ME-573 Homework 5

(Note: Do all problems for full credit (20 points each))

Problem 1:

For the 2-D stress state of plane stress (σ_z=0) the principal stress relation can be written in terms of the coordinate stresses such that:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Derive the estimator functions for the statistics (mean and standard deviation) of σ_1 and σ_2 in terms of σ_x, σ_y and τ_{xy}. (Hint: use truncated Taylor series expansions)

Problem 2:

For the 2-D plane stress state shown, estimate the statistics for the principal normal stresses (recall the number of principal normal stresses which exist in 3-D space).

$$\left[\bar{\sigma}, s_\sigma\right] = \begin{bmatrix} \sigma_x = 50, 4.5 & \tau_{xy} = 60, 2.2 \\ \tau_{xy} = 60, 2.2 & \sigma_y = 100, 12 \end{bmatrix} \text{MPa}$$

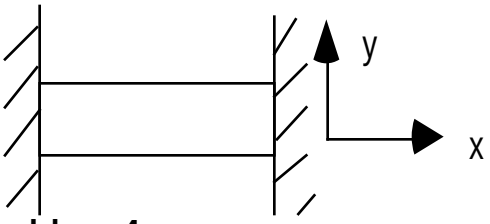
Comment on the significance of the mean value estimators of the principal stresses. Can the principal stresses be ordered in the classical sense using the mean value estimators? Given the nature of the principal stresses as random variables, is ordering of the principal stress valid in a strict sense? Comment.

Problem 3:

An elastic member is rigidly constrained only in the longitudinal direction and subjected to a temperature change of $\Delta\bar{T}, s_T$ (225,11.5) °C. If the material is 2014-T651 aluminum alloy determine the statistics for the following:

a) strain in the longitudinal direction, x.
b) stress in the x direction.
c) strain in the y and z directions.
d) stresses in the y and z directions.

Hint: Use Table 5.1 for the statistics of E and estimate the statistics for CTE as ±5% CV variability.



Problem 4:

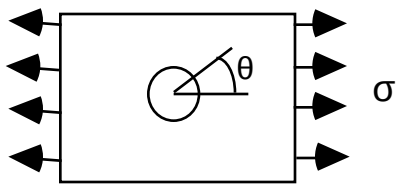
A solid shaft with one end fixed resists a torque applied at the other end of $\bar{M}_T, s_{M_T}$ (312,59) N-m. If the diameter of the shaft is $\bar{d}, s_d$ (19.98,0.35) mm. If the material is a Ti-4Al-3Mo-1V alloy, determine the statistics for the following:

a) maximum shear stress at the surface.
b) maximum angle of twist per unit length.

Problem 5:

At the edge of a circular hole (radius = a) in an infinite plate subjected to uniaxial tension, the elasticity solution for the polar coordinate theta stress is:

$$\sigma_\theta = \frac{\sigma}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta$$



If θ is measured from the direction of the nominal applied tensile stress, σ, determine the statistics for the stress concentration factor, K_t at $\theta=\pi/2$ for the following locations from the center of the hole with radius $\bar{a}, s_a$ (25.0,0.12) mm: a) $\bar{r}, s_r$ (25.1,0.1) mm, b) $\bar{r}, s_r$ (50,0.5) mm c) $\bar{r}, s_r$ (75,1.0) mm.

Plot the mean and ±3s.d. envelope of K_t versus normalized distance from the hole, r/a.