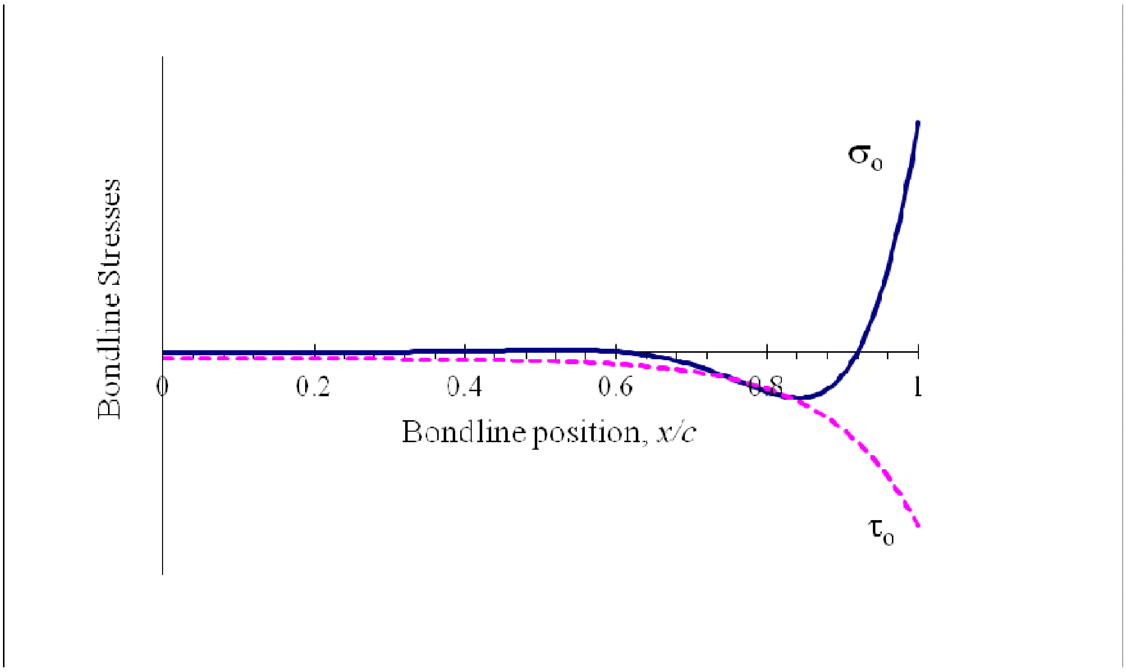
ME 553 Adhesion Mechanics - Homework Assignment #6 - Due Monday May 21

Develop a computer program to calculate and plot the stress distribution in a user-defined single-lap joint, based on the Goland and Reissner solution. That is, set up a procedure using any appropriate computer "language" (EXCEL, MATLAB, MAPLE, MATHEMATICA, FORTRAN, etc) to obtain the peel and shear stresses within a single lap joint. The steps in calculating this solution are summarized on the next page.

The validity of your program will be judged based on a plot of predicted peel and shear stress distributions for a single lap joint consisting of aluminum adherends (E = 10 Msi, ν = 0.33) and a rubber-modified epoxy adhesive (E = 0.40 Msi, ν = 0.45), using the following dimensions and loading.

c = 0.60 in $\quad$ t = 0.120 in $\quad$ T = 0.025 in $\quad$ F = 1000 lbf/in $\quad$ l = 8 in

The shape of your plots should *resemble* the plot shown below (i.e., your plots should similar to Figures 3.14 and 3.15 of the Pocius summary of the Goland and Reissner solution). Since your grade for this problem will be based on these plots, *be sure to plot enough points (at least 100) so that the shape of the stress distributions are clearly identified*.



Goland and Reissner Solution for A Single Lap Joint : Summary of Calculation Steps

1) Define problem. That is, specify:

(a) F = applied load/unit width
(b) l = adherend length from load point to bonded region
(c) t = adherend thickness
(d) c = (bonded length)/2
(e) T = adhesive bondline thickness
(f) E, ν = elastic properties of the adherends
(g) E_A, ν_A = elastic properties of the adhesive

2) Perform initial calculations:

(a) $R_1 = \dfrac{Et^3}{12(1-\nu^2)}$ $\qquad R_2 = \dfrac{E(2t)^3}{12(1-\nu^2)} = 8R_1$

(b) $\xi_1 = \sqrt{\dfrac{F}{R_1}}$ $\qquad \xi_2 = \sqrt{\dfrac{F}{R_2}}$

(c) Calculate: $\Lambda = \xi_2 c$

(d) Calculate: $k = \dfrac{\cosh(\Lambda)}{\cosh(\Lambda) + 2\sqrt{2}\sinh(\Lambda)}$

(e) Calculate: $k' = ck\sqrt{\dfrac{3(1-\nu^2)F}{Et^3}}$

(f) Calculate: $\lambda = \dfrac{c}{t}\left(6\dfrac{tE_A}{ET}\right)^{1/4}$

(g) Calculate: $\Delta = \dfrac{1}{2}\left[\sinh(2\lambda) + \sin(2\lambda)\right]$

(h) Calculate: $K_1 = \cosh(\lambda)\sin(\lambda) + \sinh(\lambda)\cos(\lambda)$

(i) Calculate: $K_2 = \sinh(\lambda)\cos(\lambda) - \cosh(\lambda)\sin(\lambda)$

(j) Calculate: $\beta = \sqrt{\dfrac{8tG_A}{ET}}$

3) Calculate and plot normal "peel stress" (σ_o) and shear stress (τ_o) at "many" (at least 100) positions along the length of the bondline, from $0 \le x \le c$ (or, equivalently, from $0 \le x/c \le 1$):

$$\sigma_o = \frac{Ft}{c^2\Delta}\left[\left\{K_2\lambda^2\left(\frac{k}{2}\right) + \lambda k'\cosh(\lambda)\cos(\lambda)\right\}\cosh(\lambda x/c)\cos(\lambda x/c) \;+\; \left\{K_1\lambda^2\left(\frac{k}{2}\right) + \lambda k'\sinh(\lambda)\sin(\lambda)\right\}\sinh(\lambda x/c)\sin(\lambda x/c)\right]$$

$$\tau_o = \frac{-F}{8c}\left\{\frac{\beta c}{t}(1+3k)\left(\frac{\cosh\left(\frac{\beta x}{t}\right)}{\sinh\left(\frac{\beta c}{t}\right)}\right) + 3(1-k)\right\}$$