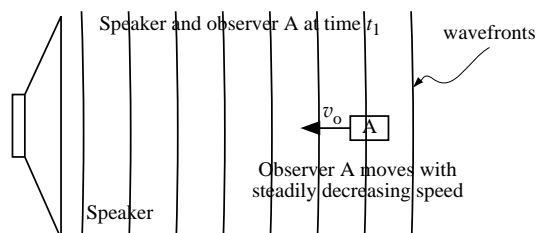


1. [25 pts] A speaker at rest on the ground emits a tone with frequency f_s .

- A. [8 pts] Observer A moves directly toward the speaker with steadily decreasing speed. At time t_1 , observer A has a speed of v_o , as shown in the figure at right.



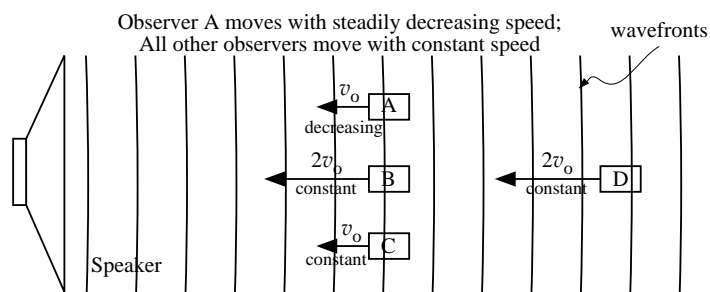
- i. [4pts] Is the frequency measured by observer A at time t_1 *greater than, less than, or equal to* f_s ? Explain.

As the observer approaches the speaker, the observer passes over more wavefronts in a given amount of time than the speaker is emitting (or than the observer would receive if at rest relative to the speaker), so the measured frequency is greater than f_s . (This is the Doppler shift: $f' = f [(v_{\text{sound}} + v_{\text{obs}}) / v_{\text{sound}}]$.)

- ii. [4 pts] Is the frequency measured by observer A at time t_1 *greater than, less than, or equal to* the frequency measured by observer A a short time later, at time t_2 ? Explain.

The measured frequency depends on the *speed* of the observer; the higher the speed of the approaching observer, the higher the measured frequency. Since observer A has a greater speed at time t_1 , the frequency at time t_1 is greater than that at time t_2 .

- B. [12 pts] In addition to observer A, observers B, C, and D each move *with a constant speed* directly toward the speaker, as shown in the figure at right.



- i. [4 pts] At time t_1 , is the frequency measured by observer A *greater than, less than, or equal to* the frequency measured by observer C? Explain.

At time t_1 , observers A and C have the same speed, so the frequencies they measure are equal to one another. (Note: the fact that observer A's speed is changing does not affect the result.)

- ii. [4 pts] At time t_1 , is the frequency measured by observer B *greater than, less than, or equal to* the frequency measured by observer C? Explain.

At time t_1 , observer B is moving with a *greater speed* than observer C, so B measures a frequency greater than what C measures. (Note: the fact that observers B and C have the same *position* relative to the speaker does not affect the result.)

- iii. [4 pts] At time t_1 , is the frequency measured by observer B *greater than, less than, or equal to* the frequency measured by observer D? Explain.

At time t_1 , observers B and D have the same speed, so the frequencies they measure are equal to one another. (Note: the fact that observers B and D have different *positions* relative to the speaker does not affect the result.)

- C. [5 pts] Observer C is traveling with a speed of 25 m/s. She measures the frequency from the speaker to be 440 Hz, a perfect concert A. . (The speed of sound in air is 343 m/s.)

Find f_s . Show your work and explain your reasoning.

$f' = f [(v_{\text{sound}} + v_{\text{obs}}) / v_{\text{sound}}]$, where f is the frequency in the rest frame of the source and f' is the frequency in the rest frame of (i.e., measured by) the observer. We want to solve for f . Rearranging terms to solve for f gives

$$f = f' [(v_{\text{sound}} / (v_{\text{sound}} + v_{\text{obs}}))] \quad f' = 440 \text{ Hz}, \quad v_{\text{sound}} = 343 \text{ m/s}, \quad v_{\text{obs}} = 25 \text{ m/s}$$

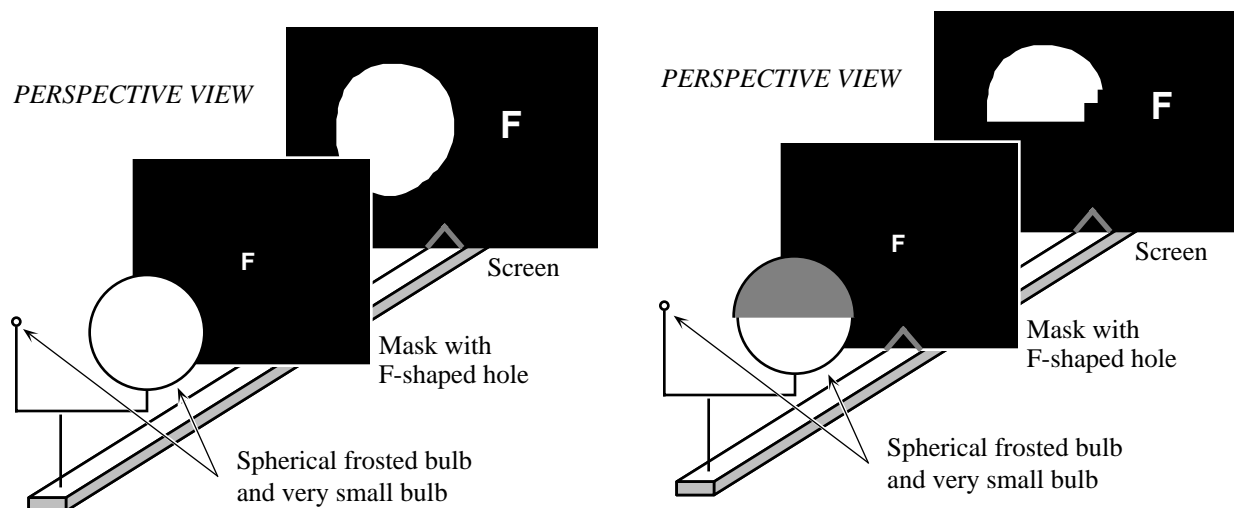
$$f_s = 440 \text{ Hz} [(343 \text{ m/s}) / (368 \text{ m/s})]$$

$$f_s = 410 \text{ Hz}$$

2. [25 points total] Parts A and B of this problem are independent.

- A. In two different experiments, a very small bulb is held next to a spherical frosted bulb, as shown below. (A frosted bulb is uniformly lit over its entire surface.) In each case, a mask with an F-shaped hole in it is placed between the bulbs and the screen. In the second experiment, the top half of the frosted bulb is covered with an opaque covering.

[16 pts] On each diagram below, sketch what you would see on the screen in that case when the bulbs are turned on. No explanation is required.

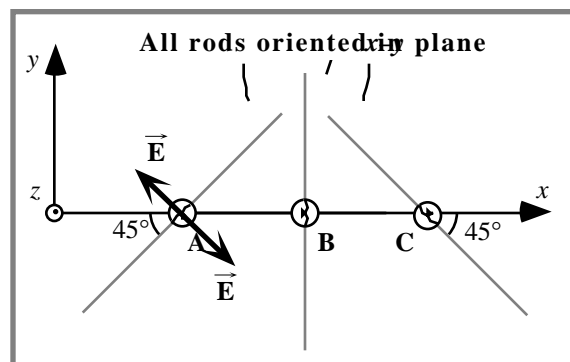


In the experiment on the left, the small bulb makes an F on the screen. The frosted bulb can be modeled as many small bulbs arranged in a circular shape; the shape on the screen can be thought of as a large number of F's arranged in the shape of a circle, for an overall shape that is round and slightly irregular. Similar reasoning gives the shape in the second experiment.

- B. An EM plane wave travels in the $+z$ **direction**. Three identical bulbs (A–C), each connected to identical, long conducting rods, are positioned so that the rods all lie in the x – y plane. (The EM plane wave is *not* shown.) **It is observed that bulbs B and C glow, but not bulb A.**

[9 pts] Is bulb B *brighter than*, *dimmer than*, or *the same brightness as* bulb C? Explain your reasoning.

The greater the component of the electric field along the direction of the device, the greater the brightness of the bulb. If bulb A does not glow, the electric field must be oriented perpendicular to the device containing bulb A, that is, in one of the two directions shown. The device containing bulb B makes a 45° angle with the electric field, and the device containing bulb C is parallel to the electric field, so bulb B is dimmer than bulb C.



SIDE VIEW DIAGRAM

Name (please print):

SOLUTION

(last)

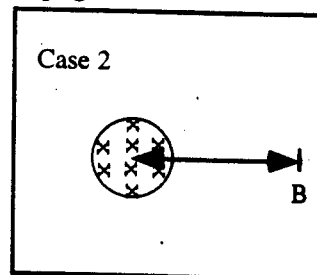
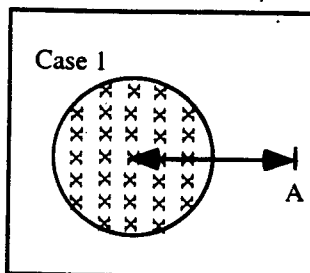
(first)

Total Points: _____

3. A. [10 pts] Two capacitors are made of parallel circular plates separated by the same distance. In case 1, the plates have radius $2R$; in case 2 the plates have radius R . The voltage across each capacitor is given by $V = V_0 + \alpha t$ (where V_0 and α are positive quantities.)

Two points, A and B, lie in a plane parallel to the capacitor plates that passes midway between them. Point A is $4R$ from the center of capacitor 1; point B is $4R$ from the center of capacitor 2. The diagram above shows the direction of the electric field between the plates for both cases. Ignore fringing fields (i.e., assume that the electric field inside the capacitor is uniform and that it is zero outside).

× denotes a vector into the page.

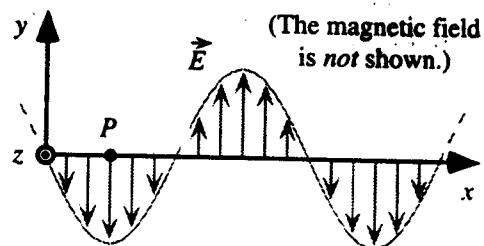


- i. What is the direction of the magnetic field at point A at the instant shown? Explain how you determined your answer. V is increasing $\Rightarrow E$ is increasing \Rightarrow flux through a circular Amperian loop that passes through point A is increasing $\Rightarrow \frac{d\Phi_E}{dt}$ is +ve so B field is non-zero and is directed DOWN at A ($\oint \vec{B} \cdot d\vec{s} > 0$)

- ii. Is the magnitude of the magnetic field at point A greater than less than or equal to the magnitude of the magnetic field at point B? Explain how you determined your answer.

In each case $B(2\pi(4R)) = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ $\Phi_E = EA$ where in this case $A = \text{area of cap}$
 $\frac{d\Phi_E}{dt} = A \frac{dE}{dt}$; $A_1 > A_2 \Rightarrow \frac{d\Phi_{E1}}{dt} > \frac{d\Phi_{E2}}{dt} \Rightarrow B_1 > B_2$

- B. An electromagnetic plane wave of wavelength 15 cm is propagating in the negative x-direction. The diagram at right is a representation of the electric field in a region near point P, where the electric field is in the negative y-direction at the instant shown and the intensity is 450 W/m^2 . The magnetic field is not shown.



- i. [3 pts] What is the direction of the magnetic field at point P at the instant shown? Explain how you determined your answer.

Poynting vector \vec{S} gives dir'n of propagation

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ \vec{S} is in -ve x dir'n & \vec{E} is in -ve y dir'n at P \Rightarrow B must be in +ve z dir'n

- ii. [3 pts] What is the magnitude of the magnetic field at point P at the instant shown?

$$I = \frac{1}{2\mu_0} E_m B_m \quad \therefore E_m = c B_m \Rightarrow I = \frac{c B_m^2}{2\mu_0} \Rightarrow B_m = \sqrt{\frac{2\mu_0 I}{c}} \quad (\text{either } 1.3 \text{ or } 1.9 \times 10^{-6} \text{ T})$$

$$\text{OR } = \boxed{1.9 \times 10^{-6} \text{ T}}$$

- ii. [9 pts] Write a set of equations that describe the x-, y-, and z- components of the magnetic field at point P as a function of time. Where possible, show values rather than variable names.

$$B_x(t) = 0$$

$$B_y(t) = 0$$

$$B_z(t) = B_m \cos \omega t \quad \text{where } B_m = 1.9 \times 10^{-6} \text{ T}$$

$$\omega = 1.3 \times 10^{10} \text{ s}^{-1}$$

dir'n of integration \Rightarrow area vector in finding flux is INTO page

$$-\hat{j} \times \hat{k} = -\hat{i}$$

$$\lambda v = c$$

$$\Rightarrow \lambda \frac{\omega}{2\pi} = c \Rightarrow \omega = \frac{2\pi c}{\lambda} = 1.3 \times 10^{10}$$

Name (please print): SOLUTIONS (JTW)
 (last) (first)

total points _____

4. [25 points] Refraction (Figs. 1a, 1b, 2a, and 2b all show cross-sectional top views.)

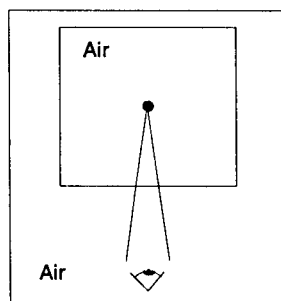


Fig. 1a

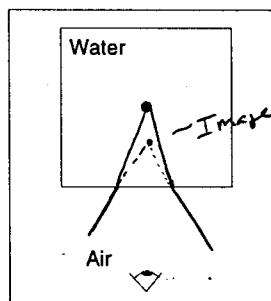


Fig 1b

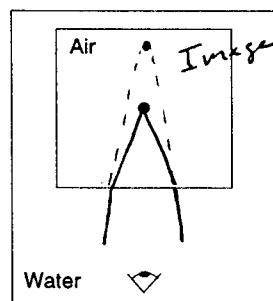


Fig. 2a

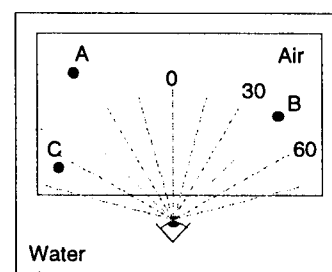


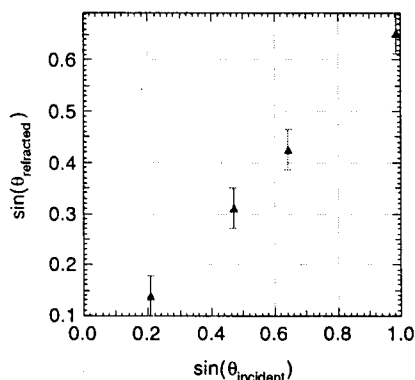
Fig 2b

A. [4 pts] A pin is held vertically in the middle of a clear square container (for simplicity ignore the container walls). Fig. 1a is an EXAMPLE that shows the path of light rays that pass from the pin to near an observer's eye, where the box and the observer are both in air. For Fig. 1b, where the inner box is filled with water and the observer is in air, draw the path of the light rays inside and outside the box and qualitatively show where one would expect to observe the image of the pin.

B. [8 pts] As shown in Figs. 2a and 2b, imagine that the observer lives in a water world, and that the boxes are filled with air. For Fig. 2a draw the path of the light rays and qualitatively show where one would expect to observe the image of the pin. In Fig 2b., if pins were placed at locations A, B, and C, would the observer be able to see these pins? Explain your answer.

Total Internal Reflection only occurs when going from low n to high n . You would observe all 3 points, A, B & C.

C. 5 pts) The plot shows $\sin(\theta_{\text{incident}})$ of light rays in water ($n_w=1.33$) vs $\sin(\theta_{\text{refracted}})$ for the rays refracted in a transparent material of unknown composition. What is the index of refraction of the unknown material?



From the graph the slope of the line = 1.5

$$n_i \sin \theta_i = \sin \theta_r$$

or

$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{n_r}{n_i} = 1.5$$

$$n_r = n_i \times 1.5 = 1.33 \times 1.5 = 2.0$$

D. 8 pts) For a light ray in water ($n_w=1.33$), incident at 35 degrees towards the normal of the surface of a flat diamond plate ($n_d=2.42$), what will be the angle of the ray be when it enters the diamond? For a rays originating in the diamond plate and entering the water, what is the critical angle of internal reflection?

$$n_w \sin \theta_w = n_d \sin \theta_d$$

$$\sin \theta_d = \frac{n_w}{n_d} \sin \theta_w = \frac{1.33}{2.42} \sin(35) = .315$$

$$\theta_d = \arcsin(.315) = 18.4^\circ$$