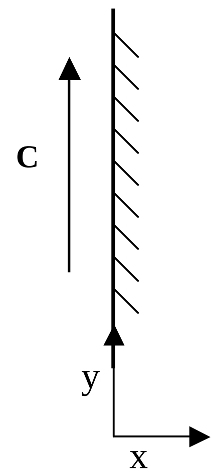
Kelvin Waves

Kelvin waves are important in tidal wave propagation along boundaries, in wind-driven variability in the coastal ocean, and in El Niño. The mathematical derivation for Kelvin waves can be found in Knauss. Consider the eastern boundary of a sea. The basic dynamics of Kelvin waves the same as for Poincare waves

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$



For the Kelvin wave, we need a solution such that the velocity into the wall is identically zero everywhere

$$u = 0$$

which reduces the force balance to

$$-fv = -g\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y}$$

Notice that the cross-shore (x) momentum balance is *geostrophic,* while the along shore momentum balance (y) is the same as that for shallow water gravity waves. The solution for the sea surface height becomes

$$\eta = \eta_o \exp(x/L_D)\cos(ky - \omega t)$$

The wave propagates along the boundary (*poleward* along an eastern boundary, *equatorward* along a western boundary) with its maximum amplitude at the boundary.

The wave amplitude decays offshore with a scale equal to the deformation radius $L_D = \frac{\sqrt{gH}}{f}$. The wave is *trapped* to the boundary.

The *dispersion relation* is

$$\omega = k\sqrt{gH}$$

the same as for the shallow water wave. The wave is *non-dispersive* and travels at the shallow water surface gravity wave speed. There are also *internal* Kelvin waves that travel at the internal gravity wave speed and whose decay scale offshore is the *internal* deformation radius.

$$L_D = \frac{\sqrt{g'H}}{f}$$

where now H is mean depth of the thermocline.

For an ocean 4000m thick, the Kelvin waves propagates at a speed of about 200 m/s. The tide travels as a Kelvin wave around the basin, and this is the speed with which it travels.

For an internal Kelvin waves travels.
$g' = g\dfrac{\Delta\rho}{\rho} = 0.03 m/s^2$, H=200m. At 20 N, $f = 2.5\times10^{-4}/s$
we get a Rossby radius of about 50km (it is less at higher latitude) and a phase speed of about 2.4 m/s.

Equatorial Kelvin waves

On the Equator, they are trapped to a boundary (in this case the "boundary" between positive and negative Coriolis parameter, the equator) and propagate to the east to flatten the thermocline. This results in a deeper thermocline and warmer surface temperatures in the east. As the sea surface temperatures increase in the central Pacific, the region of rising motion associated with deep convection also moves to the east, consistent with the negative anomaly in the SOI and the shifting of the Walker circulation to the east. As more westerly wind events occur, the thermocline flattens further still.

The Kelvin wave has a Gaussian shape, symmetric about the equator

$$\eta(x,t) = \exp(-y^2/L_D^2)\cos(kx-\omega t)$$

It propagates eastward with the shallow water gravity wave speed. The decay length scale is the ***internal equatorial deformation radius***

$$L_R{}^2 = \frac{\sqrt{g' H_1}}{2\beta}$$

where

$$\beta = \frac{2\Omega\cos\theta}{R_e} = \frac{df}{dy}$$

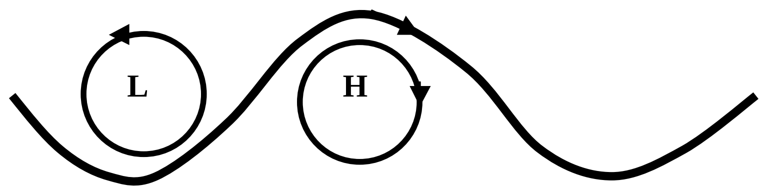
and *Re* is the radius of the Earth. So at the equator, we find that $\beta = 2.3\times10^{-11}/m/s$. If g'=0.03, and $H_1 = 200m$, then the internal equatorial deformation radius will be about 200 km. The internal Kelvin was would travel at speed 2.4 m/s.

Rossby Waves

Carl Gustav Rossby originally discovered these waves, which he called **planetary waves**, in his investigation of disturbances in the midlatitude westerly winds. The large-scale meanders in the jet stream that are familiar from weather maps are Rossby waves.

Initially sinusoidal wiggles of the jet stream in the *xy*- plane amplify nonlinearly to become the alternating high and low pressure patterns that are associated with storm systems.

In the ocean, the analog is a meandering western boundary current. Through a dynamically similar process (baroclinic instability) the initially sinusoidal, smooth Rossby waves amplify to pinch off rings and generate the eddy fluxes that cause recirculation gyres. Rossby waves are also responsible for much of the subinertial variability near topography. Rossby waves of very long wavelength are important in communicating changes in the large-scale circulation to the gyre interior. Kelvin waves can run around the boundaries of an ocean and transmit information about changes in large-scale forcing or boundary conditions eastward along the equator. Communication of that information into the interior is accomplished by Rossby waves.



Restoring Force

The restoring force is the north-south gradient of ***potential vorticity***,

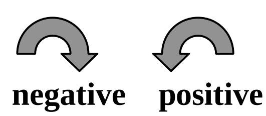
$$\mathrm{PV} = (f+\zeta)/H.$$

which is a conserved property of the fluid parcel associated with rotation. In the numerator is the Coriolis parameter (or ***planetary vorticity***)

$$f = 2\,\Omega \sin\theta$$

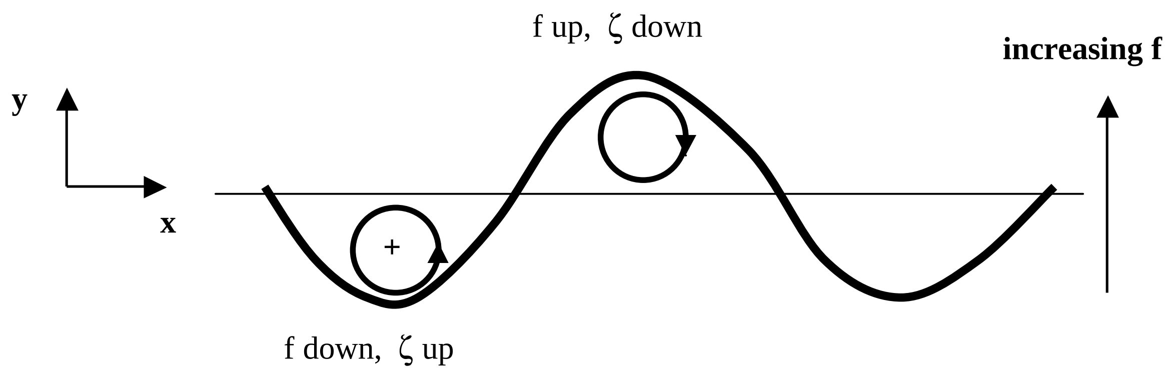
which describes the vertical component of the Earth's rotation and varies with latitude. The other component is ***relative vorticity*** ζ, which describes the rotation of the fluid motion itself

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$



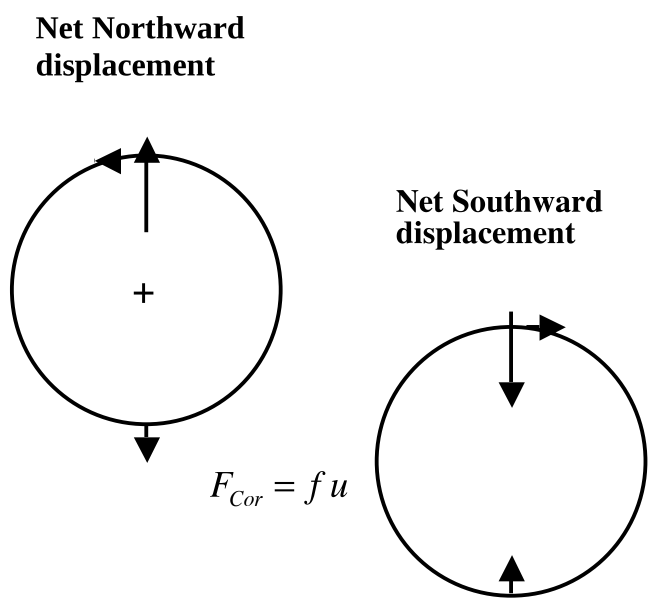
The depth H is the *ocean depth* for barotropic motion and a layer depth for baroclinic motion. The depth enters into the description of rotation in that, as a column of fluid is squashed (H decreases), its PV increases, or if it is stretched, its PV decreases. Since fluid columns want to preserve their PV, they resist changes in f (latitude) or changes in water (or layer) depth in the same wave that a parcel of fluid resists changes in density (depth). So the PV gradient can be due to either the variation in f with latitude, or to a variation in H (resulting in "topographic Rossby waves" or, when trapped near the coast, "continental shelf waves"). So far we've considered only waves where the restoring force was gravity, which oscillated in the vertical direction. Here the restoring force is horizontal, and *water parcels oscillate in the horizontal plane*.

Consider a line of water parcels in a barotropic ocean, initially lying along a line of constant latitude that are displaced into a sinusoidal wrinkle in the *xy*-plane and released. Since the ocean is barotropic, the whole water column moves together. The water parcels conserve PV = $(f+\zeta)/H$. What will happen? As a water column moves into a region of higher f, its relative vorticity will decrease in order to keep PV constant. This results in a clockwise rotation. Likewise, if it is displaced towards lower f (southward), it will increase its relative vorticity -- a tendency towards counterclockwise motion.



Now let's look at the Coriolis forces on the induced circulation pattern. For a given water parcel speed, the Coriolis force is larger at the northern end of the orbit, where f is larger. This difference between the Coriolis force at the north end of the orbit and at the south end results in a net restoring force back towards the equilibrium latitude. The strength of the restoring force is proportional to the spatial gradient in the Coriolis parameter,

$$\beta = \partial f / \partial y = \frac{2\Omega \cos\theta}{R_E}$$



Dynamical balance

The forces at work are the same as for rotating surface gravity waves (acceleration, Coriolis force, pressure gradient force) but now f is a function of latitude. For example, the east-west (x) momentum equation is:

$$\frac{\partial u}{\partial t} - (f_0 + \beta y)v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

where now we have explicitly written out the dependence of the Coriolis parameter *f* on the meridional direction y.

Dispersion relation

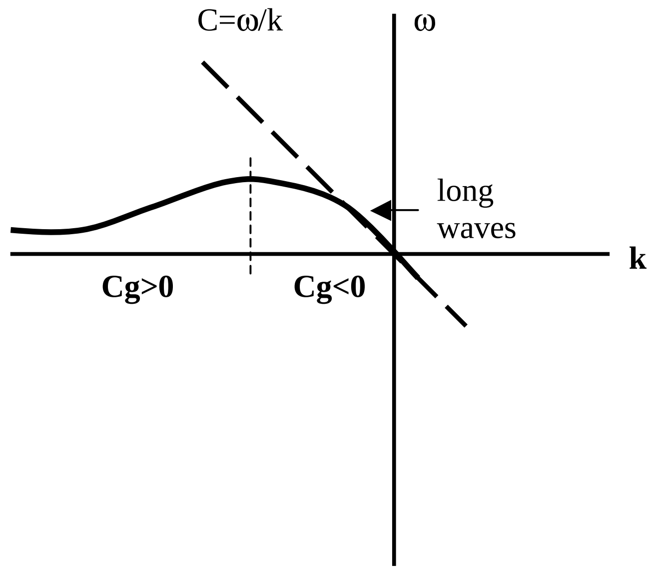
The dispersion relation includes two horizontal components of the wavenumber vector

$$\omega = \frac{-\beta k}{\frac{1}{L_D^2} + k^2 + l^2}$$

l = north-south wavenumber
k = east-west wavenumber
L_D = Rossby radius
(barotropic $L_D = \sqrt{gH}/f$ or baroclinic $L_D = \sqrt{g'H}/f$ as appropriate)

$$\beta = \partial f/\partial y = \frac{2\Omega\cos\theta}{R_E}$$

Here is what the dispersion relation looks like, where we have used the convention of using negative wavenumber k because of the minus sign in the equation. We have plotted the curve versus the x-component of wavenumber only.



Phase Speed.

These waves propagate in two-dimensions so **C** = (C_x, C_y). Note that the ratio $C_x = \omega/k$ is always negative, so these Rossby wave crests always propagate *westward*.
Actually, Rossby waves can propagate vertically, but here we are only going to consider surface waves such as occur on the sea surface (barotropic) or on the thermocline (baroclinic).

Energy propagation (group velocity).

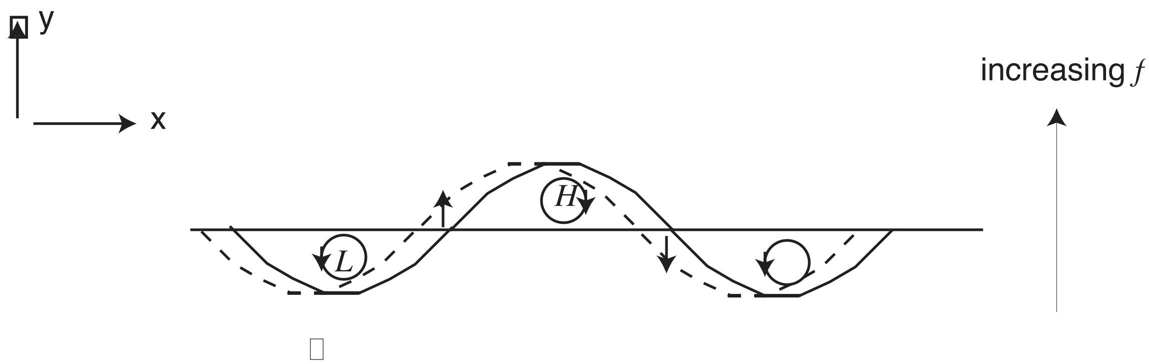
The slope of the dispersion curve $C_{gx} = \partial\omega/\partial k$ can be either positive or negative:

$C_{gx} < 0$ (westward) for *long* waves (low $|k|$) and $C_{gx} > 0$ (eastward) for *short* waves (large $|k|$).

Maximum frequency $\omega = \beta L_D/2$. For barotropic waves this is about 2 x 10^{-11} * 2000 km = 4 x 10^{-5} s^{-1} , i.e., a period of about 2 days. Baroclinic Rossby waves have much lower maximum frequencies because L_D is much smaller, and the typical period is months.

Long Rossby waves. An important limit of the dispersion relationship is found near the origin of the graph, where k (and l) are small. In this limit Rossby waves are nondispersive (i.e., ω = constant * k) and propagate exactly westward at the speed $C = C_g = -\beta L_D^2$. This is the limit that is important in communicating large-scale circulation changes to the interior. Maximum phase speed is found at the equator.

Solution form



Pressure and velocity. Rossby waves are *nearly geostrophic*. The displacements are associated with highs and lows in the sea surface with high on the right of the wave motion (N Hem). For the meandering western boundary current extension (and jet stream) there is a current that moves through the sinusoidal pattern … think of the pattern shown above as a perturbation that is added to the mean flow.

Westward propagation of crests and troughs in pressure. A way to visualize why the wave only propagates westward is to imagine the effect of the induced water parcel motion on the sinusoidal line. To the east of the high pressure "crest", as far as the next trough, the induced relative vorticity is such as to produce southward motion (look at the sense of both orbits, and imagine something being drawn through these counter-rotating rollers). To the west of the high-pressure crest, northward motion results. Let the pattern shift with the velocities, and you can see intuitively why phase propagates westward.

Examples

Shown below are time/longitude plots of sea surface height in the Pacific Ocean. You can see propagation to the west at all latitudes and the propagation speed is slower at higher latitudes. These are long waves and therefore have dispersion relation

$$\omega = -\beta L_D^2 k$$

Note that these waves are nondispersive. Once again, the waves can be either barotropic or baroclinic (external or internal). The waves propagate at the internal Rossby wave speed.

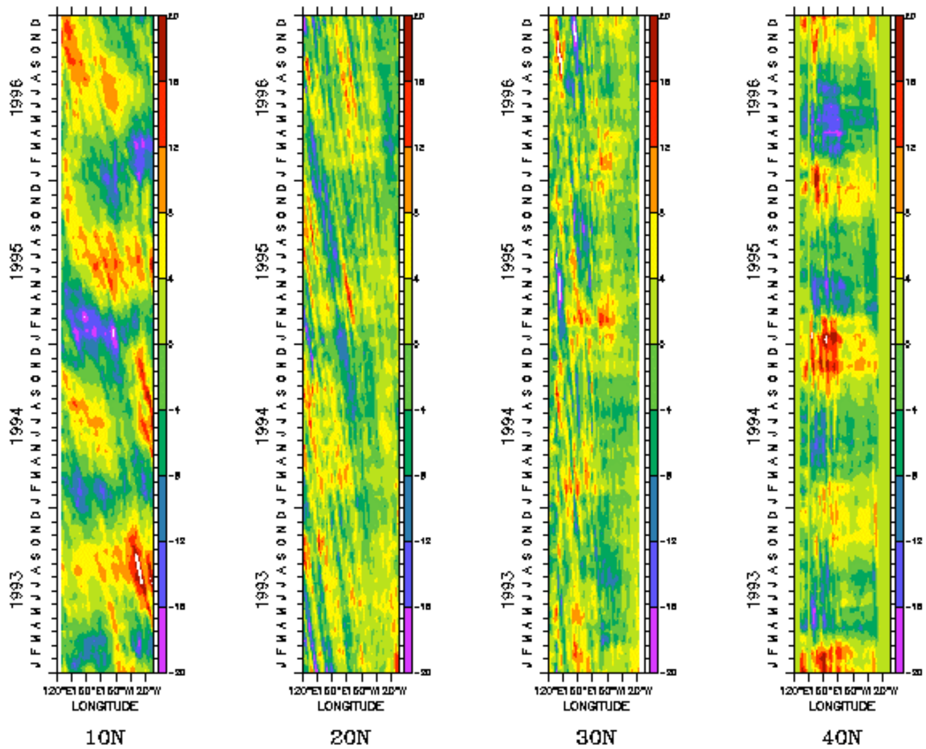
$c = -\beta L_D^2$.

For typical values of the parameters

$g' = g\dfrac{\Delta\rho}{\rho} = 0.03 m/s^2$, H=200m. At 20 N $\beta = 2\times10^{-11}/m/s$, $f = 2.5\times10^{-4}/s$

we get a Rossby radius of about 50km (it is less at higher latitude).

Then, the phase speed would be 5 cm/s. It would take about 6.5 years to go 10,000 km.



There are also equatorially trapped Rossby waves with meridional decale scale the equatorial deformation radius, with maximum thermocline deivations located off the equator and the phase speed given by

$c = -\sqrt{g'H}\ /\ 3$

Thus, if the phase speed for the Kelvin wave is 2.4 m/s, then the Rossby wave speed would be 0.8 m/s

The time it takes a Kelvin wave to cross the equator in about 50 days (2 months), while the Rossby wave take three times longer (6 months).