Name SOLUTION
Last First

Total Points ________

1. [33 pts] One of the four strings of a cello is plucked and begins to vibrate in its fundamental (lowest frequency) standing-wave mode. The frequency of this mode is 220 Hz. The string is fixed at both ends, has length $L = 60$ cm and *total* mass $m = 5$ g.

A. [6 pts] What is the tension in this string?

$\frac{1}{2}\lambda = L, \quad f\lambda = V = \sqrt{T/\mu}$

$\therefore f\,2L = \sqrt{T/\mu}$ so $T = \mu f^2 2^2 L^2 = m/L\,(2fL)^2 = 5.8\times10^2$ N

B. [6 pts] Determine the wavelengths (λ_1, λ_2, and λ_3) corresponding to the lowest three standing wave modes.

λ_1 is shown above and equals $2L$

$\lambda_2 = L$

$\frac{3}{2}\lambda_3 = L \therefore \lambda_3 = \frac{2}{3}L$

$\therefore \lambda_1 = 120$cm, $\lambda_2 = 60$cm, $\lambda_3 = 40$ cm

C. [6 pts] For the fundamental mode, what distance along the string corresponds to a phase change of $\pi/3$?

$0 \quad \pi/3 \quad 2\pi/3 \quad \pi$

$\frac{1}{3}$ of the length of the string

$\frac{1}{3}L = 20$ cm

D. [7 pts] Let $y(x, t)$ represent the transverse displacement of the string in the fundamental mode. The fixed ends of the string are at $x = 0$ and $x = L$. We can then write: $y(x, t) = y_0 \sin(kx)\cos(\omega t)$.

i. Express the constant k in terms of L.

$k = 2\pi/\lambda \therefore$ and $\lambda_1 = 2L$ so

$k = \frac{2\pi}{2L} = \frac{\pi}{L}$

ii. At what values of x will the string experience the maximum transverse:

(a) displacement?

$\sin(x)$ is max at $x = \frac{\pi}{2}$, $\frac{3\pi}{2}$

so $kx = \frac{\pi}{2}, \frac{3\pi}{2}$ so $x = \frac{\pi}{2k}$, $\frac{3\pi}{2k}$

(b) velocity?

$\dot{y} = -y_0 \sin kx \sin(\omega t)\,\omega \therefore kx = \frac{\pi}{2}, \frac{3\pi}{2} \therefore x = \frac{\pi}{2k}$

(c) acceleration?

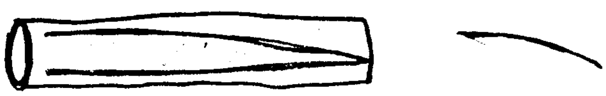
$\ddot{y} = -y_0 \sin kx \cos(\omega t)(\omega^2) \therefore$ same values

using fact $k = \frac{\pi}{L}$ we get $x = \frac{L}{2}$

iii. If the maximum transverse velocity u of the string at the location in ii.(b) is found to be 691 mm/s, what is the amplitude y_0 in the above equation?

$\dot{y} = -y_0 \underbrace{(1)(1)}_{\text{MAX VALS}}\,\omega \quad \therefore \frac{691\text{ mm/s}}{\omega} = \frac{691\text{ mm/s}}{2\pi(220\text{ htz})} = .5$ mm/s

E. [8 pts] A nearby hollow tube is open at one end and closed at the other. After the cello string is plucked, it is observed that a standing wave of sound in the fundamental mode is excited in the tube. What is the length of the tube? The speed of sound in air is 330 m/s.



Fundamental in tube would mean

$\frac{1}{4}\lambda_1 = L \quad \lambda_0 = 4L$

$\therefore f\lambda_1 = V_{Air}$

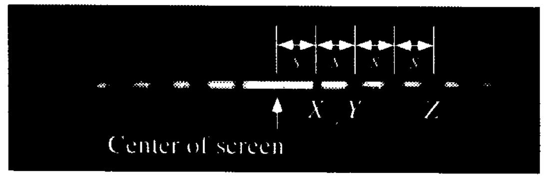
$f4L = V_{Air} \therefore \frac{V_{Air}}{4f} = L \quad \frac{330\text{ m/s}}{4(220\text{ htz})} = L$

$= .38$ m

Name (please print): ________ Solution ________ total points: ________

(last) (first)

2 [35 points] A mask with a single vertical slit of width *a* is placed between a laser and a distant screen. The photograph shows the resulting pattern on the screen. *s* is the distance between the center of the pattern and the first dark spot to the right of the center.



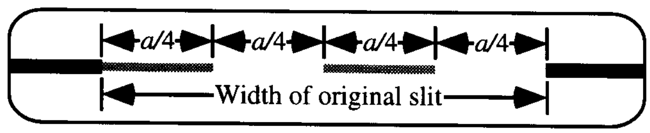
A. [7 pts] Is the slit width *greater than, less than,* or *equal to* λ, the wavelength of the laser light? Explain.

For a minimum on the screen, the path length difference from the two edges of the slit to the point on the screen is equal to one wavelength. Since the width of the slit is always greater than the path length difference, the slit width must be greater than the wavelength.

B. [7 pts] The pattern at the top of the page was produced by a laser that emits red light. If the laser were replaced by a laser that emits green light (*i.e.,* light with a shorter wavelength), would the distance from the center of the screen to the first dark spot to the right *increase, decrease,* or *stay the same?* Explain.

If light with a shorter wavelength were used, the path length difference (between the two edges of the slit) to the first minimum on the screen would decrease. Thus, the first minimum would be closer to the center of the screen.

C. Two quarters of the slit are now covered as shown in the magnified view of the slit.



Magnified view of slit

i. [7 pts] Does the brightness at point *X* *increase, decrease,* or *remain the same?* Explain.

For point X, the first (diffraction) minimum in the original pattern, the light from each point on the left half of the slit cancels the light from a point located a distance a/2 away, on the right half of the slit. After the first and third quarter of the slit are covered, each point on the second quarter of the slit is still canceled by the corresponding point on the fourth quarter of the slit. This results in an (interference) minimum on the screen, so the brightness remains the same (*i.e.,* zero).

ii. [7 pts] Does the brightness at point *Y* *increase, decrease,* or *remain the same?* Explain.

For point Y, the second (diffraction) minimum in the original pattern, the light from the two edges of the original slit has a path length difference of 2λ. Light from the two edges of each new slit (of width a/4) has a path length difference of $\lambda/2$ which does not correspond to a diffraction miminum. The centers of the new slits are a/2 apart, resulting in a path length difference of λ. Light from the two slits therefore interferes constructively (*i.e.,* there is an interference maximum), and the brightness at point Y increases.

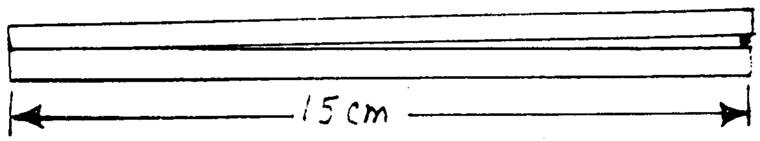
iii. [7 pts] Does the brightness at point *Z* *increase, decrease,* or *remain the same?* Explain.

For point Z, the fourth (diffraction) minimum in the original pattern, the light from the two edges of the original slit has a path length difference of 4λ. Light from the two edges of each new slit (of width a/4) has a path length difference of one wavelength. This results in a diffraction minimum for each of the two new slits, so the brightness remains the same (*i.e.,* zero).

Name: SOLUTION Total points: ____________

(last) (first)

Prob. ~~1~~ 3) Two flat rectangular glass plates (index of refraction n=1.52) are separated at one edge by a 10 μm diameter wire as shown below. The plates are illuminated from above by a sodium lamp (λ =589 nm). You study the light reflected from the film.



a. (4 pts.) Will there be a dark fringe or a bright fringe at the edge where the plates touch? Explain. $\boxed{\text{DARK FRINGE}}$

reflected wave from bottom of upper plate ($n=1.52 \rightarrow n=1$) is out of phase with the reflected wave from top of lower plate ($n=1 \rightarrow n=1.52$). These 2 waves cancel

b. (8 pts.) What is the total number of bright fringes you will observe?

get a bright fringe every time the path length up & down between the plates increases by λ

$$\therefore N = \frac{2(10\,\mu m)}{\lambda} = \frac{20\,000\ nm}{589\ nm} = \boxed{34}$$

The wedge-shaped gap is now filled with soapy water (index n=1.33).

c. (4 pts.) Are the fringes brighter or fainter than before the water was in place? Explain.

$\boxed{\text{FAINTER}}$ The reflection coefficient is related to the change in n.

For air/glass $\Delta n = 0.52$, for water/glass $\Delta n = 1.52 - 1.33 = 0.19$

d. (4 pts.) Are the fringes closer together or farther apart than before the water was in place? Explain. $\boxed{\text{CLOSER TOGETHER}}$

Wavelength in water is shorter than in air $\left(\lambda' = \frac{589\ nm}{1.33}\right)$ so a shorter distance gives a phase of 2π.

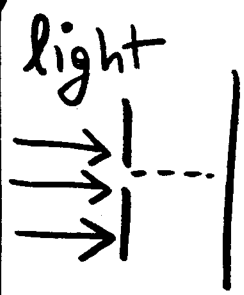
e. (5 pts.) A thin soap film, illuminated by white light, is held vertically and you observe the light reflected from the film. After a minute, the top of the film appears dark. What color do you see just below the dark region? Explain.

VIOLET (OR BLUE) The 1st bright fringe below the dark spot occurs when the film thickness $d = \lambda/4n$. Since the film is thinnest at the top the first bright fringe will occur for the shortest visible wavelength.

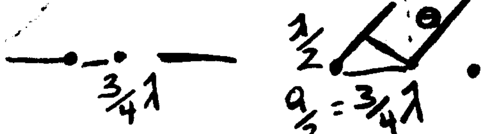
Name (please print): SOLUTION Total Points ________
last first

4. [34 pts] Parts A and B are independent.

A. [14 pts] Coherent light is incident on a mask with single hole that is $^{3}/_{2}\lambda$ in width. The light then falls on a distant screen.



i. Would there be any nodal lines for this arrangement? If so, find the angle θ (measured from the dashed line) at which the first nodal line lies. If there are no nodal lines, explain why not. yes



$\frac{a}{2} = \frac{3}{4}\lambda$

$\frac{a}{2}\sin\theta = \frac{\lambda}{2}$ $\quad \sin\theta = \frac{\lambda}{a}$

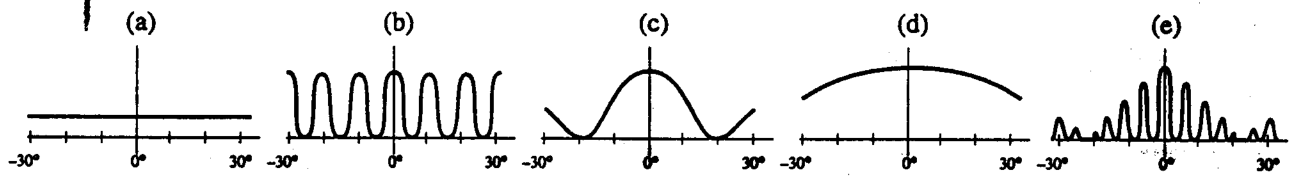
$\sin\theta = \frac{\lambda/2}{3/4\,\lambda} = \frac{4}{6} = \frac{2}{3}$

$\boxed{\theta = 42°}$

ii. Which of the diagrams below best represents the intensity pattern that would be observed on the screen? Explain.

(d.) d has the 1st min at 42° and a max at the center.

Intensity diagrams for screen (Note the screen spans only 30° on either side of sources)

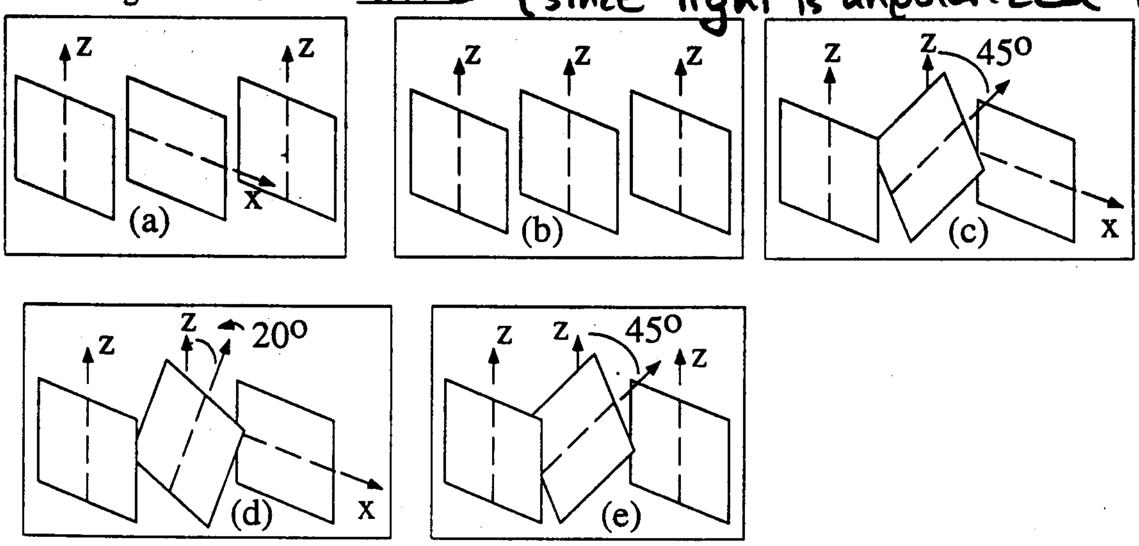


B. [20 pts] In the sketches below, three flat identical ideal polaroid sheets are arranged in five combinations, each with **unpolarized** light incident from the left. The transmission axis of each polaroid sheet is shown (dashed line) and is referenced to *xyz* coordinates. For each question, indicate **all** combinations (a b c d e) that satisfy the stated condition on the light transmitted by the combination. (Include all correct combinations and no incorrect combinations for full credit on each question.)

i. No light is transmitted a, [illegible] ii. Transmitted light is polarized along the x axis d, c

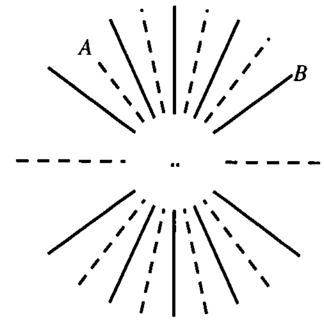
iii. Half of the incident light intensity is transmitted b

iv. All light is transmitted none (since light is unpolarized at first so ½ will go through 1st polarizer



Name (please print) ____________________ Total __________
last first

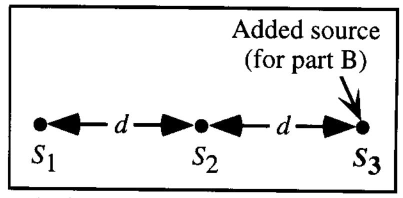
5. [30 pts total] The diagram at right shows all nodal lines (dashed) and all lines of maximum constructive interference (solid) due to two sources of periodic circular water waves that are in phase and a distance *d* apart.



A. [9 pts] Determine *d*, the distance between the sources, in terms of λ. Explain how you can tell from the diagram.

For points along the line the contains the sources, ΔD is equal to the source separation [$\Delta D = d \sin \theta = d$ for $\theta = 90°$]. The nodal lines and lines of maximum constructive interference correspond to the values of ΔD shown in the diagram at right. Thus, $d = 5\lambda/2$.

B. Suppose that a third source, in phase with S_1 and S_2, is placed to the right of S_2 *without moving S_1 or S_2*, as shown at right.



Enlarged view of three sources

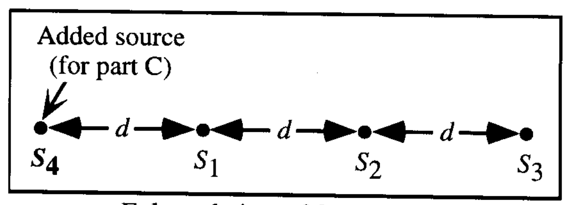
i. [7 pts] After the third source is added, will there still be a nodal line at the same angle as line *A* above? Explain your

No, the waves from the first two sources add to zero at points on line A [line A is a nodal line]. Using superposition, adding a third [non-zero] wave to the "zero" produced by the waves from the first two sources means that there will no longer be a nodal line at the location of line A.

ii. [7 pts] After the third source is added, will there still be maximum constructive interference at the same angle as line *B* above? Explain your reasoning.

Yes. Since S_2 and S_3 are the same distance apart as S_1 and S_2, the difference in distances from S_2 and S_3 to a distant point on line B is the same as the difference in distances from that point to S_1 and S_2. [$\Delta D = d \sin \theta$] Since the waves from S_1 and S_2 are in phase at distant points on line B, the waves from S_2 and S_3 are also in phase there. Thus the waves from all three sources are in phase there, and there will still be maximum constructive interference at the same angle as line B.

C. Suppose that a fourth source, in phase with the other three, is placed to the left of S_1, *without moving the other sources,* as shown.



Enlarged view of four sources

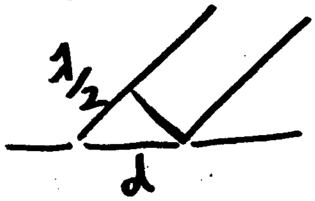
[7 pts] After the fourth source is added, will there be *maximum constructive interference*, *complete destructive interference*, or *neither* at the same angle as line *A* above? Explain your reasoning.

At this angle, waves from S_1 and S_2, a distance *d* apart, add to zero. Thus, the waves from S_4 and S_1, a distance *d* apart, add to zero at this angle and the waves from S_2 and S_3, a distance *d* apart also add to zero at this angle. Thus, the waves from all four sources add to zero at points on line A; there will be complete destructive interference on line A.

Name (please print): SOLUTION total points: ________
(last) (first)

6. [33 pts total] Parts A and B are independent.

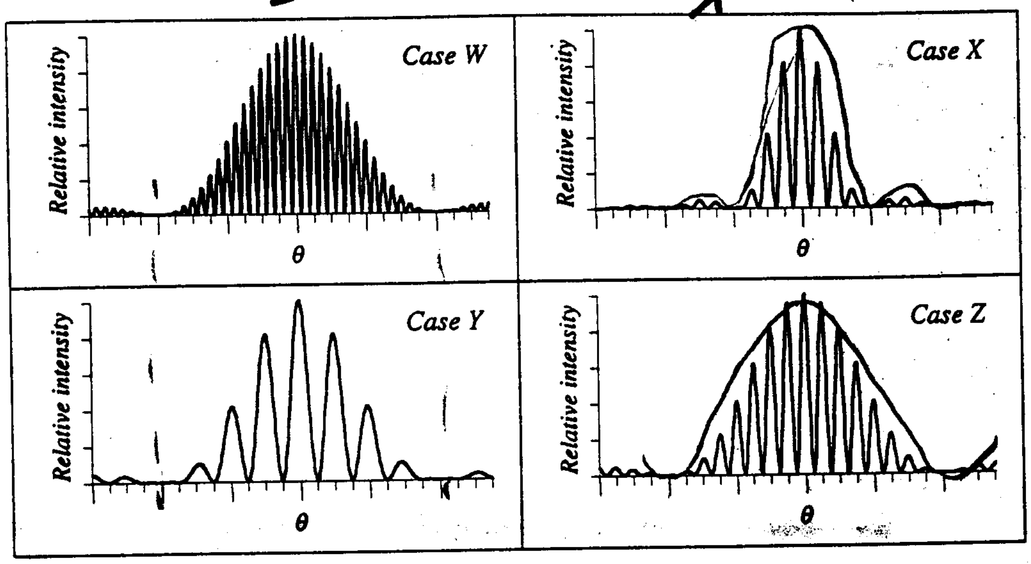
A. [9 pts] Light of wavelength $\lambda = 500$ nm is incident on a mask containing a pair of very narrow slits separated by a distance $d = 30$ μm. The distance between the center of the screen and the first dark fringe is $x = 1.5$ cm. Determine the distance L between the mask and the screen.



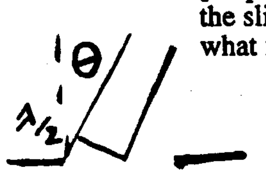
$d\sin\theta = \lambda/2$ and $\sin\theta \sim y/L$

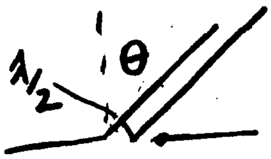
so $d\,y/L = \lambda/2$ ∴ $L = \dfrac{2dy}{\lambda} = 1.8\,\text{m}$

B. Each of the graphs at right shows *relative intensity* vs. θ for a double-slit experiment. The horizontal scale is the same for all graphs. The only possible differences among the four cases are the values of slit width and/or distance between the centers of the slits.



i. [12 pts] Rank the cases according to the ***width of the slits,*** from largest to smallest. If the width of the slits is the same in any two cases, state that explicitly. Explain your reasoning, clearly indicating what feature(s) of the graphs you used in making your ranking.



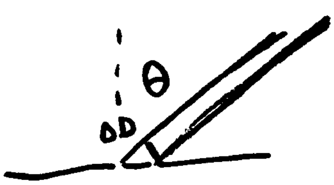


The larger the width the smaller the θ to the mins for the diffraction envelope

W = Y = Z < X

To have the same path diff. θ must be larger for smaller a

ii. [12 pts] Rank the cases according to the ***distance between the centers of the slits,*** from largest to smallest. If the distance between the centers of the slits is the same in any two cases, state that explicitly. Explain your reasoning, clearly indicating what feature(s) of the graphs you used in making your ranking.



The distance between the centers will determine the 2 source mins – the farther the 2 slits are from one another the smaller the angle θ to the 1st min. W > X = Z > Y

Again, to have the same path diff. θ must be larger for smaller a

3/17/98 (4)