economy, and reasonable accuracy for a wide range of turbulent flows explain its popularity in industrial flow and heat transfer simulations. It is a semi-empirical model, and the derivation of the model equations relies on phenomenological considerations and empiricism.

As the strengths and weaknesses of the standard k-ϵ model have become known, improvements have been made to the model to improve its performance. Two of these variants are available in FLUENT: the RNG k-ϵ model [408] and the realizable k-ϵ model [330].

The standard k-ϵ model [196] is a semi-empirical model based on model transport equations for the turbulence kinetic energy (k) and its dissipation rate (ϵ). The model transport equation for k is derived from the exact equation, while the model transport equation for ϵ was obtained using physical reasoning and bears little resemblance to its mathematically exact counterpart.

In the derivation of the k-ϵ model, the assumption is that the flow is fully turbulent, and the effects of molecular viscosity are negligible. The standard k-ϵ model is therefore valid only for fully turbulent flows.

Transport Equations for the Standard k-ϵ Model

The turbulence kinetic energy, k, and its rate of dissipation, ϵ, are obtained from the following transport equations:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k + G_b - \rho\epsilon - Y_M + S_k \qquad (12.4\text{-}1)$$

and

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_i}(\rho \epsilon u_i) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] + C_{1\epsilon}\frac{\epsilon}{k}\left(G_k + C_{3\epsilon}G_b\right) - C_{2\epsilon}\rho\frac{\epsilon^2}{k} + S_\epsilon \qquad (12.4\text{-}2)$$

In these equations, G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients, calculated as described in Section 12.4.4: Modeling Turbulent Production in the k-ϵ Models. G_b is the generation of turbulence kinetic energy due to buoyancy, calculated as described in Section 12.4.5: Effects of Buoyancy on Turbulence in the k-ϵ Models. Y_M represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate, calculated as described in Section 12.4.6: Effects of Compressibility on Turbulence in the k-ϵ Models. $C_{1\epsilon}$, $C_{2\epsilon}$, and $C_{3\epsilon}$ are constants. σ_k and σ_ϵ are the turbulent Prandtl numbers for k and ϵ, respectively. S_k and S_ϵ are user-defined source terms.

Modeling the Turbulent Viscosity

The turbulent (or eddy) viscosity, μ_t, is computed by combining k and ϵ as follows:

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \qquad (12.4\text{-}3)$$

where C_μ is a constant.

Model Constants

The model constants $C_{1\epsilon}, C_{2\epsilon}, C_\mu, \sigma_k$, and σ_ϵ have the following default values [196]:

$$C_{1\epsilon} = 1.44, \quad C_{2\epsilon} = 1.92, \quad C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3$$

These default values have been determined from experiments with air and water for fundamental turbulent shear flows including homogeneous shear flows and decaying isotropic grid turbulence. They have been found to work fairly well for a wide range of wall-bounded and free shear flows.

Although the default values of the model constants are the standard ones most widely accepted, you can change them (if needed) in the Viscous Model panel.

12.4.2 RNG k-ϵ Model

Overview

The RNG k-ϵ model was derived using a rigorous statistical technique (called renormalization group theory). It is similar in form to the standard k-ϵ model, but includes the following refinements:

- The RNG model has an additional term in its ϵ equation that significantly improves the accuracy for rapidly strained flows.
- The effect of swirl on turbulence is included in the RNG model, enhancing accuracy for swirling flows.
- The RNG theory provides an analytical formula for turbulent Prandtl numbers, while the standard k-ϵ model uses user-specified, constant values.
- While the standard k-ϵ model is a high-Reynolds-number model, the RNG theory provides an analytically-derived differential formula for effective viscosity that accounts for low-Reynolds-number effects. Effective use of this feature does, however, depend on an appropriate treatment of the near-wall region.

These features make the RNG k-ϵ model more accurate and reliable for a wider class of flows than the standard k-ϵ model.

The RNG-based k-ϵ turbulence model is derived from the instantaneous Navier-Stokes equations, using a mathematical technique called "renormalization group" (RNG) methods. The analytical derivation results in a model with constants different from those in the standard k-ϵ model, and additional terms and functions in the transport equations for k and ϵ. A more comprehensive description of RNG theory and its application to turbulence can be found in [59].

Transport Equations for the RNG k-ϵ Model

The RNG k-ϵ model has a similar form to the standard k-ϵ model:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j}\left(\alpha_k \mu_{\rm eff} \frac{\partial k}{\partial x_j}\right) + G_k + G_b - \rho\epsilon - Y_M + S_k \tag{12.4-4}$$

and

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_i}(\rho \epsilon u_i) = \frac{\partial}{\partial x_j}\left(\alpha_\epsilon \mu_{\rm eff} \frac{\partial \epsilon}{\partial x_j}\right) + C_{1\epsilon}\frac{\epsilon}{k}\left(G_k + C_{3\epsilon} G_b\right) - C_{2\epsilon}\rho\frac{\epsilon^2}{k} - R_\epsilon + S_\epsilon \tag{12.4-5}$$

In these equations, G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients, calculated as described in Section 12.4.4: Modeling Turbulent Production in the k-ϵ Models. G_b is the generation of turbulence kinetic energy due to buoyancy, calculated as described in Section 12.4.5: Effects of Buoyancy on Turbulence in the k-ϵ Models. Y_M represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate, calculated as described in Section 12.4.6: Effects of Compressibility on Turbulence in the k-ϵ Models. The quantities α_k and α_ϵ are the inverse effective Prandtl numbers for k and ϵ, respectively. S_k and S_ϵ are user-defined source terms.

Modeling the Effective Viscosity

The scale elimination procedure in RNG theory results in a differential equation for turbulent viscosity:

$$d\left(\frac{\rho^2 k}{\sqrt{\epsilon\mu}}\right) = 1.72 \frac{\hat{\nu}}{\sqrt{\hat{\nu}^3 - 1 + C_\nu}} d\hat{\nu} \tag{12.4-6}$$

where

$$\begin{aligned} \hat{\nu} &= \mu_{\rm eff}/\mu \\ C_\nu &\approx 100 \end{aligned}$$

Equation 12.4-6 is integrated to obtain an accurate description of how the effective turbulent transport varies with the effective Reynolds number (or eddy scale), allowing the model to better handle low-Reynolds-number and near-wall flows.

In the high-Reynolds-number limit, Equation 12.4-6 gives

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \tag{12.4-7}$$

with $C_\mu = 0.0845$, derived using RNG theory. It is interesting to note that this value of C_μ is very close to the empirically-determined value of 0.09 used in the standard k-ϵ model.

In FLUENT, by default, the effective viscosity is computed using the high-Reynolds-number form in Equation 12.4-7. However, there is an option available that allows you to use the differential relation given in Equation 12.4-6 when you need to include low-Reynolds-number effects.

RNG Swirl Modification

Turbulence, in general, is affected by rotation or swirl in the mean flow. The RNG model in FLUENT provides an option to account for the effects of swirl or rotation by modifying the turbulent viscosity appropriately. The modification takes the following functional form:

$$\mu_t = \mu_{t0} \; f\left(\alpha_s, \Omega, \frac{k}{\epsilon}\right) \tag{12.4-8}$$

where μ_{t0} is the value of turbulent viscosity calculated without the swirl modification using either Equation 12.4-6 or Equation 12.4-7. Ω is a characteristic swirl number evaluated within FLUENT, and α_s is a swirl constant that assumes different values depending on whether the flow is swirl-dominated or only mildly swirling. This swirl modification always takes effect for axisymmetric, swirling flows and three-dimensional flows when the RNG model is selected. For mildly swirling flows (the default in FLUENT), α_s is set to 0.07. For strongly swirling flows, however, a higher value of α_s can be used.

Calculating the Inverse Effective Prandtl Numbers

The inverse effective Prandtl numbers, α_k and α_ϵ, are computed using the following formula derived analytically by the RNG theory:

$$\left|\frac{\alpha - 1.3929}{\alpha_0 - 1.3929}\right|^{0.6321} \left|\frac{\alpha + 2.3929}{\alpha_0 + 2.3929}\right|^{0.3679} = \frac{\mu_{\rm mol}}{\mu_{\rm eff}} \tag{12.4-9}$$

where $\alpha_0 = 1.0$. In the high-Reynolds-number limit ($\mu_{\rm mol}/\mu_{\rm eff} \ll 1$), $\alpha_k = \alpha_\epsilon \approx 1.393$.

The R_ϵ Term in the ϵ Equation

The main difference between the RNG and standard k-ϵ models lies in the additional term in the ϵ equation given by

$$R_\epsilon = \frac{C_\mu \rho \eta^3 (1 - \eta/\eta_0)}{1 + \beta\eta^3} \frac{\epsilon^2}{k} \tag{12.4-10}$$

where $\eta \equiv Sk/\epsilon$, $\eta_0 = 4.38$, $\beta = 0.012$.

The effects of this term in the RNG ϵ equation can be seen more clearly by rearranging Equation 12.4-5. Using Equation 12.4-10, the third and fourth terms on the right-hand side of Equation 12.4-5 can be merged, and the resulting ϵ equation can be rewritten as

$$\frac{\partial}{\partial t}(\rho\epsilon) + \frac{\partial}{\partial x_i}(\rho\epsilon u_i) = \frac{\partial}{\partial x_j}\left(\alpha_\epsilon \mu_{\text{eff}} \frac{\partial \epsilon}{\partial x_j}\right) + C_{1\epsilon}\frac{\epsilon}{k}\left(G_k + C_{3\epsilon}G_b\right) - C^*_{2\epsilon}\rho\frac{\epsilon^2}{k} \tag{12.4-11}$$

where $C^*_{2\epsilon}$ is given by

$$C^*_{2\epsilon} \equiv C_{2\epsilon} + \frac{C_\mu \eta^3 (1 - \eta/\eta_0)}{1 + \beta\eta^3} \tag{12.4-12}$$

In regions where $\eta < \eta_0$, the R term makes a positive contribution, and $C^*_{2\epsilon}$ becomes larger than $C_{2\epsilon}$. In the logarithmic layer, for instance, it can be shown that $\eta \approx 3.0$, giving $C^*_{2\epsilon} \approx 2.0$, which is close in magnitude to the value of $C_{2\epsilon}$ in the standard k-ϵ model (1.92). As a result, for weakly to moderately strained flows, the RNG model tends to give results largely comparable to the standard k-ϵ model.

In regions of large strain rate ($\eta > \eta_0$), however, the R term makes a negative contribution, making the value of $C^*_{2\epsilon}$ less than $C_{2\epsilon}$. In comparison with the standard k-ϵ model, the smaller destruction of ϵ augments ϵ, reducing k and, eventually, the effective viscosity. As a result, in rapidly strained flows, the RNG model yields a lower turbulent viscosity than the standard k-ϵ model.

Thus, the RNG model is more responsive to the effects of rapid strain and streamline curvature than the standard k-ϵ model, which explains the superior performance of the RNG model for certain classes of flows.

Model Constants

The model constants $C_{1\epsilon}$ and $C_{2\epsilon}$ in Equation 12.4-5 have values derived analytically by the RNG theory. These values, used by default in FLUENT, are

$$C_{1\epsilon} = 1.42, \quad C_{2\epsilon} = 1.68$$

12.4.3 Realizable k-ϵ Model

Overview

The realizable k-ϵ model [330] is a relatively recent development and differs from the standard k-ϵ model in two important ways:

- The realizable k-ϵ model contains a new formulation for the turbulent viscosity.
- A new transport equation for the dissipation rate, ϵ, has been derived from an exact equation for the transport of the mean-square vorticity fluctuation.

The term "realizable" means that the model satisfies certain mathematical constraints on the Reynolds stresses, consistent with the physics of turbulent flows. Neither the standard k-ϵ model nor the RNG k-ϵ model is realizable.

An immediate benefit of the realizable k-ϵ model is that it more accurately predicts the spreading rate of both planar and round jets. It is also likely to provide superior performance for flows involving rotation, boundary layers under strong adverse pressure gradients, separation, and recirculation.

To understand the mathematics behind the realizable k-ϵ model, consider combining the Boussinesq relationship (Equation 12.2-5) and the eddy viscosity definition (Equation 12.4-3) to obtain the following expression for the normal Reynolds stress in an incompressible strained mean flow:

$$\overline{u^2} = \frac{2}{3}k - 2\,\nu_t \frac{\partial U}{\partial x} \tag{12.4-13}$$

Using Equation 12.4-3 for $\nu_t \equiv \mu_t/\rho$, one obtains the result that the normal stress, $\overline{u^2}$, which by definition is a positive quantity, becomes negative, i.e., "non-realizable", when the strain is large enough to satisfy

$$\frac{k}{\epsilon}\frac{\partial U}{\partial x} > \frac{1}{3C_\mu} \approx 3.7 \tag{12.4-14}$$

Similarly, it can also be shown that the Schwarz inequality for shear stresses ($\overline{u_\alpha u_\beta}^2 \leq \overline{u_\alpha^2 u_\beta^2}$; no summation over α and β) can be violated when the mean strain rate is large. The most straightforward way to ensure the realizability (positivity of normal stresses and Schwarz inequality for shear stresses) is to make C_μ variable by sensitizing it to the mean flow (mean deformation) and the turbulence (k, ϵ). The notion of variable C_μ is suggested by many modelers including Reynolds [303], and is well substantiated by experimental evidence. For example, C_μ is found to be around 0.09 in the inertial sublayer of equilibrium boundary layers, and 0.05 in a strong homogeneous shear flow.

Both the realizable and RNG k-ϵ models have shown substantial improvements over the standard k-ϵ model where the flow features include strong streamline curvature, vortices, and rotation. Since the model is still relatively new, it is not clear in exactly which instances the realizable k-ϵ model consistently outperforms the RNG model. However, initial studies have shown that the realizable model provides the best performance of all the k-ϵ model versions for several validations of separated flows and flows with complex secondary flow features.

One of the weaknesses of the standard k-ϵ model or other traditional k-ϵ models lies with the modeled equation for the dissipation rate (ϵ). The well-known round-jet anomaly (named based on the finding that the spreading rate in planar jets is predicted reasonably well, but prediction of the spreading rate for axisymmetric jets is unexpectedly poor) is considered to be mainly due to the modeled dissipation equation.

The realizable k-ϵ model proposed by Shih et al. [330] was intended to address these deficiencies of traditional k-ϵ models by adopting the following:

- A new eddy-viscosity formula involving a variable C_μ originally proposed by Reynolds [303].
- A new model equation for dissipation (ϵ) based on the dynamic equation of the mean-square vorticity fluctuation.

One limitation of the realizable k-ϵ model is that it produces non-physical turbulent viscosities in situations when the computational domain contains both rotating and stationary fluid zones (e.g., multiple reference frames, rotating sliding meshes). This is due to the fact that the realizable k-ϵ model includes the effects of mean rotation in the definition of the turbulent viscosity (see Equations 12.4-17–12.4-19). This extra rotation effect has been tested on single rotating reference frame systems and showed superior behavior over the standard k-ϵ model. However, due to the nature of this modification, its application to multiple reference frame systems should be taken with some caution. See Section 12.4.3: Modeling the Turbulent Viscosity for information about how to include or exclude this term from the model.

Transport Equations for the Realizable k-ϵ Model

The modeled transport equations for k and ϵ in the realizable k-ϵ model are

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho k u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + G_k + G_b - \rho\epsilon - Y_M + S_k \tag{12.4-15}$$

and

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_j}(\rho \epsilon u_j) = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] + \rho\, C_1 S\epsilon - \rho\, C_2 \frac{\epsilon^2}{k + \sqrt{\nu\epsilon}} + C_{1\epsilon}\frac{\epsilon}{k} C_{3\epsilon} G_b + S_\epsilon \tag{12.4-16}$$

where

$$C_1 = \max\left[0.43, \frac{\eta}{\eta + 5}\right], \qquad \eta = S\frac{k}{\epsilon}, \qquad S = \sqrt{2 S_{ij} S_{ij}}$$

In these equations, G_k represents the generation of turbulence kinetic energy due to the mean velocity gradients, calculated as described in Section 12.4.4: Modeling Turbulent Production in the k-ϵ Models. G_b is the generation of turbulence kinetic energy due to buoyancy, calculated as described in Section 12.4.5: Effects of Buoyancy on Turbulence in the k-ϵ Models. Y_M represents the contribution of the fluctuating dilatation in compressible turbulence to the overall dissipation rate, calculated as described in Section 12.4.6: Effects of Compressibility on Turbulence in the k-ϵ Models. C_2 and $C_{1\epsilon}$ are constants. σ_k and σ_ϵ are the turbulent Prandtl numbers for k and ϵ, respectively. S_k and S_ϵ are user-defined source terms.

Note that the k equation (Equation 12.4-15) is the same as that in the standard k-ϵ model (Equation 12.4-1) and the RNG k-ϵ model (Equation 12.4-4), except for the model constants. However, the form of the ϵ equation is quite different from those in the standard and RNG-based k-ϵ models (Equations 12.4-2 and 12.4-5). One of the noteworthy features is that the production term in the ϵ equation (the second term on the right-hand side of Equation 12.4-16) does not involve the production of k; i.e., it does not contain the same G_k term as the other k-ϵ models. It is believed that the present form better represents the spectral energy transfer. Another desirable feature is that the destruction term (the next to last term on the right-hand side of Equation 12.4-16) does not have any singularity; i.e., its denominator never vanishes, even if k vanishes or becomes smaller than zero. This feature is contrasted with traditional k-ϵ models, which have a singularity due to k in the denominator.

This model has been extensively validated for a wide range of flows [183, 330], including rotating homogeneous shear flows, free flows including jets and mixing layers, channel and boundary layer flows, and separated flows. For all these cases, the performance of the model has been found to be substantially better than that of the standard k-ϵ model. Especially noteworthy is the fact that the realizable k-ϵ model resolves the round-jet anomaly; i.e., it predicts the spreading rate for axisymmetric jets as well as that for planar jets.

Modeling the Turbulent Viscosity

As in other k-ϵ models, the eddy viscosity is computed from

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \tag{12.4-17}$$

The difference between the realizable k-ϵ model and the standard and RNG k-ϵ models is that C_μ is no longer constant. It is computed from

$$C_\mu = \frac{1}{A_0 + A_s \frac{kU^*}{\epsilon}} \tag{12.4-18}$$

where

$$U^* \equiv \sqrt{S_{ij}S_{ij} + \tilde{\Omega}_{ij}\tilde{\Omega}_{ij}} \tag{12.4-19}$$

and

$$\begin{aligned} \tilde{\Omega}_{ij} &= \Omega_{ij} - 2\epsilon_{ijk}\omega_k \\ \Omega_{ij} &= \overline{\Omega_{ij}} - \epsilon_{ijk}\omega_k \end{aligned}$$

where $\overline{\Omega_{ij}}$ is the mean rate-of-rotation tensor viewed in a rotating reference frame with the angular velocity ω_k. The model constants A_0 and A_s are given by

$$A_0 = 4.04, \quad A_s = \sqrt{6}\cos\phi$$

where

$$\phi = \frac{1}{3}\cos^{-1}(\sqrt{6}W), \quad W = \frac{S_{ij}S_{jk}S_{ki}}{\tilde{S}^3}, \quad \tilde{S} = \sqrt{S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right)$$

It can be seen that C_μ is a function of the mean strain and rotation rates, the angular velocity of the system rotation, and the turbulence fields (k and ϵ). C_μ in Equation 12.4-17 can be shown to recover the standard value of 0.09 for an inertial sublayer in an equilibrium boundary layer.

i In FLUENT, the term $-2\epsilon_{ijk}\omega_k$ is, by default, not included in the calculation of $\tilde{\Omega}_{ij}$. This is an extra rotation term that is not compatible with cases involving sliding meshes or multiple reference frames. If you want to include this term in the model, you can enable it by using the `define/models/viscous/turbulence-expert/rke-cmu-rotation-term?` text command and entering `yes` at the prompt.

Model Constants

The model constants C_2, σ_k, and σ_ϵ have been established to ensure that the model performs well for certain canonical flows. The model constants are

$$C_{1\epsilon} = 1.44, \quad C_2 = 1.9, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.2$$

12.4.4 Modeling Turbulent Production in the k-ϵ Models

The term G_k, representing the production of turbulence kinetic energy, is modeled identically for the standard, RNG, and realizable k-ϵ models. From the exact equation for the transport of k, this term may be defined as

$$G_k = -\rho \overline{u_i' u_j'} \frac{\partial u_j}{\partial x_i} \tag{12.4-20}$$

To evaluate G_k in a manner consistent with the Boussinesq hypothesis,

$$G_k = \mu_t S^2 \tag{12.4-21}$$

where S is the modulus of the mean rate-of-strain tensor, defined as

$$S \equiv \sqrt{2 S_{ij} S_{ij}} \tag{12.4-22}$$

i When using the high-Reynolds number k-ϵ versions, μ_{eff} is used in lieu of μ_t in Equation 12.4-21.

12.4.5 Effects of Buoyancy on Turbulence in the k-ϵ Models

When a non-zero gravity field and temperature gradient are present simultaneously, the k-ϵ models in FLUENT account for the generation of k due to buoyancy (G_b in Equations 12.4-1, 12.4-4, and 12.4-15), and the corresponding contribution to the production of ϵ in Equations 12.4-2, 12.4-5, and 12.4-16.

The generation of turbulence due to buoyancy is given by

$$G_b = \beta g_i \frac{\mu_t}{\mathrm{Pr}_t} \frac{\partial T}{\partial x_i} \tag{12.4-23}$$

where Pr_t is the turbulent Prandtl number for energy and g_i is the component of the gravitational vector in the ith direction. For the standard and realizable k-ϵ models, the default value of Pr_t is 0.85. In the case of the RNG k-ϵ model, $\mathrm{Pr}_t = 1/\alpha$, where α is given by Equation 12.4-9, but with $\alpha_0 = 1/\mathrm{Pr} = k/\mu c_p$. The coefficient of thermal expansion, β, is defined as

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p \tag{12.4-24}$$

For ideal gases, Equation 12.4-23 reduces to

$$G_b = -g_i \frac{\mu_t}{\rho \mathrm{Pr}_t} \frac{\partial \rho}{\partial x_i} \tag{12.4-25}$$

It can be seen from the transport equations for k (Equations 12.4-1, 12.4-4, and 12.4-15) that turbulence kinetic energy tends to be augmented ($G_b > 0$) in unstable stratification. For stable stratification, buoyancy tends to suppress the turbulence ($G_b < 0$). In FLUENT, the effects of buoyancy on the generation of k are always included when you have both a non-zero gravity field and a non-zero temperature (or density) gradient.

While the buoyancy effects on the generation of k are relatively well understood, the effect on ϵ is less clear. In FLUENT, by default, the buoyancy effects on ϵ are neglected simply by setting G_b to zero in the transport equation for ϵ (Equation 12.4-2, 12.4-5, or 12.4-16).

However, you can include the buoyancy effects on ϵ in the Viscous Model panel. In this case, the value of G_b given by Equation 12.4-25 is used in the transport equation for ϵ (Equation 12.4-2, 12.4-5, or 12.4-16).

The degree to which ϵ is affected by the buoyancy is determined by the constant $C_{3\epsilon}$. In FLUENT, $C_{3\epsilon}$ is not specified, but is instead calculated according to the following relation [140]:

$$C_{3\epsilon} = \tanh \left| \frac{v}{u} \right| \qquad (12.4\text{-}26)$$

where v is the component of the flow velocity parallel to the gravitational vector and u is the component of the flow velocity perpendicular to the gravitational vector. In this way, $C_{3\epsilon}$ will become 1 for buoyant shear layers for which the main flow direction is aligned with the direction of gravity. For buoyant shear layers that are perpendicular to the gravitational vector, $C_{3\epsilon}$ will become zero.

12.4.6 Effects of Compressibility on Turbulence in the k-ϵ Models

For high-Mach-number flows, compressibility affects turbulence through so-called "dilatation dissipation", which is normally neglected in the modeling of incompressible flows [403]. Neglecting the dilatation dissipation fails to predict the observed decrease in spreading rate with increasing Mach number for compressible mixing and other free shear layers. To account for these effects in the k-ϵ models in FLUENT, the dilatation dissipation term, Y_M, is included in the k equation. This term is modeled according to a proposal by Sarkar [315]:

$$Y_M = 2\rho\epsilon \mathrm{M}_t^2 \qquad (12.4\text{-}27)$$

where M_t is the turbulent Mach number, defined as

$$\mathrm{M}_t = \sqrt{\frac{k}{a^2}} \qquad (12.4\text{-}28)$$

where a ($\equiv \sqrt{\gamma RT}$) is the speed of sound.

This compressibility modification always takes effect when the compressible form of the ideal gas law is used.

12.4.7 Convective Heat and Mass Transfer Modeling in the k-ϵ Models

In FLUENT, turbulent heat transport is modeled using the concept of Reynolds' analogy to turbulent momentum transfer. The "modeled" energy equation is thus given by the following:

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_i}[u_i(\rho E + p)] = \frac{\partial}{\partial x_j}\left(k_{\text{eff}}\frac{\partial T}{\partial x_j} + u_i(\tau_{ij})_{\text{eff}}\right) + S_h \tag{12.4-29}$$

where E is the total energy, k_{eff} is the effective thermal conductivity, and

$(\tau_{ij})_{\text{eff}}$ is the deviatoric stress tensor, defined as

$$(\tau_{ij})_{\text{eff}} = \mu_{\text{eff}}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) - \frac{2}{3}\mu_{\text{eff}}\frac{\partial u_k}{\partial x_k}\delta_{ij}$$

The term involving $(\tau_{ij})_{\text{eff}}$ represents the viscous heating, and is always computed in the density-based solvers. It is not computed by default in the pressure-based solver, but it can be enabled in the Viscous Model panel.

Additional terms may appear in the energy equation, depending on the physical models you are using. See Section 13.2.1: Heat Transfer Theory for more details.

For the standard and realizable k-ϵ models, the effective thermal conductivity is given by

$$k_{\text{eff}} = k + \frac{c_p \mu_t}{\text{Pr}_t}$$

where k, in this case, is the thermal conductivity. The default value of the turbulent Prandtl number is 0.85. You can change the value of the turbulent Prandtl number in the Viscous Model panel.

For the RNG k-ϵ model, the effective thermal conductivity is

$$k_{\text{eff}} = \alpha c_p \mu_{\text{eff}}$$

where α is calculated from Equation 12.4-9, but with $\alpha_0 = 1/\text{Pr} = k/\mu c_p$.

The fact that α varies with $\mu_{\text{mol}}/\mu_{\text{eff}}$, as in Equation 12.4-9, is an advantage of the RNG k-ϵ model. It is consistent with experimental evidence indicating that the turbulent Prandtl number varies with the molecular Prandtl number and turbulence [175]. Equation 12.4-9 works well across a very broad range of molecular Prandtl numbers, from liquid metals ($\text{Pr} \approx 10^{-2}$) to paraffin oils ($\text{Pr} \approx 10^3$), which allows heat transfer to be calculated in low-Reynolds-number regions. Equation 12.4-9 smoothly predicts the variation of effective

Prandtl number from the molecular value ($\alpha = 1/\mathrm{Pr}$) in the viscosity-dominated region to the fully turbulent value ($\alpha = 1.393$) in the fully turbulent regions of the flow.

Turbulent mass transfer is treated similarly. For the standard and realizable k-ϵ models, the default turbulent Schmidt number is 0.7. This default value can be changed in the Viscous Model panel. For the RNG model, the effective turbulent diffusivity for mass transfer is calculated in a manner that is analogous to the method used for the heat transport. The value of α_0 in Equation 12.4-9 is $\alpha_0 = 1/\mathrm{Sc}$, where Sc is the molecular Schmidt number.

12.5 Standard and SST k-ω Models Theory

This section presents the standard [403] and shear-stress transport (SST) [237] k-ω models. Both models have similar forms, with transport equations for k and ω. The major ways in which the SST model [238] differs from the standard model are as follows:

- gradual change from the standard k-ω model in the inner region of the boundary layer to a high-Reynolds-number version of the k-ϵ model in the outer part of the boundary layer
- modified turbulent viscosity formulation to account for the transport effects of the principal turbulent shear stress

The transport equations, methods of calculating turbulent viscosity, and methods of calculating model constants and other terms are presented separately for each model.

12.5.1 Standard k-ω Model

Overview

The standard k-ω model in FLUENT is based on the Wilcox k-ω model [403], which incorporates modifications for low-Reynolds-number effects, compressibility, and shear flow spreading. The Wilcox model predicts free shear flow spreading rates that are in close agreement with measurements for far wakes, mixing layers, and plane, round, and radial jets, and is thus applicable to wall-bounded flows and free shear flows. A variation of the standard k-ω model called the SST k-ω model is also available in FLUENT, and is described in Section 12.5.2: Shear-Stress Transport (SST) k-ω Model.

The standard k-ω model is an empirical model based on model transport equations for the turbulence kinetic energy (k) and the specific dissipation rate (ω), which can also be thought of as the ratio of ϵ to k [403].

As the k-ω model has been modified over the years, production terms have been added to both the k and ω equations, which have improved the accuracy of the model for predicting free shear flows.