

## Chapter 4

# Known particles

### 4.1 Ordinary matter

What are you made of? Blood and guts and bone and muscle is a little more accurate than the traditional mother goose rhyme. Your tissues are made of cells, which are little bags of chemicals: proteins, nucleic acids, lipids, water and other molecules. Each molecule is a specific assembly of atoms. And each atom contains an atomic nucleus surrounded by some number of electrons.

This should all sound familiar. But stop for a minute and ask how this is known. You can see cells in a microscope. But for objects smaller than cells direct observation gets more difficult. How do you know that atoms and molecules, or electrons and nuclei, exist? Is it just because someone told you so? What's the evidence?

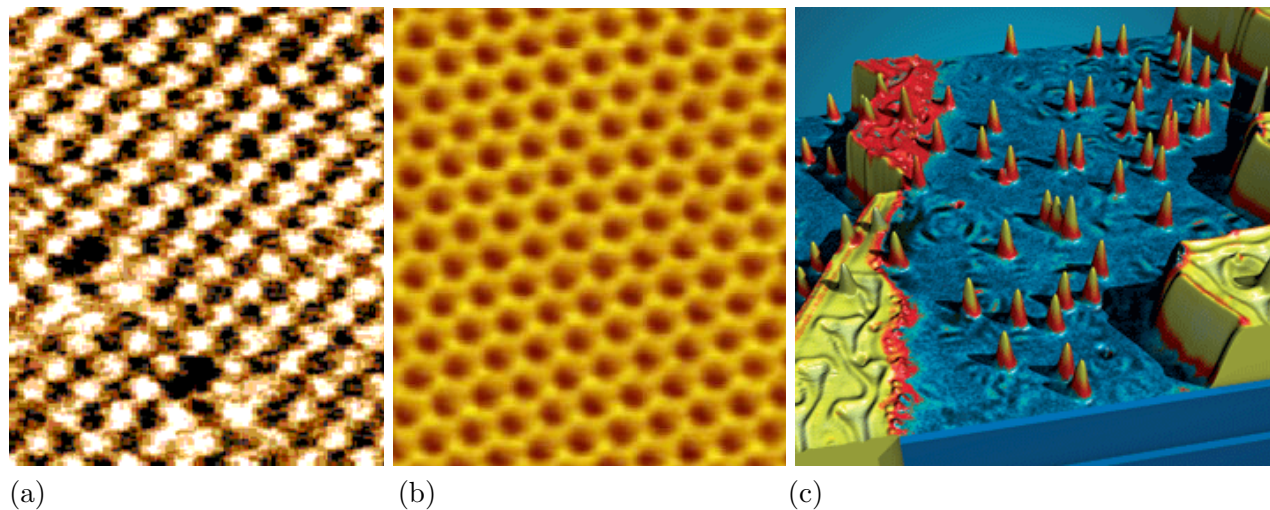


Figure 4.1: Three examples of modern atomic scale imaging. Image (a) shows the surface of sodium chloride, imaged by atomic force microscopy (AFM). Note the two surface defects. Image (b) (courtesy of E. Andrei) is a scanning tunneling microscope (STM) image of a freely suspended graphene sheet — a single atomic layer of graphite. The hexagonal structure, reflecting the  $sp^2$  hybridization of valence electrons in the carbon atoms, is obvious. Image (c) (from the cover of the April 4, 2008 issue of *Science*) shows single cobalt atoms on a platinum surface with steps, imaged with spin-polarized scanning tunneling microscopy. Blue areas show the platinum substrate; red and yellow regions in front of the steps show adsorbed cobalt monolayer stripes with magnetization up (yellow) or down (red).

The historical basis for the atomic structure of matter owes much to the development of the kinetic theory of gases, the understanding of Brownian motion, and chemistry. From a more modern perspective, two compelling types of experimental evidence for the existence of atoms can be summed up as (i) chemistry works, and (ii) individual atoms and molecules can be imaged using a variety of modern techniques, such as scanning tunneling microscopy and atomic force microscopy. A few examples of atomic scale imaging are shown in Figure 4.1.

The existence of electrons has been known experimentally since the work of J.J. Thomson who, in 1897, studied the behavior of particles that pass through a cathode ray tube (the precursor of televisions) when a suitably large voltage is applied. Thomson found that these particles have a mass to charge ratio which is independent of the type of material forming the cathode or the gas in the tube, and this ratio is about 2000 times smaller than the mass to charge ratio of a hydrogen ion. Measurement of the charge-to-mass ratio involves observing the deflection of a moving particle produced by a magnetic field. The charge of a single electron can be measured using the approach of Millikan and Fletcher's famous oil drop experiment. Based on refinements of such measurements, the magnitude of the electron charge is now known to a precision of a few parts in  $10^8$ ,

$$|-e| = 1.602\,176\,487\,(40) \times 10^{-19} \text{ C}. \quad (4.1.1)$$

(The number in parentheses indicates the uncertainty in the last two digits.) In other words, a Coulomb, whose definition is based on macroscopic measurements of current plus the definition of a second, is equal in magnitude to  $6.241\,509\,65\,(15) \times 10^{18}$  electron charges. The mass of the electron is also known to a similar precision,

$$m_e = 0.510\,998\,910\,(13) \text{ MeV}/c^2 = 9.109\,382\,15(45) \times 10^{-31} \text{ kg}. \quad (4.1.2)$$

One MeV ( $= 10^6$  eV) is the energy acquired by an electron passing through a potential difference of one million volts.

A few angstroms ( $1 \text{ \AA} = 10^{-10} \text{ m}$ ) is the size of individual atoms, whereas nuclear sizes are naturally measured in units of the fermi (or femtometer), where  $1 \text{ fm} = 10^{-15} \text{ m}$ . (In natural units,  $1 \text{ fm} \approx 5 \text{ GeV}^{-1}$  while  $1 \text{ \AA} \approx 500,000 \text{ GeV}^{-1}$ .) Direct evidence of the size of atomic nuclei comes from scattering experiments, specifically measurements of the momentum dependence of the scattering cross section. This will be discussed more fully in a later chapter. For now, it suffices to note that in order to learn about the structure of some object like an atomic nucleus, one must use some probe [such as photons (light), electrons, or other nuclei] whose wavelength is *smaller* than the size of the object of interest.

Atomic nuclei are known to be bound states of more fundamental particles, protons and neutrons (except for the lightest nucleus of hydrogen, which is just a single proton). This information again comes from scattering experiments: one can bombard nuclei with various projectiles, such as electrons or other nuclei, and observe individual protons or neutrons knocked out of the target nucleus. Just as atoms come in different types, which are usefully organized in the traditional periodic table and characterized by their differing chemical interactions, there are many different atomic nuclei distinguished by the numbers of neutrons and protons that they contain. It is conventional to label nuclei with the atomic symbol for the corresponding element, with a preceding *superscript* indicating the atomic number  $A$ , equal to the number of protons plus neutrons, and a preceding *subscript*  $Z$  indicating the number of protons. For example, the lithium-7 nucleus,  ${}^7_3\text{Li}$ , is a bound state of three protons and four neutrons. Figure 4.2 shows a plot of known nuclear species (or *nuclides*), color coded

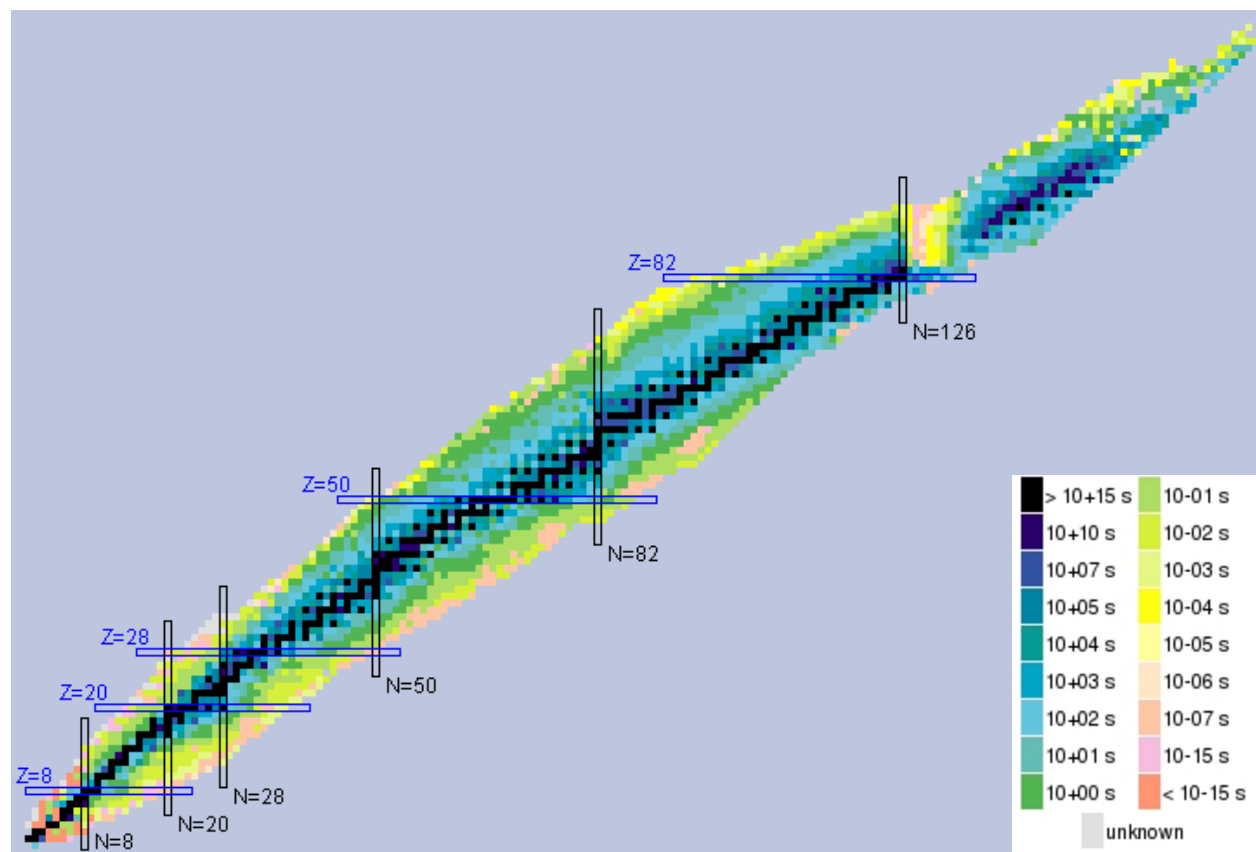


Figure 4.2: Chart of the nuclides (from the National Nuclear Data Center at Brookhaven National Laboratory). The number of protons,  $Z$ , is plotted vertically and the number of neutrons,  $N$ , horizontally. The color coding indicates the lifetime, with stable nuclei in black and lighter colors corresponding, as shown in the legend, to progressively shorter lifetimes. The rows and columns labeled with specific values of  $Z$  and  $N$  are so-called “magic” numbers where nuclei have enhanced stability.

according to their stability. Useful interactive online versions may be found at [www.nndc.bnl.gov](http://www.nndc.bnl.gov) and [atom.kaeri.re.kr](http://atom.kaeri.re.kr).<sup>1</sup>

Protons have charge  $+e$ , equal in magnitude but opposite in sign to the electron. This apparent *exact* equality, except for sign, between the proton’s electric charge and that of the electron has been tested to a precision of better than one part in  $10^{21}$  and is a crucial feature of our universe. Because atoms have zero net charge, electrostatic repulsion does not prevent the assembly of macroscopic objects, such as your body or the Earth.<sup>2</sup>

<sup>1</sup>Note that most tables of nuclides, including the one at [atom.kaeri.re.kr](http://atom.kaeri.re.kr), list *atomic* masses, not nuclear masses, the distinction being that the atomic mass is the mass of the neutral atom. In other words, the atomic mass includes the rest mass of all the electrons plus the mass of the nucleus, as well as the (negative) atomic binding energy.

<sup>2</sup>The observed exact equality, up to sign, of the electron and proton charge calls out for some more fundamental explanation — which is not currently known. Some hypothetical theories (called “grand unified theories” or GUTs) of possible physics beyond the Standard Model provide such an explanation by postulating a “grand” underlying symmetry which relates the electron and the constituents of the proton (*i.e.*, leptons and quarks, discussed below). The existence of such an underlying symmetry would lead to new interactions, beyond those discussed in the next section, which would allow the proton to decay at a very slow rate. Although the search continues, no such decay has yet been observed.

The mass of a proton is measured to be

$$m_p = 0.938\,272\,013(23) \text{ GeV}/c^2. \quad (4.1.3)$$

This is about 2000 times larger than the mass of an electron. Neutrons, which are neutral (zero electric charge) particles, are slightly heavier than protons,

$$m_n = 0.939\,565\,36(8) \text{ GeV}/c^2. \quad (4.1.4)$$

Neutrons, protons, and electrons are all spin 1/2 particles, where the spin is measured in units of the fundamental quantum of spin,  $\hbar$ .

Protons and neutrons are collectively referred to as *nucleons*. Nucleons are known to have internal structure: they may be regarded as bound states of three *quarks*. We will later be discussing quarks, and their possible bound states, in much greater detail. For now, we simply note that the observational evidence for quarks is necessarily somewhat indirect. It turns out that scattering experiments with nucleons cannot liberate free quarks.<sup>3</sup> Consequences of this peculiar experimentally-observed phenomena will be examined in the next chapter.

Today, the story of successive levels of microscopic structure — molecules made of atoms, atoms made of electrons and nuclei, nuclei made of nucleons, nucleons made of quarks — ends here. No evidence for internal structure within quarks, or electrons, has yet been found. If in the future these particles are discovered to be composite objects, bound states of some not-yet-known more fundamental constituents, then the length scale on which this binding occurs must be at least three orders of magnitude smaller than the femtometer scale of nucleons. This limit on the length scale is set by the corresponding energy scale (TeV) of experimental measurements at the LHC and the Fermilab Tevatron particle accelerators, which have not yet exhibited any evidence of internal structure for quarks or electrons.

## 4.2 Known interactions

Four different fundamental types of interactions, or forces, between particles are known today: *strong*, *weak*, *electromagnetic*, and *gravitational*. The relative strength and range of these interactions is summarized in Table 4.1: All particles participate in gravitational interactions, but this interaction is extremely weak and will largely be ignored in this class.<sup>4</sup> Particles with a nonzero electric charge participate in electromagnetic interactions. Nearly all particles participate in the weak interactions, which are stronger than gravity but weaker than electromagnetic and strong interactions. Weak interactions are responsible for some forms of radioactivity and many nuclear and particle decays.

<sup>3</sup>Scattering experiments which attempt to disrupt a proton and liberate its constituents produce nucleons, pions, kaons and other particles we will discuss in the next chapter, but not free quarks. On spatial resolution scales larger than a fermi, quarks always appear “confined” inside nucleons or other strongly interacting particles. Understanding how this phenomena emerges from the mathematical details of the Standard Model has been a major intellectual challenge in theoretical particle physics.

<sup>4</sup>The assertion that gravity is weak may seem surprising since, on scales of the solar system and larger, gravity is the dominant interaction. This reflects the fact that strong and weak interactions act only over short ranges, and electromagnetic forces between electrically neutral objects fall off with distance much faster than does gravity. Hence, even though gravity is much weaker than the other interactions in terms of its effects on fundamental particles, it becomes the dominant interaction at large distances because its effects are long-ranged and add coherently in bulk matter.

Force	Relative Strength	Range
Strong	1	$\approx 1$ fm
Weak	$10^{-6}$	$\approx 10^{-3}$ fm
Electromagnetic	$10^{-2}$	Infinite ( $\propto 1/r^2$ )
Gravitational	$10^{-43}$	Infinite ( $\propto 1/r^2$ )

Table 4.1: Known forces of the Standard Model. The second column shows the (approximate) relative strength of the respective forces when acting between fundamental charged particles such as protons.

Finally, nucleons and other particles made from quarks (collectively known as *hadrons*) participate in the strong interactions. These are the interactions which cause quarks to become bound into composite particles. While gravitational and electromagnetic interactions (between charged particles) decrease slowly with distance like  $1/r^2$ , both weak and strong interactions are short-ranged, becoming negligible beyond a characteristic distance. This distance is about 1 fm for strong interactions, and roughly a thousand times shorter,  $10^{-3}$  fm, for weak interactions.

### 4.3 Stability of particles

Are protons, or electrons, or hydrogen atoms stable? Or can they spontaneously decay? In other words, if one of these particles (or atoms) is completely isolated, in a vacuum, will it eventually, spontaneously fall apart? Phrased in this manner, this is a “bad” question. It is fundamentally unanswerable — because feasible experiments must necessarily last a finite length of time. If there is no known evidence that a certain type of particle *can* decay, then the question one should ask is what *limits* can be placed on the stability of the particle.

For protons and electrons, we have no evidence whatsoever that these particles are unstable, and experimental bounds on the lifetimes of these particles, if they do decay, are very long,

$$\text{proton lifetime } \tau_p > 2.1 \times 10^{29} \text{ yr}, \quad (4.3.1)$$

$$\text{electron lifetime } \tau_e > 4.6 \times 10^{26} \text{ yr}. \quad (4.3.2)$$

You should be impressed with these limits, considering that they vastly exceed the age of the Earth (a mere 4.5 billion years). Suppose, hypothetically, that protons do decay with a lifetime of  $10^{30}$  years. How could one ever know? The direct approach of watching one particle for  $10^{30}$  years is obviously impossible. But if you can watch many identical particles simultaneously, and detect if (and when) a single one of them decays, then extremely long lifetimes can be measured.<sup>5</sup> A cubic meter of water contains  $2.7 \times 10^{29}$  protons (and the same number of electrons). So if  $\tau_p = 10^{30}$  yrs, then within a tank holding 100 cubic meters of water, 27 protons (on average) will decay every year. The challenge is in designing and operating an experiment which can detect the decay of individual protons within a large quantity of material.<sup>6</sup>

<sup>5</sup>The lifetime  $\tau$  of an unstable particle is, by definition, the time interval (in its rest frame) for which the probability of the particle decaying is  $1/e$ . If you start with  $N_0$  identical particles, then the mean number of particles which will remain after time  $t$  is given by  $N(t) = N_0 e^{-t/\tau}$ . If  $N_0 \gg 1$  then, on average, one particle will have decayed by the time  $t_1 = \tau/N_0$ , since  $N(t_1) \approx N_0 - 1$ .

<sup>6</sup>The development of such large detectors, essentially instrumented large tanks of water, has not yet led to any

Next, consider neutrons, the other basic constituent of nuclei besides protons. Unlike protons, an isolated neutron is known to be unstable, with a lifetime of about 15 minutes. The products of the decay are a proton, an electron, and a less familiar particle called an *electron antineutrino*, denoted  $\bar{\nu}_e$ ,

$$n \longrightarrow p + e^- + \bar{\nu}_e. \quad (4.3.3)$$

This decay is referred to as a *beta decay*,<sup>7</sup> and is a consequence of *weak interactions*, which will be discussed more fully in a later chapter. Neutrinos are nearly massless, spin-1/2 particles which interact extremely weakly with ordinary matter and as a result are very difficult to detect. They come in several different types, and exhibit interesting quantum-mechanical phenomena which we will examine later.

Although a single free neutron is unstable, when neutrons bind with protons to form nuclei the resulting bound states are, in many cases, effectively stable (meaning that their lifetimes, if finite, are in excess of billions of years). Such stable nuclei include deuterium ( ${}^2_1\text{H}$ ) which is a bound state of one proton with one neutron, helium-3 ( ${}^3_2\text{He}$ ) which contains two protons and one neutron, helium-4 ( ${}^4_2\text{He}$ ) consisting of two protons and two neutrons, and many progressively heavier nuclei (recall Figure 4.2) up to bismuth-209 ( ${}^{209}_{83}\text{Bi}$ ) which is the heaviest (known) nucleus that is essentially stable.<sup>8</sup>

## 4.4 Nuclear decays

In addition to (apparently) stable bound states, there are many more unstable nuclei with lifetimes that range from very long, billions of years, down to very short, less than femtoseconds. Stable nuclei have roughly the same number of protons and neutrons (or in heavier nuclei, somewhat more neutrons than protons). Many nuclei with an excess of neutrons, relative to the number of protons, undergo beta decay. This converts a neutron within the nucleus into a proton, while emitting an electron and an antineutrino. For example,

$\beta$ decay	lifetime
${}^3_1\text{H} \longrightarrow {}^3_2\text{He} + e^- + \bar{\nu}_e$	17.8 yr
${}^6_2\text{He} \longrightarrow {}^6_3\text{Li} + e^- + \bar{\nu}_e$	1.16 s
${}^{10}_4\text{Be} \longrightarrow {}^{10}_5\text{B} + e^- + \bar{\nu}_e$	2.18 Myr
${}^{14}_5\text{B} \longrightarrow {}^{14}_6\text{C} + e^- + \bar{\nu}_e$	18.0 ms

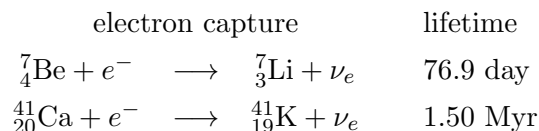
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observation of proton decay. But it has resulted in detectors capable of measuring neutrinos emitted from our Sun or supernovae occurring in our galaxy (or neighboring dwarf galaxies), as well as terrestrial nuclear power plants. This is a nice example of the synergies that drive science. Experiments and detector technology that was developed to observe or improve limits on proton decay played a critical role in the unexpected and fundamental discovery that neutrinos are not exactly massless.

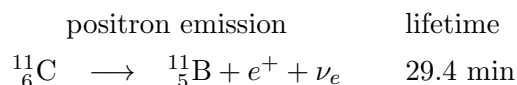
<sup>7</sup>This is a historical name which dates from the early 1900s, when three distinct types of radioactive decay, called  $\alpha$ ,  $\beta$ , and  $\gamma$ , had been identified. The different decay types were distinguished by the degree to which the particles emitted in the decay could penetrate ordinary matter. Alpha decays produce particles with very little penetrating power which were later identified as helium-4 nuclei. Gamma decays produce extremely penetrating particles, later identified to be high energy photons (“gamma rays”). Beta decays produce particles which penetrate farther than alphas, but less than gammas. These were subsequently identified to be electrons.

<sup>8</sup>In fact, bismuth-209 has recently been found to alpha decay with a lifetime of  $2 \times 10^{19}$  yr.

Some nuclei with an excess of protons, relative to the number of neutrons, can convert a proton into a neutron by capturing an electron from the cloud of electrons surrounding the nucleus, and then emitting a neutrino which carries off the excess energy,



This mode of decay is only possible if the atom is not fully ionized, so that one or more electrons are bound to the nucleus. If that is not the case, neutron-poor nuclei can convert a proton into a neutron via *positron emission*. A *positron*, denoted  $e^+$ , is a particle with the same mass as an electron, but with charge  $+e$  instead of  $-e$ . It is an example of an *antiparticle*, discussed below. The carbon-11 nucleus preferentially decays via positron emission even when it has an orbital electron it could otherwise capture,



Certain nuclei have multiple modes of decay with measurable rates. For example, potassium-40 ( ${}^{40}_{19}\text{K}$ ) has a lifetime of 1.8 billion years. In 89% of its decays, potassium-40 undergoes beta-decay to calcium-40, but in the remaining 11% of its decays, potassium-40 decays to argon-40 via electron capture or positron emission.

In addition to the above types of nuclear decay, in which a neutron is converted into a proton or vice-versa, some nuclei which are very proton-rich decay by simply ejecting a proton, or in some cases, an alpha particle. And some very neutron-rich nuclei simply eject a neutron.

Many excited states of nuclei decay to their ground states by emitting photons (just like excited atomic states). But in the case of nuclei, excited state energies are typically in the range of several MeV, so the photons emitted in nuclear decays are in the *gamma ray* portion of the electromagnetic spectrum.

## 4.5 Photons

One other elementary particle which plays a major role in innumerable aspects of everyday life is the *photon*. Photons are quantized excitations of the electromagnetic field and are the “force carriers” of the electromagnetic interaction. (One may regard electromagnetic interactions between charged particles as occurring via the exchange of photons.) Photons have no rest mass, unlike the other particles we have discussed so far. This means that the energy of a photon is directly proportional to its momentum,  $E = c|\vec{p}|$ . This energy can be arbitrarily small, unlike a massive particle whose energy is always greater than (or equal to) its rest energy  $mc^2$ . The de Broglie wavelength of any particle is inversely related to its spatial momentum and thus, for photons, wavelength and energy (or frequency) are inversely related,

$$\lambda = \frac{2\pi\hbar}{|\vec{p}|} = \frac{2\pi\hbar c}{E} = \frac{2\pi c}{\omega} = \frac{c}{\nu}. \quad (4.5.1)$$

The familiar breadth of the classical electromagnetic spectrum, ranging from arbitrarily low frequency (radio waves) to extremely high (gamma rays) is a direct reflection of the fact that photons, as massless particles, can have energies which are arbitrarily small or large.

Photons are spin-1 particles, meaning that they carry one unit of angular momentum (in units of  $\hbar$ ).

In everyday life, quantum aspects of the electromagnetic field are not readily apparent. For a great many applications, a classical treatment of electromagnetism suffices. This is the case when the number of photons is enormous and one cannot readily detect the emission or absorption of a single photon. But the quantized nature of light is revealed in phenomena such as the photoelectric effect, the presence of stimulated emission in lasers and masers, and the operation of sensitive photo-diodes which can detect *single* photons.<sup>9</sup>

## 4.6 Antiparticles

Early studies of cosmic rays revealed the existence of *positrons*, particles with the same mass as electrons but opposite charge. When a positron collides with an ordinary electron, they can both annihilate and produce photons,

$$e^+ + e^- \longrightarrow \gamma + \gamma.$$

Accelerator-based scattering experiments have also revealed the existence of *antiprotons* and *antineutrons*, denoted  $\bar{p}$  and  $\bar{n}$ , respectively. They can similarly annihilate with their ordinary partners to produce photons,

$$p + \bar{p} \longrightarrow \gamma + \gamma,$$

$$n + \bar{n} \longrightarrow \gamma + \gamma.$$

When one combines quantum mechanics and special relativity (leading to relativistic quantum field theory), a remarkable theoretical prediction is that antiparticles *must* exist. Charged particles must have distinct antiparticles with exactly the same mass and spin, but opposite electric charge. For certain neutral particles, such as the photon, there is no distinction between particle and antiparticle — one can say that the photon is its own antiparticle. At the moment, it is not known when the neutrino, the other fundamental electrically neutral particle, is its own antiparticle or not.<sup>10</sup>

Although antimatter is not present in everyday life (why this is so is another mystery), antiparticles do exist and the laws of nature are almost, but not quite, symmetric under the interchange of ordinary matter and antimatter. We will discuss this further in a later chapter.

## 4.7 Leptons

Electrons ( $e^-$ ) and electron neutrinos ( $\nu_e$ ) are members of a class of particles known as *leptons*. Their antiparticles, the positron ( $e^+$ ) and electron antineutrino ( $\bar{\nu}_e$ ), are *antileptons*. Leptons (and

<sup>9</sup>Human vision, when fully dark-adapted, can nearly detect single photons of visible light. See, for example, the classic paper *Energy, Quanta, and Vision* by Hecht, Shlaer and Pirenne.

<sup>10</sup>If neutrinos are their own antiparticle, then certain rare nuclear processes known as neutrino-less double beta decay are possible. Such decays have not yet been observed, but quite a few experiments aiming to detect neutrino-less double beta (including CUORE, EXO, SNO+, MAJORANA and others) are currently running or under development.



particle	rest energy	lifetime	dominant decay	charge	$L$
$\nu_e$	$< 2$ eV	$\approx$ stable	—	0	1
$\nu_\mu$	$< 2$ eV	$\approx$ stable	—	0	1
$\nu_\tau$	$< 2$ eV	$\approx$ stable	—	0	1
$e^-$	0.511 MeV	stable	—	-1	1
$\mu^-$	105.7 MeV	$2 \mu\text{s}$	$e^- \bar{\nu}_e \nu_\mu$	-1	1
$\tau^-$	1777 MeV	0.3 ps	$\pi^- \pi^0 \nu_\tau, e^- \bar{\nu}_e \nu_\tau, \mu^- \bar{\nu}_\mu \nu_\tau$	-1	1
$\bar{\nu}_e$	$< 2$ eV	$\approx$ stable	—	0	-1
$\bar{\nu}_\mu$	$< 2$ eV	$\approx$ stable	—	0	-1
$\bar{\nu}_\tau$	$< 2$ eV	$\approx$ stable	—	0	-1
$e^+$	0.511 MeV	stable	—	+1	-1
$\mu^+$	105.7 MeV	$2 \mu\text{s}$	$e^+ \nu_e \bar{\nu}_\mu$	+1	-1
$\tau^+$	1777 MeV	0.3 ps	$\pi^+ \pi^0 \bar{\nu}_\tau, e^+ \nu_e \bar{\nu}_\tau, \mu^+ \nu_\mu \bar{\nu}_\tau$	+1	-1

Table 4.2: Leptons and antileptons.

antileptons) are spin 1/2 particles. All leptons participate in the weak interactions (*leptos* is Greek for ‘weak’) and the electrically charged leptons also participate in electromagnetic interactions. However, leptons do not participate in strong interactions; leptons are *not* bound states of quarks.

In addition to the electron, two other charged leptons are known: the muon ( $\mu^-$ ) and the tau ( $\tau^-$ ). As the superscripts indicate, these particles are negatively charged; their electric charge is (apparently) *identical* to that of the electron. Their antiparticles are the antimuon ( $\mu^+$ ) and antitau ( $\tau^+$ ). There are distinct neutrinos associated, via the weak interactions, with each charged lepton. In addition to the electron neutrino, there is a muon neutrino ( $\nu_\mu$ ) and a tau neutrino ( $\nu_\tau$ ), as well as the corresponding antineutrinos ( $\bar{\nu}_\mu, \bar{\nu}_\tau$ ). Why there are three different charged leptons, each with its corresponding neutrino, is another question for which no satisfying answer is currently known.

Basic properties of leptons are summarized in Table 4.2. The electric charge listed is in units of  $|e|$ . Neutrinos have much smaller rest masses than the charged leptons, so much smaller that it is extraordinarily difficult to measure neutrino masses (and no one has yet succeeded). However, the observation of neutrino oscillations, which will be discussed in a later chapter, implies that neutrinos have non-zero masses. But at the moment only an upper bound on the actual values of the neutrino masses is known.

As indicated in Table 4.2, heavier (*i.e.*, more massive) leptons decay into lighter ones. These are weak interaction processes. The muon decays into an electron plus an electron antineutrino and a muon neutrino. The heavier tau has more options, decaying to both electrons and muons, and into a final state with hadrons (two pions) and just a single light lepton, the tau neutrino. In all these processes *lepton number*, denoted  $L$ , is conserved. Lepton number is defined as the total number of leptons *minus* antileptons,

$$L \equiv (\# \text{ leptons}) - (\# \text{ antileptons}). \quad (4.7.1)$$

All known interactions conserve lepton number.<sup>11</sup> Nearly all processes (but not neutrino oscillations)

<sup>11</sup>Actually, this is not quite true. The theory of weak interactions predicts extremely rare processes which change lepton number. However, the rate of these processes is so small that lepton number violation is completely unobservable.

conserve lepton number separately for each species or “flavor” of lepton: electron, muon, and tau. The electron-type lepton number  $L_e$  counts the number of electrons and electron neutrinos, minus the number of positrons and electron antineutrinos. Likewise,  $L_\mu$  counts muons and muon neutrinos (minus their antiparticles), and  $L_\tau$  counts taus and tau neutrinos (minus their antiparticles). In the decay  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ , for example, both initial and final states have  $L_e = 0$  and  $L_\mu = +1$ . Similarly, each of the final states listed in Table 4.2 for the decay of a  $\tau^-$  has  $L_\tau = +1$  while  $L_e = L_\mu = 0$ .

## 4.8 Spin and statistics

Important attributes of particles, which will play essential roles in understanding possible interactions and decays, include their mass, spin, electric charge, lepton number, and a few other “quantum numbers” which will be introduced later. One very basic property concerns the value of a particle’s spin. As indicated above, many fundamental particles (including protons, neutrons, and all the leptons) have half-integer values of spin (in units of  $\hbar$ ).<sup>12</sup> Such particles are called *fermions*. Other particles (such as the photon, and nuclei such as  ${}^2_1\text{H}$ ,  ${}^4_2\text{He}$  and  ${}^{12}_6\text{C}$ ) have integer values of spin. These are called *bosons*.

The spin of a particle determines how its quantum states behave under rotations. (You should be reasonably familiar with the case of spin 1/2.) In particular, how a particle behaves under a  $2\pi$  (*i.e.*,  $360^\circ$ ) rotation is determined by whether its spin is an integer, or a half-integer. Bosons, with integer spin, are *unchanged* after a  $2\pi$  rotation, the same as one would expect from classical physics. Fermions, with half-integer spin, have quantum states which, after a  $2\pi$  rotation, come back to *minus* the initial state. In other words, the phase of the state of any fermion changes by  $\pi$  under a  $360^\circ$  rotation.

Of course, such an overall change of phase only matters in the context of quantum mechanics. The intrinsic difference between bosons and fermions is especially clear in quantum field theory (QFT), where one defines operators to represent the creation or removal of particles. For bosons, such operators commute ( $[A, B] \equiv AB - BA = 0$ , so  $AB = BA$ ), while for fermions these operators must *anti-commute* ( $\{A, B\} \equiv AB + BA = 0$ , or  $AB = -BA$ ). Consequently, when one builds multiparticle states out of fermions, the result must be *anti-symmetric* under the interchange of any pair of identical fermions. This immediately leads to the Pauli Exclusion principle — no two identical fermions can reside in the same quantum state, since such a situation would necessarily be symmetric under interchange. On the other hand, multiparticle states constructed from bosons must be symmetric under the interchange of any two identical bosons. Consequently, any number of identical bosons can reside in the same quantum state (and lasers really do produce beams of coherent photons). This connection between spin and interchange symmetry is referred to as the Spin-Statistics Theorem.

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<sup>12</sup>“Half-integer” is shorthand for any integer plus 1/2.