ME-573 Homework 1

(Note: Do all problems for full credit (20 points each))

Problem 1:
The following four sample sets of data were obtained for the shear strength of a certain ferrous alloy where the values are in MPa.

a) Plot histograms for each sample set, selecting a class interval such that 10-20 classes will result for the range of data and which will 'smooth' the data to provide a clear picture of the underlying distributions.

b) Assume a central tendency for the sets of data and determine the mean, median, mode and standard deviation for each sample set, A, B, C, and D. Determine the mean, median, mode and standard deviation for combined sample sets, A+B and A+B+C. Which measure of 'average,' $\bar{x}$, best represents each sample set? Are any of the sets skewed?

A	B	C	D
146	162	150	148
148	160	144	154
146	146	152	152
140	156	154	152
150	150	138	154
146	146	154	154
146	154	160	158
142	150	160	158
152	148	152	156
148	148	150	160
158	148	150	162
152	150	150	154
146	152	152	148
156	150	158	152
152	152	152	148
150	152	156	158
150	152	156	162
146	148	152	160
148	152	154	160
152	146	154	158

Problem 2:
Combine the four sample sets of data in Problems 1 and assume the resulting data set represents the population.

a) Plot a histogram of the population set, selecting a class interval such that 10-20 classes will result for the range of data and which will 'smooth' the data to provide a clear picture of the underlying distributions.

b) Assume a central tendency for the sets of data and determine the mean, median, mode and standard deviation for each sample set. Which measure of 'average,' μ, best represents the population?

c) If the error, E, is defined as the difference between, $\bar{x}$, and μ, determine errors for the following: $\bar{X}_A, \bar{X}_B, \bar{X}_C, \bar{X}_D, \bar{X}_{A+B}, \bar{X}_{A+B+C}$. What happens as the size of the sample set increases?

Problem 3:
Assume the normal (Gaussian) distribution applies to the shear strength population. The standard normal variable is defined as $z = \frac{x_i - \mu}{s.d.}$ and the probability density function for the normal distribution is $f(x_i) = \frac{e^{\left(-\frac{z_i^2}{2}\right)}}{s.d.\sqrt{2\pi}}$. Plot the assumed normal distribution function on the same graph as your histogram for the population set where the calculated frequency for your assumed intervals is $f = f(X_i)\,n\,I$ where n is the number observations in the set and I is the class interval. How well does the assumed and continuous normal distribution function fit the discrete data of the histogram? Is a normal distribution a good description of this data set?

Problem 4:
Somehow, at a particular instant, the following plane stress state was determined at the surface of a component.

$$\begin{bmatrix} \sigma_x = 50 & \tau_{xy} = 60 \\ \tau_{xy} = 60 & \sigma_y = 100 \end{bmatrix} \text{MPa}$$

a) Determine the principal normal stresses and the maximum shear stress.
b) Using a deterministic approach and appropriate conventional failure criteria (maximum normal stress fracture criterion or maximum shear stress yield criterion), determine for this stress state the factors of safety if the following materials are used:
i) SiC (a brittle advanced ceramic) with $\bar{S}_{uts}$=280 MPa (s.d.=40 MPa) for an effective volume of 1100 mm^3.
ii) 6061-T6 aluminum (a ductile metal) with $\bar{S}_{yp}$=275 MPa (s.d.=20 MPa), $\bar{S}_{uts}$=310 MPa (s.d.=10 MPa).
c) What is the assumed reliability for the deterministic approach which uses discrete, single valued parameters?

Problem 5:
Redo Problem 4 for each material but assume a first order approximation of a probabilistic design approach with strength and stress normally distributed. Assume the stress distributions (principal normal stresses or maximum shear stress) as calculated are mean values and have standard deviations of 20 MPa.

a) Recall that $\bar{Q} = \bar{S} - \bar{\sigma}$ and $s.d._Q = \sqrt{s.d._S^2 + s.d._\sigma^2}$ and that $z = \frac{x_i - \mu}{s.d.}$ for the critical condition of Q≤0. Determine the area of the cumulative distribution funtion ($CDF = \int_{-\infty}^{z} f(z)dz$) from -∞ to z for Q=0. (See problem 3 for more on the normal distribution).

b) If the area of the CDF determined in a) is the probability of failure, P_f, what is the reliability for this design? Is this reliability acceptable? Which method, deterministic or probabilistic, better reflects reality?