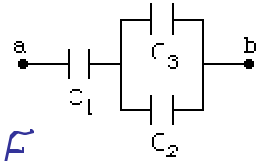


Part I. [24 points] Consider the capacitor network at left, with  $C_1 = 10 \mu\text{F}$ ,  $C_2 = 5 \mu\text{F}$ , and  $C_3 = 2 \mu\text{F}$ .



1. [4 points] Calculate the equivalent capacitance ( $C_{eq}$ ) of the network shown in the diagram.

- A.  $4.1 \mu\text{F}$
- B.  $7.0 \mu\text{F}$
- C.  $11 \mu\text{F}$
- D.  $17 \mu\text{F}$
- E.  $243000 \text{ F}$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2 + C_3}} = 4.1 \mu\text{F}$$

A  $17 \text{ V}$  battery is connected across the terminals  $a-b$  of this network.

2. [5 points] Calculate the energy stored in capacitor  $C_1$  when fully charged.

- A.  $2.5 \times 10^{-4} \text{ J}$
- B.  $7.1 \times 10^{-4} \text{ J}$
- C.  $1.9 \times 10^{-3} \text{ J}$
- D.  $4.2 \times 10^{-3} \text{ J}$
- E.  $8.5 \times 10^{17} \text{ J}$

$U_1 = \frac{1}{2} C_1 V_1^2$  because in series.

$Q_1 = C_1 V_1$

$Q_1 = C_{eq} V$

$\Rightarrow V_1 = \frac{2}{2} C_1 \left( \frac{Q_1}{C_1} \right)^2$

$= \frac{1}{2} C_1 \left( \frac{C_{eq} V}{C_1} \right)^2$

$= \frac{1}{2} \frac{C_{eq}^2 V^2}{C_1} = 2.45 \times 10^{-4} \text{ J}$

3. [5 points] What is the energy stored in capacitor  $C_2$ ?

- A.  $1.5 \times 10^{-5} \text{ J}$
- B.  $6.5 \times 10^{-5} \text{ J}$
- C.  $2.5 \times 10^{-4} \text{ J}$
- D.  $3.5 \times 10^{-4} \text{ J}$
- E.  $4.3 \times 10^{17} \text{ J}$

$U_2 = \frac{1}{2} C_2 V_2^2$

$V_2 = V - V_1$

$V_1 = \frac{C_{eq} V}{C_1} = 7 \text{ V} \Rightarrow V_2 = 10 \text{ V}$

$U_2 = 2.5 \times 10^{-4} \text{ J}$

4. [5 points] A new battery is hooked up to the capacitor network. The capacitors are shown to hold  $1.2 \text{ J}$  of energy. What is the EMF of the battery?

- A.  $0.0031 \text{ V}$
- B.  $380 \text{ V}$
- C.  $458 \text{ V}$
- D.  $586 \text{ V}$
- E.  $760 \text{ V}$

$U = \frac{1}{2} C_{eq} V^2$

$V = \sqrt{\frac{2U}{C_{eq}}} = 764.1 \text{ V}$

5. [5 points] A dielectric of  $k=1.5$  is inserted to  $C_1$  while the network is still connected to the  $17 \text{ V}$  battery. What is  $C_{eq}$  now?

- A.  $5.0 \mu\text{F}$
- B.  $7.0 \mu\text{F}$
- C.  $18 \mu\text{F}$
- D.  $24 \mu\text{F}$
- E.  $2.0 \times 10^5 \text{ F}$

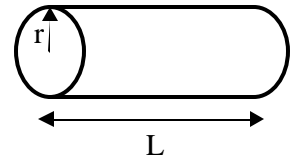
$C_1 \rightarrow kC_1$

$$C_{eq} = \frac{1}{\frac{1}{kC_1} + \frac{1}{C_2 + C_3}} = 4.56 \mu\text{F}$$

This question is bad! The answer is  $\rho/4$ , not  $4\rho$ ! It is being removed from grading!

Part II. [31 Points]

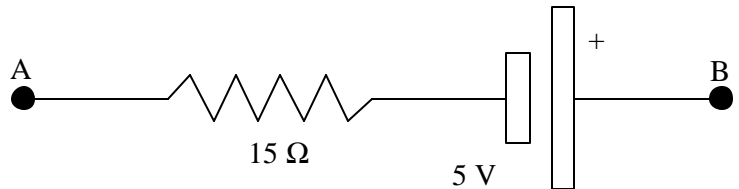
6. [5 points] RESISTOR, Inc., has made a cylindrical resistor out of a material with resistivity  $\rho = 9.71 \times 10^{-8} \Omega\text{m}$  (shown at right). The resistor is  $15 \Omega$ . If RESISTOR, Inc., wants to halve the diameter but keep the same resistance, they will have to find a material with a resistivity of



- A.  $2.4 \times 10^{-9} \Omega\text{m}$
- B.  $4.9 \times 10^{-8} \Omega\text{m}$
- C.  $9.7 \times 10^{-8} \Omega\text{m}$
- D.  $1.9 \times 10^{-7} \Omega\text{m}$
- E.  $3.9 \times 10^{-7} \Omega\text{m}$

$r \rightarrow r/2$   
 $R = \rho \frac{L}{A}$      $A \rightarrow A/4$   
 $\therefore \rho \rightarrow 4\rho = 3.88 \times 10^{-7} \Omega\text{m}$

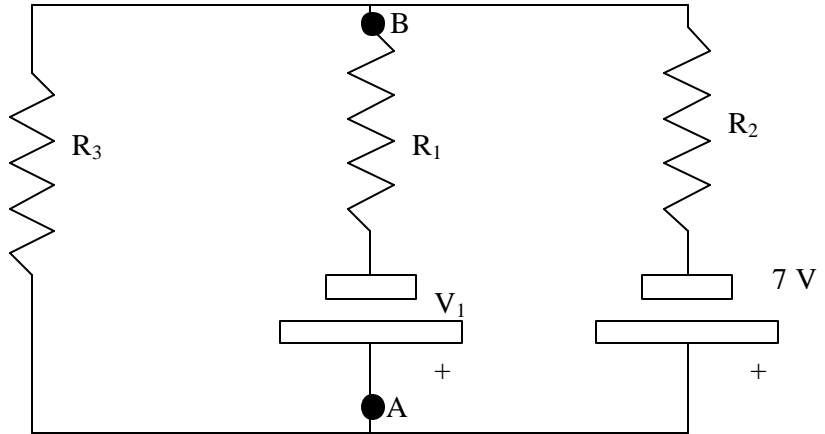
7. [4 points] USER, Inc., has purchased the resistor and placed the resistor in the circuit diagram shown. What is the measured voltage between points A and B? All USER, Inc. components are built and tested under vacuum.



- A. 1 V
- B. 3 V
- C. 5 V
- D. 20 V
- E. Not enough information to calculate

← No current  $\therefore$  no flow!!

USER, Inc., now constructs the circuit at left.  $R_1$  and  $V_1$  are adjusted so there no current flows through  $R_1$ . The resistors labeled  $R_2$  and  $R_3$  are  $500\Omega$ .



8. [5 points] What is the absolute value of the voltage drop across resistor  $R_1$ ?

- A. 0 V
- B. 1 V
- C. 3.5 V
- D. 5 V
- E. 7 V

*no current !!*

9. [5 points] What current flows through  $R_2$ ?

- A. 7 mA
- B. 28 mA
- C. 34 mA
- D. 140 A
- E. 7000 A

*only outer loop matters*

$\Rightarrow 7V = I \cdot (R_1 + R_2), I = 7 \times 10^{-3} A$

10. [4 points] What is the absolute value of the voltage difference between points A and B?

- A. 0 V
- B. 1 V
- C. 3.5 V
- D. 5 V
- E. 7 V

*Must be the same as the drop across  $R_3$  (or  $R_2$ )  $\rightarrow \frac{1}{2}$  the voltage (since  $R_1 = R_2$ )*

11. [4 points] What is the value of  $V_1$ ?

- A. 0 V
- B. 3.5 V
- C. 5 V
- D. 7 V
- E. Not enough information to determine.

*otherwise none of this would work.*

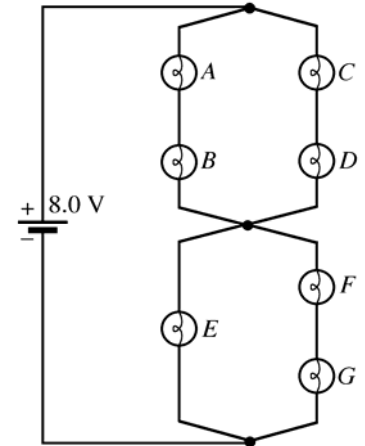
12. [4 points] What is the value of  $R_1$ ?

- A. 10  $\Omega$
- B. 150  $\Omega$
- C. 310  $\Omega$
- D. 500  $\Omega$
- E. Not enough information to determine.

*no current  $\rightarrow$  could be anything!*

## 122Z Lab Question

III. [25 pts total] Consider the battery and bulb circuit shown to the right. The battery is assumed to be ideal (that is, it can deliver any amount of current without changing its 8.0 volt output). All the bulbs are identical, but they will not perform like fixed-value resistors. Use the model for electric current that you developed in the Circuits I lab to answer the following questions.



- a. [5pts] Is the brightness of bulb A *greater* than, *equal* to, or *less* than the brightness of bulb D? Explain your answer.

*Bulb A is equal in brightness to bulb D because the two branches are identical with the same resistance to the flow of current. Therefore, the same current flows through both branches and therefore each has the same brightness.*

- b. [5 pts] Is the brightness of bulb F *greater* than, *equal* to, or *less* than the brightness of bulb G? Explain your answer.

*The brightness of bulb F is equal to the brightness of bulb G because these two bulbs are in series, which means that the same current must flow through both bulbs. If the same current flows through both, they have the same brightness*

- c. [5 pts] Is the brightness of bulb E *greater* than, *equal* to, or *less* than the brightness of bulb F? Explain your answer.

*The current that returns to the battery must split unequally between the two branches since the resistance of one bulb must be less than the resistance of two bulbs in series. Therefore more current goes through E as compared to bulb F and bulb G. Thus, the brightness of bulb E is greater than the brightness of bulb F.*

- d. [5 pts] Is the brightness of bulb C *greater* than, *equal* to, or *less* than the brightness of bulb F? Explain your answer.

*Since the two upper branches are equal, they have the same current (as explained in part-a). This same current recombines and flows down through the two branches of the lower loop which must have unequal currents (as stated in part-c). Since less than half the current flows through the branch with bulbs F and G, the brightness of bulb C is greater than the brightness of bulb F.*

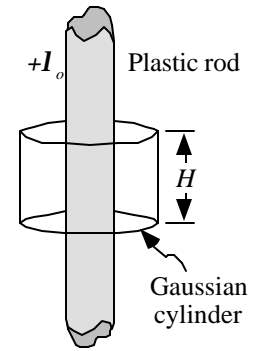
- e. [5 pts] Does the brightness of bulb B *increase*, *decrease*, or *remain the same* if bulb E is removed from its socket? Explain your answer.

*The resistance to the flow of current is less resistance in the lower loop when both branches have their bulbs in their respective sockets (i.e. there are two paths for current to flow through). When bulb E is removed, the lower loop now has more resistance to the flow of current. Therefore the current through the battery will reduce and thus the half going through B will reduce, making it less bright.*

IV. [20 pts] Shown at right is a small portion near the center of a very long plastic rod of radius  $a$ . The rod has a positive charge per unit length,  $+I_o$ . A Gaussian cylinder concentric with the rod has a radius  $R$  and a height  $H$ .

13. [4 pts] Find the charge enclosed by the Gaussian cylinder in terms of the given variables.

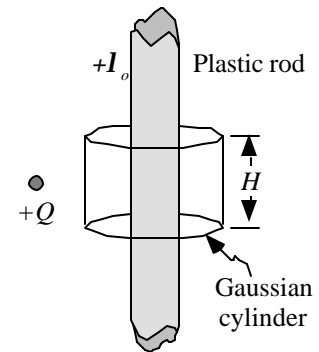
$$q_{enc} = IH$$



A positive point charge is now placed to the left of the Gaussian cylinder.

14. [4 pts] Is the net flux through the Gaussian cylinder *greater than, less than, or the same as* it was before the point charge was introduced? Explain.

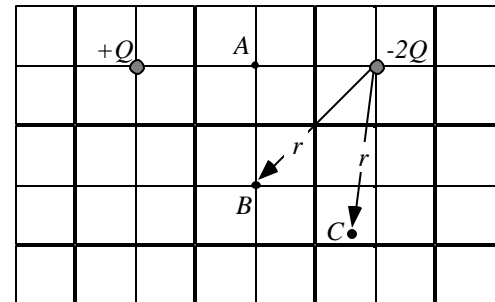
The charge enclosed hasn't changed, and the net flux is proportional to the charge enclosed, so the net flux through the Gaussian cylinder remains the same.



15. [4 pts] Is the flux through the curved side wall of the Gaussian cylinder *greater than, less than, or the same as* it was before the point charge was introduced? Explain.

The flux through the top and bottom of the cylinder was zero because the electric field was perpendicular to the area vectors for these locations. Now, however, there is a component of the electric field upwards at the top of the cylinder and downwards at the bottom of the cylinder. Thus there is a positive flux through these surfaces. Since the net flux through the cylinder is unchanged, the flux through the remaining surface of the cylinder, the curved side wall, must have decreased.

A  $+Q$  point charge and a  $-2Q$  point charge are fixed to a grid as shown. Points  $B$  and  $C$  are both a distance  $r$  from the  $-2Q$  charge.



16. [4 pts] Is the electric potential at point  $A$  *greater than, less than, or equal to* the electric potential at point  $B$ ? Explain.

The net electric field along a straight-line path from  $A$  and  $B$  has a component pointing toward the top of the page. Since the angle between the displacement vector and the electric field vector is greater than  $90^\circ$ , the net work done on a positive test charge by the field is negative.  $\Delta V = -W / q_{test}$  so the electric potential increases. Hence, the electric potential at  $A$  is less than at  $B$ .

17. [4 pts] Is the electric potential at point  $B$  *greater than, less than, or equal to* the electric potential at point  $C$ ? Explain.

The electric field due to the  $-2Q$  charge is perpendicular to a circular path from  $B$  to  $C$ , so there is no work done by this electric field. The electric field due to the  $+Q$  charge has a component in the same direction as the circular path, so the work done by this electric field is positive.  $\Delta V = -W / q_{test}$  so the electric potential decreases. Hence, the electric potential at  $B$  is greater than at  $C$ .