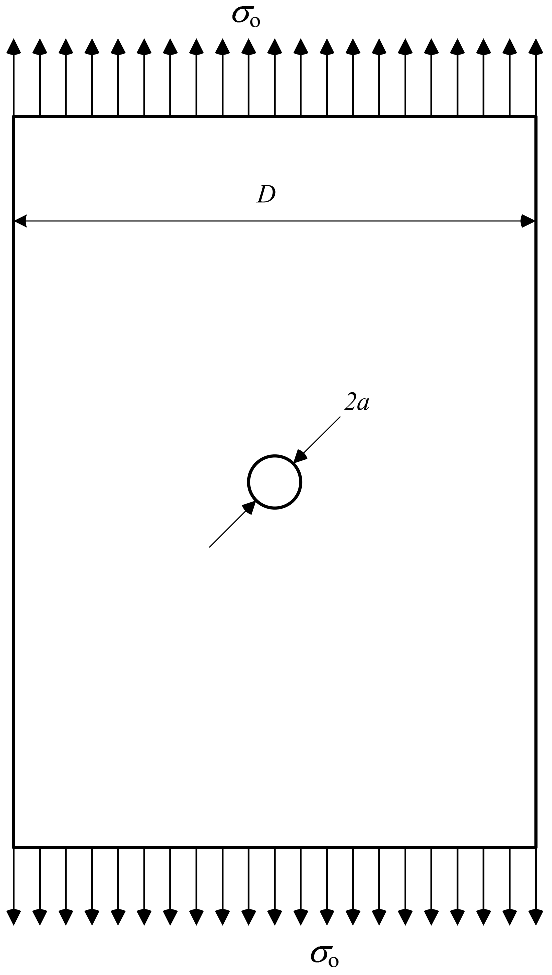
Stress Concentrations

- A “stress concentration” refers to an area in a object where stress increases over a very short distance (i.e., where a high stress gradient exists)
- Stress concentrations typically occur due to some localized change in geometry (near holes, filets, corners, grooves, cracks, etc)
- These changes in geometry are often called “stress risers”

Stress Concentration Near a Circular Hole

- In 1898 Ernst Kirsch (a German engineer) published a solution for the elastic stresses near a circular hole in an isotropic “infinitely large” thin plate (the Kirsch solution is derived in Sec 3.13 of the Shukla and Dally textbook)
- In practice, a thin plate can be considered to be “infinitely large” if the hole diameter is small compared to the in-plane plate dimensions (if $a/D < \sim 0.005$, say)



Stress Concentration Near a Circular Hole

- Stresses along the x-axis in an infinite plate predicted by the Kirsch solution:

$$\sigma_{rr} = \sigma_{xx} = \frac{\sigma_o}{2}\left(1 - \frac{a^2}{x^2}\right)\frac{3a^3}{x^2}$$

$$\sigma_{\theta\theta} = \sigma_{yy} = \frac{\sigma_o}{2}\left(2 + \frac{a^2}{x^2} + \frac{3a^4}{x^4}\right)$$

$$\tau_{r\theta} = \tau_{xy} = 0$$

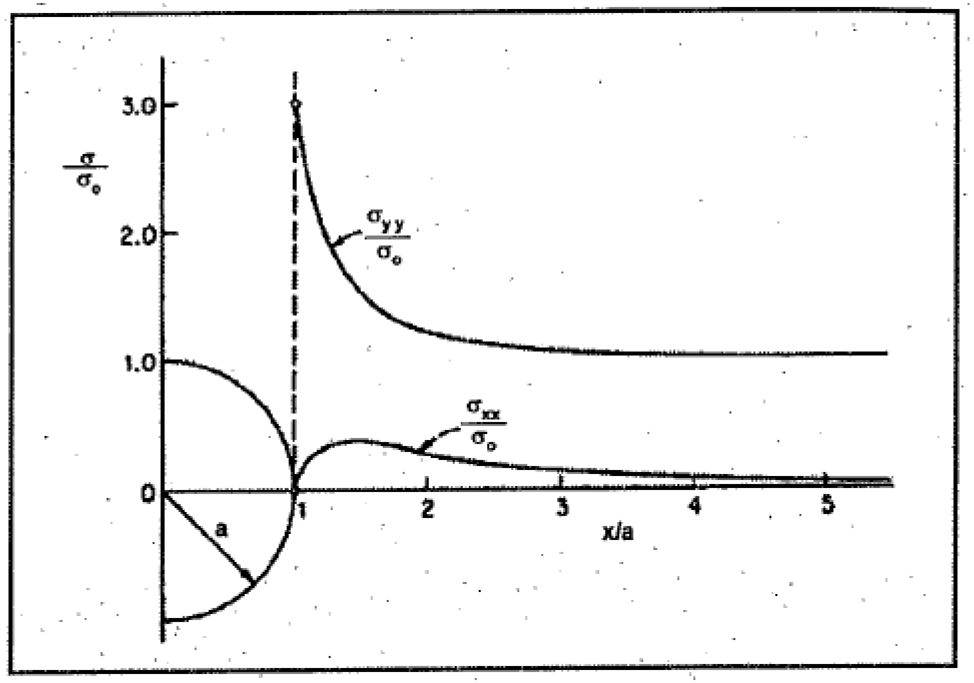


Figure 3.6: Distribution of σ_{xx}/σ_o and σ_{yy}/σ_o along the x-axis

Stress Concentration Near a Circular Hole

- Stresses at the edge of the hole (at *x* = *a*):

$$\sigma_{rr} = \sigma_{xx} = 0$$

$$\sigma_{\theta\theta} = \sigma_{yy} = 3\sigma_o$$

$$\tau_{r\theta} = \tau_{xy} = 0$$

- The stress concentration factor for a circular hole in an infinite plate:

$$K_t = \frac{\sigma_{yy}}{\sigma_o} = 3$$

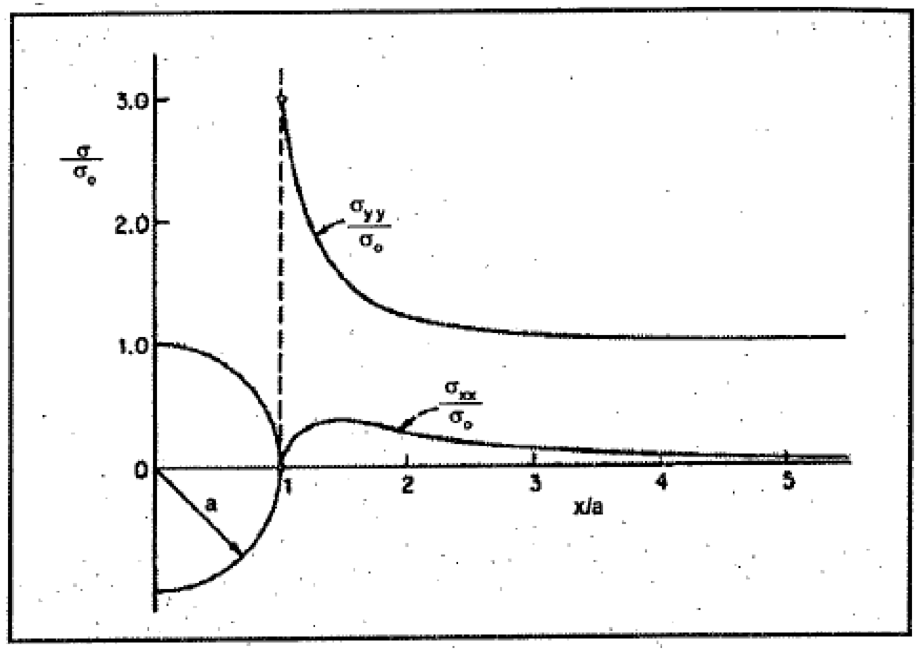
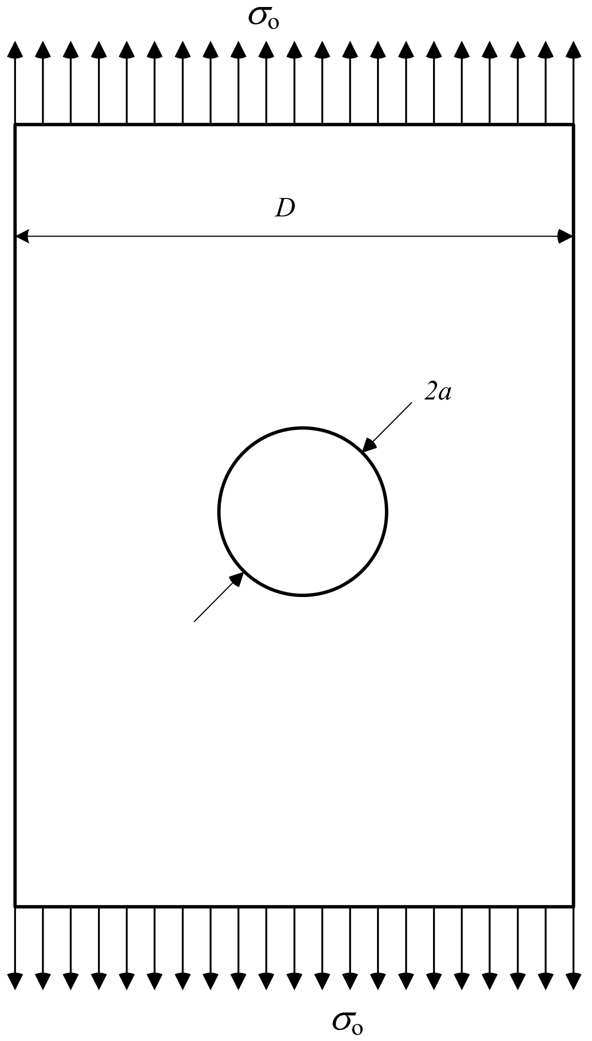


Figure 3.6: Distribution of σ_{xx}/σ_o and σ_{yy}/σ_o along the x-axis

Stress Concentration Near a Circular Hole

- If $a/D >$ 0.005 then the plate is “finite” and the Kirsch solution is no longer valid
- Stress concentration factors for a circular holes in finite plates have been measured experimentally for a range of a/D ratios (usually using photoelasticity), and tabulated in the form of curve-fits in reference handbooks …required several years and many contributors



Stress Concentration Near a Circular Hole

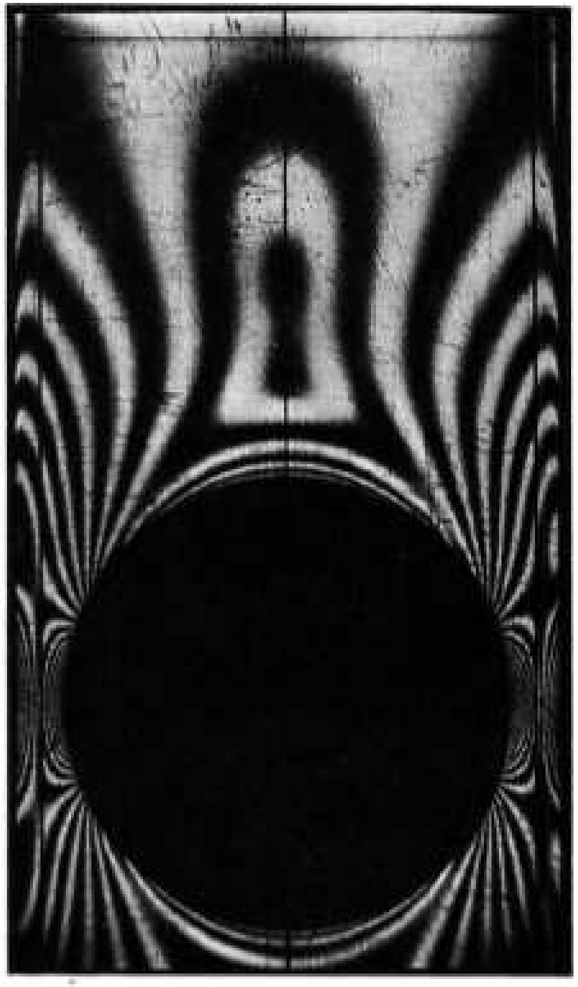


Fig. 8 Bar With Large Hole

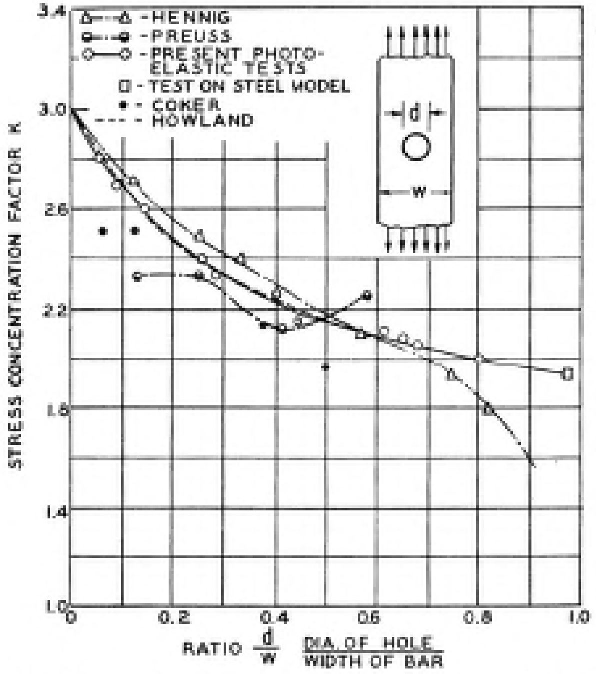


Fig. 10 Stress Concentration Factor Vs. d/w for Flat Bar With Hole

- Example: Wahl, A.M., and Beeuwkes, R., “Stress Concentration Produced by Holes and Notches”, Transaction of the ASME; Applied Mechanics, Vol 56 (11) , 1930

Stress Concentration Near a Circular Hole

- Two different definitions of the stress concentration factors are in common use:

  -based on the gross stress :

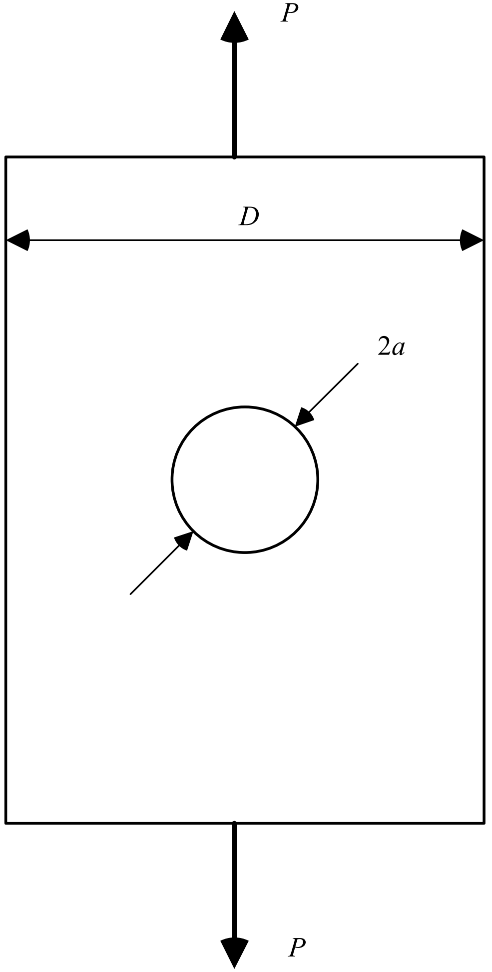
$$K_t^g = \frac{\sigma_{yy}^{\max}}{\sigma_g} \quad \text{where} \quad \sigma_g = \frac{P}{t * D}$$

(σ_g remains constant as *a* increases)

  -based on the net stress:

$$K_t^n = \frac{\sigma_{yy}^{\max}}{\sigma_n} \quad \text{where} \quad \sigma_n = \frac{P}{t * (D - 2a)}$$

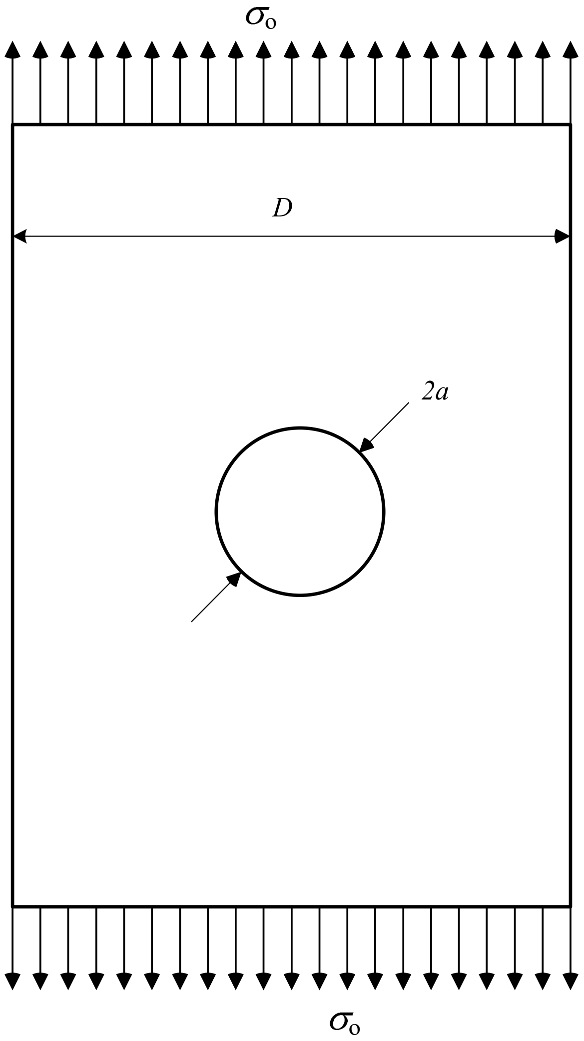
(σ_n increases as *a* increases)



Stress Concentration Near a Circular Hole

- Example: from Roark's Formulas for Stress and Strain (2002):

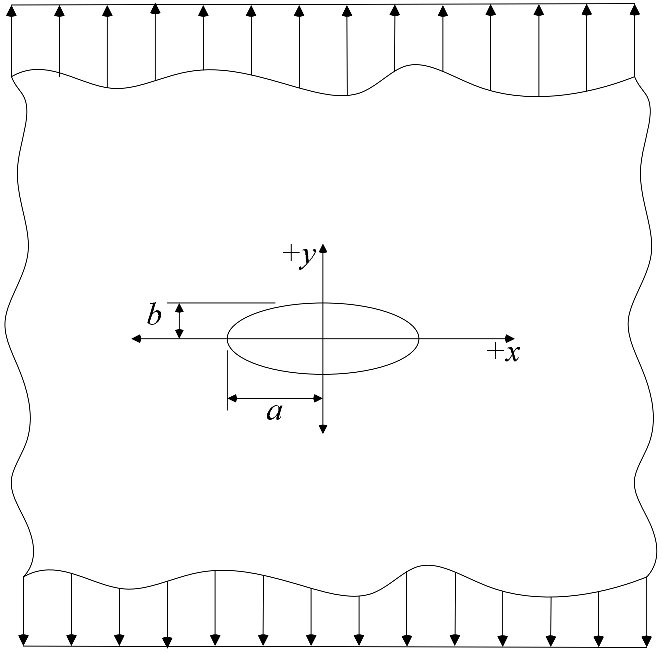
$$K_t^n = 3.00 - 3.14\left(\frac{2a}{D}\right) + 3.67\left(\frac{2a}{D}\right)^2 - 1.53\left(\frac{2a}{D}\right)^3$$



Stress Concentration Near an Elliptical Hole

- In 1913 Charles Inglis (a British mathematician) published a solution for the elastic stresses near an elliptical hole in an isotropic infinitely large thin plate (the Inglis solution is discussed in Sec 4.2)

- In this case the stress concentration depends on both the aspect ratio of the hole (*a*/*b*) and on the size of the plate



Stress Concentration Near an Elliptical Hole

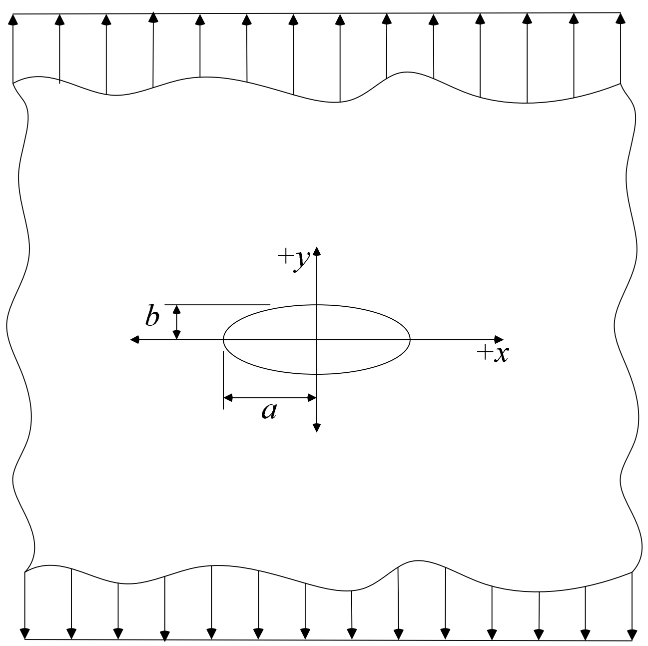
- For an infinite plate the stresses along the *x*-axis (i.e., for $x \geq a$, $y = 0$) are given by:

$$\sigma_{xx}(x) = F_{1(s)} - F_{2(s)}$$

$$\sigma_{yy}(x) = F_{1(s)} + F_{2(s)}$$

$$\tau_{xy}(x) = 0$$

where:

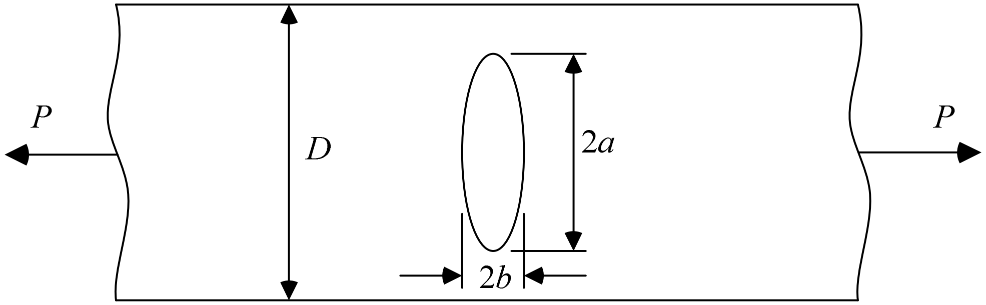


σ = uniaxial stress applied to infinitely large plate

$$F_{1(s)} = \frac{\sigma}{2}\left[1 + \frac{2(1+m)}{s^2 - m}\right] \qquad F_{2(s)} = \frac{\sigma}{2}\left\{1 + \frac{m^2 - 1}{s^2 - m}\left[1 + \left(\frac{m-1}{s^2 - m}\right)\left(\frac{3s^2 - m}{s^2 - m}\right)\right]\right\}$$

$$s = \frac{x}{2B} + \sqrt{\left(\frac{x}{2B}\right)^2 - m} \qquad B = \frac{1}{2}(a+b) \qquad m = \frac{a-b}{a+b}$$

Stress Concentration Near an Elliptical Hole



- For a finite plate:

$$K_t^n = C_1 + C_2\left(\frac{2a}{D}\right) + C_3\left(\frac{2a}{D}\right)^2 + C_4\left(\frac{2a}{D}\right)^3$$

$$C_1 = 1.000 + 0.000\sqrt{a/b} + 2.000a/b$$

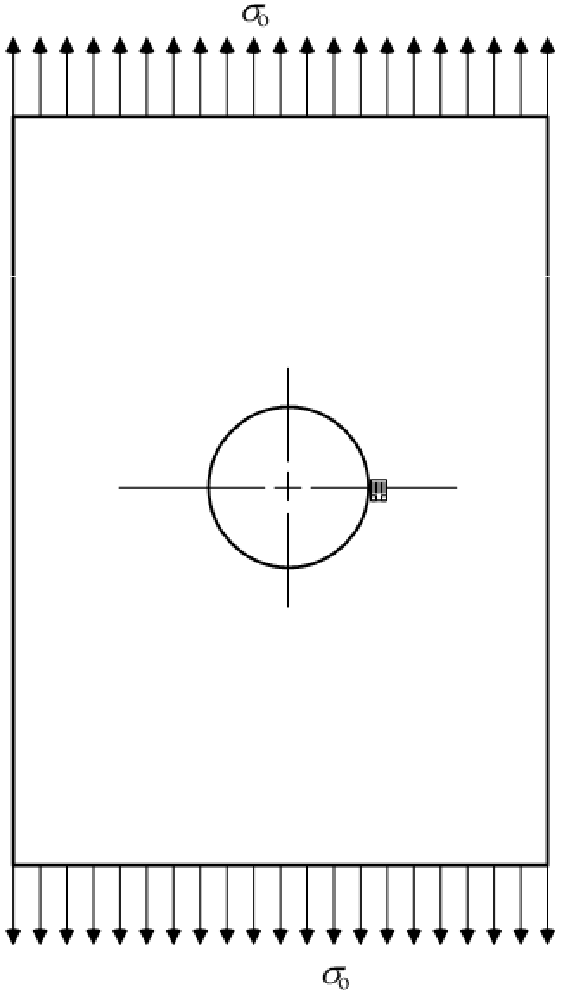
$$C_2 = -0.351 - 0.021\sqrt{a/b} - 2.483a/b$$

$$C_3 = 3.621 - 5.183\sqrt{a/b} + 4.494a/b$$

$$C_4 = -2.270 + 5.204\sqrt{a/b} - 4.011a/b$$

Measuring Stress Concentrations Using Strain Gages

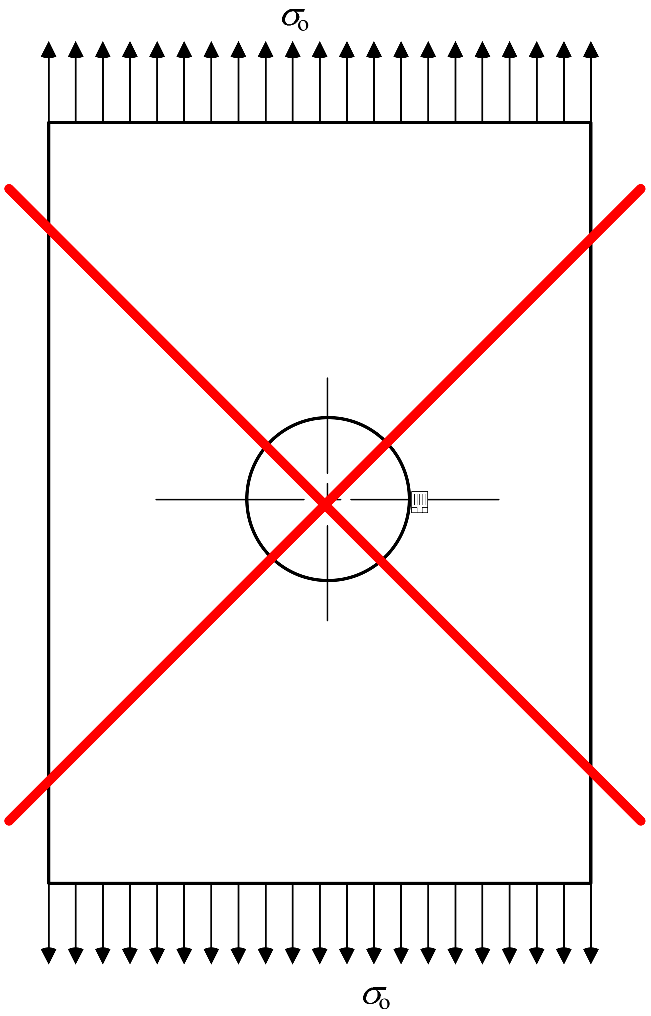
- In general, even the smallest of commercial resistance strain gages are too large to measure strain concentrations near stress risers:



Measuring Stress Concentrations Using Strain Gages

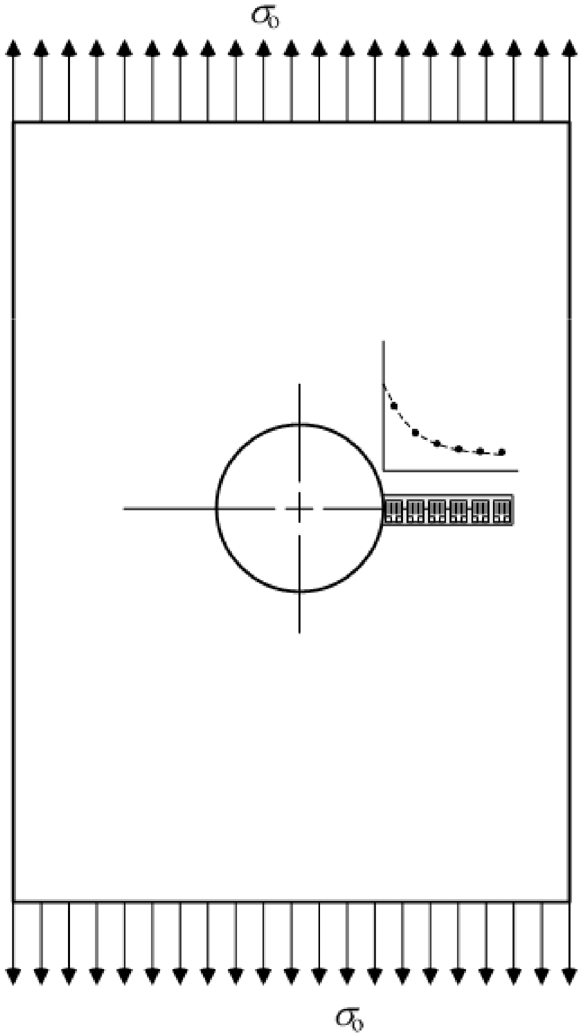
- In general, even the smallest of commercial resistance strain gages are too large to measure strain concentrations near stress risers:

  …a poor experimental approach

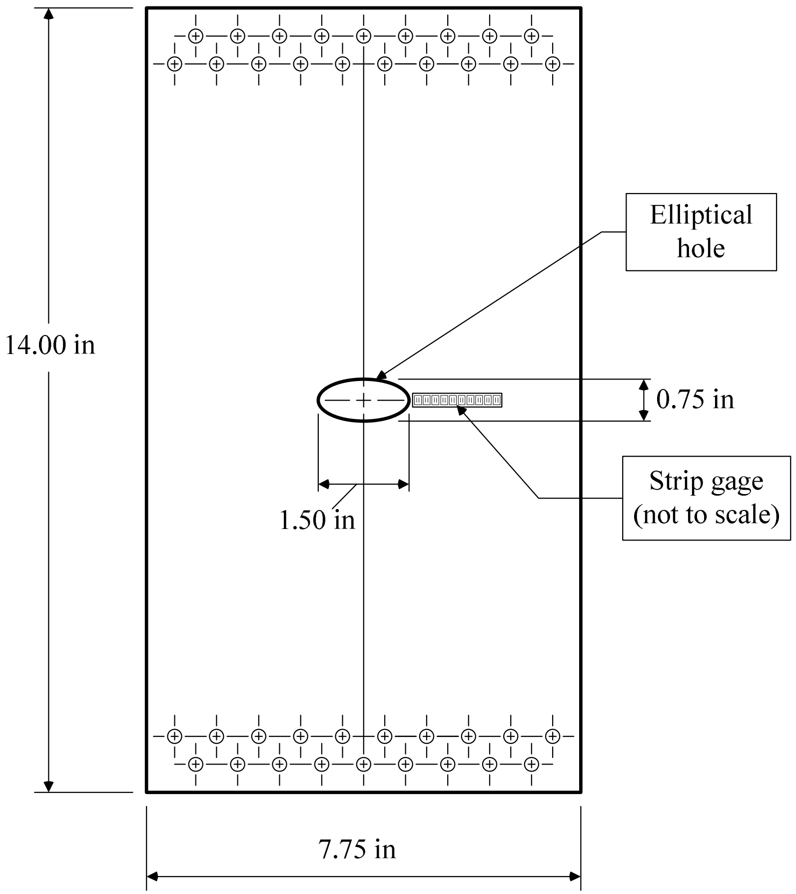


Measuring Stress Concentrations Using Strain Gages

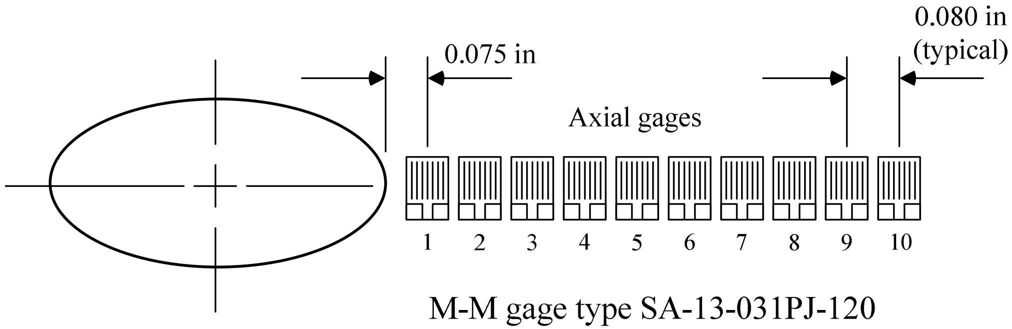
- Instead, use a commercial "strip gage" and extrapolate experimental measurements to edge of stress riser



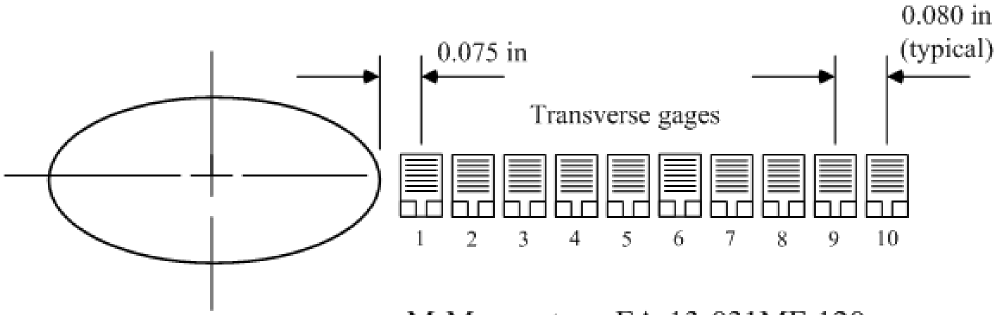
Lab #5: Stress and Strain Concentrations



Lab #5: Stress and Strain Concentrations



M-M gage type SA-13-031PJ-120
Gage factor = 2.12, K_t = 2.0%



M-M gage type EA-13-031MF-120
Gage factor = 2.09, K_t = 1.2%

Corrections for Biaxial Rosettes With Differing Transverse Sensitivity Coefficients

$$\varepsilon_x = \frac{(1-\nu_o K_t^x)\varepsilon_{mx} - (1-\nu_o K_t^y)K_t^x \varepsilon_{my}}{1-K_t^x K_t^y}$$

$$\varepsilon_y = \frac{(1-\nu_o K_t^y)\varepsilon_{my} - (1-\nu_o K_t^x)K_t^y \varepsilon_{mx}}{1-K_t^x K_t^y}$$

$\varepsilon_{mx}, \varepsilon_{my} =$ strains measured in the *x*- and *y*- directions

$K_t^x, K_t^y =$ Transverse sensitivity coefficients for gages in the *x*- and *y*- directions

MM Tech-Note 509 “Errors Due to Transverse Sensitivity in Strain Gages”

Lab #5: Stress and Strain Concentrations

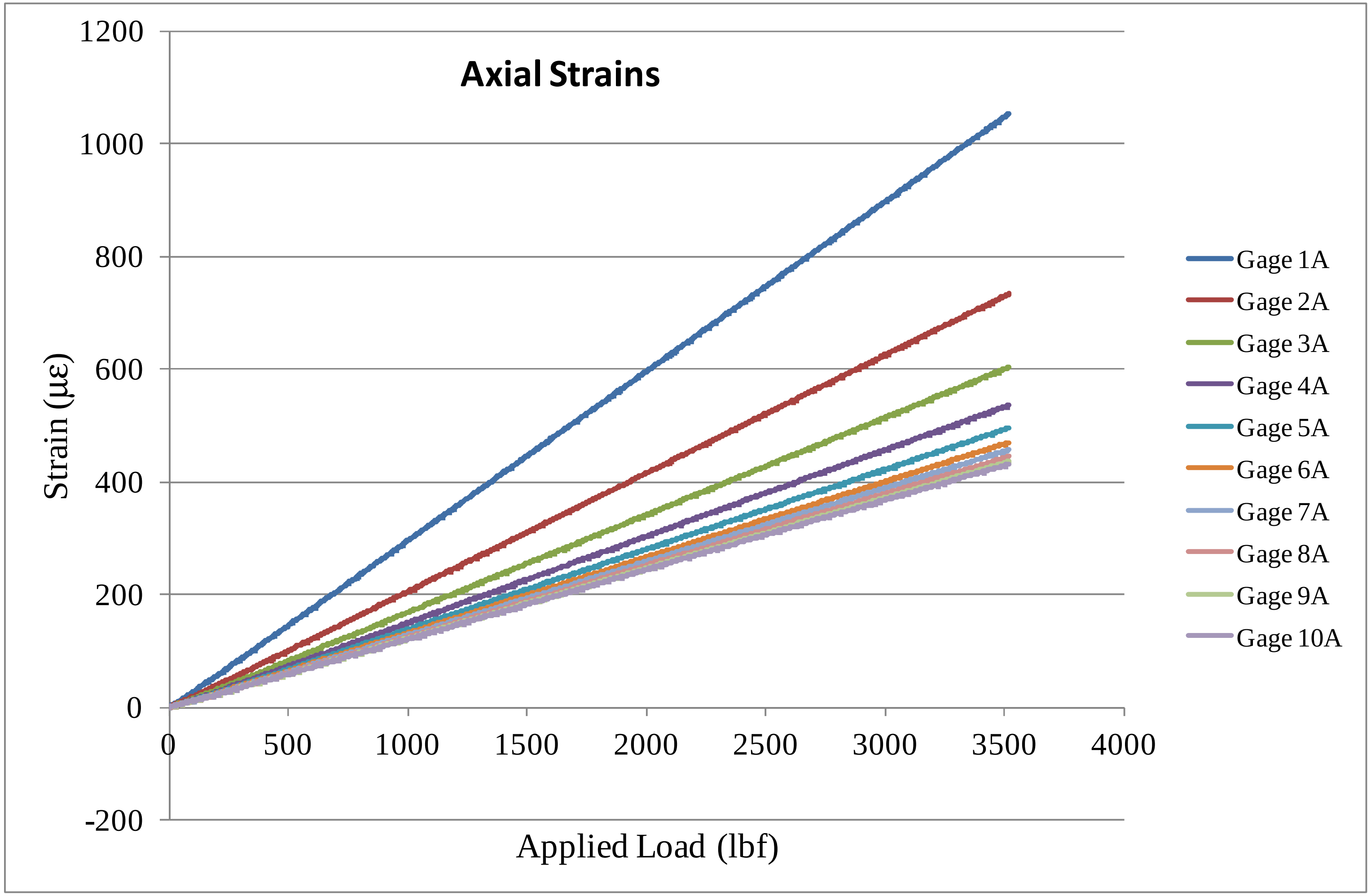
Goals:

- To compare stress distributions measured near an elliptical hole in a finite thin plate to those predicted for an infinite thin plate, and

- To compare the stress concentration factor measured for an elliptical hole in a finite thin plate to the value expected from a reference handbook.

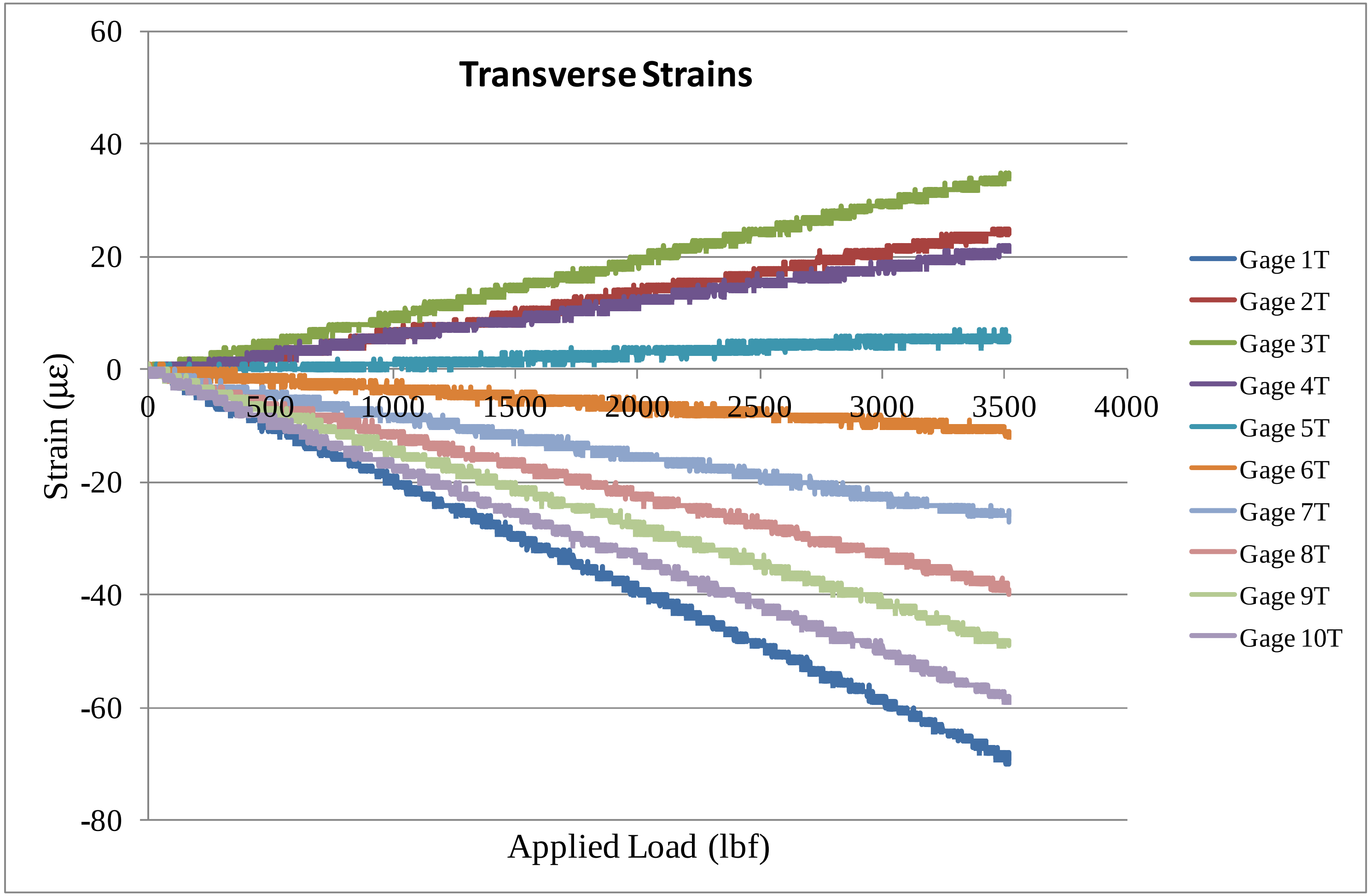
Lab #5: Stress and Strain Concentrations

“Official Data” – Axial Strains



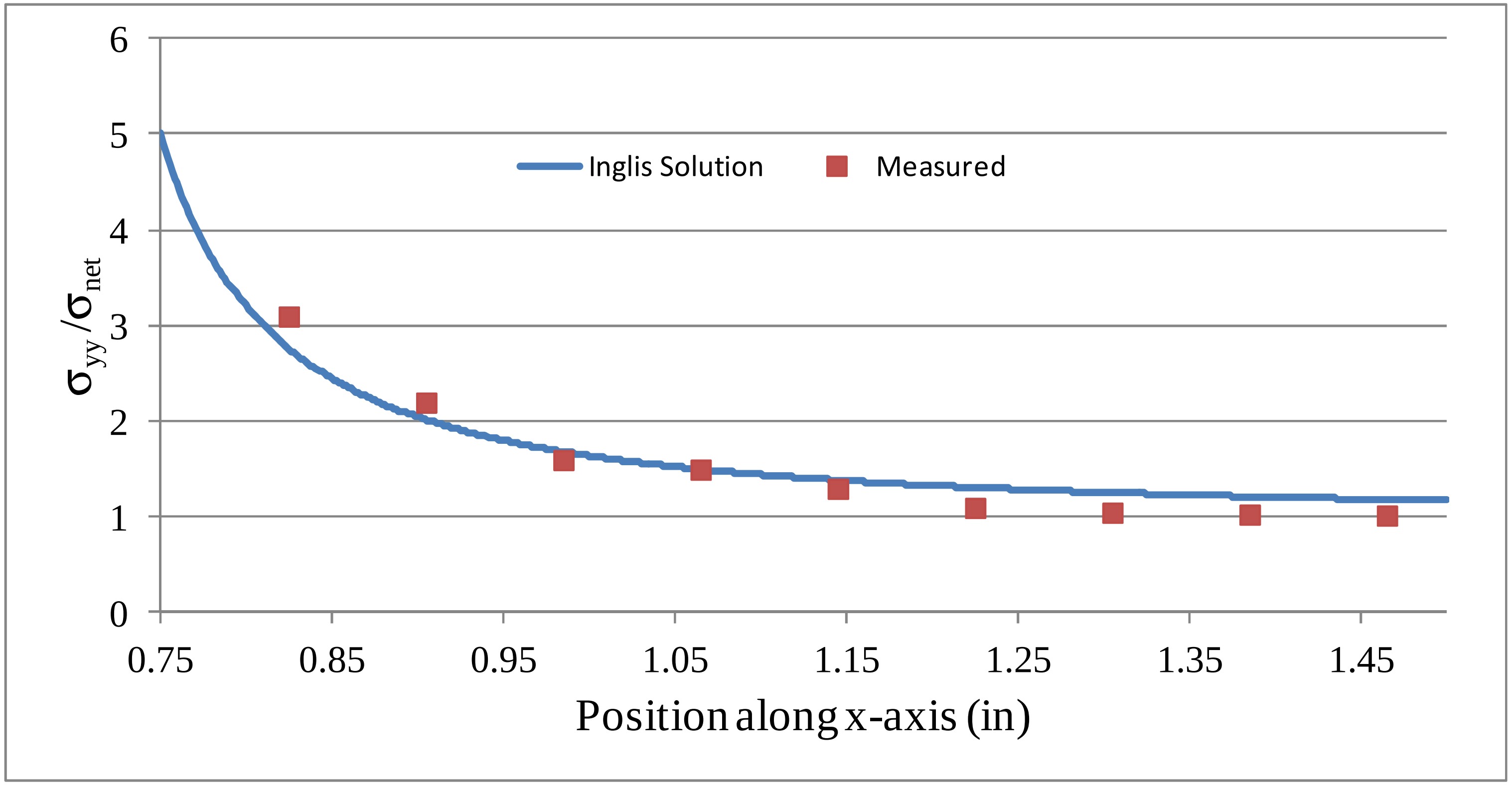
Lab #5: Stress and Strain Concentrations

“Official Data” – Transverse Strains



Lab #5: Stress and Strain Concentrations

Goal 1: Compare stress distributions measured near an elliptical hole in a finite thin plate to those predicted for an infinite thin plate (Suggestion: compare normalized stresses)

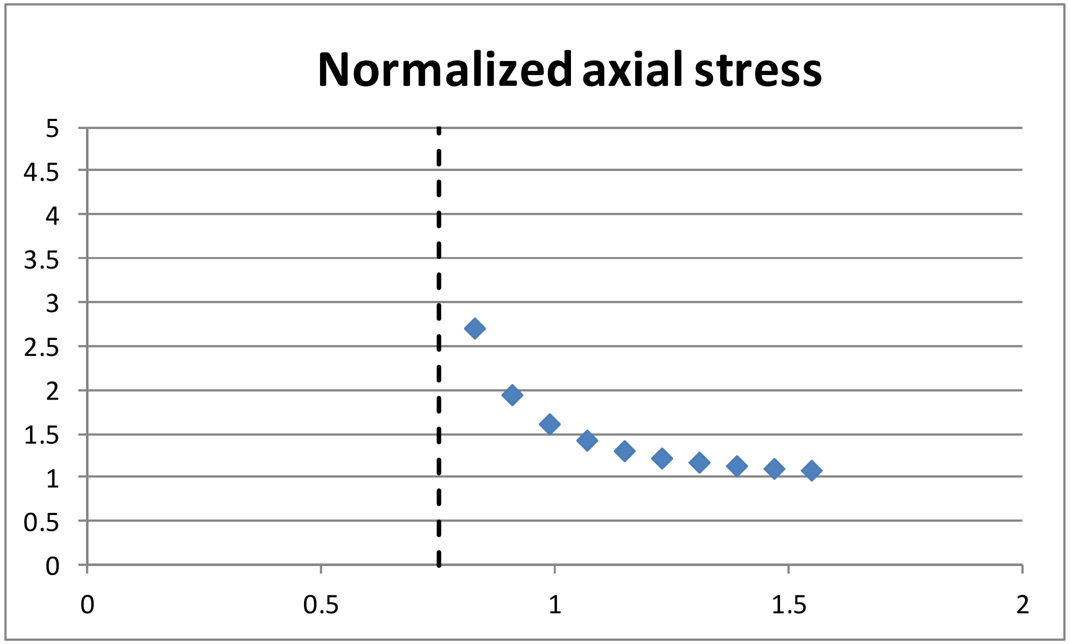


Note: as mentioned during class lecture, the measured values shown here are fictitious

Lab #5: Stress and Strain Concentrations

Goal 2: To compare the stress concentration factor measured for an elliptical hole in a finite thin plate to the value expected from a reference handbook.

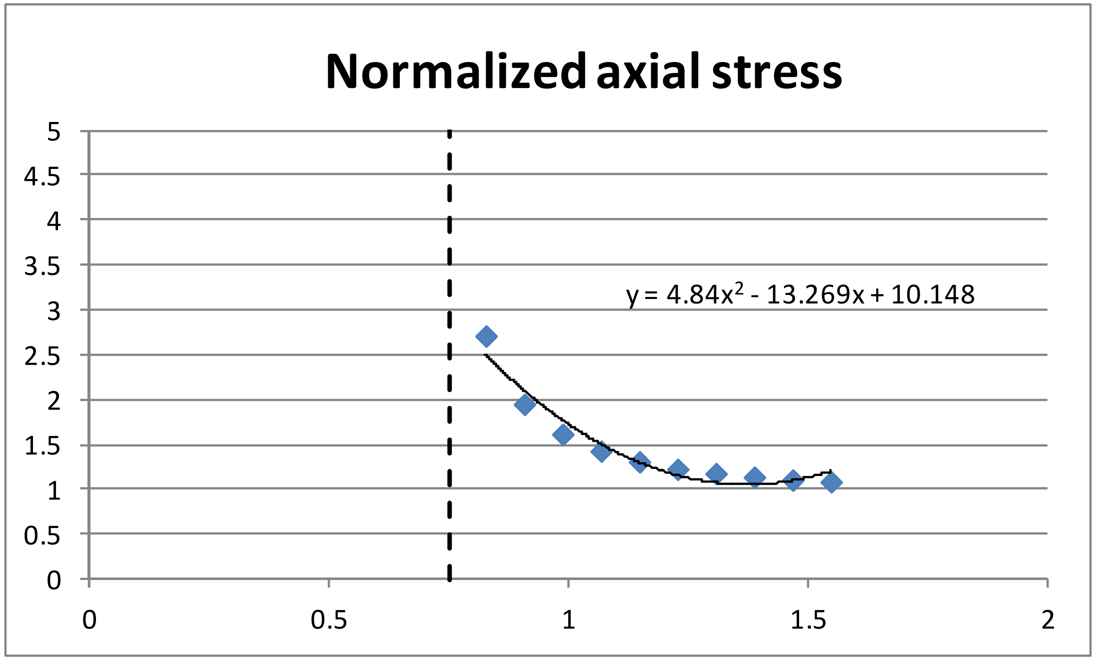
(Suggestion: extrapolate a curve fit)



Lab #5: Stress and Strain Concentrations

Goal 2: To compare the stress concentration factor measured for an elliptical hole in a finite thin plate to the value expected from a reference handbook.

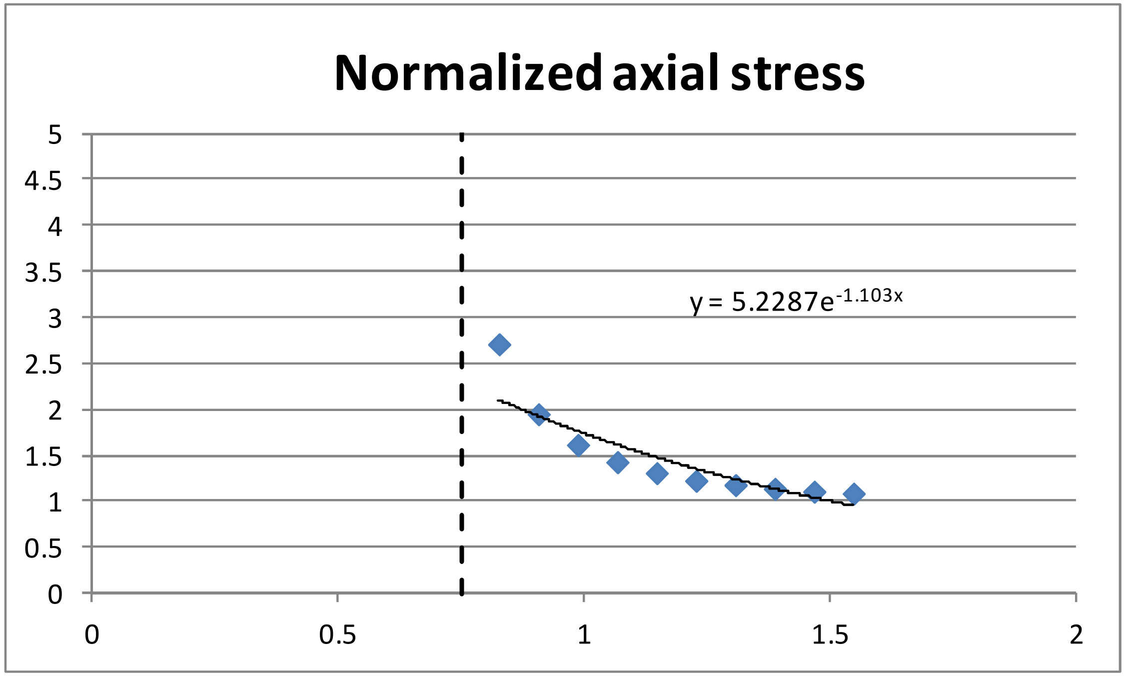
….fit using a 2nd-order polynomial:



Lab #5: Stress and Strain Concentrations

Goal 2: To compare the stress concentration factor measured for an elliptical hole in a finite thin plate to the value expected from a reference handbook.

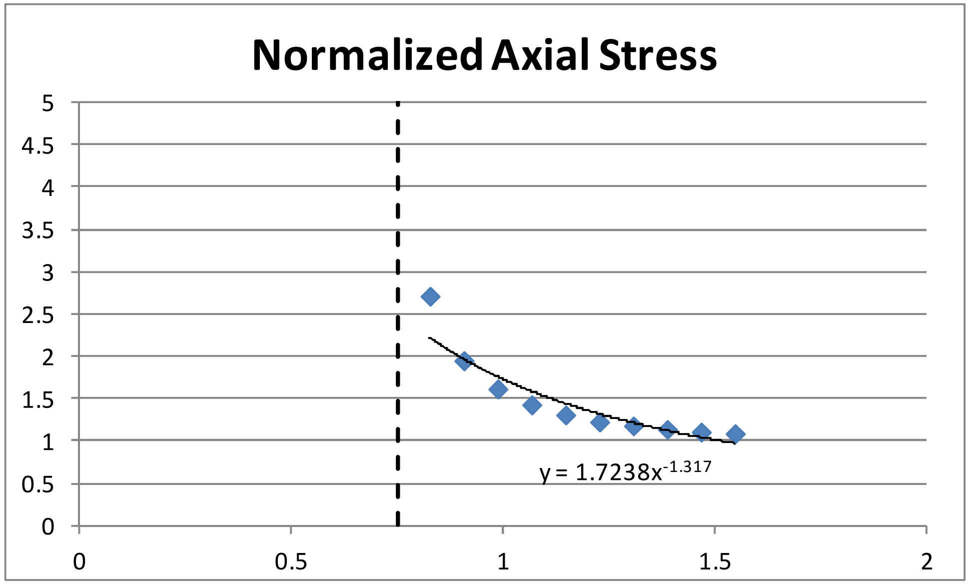
….fit using an exponential:



Lab #5: Stress and Strain Concentrations

Goal 2: To compare the stress concentration factor measured for an elliptical hole in a finite thin plate to the value expected from a reference handbook.

….fit using a power law



Lab #5: Stress and Strain Concentrations

Goal 2: To compare the stress concentration factor measured for an elliptical hole in a finite thin plate to the value expected from a reference handbook.

….fit using an polynomial and (1/normalized stress)