pt to prove the earth immovable do not argue against any motion than this diurnal one, though Aristotle does drop a gainst another motion ascribed to it by an ancient writer, of we shall speak in the proper place.

I am quite convinced of the force of your argument, but it a question for me from which I do not know how to free , and it is this: Copernicus attributed to the earth another n than the diurnal. By the rule just affirmed, this ought to n imperceptible to all observations on the earth, but be visible rest of the universe. It seems to me that one may deduce as essary consequence either that he was grossly mistaken in ing to the earth a motion corresponding to no appearance heavens generally, or that if the correspondent motion does then Ptolemy was equally at fault in not explaining it away, explained away the other.

This is very reasonably questioned, and when we come to f the other movement you will see how greatly Copernicus sed Ptolemy in acuteness and penetration of mind by seeing he latter did not—I mean the wonderful correspondence hich such a movement is reflected in all the other heavenly . But let us postpone this for the present and return to the nsideration, with respect to which I shall set forth, com- ng with the most general things, those reasons which seem r the earth's motion, so that we may then hear their refu- from Simplicio.

, let us consider only the immense bulk of the starry sphere rast with the smallness of the terrestrial globe, which is con- in the former so many millions of times. Now if we think of ocity of motion required to make a complete rotation in a day and night, I cannot persuade myself that anyone could nd who would think it the more reasonable and credible nat it was the celestial sphere which did the turning, and the ial globe which remained fixed.

f, throughout the whole variety of effects that could exist re as dependent upon these motions, all the same conse- s followed indifferently to a hairsbreadth from both posi- till my first general impression of them would be this: I think that anyone who considered it more reasonable for le universe to move in order to let the earth remain fixed e more irrational than one who should climb to the top of

DAY TWO (EXCERPTS)

your cupola just to get a view of the city and its environs, and then demand that the whole countryside should revolve around him so that he would not have to take the trouble to turn his head. Doubtless there are many and great advantages to be drawn from the new theory and not from the previous one (which to my mind is comparable with or even surpasses the above in absurdity), making the former more credible than the latter. But perhaps Aristotle, Ptolemy, and Simplicio ought to marshal their advantages against us and set them forth, too, if such there are; otherwise it will be clear to me that there are none and cannot be any.

. . . . .

SAGR. If we do not want to repeat what happened yesterday, please get back to the point; and you, Simplicio, begin producing those difficulties that seem to you to contradict this new arrangement of the universe.

SIMP. The arrangement is not new; rather, it is most ancient, as is shown by Aristotle refuting it, the following being his refutations:

"First, whether the earth is moved either in itself, being placed in the center, or in a circle, being removed from the center, it must be moved with such motion by force, for this is not its natural motion. Because if it were, it would belong also to all its particles. But every one of them is moved along a straight line toward the center. Being thus forced and preternatural, it cannot be everlasting. But the world order is eternal; therefore, etc.

"Second, it appears that all other bodies which move circularly lag behind, and are moved with more than one motion, except the *primum mobile*. Hence it would be necessary that the earth be moved also with two motions; and if that were so, there would have to be variations in the fixed stars. But such are not to be seen; rather, the same stars always rise and set in the same place without any variations.

"Third, the natural motion of the parts and of the whole is toward the center of the universe, and for that reason also it rests therein." He then discusses the question whether the motion of the parts is toward the center of the universe or merely toward that of the earth, concluding that their own tendency is to go toward the former, and that only accidentally do they go toward the latter, which question was argued at length yesterday.

Finally he strengthens this with a fourth argument taken from experiments with heavy bodies which, falling from a height, go

perpendicularly to the surface of the earth. Similarly, projectiles thrown vertically upward come down again perpendicularly by the same line, even though they have been thrown to immense height. These arguments are necessary proofs that their motion is toward the center of the earth, which, without moving in the least, awaits and receives them.

He then hints at the end that astronomers adduce other reasons in confirmation of the same conclusions—that the earth is in the center of the universe and immovable. A single one of these is that all the appearances seen in the movements of the stars correspond with this central position of the earth, which correspondence they would not otherwise possess. The others, adduced by Ptolemy and other astronomers, I can give you now if you like; or after you have said as much as you want to in reply to these of Aristotle.

SALV. The arguments produced on this matter are of two kinds. Some pertain to terrestrial events without relation to the stars, and others are drawn from the appearances and observations of celestial things. Aristotle's arguments are drawn mostly from the things around us, and he leaves the others to the astronomers. Hence it will be good, if it seems so to you, to examine those taken from earthly experiments, and thereafter we shall see to the other sort. And since some such arguments are adduced by Ptolemy, Tycho, and other astronomers and philosophers, in addition to their accepting, confirming, and supporting those of Aristotle, these may all be taken together in order not to have to give the same or similar answers twice. Therefore, Simplicio, present them, if you will; or, if you want me to relieve you of that burden, I am at your service.

SIMP. It will be better for you to bring them up, for having given them greater study you will have them readier at hand, and in great number too.

SALV. As the strongest reason of all is adduced that of heavy bodies, which, falling down from on high, go by a straight and vertical line to the surface of the earth. This is considered an irrefutable argument for the earth being motionless. For if it made the diurnal rotation, a tower from whose top a rock was let fall, being carried by the whirling of the earth, would travel many hundreds of yards to the east in the time the rock would consume in its fall, and the rock ought to strike the earth that distance away from the base of the tower. This effect they support with another experiment, which is to drop a lead ball from the top of the mast of a boat at rest,

noting the place where it hits, which is close to the foot of the mast; but if the same ball is dropped from the same place when the boat is moving, it will strike at that distance from the foot of the mast which the boat will have run during the time of fall of the lead, and for no other reason than that the natural movement of the ball when set free is in a straight line toward the center of the earth. This argument is fortified with the experiment of a projectile sent a very great distance upward; this might be a ball shot from a cannon aimed perpendicular to the horizon. In its flight and return this consumes so much time that in our latitude the cannon and we would be carried together many miles eastward by the earth, so that the ball, falling, could never come back near the gun, but would fall as far to the west as the earth had run on ahead.

They add moreover the third and very effective experiment of shooting a cannon ball point-blank to the east, and then another one with equal charge at the same elevation to the west; the shot toward the west ought to range a great deal farther out than the other one to the east. For when the ball goes toward the west, and the cannon, carried by the earth, goes east, the ball ought to strike the earth at a distance from the cannon equal to the sum of the two motions, one made by itself to the west, and the other by the gun, carried by the earth, toward the east. On the other hand, from the trip made by the ball shot toward the east it would be necessary to subtract that which was made by the cannon following it. Suppose, for example, that the journey made by the ball in itself was five miles and that the earth in that latitude traveled three miles during the flight of the ball; in the shot toward the west, the ball would fall to earth eight miles distant from the gun—that is, its own five toward the west and the gun's three to the east. But the shot toward the east would range no further than two miles, which is all that remains after subtracting from the five of the shot the three of the gun's motion toward the same place. Now experiment shows the shots to fall equally; therefore the cannon is motionless, and consequently the earth is, too. Not only this, but shots to the south or north likewise confirm the stability of the earth; for they would never hit the mark that one had aimed at, but would always slant toward the west because of the travel that would be made toward the east by the target, carried by the earth while the ball was in the air. And not merely shots along the meridians, but even those made to the east or west would not range truly; for the easterly

shots would carry high and the westerly low whenever they were aimed point-blank. Since the shots in both directions take the path of a tangent—that is, a line parallel to the horizon—and the horizon is always falling away to the east and rising in the west if the diurnal motion belongs to the earth (which is why the eastern stars appear to rise and the western stars to set), it follows that the target to the east would drop away under the shot, wherefore the shot would range high, and the rising of the western target would make the shot to the west low. Hence in no direction would shooting ever be accurate; and since experience is contrary to this, it must be said that the earth is immovable.

SIMP. Oh, these are excellent arguments, to which it will be impossible to find a valid answer.

. . . . .

SALV. Before going further I must tell Sagredo that I act the part of Copernicus in our arguments and wear his mask. As to the internal effects upon me of the arguments which I produce in his favor, I want you to be guided not by what I say when we are in the heat of acting out our play, but after I have put off the costume, for perhaps then you shall find me different from what you saw of me on the stage.

Now let us proceed. Ptolemy and his followers produce another experiment like that of the projectiles, and it pertains to things which, separated from the earth, remain in the air a long time, such as clouds and birds in flight. Since of these it cannot be said that they are carried by the earth, as they do not adhere to it, it does not seem possible that they could keep up with its swiftness; rather, it ought to look to us as if they were being moved very rapidly westward. If we, carried by the earth, pass along our parallel (which is at least sixteen thousand miles long) in twenty-four hours, how could the birds keep up on such a course? Whereas we see them fly east just as much as west or any other direction, without any detectable difference.

Besides this, if, when we travel on horseback, we feel the air strike rather strongly upon our faces, then what an east wind should we not perpetually feel when being borne in such a rapid course against the air! Yet no such effect is felt.

Here is another very ingenious argument taken from certain experiences. Circular motion has the property of casting off, scattering, and driving away from its center the parts of the moving body, whenever the motion is not sufficiently slow or the parts not

too solidly attached together. If, for example, we should very rapidly spin one of those great treadmills with which massive weights are moved by one or more men walking within them (such as huge stones used in mangles, or barges being dragged across the land from one waterway to another), then if the parts of this rapidly turned wheel were not very solidly joined, it would all come apart. Or, if many rocks or other heavy materials were strongly attached to its external surface, they would not be able to resist the impetus, and it would scatter them with great force to various places far from the wheel, and accordingly from its center. If, then, the earth were to be moved with so much greater a velocity, what weight, what tenacity of lime or mortar would hold rocks, buildings and whole cities so that they would not be hurled into the sky by such precipitous whirling? And men and beasts, none of which are attached to the earth; how would they resist such an impetus? Whereas on the contrary, we see these and the much less resistant pebbles, sand, and leaves reposing quietly upon the earth, and even falling back upon it with very slow motion.

Here, Simplicio, are the very potent arguments taken, so to speak, from terrestrial things. There remain those of the other kind; that is, those with relation to celestial appearances, which arguments tend still more to show that the earth is in the center of the universe, and consequently deprive it of the annual motion around that center as attributed to it by Copernicus. These being of rather a different nature, they can be brought forth after we have judged the strength of those already propounded.

SAGR. Well, what do you say, Simplicio? Does it seem to you that Salviati understands and knows how to explain the Ptolemaic and Aristotelian arguments? Do you think any Peripatetic understands the Copernican proofs so well?

SIMP. Had I not formed from previous arguments such a high opinion of Salviati's soundness of learning and Sagredo's sharpness of wit, with their kind permission I should wish to leave without hearing any more, as it would appear to me an impossible feat to contradict such palpable experiences. And without hearing any more, I should like to cling to my old opinion; for it seems to me that if, indeed, it is false, it may be excused on the grounds of its being supported by so many arguments of such great probability. If these are fallacies, what true demonstrations were ever more elegant?

SAGR. Yet we had better listen to Salviati's answers, which if true

must be even more beautiful; infinitely more beautiful, and the others extremely ugly, if that metaphysical proposition is correct which says that the true and the beautiful are one and the same, as are likewise the false and the ugly. Therefore, Salviati, let us not delay a moment more.

. . . . .

SALV. We may go on therefore to the fourth, with which it will be proper to deal at length, this being founded upon that experience from which most of the remaining arguments derive their force. Aristotle says, then, that a most certain proof of the earth's being motionless is that things projected perpendicularly upward are seen to return by the same line to the same place from which they were thrown, even though the movement is extrememly high. This, he argues, could not happen if the earth moved, since in the time during which the projectile is moving upward and then downward it is separated from the earth, and the place from which the projectile began its motion would go a long way toward the east, thanks to the revolving of the earth, and the falling projectile would strike the earth that distance away from the place in question. Thus we can accommodate here the argument of the cannon ball as well as the other argument, used by Aristotle and Ptolemy, of seeing heavy bodies falling from great heights along a straight line perpendicular to the surface of the earth. Now, in order to begin to untie these knots, I ask Simplicio by what means he would prove that freely falling bodies go along straight and perpendicular lines directed toward the center, should anyone refuse to grant this to Aristotle and Ptolemy.

SIMP. By means of the senses, which assure us that the tower is straight and perpendicular, and which show us that a falling stone goes along grazing it, without deviating a hairsbreadth to one side or the other, and strikes at the foot of the tower exactly under the place from which it was dropped.

SALV. But if it happened that the earth rotated, and consequently carried along the tower, and if the falling stone were seen to graze the side of the tower just the same, what would its motion then have to be?

SIMP. In that case one would have to say "its motions," for there would be one with which it went from top to bottom, and another one needed for following the path of the tower.

SALV. The motion would then be a compound of two motions; the one with which it measures the tower, and the other with which it follows it. From this compounding it would follow that the rock would no longer describe that simple straight perpendicular line, but a slanting one, and perhaps not straight.

SIMP. I don't know about its not being straight, but I understand well enough that it would have to be slanting, and different from the straight perpendicular line it would describe with the earth motionless.

SALV. Hence just from seeing the falling stone graze the tower, you could not say for sure that it described a straight and perpendicular line, unless you first assumed the earth to stand still.

SIMP. Exactly so; for if the earth were moving, the motion of the stone would be slanting and not perpendicular.

SALV. Then here, clear and evident, is the paralogism of Aristotle and of Ptolemy, discovered by you yourself. They take as known that which is intended to be proved.

SIMP. In what way? It looks to me like a syllogism in proper form, and not a *petitio principii.*

SALV. In this way: Does he not, in his proof, take the conclusion as unknown?

SIMP. Unknown, for otherwise it would be superfluous to prove it.

SALV. And the middle term; does he not require that to be known?

SIMP. Of course; otherwise it would be an attempt to prove *ignotum per aeque ignotum.*

SALV. Our conclusion, which is unknown and is to be proved; is this not the motionlessness of the earth?

SIMP. That is what it is.

SALV. Is not the middle term which must be known, the straight and perpendicular fall of the stone?

SIMP. That is the middle term.

SALV. But wasn't it concluded a little while ago that we could not have any knowledge of this fall being straight and perpendicular unless it was first known that the earth stood still? Therefore in your syllogism, the certainty of the middle term is drawn from the uncertainty of the conclusion. Thus you see how, and how badly, it is a paralogism.

. . . . .

SAGR. I am content to excuse you from this recital for the time being, on condition that this shall be one of the propositions saved, among others, for examination in some special session, since such

information is highly desirable to me. In the meanwhile let us get back to the line described by the body falling from the top of a tower to its base.

SALV. If the straight movement toward the center of the earth were uniform, and the circular motion toward the east were also uniform, the two could be compounded into a spiral line; one of those defined by Archimedes in his book about the spirals bearing his name, which are those generated when a point moves uniformly along a straight line which is being uniformly rotated about a fixed point at one of its extremities. But since the motion of the falling weight is continually accelerated, the line compounded of the two movements must have an ever-increasing ratio of successive distances from the circumference of that circle which would have been marked out by the center of gravity of the stone had it always remained on the tower. It is also required that this departure be small at the beginning—or rather minimal, even the least possible. For leaving from rest (that is, from the privation of downward motion) and entering into motion straight down, the falling weight must pass through every degree of slowness that exists between rest and any speed of motion. These degrees are infinite, as was discussed at length and decided already.

Supposing, then, that such is the progress of acceleraton; it being further true that the descending weight tends to end at the center of the earth, then the line of its compound motion must be such as to travel away from the top of the tower at an ever-increasing rate. To put it better, this line leaves from the circle described by the top of the tower because of the revolution of the earth, its departure from that circle being less *ad infinitum* according as the moving body is found to be less and less removed from the point where it was first placed. Moreover, this line of compound motion must tend to terminate at the center of the earth. Now, making these two assumptions, I draw the circle BI with A as a center and radius AB, which represents the terrestrial globe. Next, prolonging AB to C, the height of the tower BC is drawn; this, carried by the earth along the circumference BI, marks out with its top the arc CD.

Now dividing line CA at its midpoint E, and taking E as a center and EC as radius, the semicircle CIA is described, along which I think it very probable that a stone dropped from the top of the

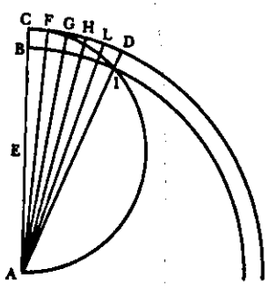

Fig. 1

tower C will move, with a motion composed of the general circular movement and its own straight one.

For if equal sections CF, FG, GH, HL are marked on the circumference CD, and straight lines are drawn to the center A from the points F, G, H, and L, the parts of these intercepted between the two circles CD and BI represent always the same tower CB, carried by the earth's globe toward DI. And the points where these lines are cut by the arc of the semicircle CA are the places at which the falling stone will be found at the various times. Now these points become more distant from the top of the tower in an ever-increasing proportion, and that is what makes its straight motion along the side of the tower show itself to be always more and more rapid. You may also see how, thanks to the infinite acuteness of the angle of contact between the two circles DC and CI, the departure of the stone from the circumference CFD (that is, from the top of the tower), is very, very small at the beginning, which is the same as saying that the downward motion is extremely slow; in fact, slower and slower *ad infinitum* according to its closeness to the point C, the state of rest. Finally, one may understand how such motion tends eventually to terminate at the center of the earth.

SAGR. I understand the whole thing perfectly, and I cannot think that the center of gravity of the falling body follows any other line but one such as this.

SALV. Hold on, Sagredo; I have also in store for you three little reflections of mine which may not displease you. The first is that if we consider the matter carefully, the body really moves in nothing other than a simple circular motion, just as when it rested on the tower it moved with a simple circular motion.

The second is even prettier; it moves not one whit more nor less than if it had continued resting on the tower; for the arcs CF, FG, GH, etc., which it would have passed through staying always on the tower, are precisely equal to the arcs of the circumference CI corresponding to the same CF, FG, GH, etc.

From this there follows a third marvel—that the true and real motion of the stone is never accelerated at all, but is always equable and uniform. For all these arcs marked equally on the circumference CD, and corresponding arcs marked on the circumference CI, are passed over in equal times. So we need not look for any other causes of acceleration or any other motions, for the moving body, whether remaining on the tower or falling, moves always in the same manner; that is, circularly, with the same rapidity, and with the same uniformity.

Now tell me what you think of these curiosities of mine.

SAGR. I tell you that I cannot find words to express the admiration they cause in me; and so far as my mind can make out at present, I do not believe that there is any other way in which these things can happen. I sincerely wish that all proofs by philosophers had half the probability of this one. Just to complete my satisfaction, I should like to hear the proof that those arcs are equal.

SALV. The demonstration is very easy. Suppose a line to be drawn from I to E; now the radius of the circle CD, that is the line CA, being double the radius CE of the circle CI, the circumference of the former will be double that of the latter, and every arc of the larger circle will be double the similar arc of the smaller. Thus half the arc of the larger circle is equal to the arc of the lesser. And since the angle CEI, made at the center E of the lesser circle and subtending the arc CI, is double the angle CAD, made at the center A of the larger circle and subtending the arc CD, then the arc CD is one-half of the arc in the larger circle similar to the arc CI. Hence the two arcs CD and CI are equal; and the same may be demonstrated in the same way for all the other parts. But that the descent of heavy bodies does take place in exactly this way, I will not at present declare; I shall only say that if the line described by a falling body is not exactly this, it is very near to it.

SAGR. Well, Salviati, there is another remarkable thing which I have just been reflecting about. It is that, according to these considerations, straight motion goes entirely out the window and nature never makes any use of it all. Even that use which you granted to

it at the beginning, of restoring to their places such integral, natural bodies as were separated from the whole and badly disorganized, is now taken away and assigned to circular motion.

DISCOURSES CONCERNING THE TWO NEW SCIENCES

THE THIRD DAY

SALV. The present does not seem to be the proper time to investigate the cause of the acceleration of natural motion concerning which various opinions have been expressed by various philosophers, some explaining it by attraction to the center, others to repulsion between the very small parts of the body, while still others attribute it to a certain stress in the surrounding medium which closes in behind the falling body and drives it from one of its positions to another. Now, all these fantasies, and others too, ought to be examined; but it is not really worth while. At present it is the purpose of our Author merely to investigate and to demonstrate some of the properties of accelerated motion (whatever the cause of this acceleration may be)—meaning thereby a motion, such that the momentum of its velocity [*i momenti della sua velocita*] goes on increasing after departure from rest, in simple proportionality to the time, which is the same as saying that in equal time-intervals the body receives equal increments of velocity; and if we find the properties [of accelerated motion] which will be demonstrated later are realized in freely falling and accelerated bodies, we may conclude that the assumed definition includes such a motion of falling bodies and that their speed [*accelerazione*] goes on increasing as the time and the duration of the motion.

SAGR. So far as I see at present, the definition might have been put a little more clearly perhaps without changing the fundamental idea, namely, uniformly accelerated motion is such that its speed increases in proportion to the space traversed; so that, for example, the speed acquired by a body in falling four cubits would be double that acquired by a body falling two cubits and this latter speed would be double that acquired in the first cubit. Because there is no doubt but that a heavy body falling from the height of six cubits has, and