MEASUREMENT OF VELOCITY OF LIGHT BY THE METHOD OF FOUCAULT

Revised September 27, 2010.

IMPORTANT INFORMATION

For this experiment only, the equipment will be set up in advance, everyone will do this experiment during the same week in one hour slots, and groups can include up to 4 students. This experiment is required even if you have done it before in Physics 231.

SOME BACKGROUND

The history of the measurement of the velocity of light is a fascinating one, both for the variety of experimental methods that have been used and for the fundamental consequences of the results to physical theory. The apparatus used in this experiment is traceable, in concept, to Galileo. A short pulse of light is generated and the time it takes to cover a path of known length is measured.

Galileo

Galileo's measurement, as it was described by him in 1638, placed two individuals A and B with lanterns on hilltops sufficiently close that each could see the other's lantern. A would uncover his lantern and when B saw the light from A's lantern he would uncover his own. A would then note the time elapsed between uncovering his lantern and seeing the return light from B's lantern. Presumably, they repeated the measurement a number of times and concluded that the speed of light c exceeded their ability to measure it by this technique. With the benefit of hindsight we can guess what lower limit they might have been able to quote. With A and B one mile apart and a time measurement accuracy of 0.1 s, the highest velocity they would have been able to detect is about 2 mi/0.1 s = 30 km/s. We can thus credit Galileo with setting a value for $c \geq 30$ km/s.

Rømer

In 1686 the Danish scientist Olaus Rømer astonished the astronomers of the day by predicting that an eclipse of Jupiter's moon, Io, by Jupiter would occur 10 minutes later than the time predicted from the eclipses observed throughout the year. It did, and he later explained the delay as the result of the finite speed of propagation of light, a speed almost universally thought to be infinite at the time. Although he did not give a value for c, his contemporary Huygens used his statement that it took approximately 22 minutes for light to travel the diameter of the earth's orbit to calculate a velocity that we can interpret today as $c \geq 232,000$ km/s. (A very entertaining account of the numerous misstatements of Rømer's results has been given by Andrzéj Wroblewski [*Am. J. Phys.* **53**, 620–630 (1985)]. Also interesting to note: Wroblewski's materials were first presented in a UW Physics Department colloquium.)

Fizeau

The 19th century brought a number of quite accurate measurements of c. The French scientist Fizeau used a rapidly rotating toothed wheel to chop a light beam and measured the time for the beam to return from a distant mirror as the time it took the wheel to rotate sufficiently for a tooth to block the returning beam. The value he reported in 1849 was $c = 3.15 \times 10^8$ m/s.

Wheatstone, Foucault, and Michelson

A proposal to measure the speed of light that used a rotating mirror to chop a light beam came from Sir Charles Wheatstone (developer of the Wheatstone Bridge) in 1834, but the best early measurement that used a rotating mirror was carried out by Foucault (better known for the Foucault pendulum) in 1862. His published value [*Compt. Rend.* **55**, 792 (1862)] was $c = 298,000$ km/s and was thought to be accurate to about 500 km/s. The definitive value of c from a rotating mirror experiment is that of A. A. Michelson (of Michelson-Morley fame). The value, published by his collaborators after his death, [Michelson, Pease, and Pearson, *Astrophys. J.* **82**, 26 (1935)] was $c = 299,774$ km/s and had a stated "average deviation" of 11 km/s. It was incorporated in a collection of physical constants and, for over a decade, the velocity of light was quoted as $c = 299,774 \pm 4$ km/s. The striking feature of this measurement and the one most relevant to the aims of this experiment is that the stated result differs from the modern value by 19 km/s, over four times the stated uncertainty.

The most precise measurements of c came from direct measurements of the frequencies, ν, of optical transitions for which the wavelengths, λ, were known precisely. The speed c follows from the relation $c = \nu\lambda$. The precision of these measurements was of the order of 1 part in 10^9 and was limited by the precision with which the meter could be determined. In 1983 the decision was made to adopt c as a new standard. Now, instead of the distance between two scratches on a platinum-iridium bar being exactly 1.000000000... meter, the velocity of light is given the defined value $c_{def} = 299\ 792\ 458.000000\ldots$ m/s. The meter then becomes the distance light travels in 1/299792458 seconds. (Should you be interested in details of the recent history of measurements of c and the adoption of c as a defined quantity, consult the extensive references in the article by Harry E. Bates, "Resource Letter RMSL-1: Recent measurements of the speed of light and the redefinition of the meter" [*Am. J. Phys.* **56** 682–687 (1988)]).

EXPERIMENTAL OBJECTIVE

Before launching into the details of the experiment, it must be emphasized that the primary objective of this experiment is to understand the operation of the apparatus well enough to derive a value for the precision with which c can be measured by the apparatus, not to obtain a value of c equal to the defined value.

In particular, you are to obtain the standard deviation σ_{meas} in your measured velocity of light, c_{meas}, as derived from a linear fit of your measurements of spot displacement vs. the rotational frequency of the mirror and a knowledge of the experimental parameters A, B and D.

In general, if your value c_{meas}, is derived from an unbiased sample (one free from systematic errors) then it should lie within the range $c_{def} \pm \sigma_{meas}$ approximately 68% of the times you make a measurement. If c_{meas} does not lie in the range $c_{def} \pm 3\sigma_{meas}$ you should be prepared (in principle, generally) either to repeat the experiment or (in practice here) identify and estimate the size of a systematic error or errors that explain the discrepancy. Note that the percentage of measurements that would lie outside the range $c_{def} \pm 3\sigma_{meas}$ by chance alone (no systematic error) is about 0.3%.

APPARATUS

The apparatus you will use in this experiment is a commercial unit made by PASCO Scientific.

A diagram of the experimental setup is shown in Fig. 1. With all the equipment properly aligned and with the rotating mirror stationary, the optical path is as follows. The parallel beam of light from the laser is focused to a point s by lens L_1. Lens L_2 is positioned so that light from the image at s is reflected from the rotating mirror M_R, and is refocused onto the fixed, spherical mirror M_F. M_F reflects the light back along the same path and L_2 focuses it at point s. In order that the reflected point image can be viewed through the microscope, a beamsplitter is placed in the optical path, so a reflected image of the returning light is also formed at point s'.

The lens and mirror optics are designed so that light coming from s and reflected from M_F is again focused at s. This must continue to be true even if M_R is rotated slightly and the beam strikes M_F at a different

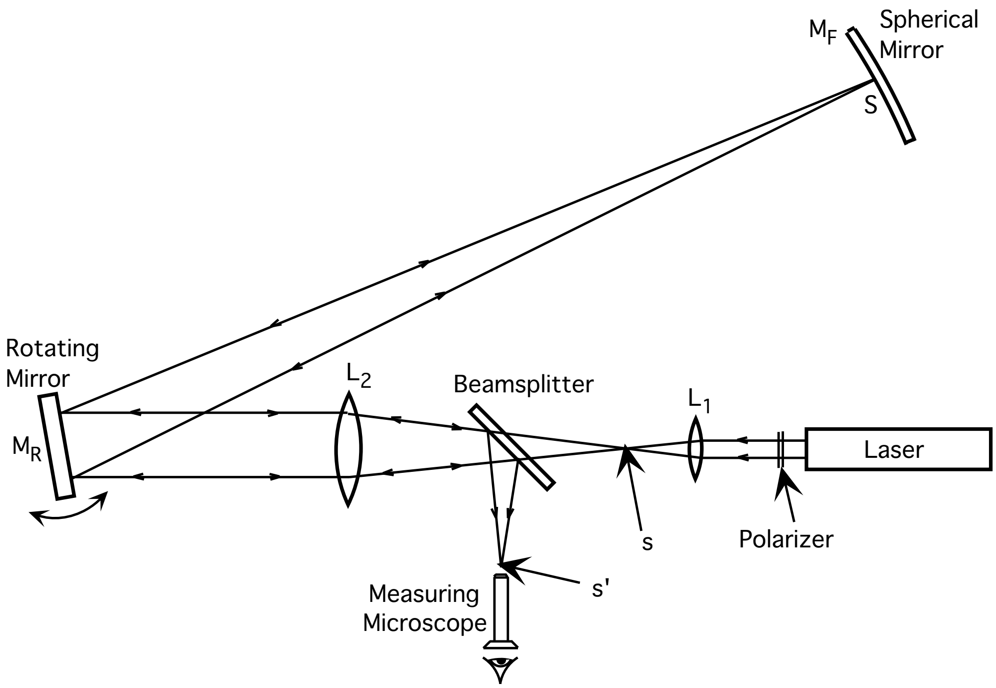

Figure 1: Schematic of the PASCO speed-of-light apparatus.

point on its surface. Because s is *focused* on M_F , the spot on M_F is the image of s. Since light rays through any optical system are reversible, light coming from the *image* on M_F will pass through the *object* at s just as light from the source at s passes through the image spot on M_F. The spherical shape of the mirror is important for two reasons. The first is that the light rays from s are directed nearly as though they came from the center of curvature of M_F. Light originating from the center of curvature will be reflected back on itself, so the return beam will again pass through L_2. The second is that the optical path length is nearly independent of the position of the spot on M_F. It helps if the length of the optical path from s to M_F is close to 13.5 m, the radius of curvature of M_F, although the shorter path available in the lab does not present a significant problem. In all rotating mirror experiments, image quality is an important issue, especially if, as was the case for Michelson, the optical path includes a mile or so of wavy air. In summary, the spherical shape of M_F, ensures that the beam will be reflected directly back toward M_R and the return image of the source point will be formed at points s and s'.

Now imagine that M_R is rotating continuously at a very high speed. In this case, the return image of the source point will no longer be formed at the same points s and s' that is was when the mirror was stationary. A light pulse that travels from M_R to M_F and back now finds M_R at a different angle when it returns than when it was first reflected. As shown in the following derivation, the velocity of light can be determined by measuring the displacement of the image point caused by the rotation of M_R.

A Quantitative Description

To begin the derivation, we will first look at a simplified form of the optical system consisting of only the rotating mirror M_R and the fixed mirror M_F, as shown in Fig. 2. A laser is aligned to create a pencil of light to hit the rotating mirror: the "incoming beam" in the figure. Now think of a small piece of the beam, or "packet" of light which strikes M_R when the mirror is at angle Θ_1. The packet is reflected about the normal to the mirror, and follows the path denoted by the thick solid lines, making incident and reflected angles α relative to the normal. The packet of light hits the fixed mirror M_F and is reflected directly back towards M_R. When it reaches M_R, the mirror has rotated to a new angle Θ_2, and so the angles of incidence and reflection off of M_R on this return trip are different; the new path is denoted by the thick dashed lines, and the new incident and reflected angles are β. Thus, because the light takes time to make the round trip between M_R and M_F, the "returning beam" of light is tilted away from the path of the "incoming beam".

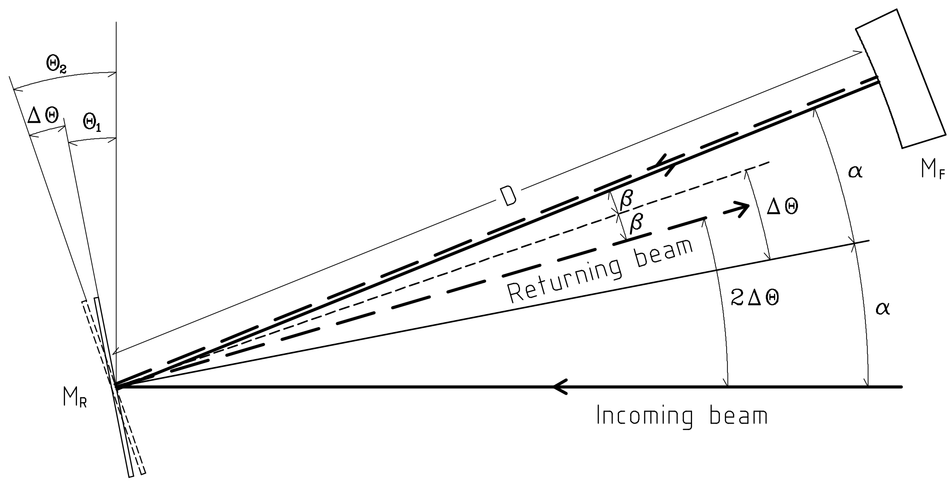

Figure 2: Simplified diagram of the speed-of-light optics showing the important angles.

The difference between the mirror's angle governing the outgoing path Θ_1 and that governing the return path Θ_2 is $\Delta\Theta$. There are a couple of ways to convince yourself that the angle between the returning beam and the incoming beam, what we will call the *deflection angle*, should be twice this angle, or $2\Delta\Theta$. First, whenever the mirror angle changes, both the incident angle and the reflected angle change by the same amount, so if the mirror angle changes by $\Delta\Theta$, then the path of the light must change by $2\Delta\Theta$. Another way to see this is by a direct calculation from Fig. 2. It is clear by inspection of the figure that the deflection angle is $2\alpha - 2\beta$. It is also clear that $\beta = \alpha - \Delta\Theta$ (looking at the upper angle α in the figure). Thus the deflection angle is

$$2\alpha - 2\beta = 2\alpha - 2(\alpha - \Delta\Theta) = 2\Delta\Theta\,. \tag{1}$$

Now, what is the relationship between the deflection angle and the speed of light? During the time interval Δt, the mirror moves by an angle $\Delta\Theta$, and the light packet travels a round trip distance of $2D$ where D is the distance between M_R and M_F. If the mirror rotation speed is ω and the speed of light is c, then we have two relations involving Δt:

$$\Delta\Theta = \omega\Delta t\ , \tag{2}$$

and

$$2D = c\Delta t\ . \tag{3}$$

When we combine these equations, we obtain for the deflection angle

$$2\Delta\Theta = \frac{4D\omega}{c}\ . \tag{4}$$

This equation says that the deflection angle will increase as we make the distance between the mirrors larger or we make the mirror rotate faster. More specifically, it says that if we measure the deflection angle $2\Delta\Theta$ for a known rotation speed ω and a known separation distance D, we can then calculate c.

In our apparatus, the distance D is conveniently measured with a tape measure, and the rotation speed is measured with an electronic tachometer that is part of the rotating-mirror device. We measure the deflection angle by means of the additional optics shown in Fig. 1. To reiterate the action of these components: Lens L_1 focuses the light from the laser to a spot located at point s. The (real) image of this spot is created by lens L_2 at the surface of M_F at point S. This image, in turn, acts as the object that is focused by L_2 back towards s. But before the light from the round trip can return to the point s, it is reflected by the beamsplitter to be focused to a spot s' in front of the measuring microscope.

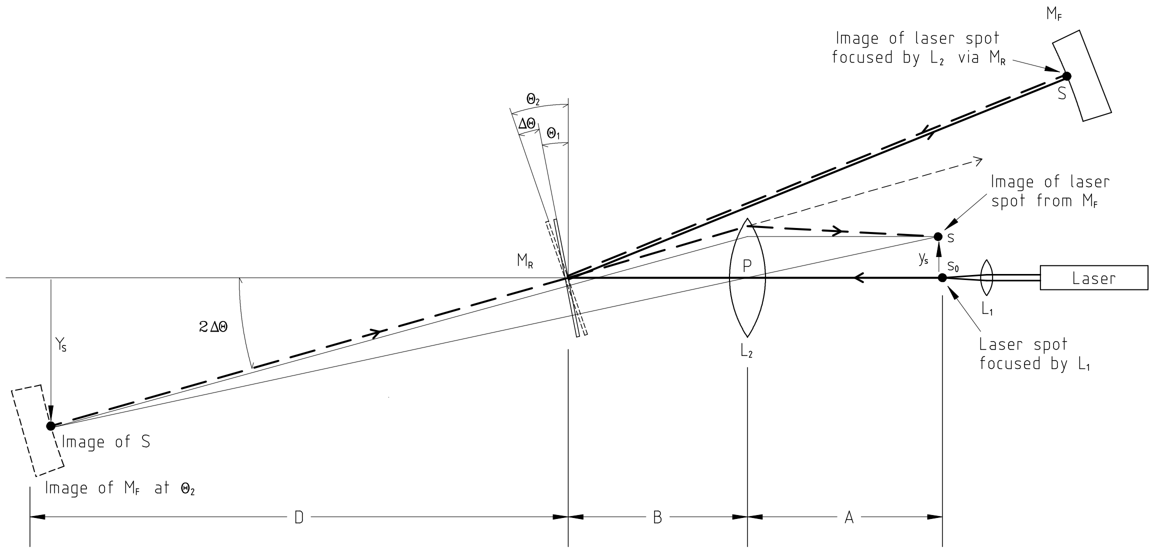

Figure 3: Diagram showing the image of the point on M_F created by M_R when it is at position Θ_2. This image is focused by L_2 at point s, as shown by the construction rays.

From the point of view of the observer looking through the microscope, the lens L_2 acts on light coming from the *image* of the fixed mirror M_F which is created by the moving mirror M_R. A diagram showing this image is given in Fig. 3. As shown in the figure, the light from the laser is focused to a point s_0, and the image of this point is created by the lens L_2 at the fixed mirror surface at S when the mirror is at Θ_1. The observer sees the returning light when the mirror is at angle Θ_2; the light appears to follow the path denoted by the thick dashed lines. The *image* of the spot at S appears to be coming from a point displaced from the horizontal by a distance Y_S. The light from this (apparent) point is refracted by the lens L_2 and creates a real image of S at the point s that is displaced from the original spot s_0 by a distance y_s. By using the beamsplitter and microscope, one can measure the displacement of the image at s from the (original) "object" at s_0.

The dimensions and angles shown in Fig. 3 are not to scale. In fact, the angle $2\Delta\Theta$ is typically very small: about $0.1°$ or less. We can use this fact to allow us to calculate the angle $2\Delta\Theta$ in terms of the vertical distance y_s. By geometry, in the small-angle limit, the vertical distance Y_S, is given by

$$Y_S = D \times 2\Delta\Theta \, . \tag{5}$$

By means of the construction rays used to locate the image of S at the point s, which are the thin lines going through the lens L_2 centered at the point P, we can see by similar triangles located about P that

$$\frac{Y_S}{D+B} = \frac{y_s}{A} \, . \tag{6}$$

By Eqs. (5) and (6), we find

$$2\Delta\Theta = y_s \frac{D+B}{AD} \ . \tag{7}$$

If we eliminate $\Delta\Theta$ by combining Eqs. (4) and (7), we obtain a formula for the speed of light in terms of measurable quantities:

$$c = \frac{4AD^2}{D+B}\left(\frac{\omega}{y_s}\right) \ . \tag{8}$$

Any change in y_s will be reflected into a change in s', the position of the spot as seen through the microscope, in an exactly proportional manner. In this experiment, we can get better results by plotting s' versus ω, and the obtaining the slope $ds'/d\omega$ by a fit to the data. This eliminates the need to know the precise value of the zero-displacement position s_0. In terms of these quantities, our final result reads

$$c = \frac{4AD^2}{\left(\frac{ds'}{d\omega}\right)(D+B)} \tag{9}$$

where:

c	=	the speed of light
A	=	the distance between lens L_2 and lens L_1 minus the focal length of L_1
B	=	the distance between lens L_2 and the rotating mirror (M_R)
D	=	the distance between the rotating mirror (M_R) and the fixed mirror (M_F)
s'	=	the position of the image point as viewed through the microscope.
ω	=	the rotational velocity of the rotating mirror (M_R)

Equation 9 was derived on the assumption that the image point is the result of a single, short pulse of light from the laser. But the displacement of the image point depends only on the *difference* in the angular position of M_R in the time it takes for the light to travel between the mirrors. The displacement does not depend on the specific mirror angles for any given pulse. If we think of the continuous laser beam as a series of infinitely short pulses, the image due to each pulse will be displaced by the same amount. All these images displaced by the same amount will, of course, result in a single image. By measuring the displacement of this image, the rate of rotation of M_R, and the relevant distances between components, the speed of light can be measured.

EXPERIMENTAL PROCEDURE

CAUTION: Do not look into the laser beam, either directly or as it reflects from either mirror. Also, when arranging the equipment, be sure the beam path does not traverse an area where someone might inadvertently look into the beam.

It should be possible to obtain the necessary data within an hour. This estimate is strongly dependent on having the apparatus in alignment before you begin your measurements. It will be aligned for the first students who use it, so it crucial that they and you and subsequent users maintain the alignment. **Take extreme care not to bump the optical bench or the spherical mirror!!!**

The procedure, in outline, is to measure the deflection of the laser spot for a number of CW (clockwise) mirror rotational speeds, then reverse the direction of rotation and repeat the measurements for CCW (counter-clockwise) rotation. This is a two-person operation, one person to set the rotational rate, read the frequency counter and record the data, and the second to measure the spot deflection. If you make the CW and CCW measurements more than once, it would be a good idea if you switched jobs (but not between CW and CCW measurements!). Finally you should measure the distances between the optical components, including the distance between the fixed mirror at the end of the room and the rotating mirror on the track.

The first order of business is to locate the spot image at s'. This should be done with the mirror rotating. While looking through the eyepiece, gradually increase the rotational rate and look for a fairly faint, fuzzy spot that moves in proportion to the rotational rate. You may see other spots in the field of view, but only the one that is reflected from the rotating mirror after return from the spherical mirror will move. Each person in the group should locate the spot even though only one or two in the group will make the actual measurements. Ask the TA to confirm that the spot focus is as it should be. If it is not good the TA or a technician will readjust the apparatus. Do not attempt this on your own!

When all have seen the spot, the person measuring the deflection should practice setting a line of the crosshairs on the spot and reading the micrometer. To do this the person setting the frequency should set it at 500 Hz and see that it stays at this value while the deflection is measured. Each member of the group should be able to read the micrometer and have an estimate of the accuracy of the reading. Therefore, when the crosshairs are first set on the spot for the 500 Hz frequency, each member of the group should read the micrometer setting and estimate the accuracy of the reading. *Record the group consensus on the micrometer reading and the accuracy of the reading. Record the frequency.* The sheet on which these data are recorded should be dated and should carry a statement that this was the first measurement and was a consensus measurement.

At this point you should get a better idea of the repeatability of the deflection measurements. *Reset the frequency to 500 Hz, reset the crosshairs on the spot and record the micrometer reading and frequency.*

It is important to set the micrometer position by turning it in the same (clockwise) direction when marking the spot direction. There is a necessary looseness in the micrometer threads that will result in a different reading for the deflection if it is approached from higher numbers (counter-clockwise) than if it is approached from lower numbers (clockwise). This “backlash” error can be minimized by always approching the final reading with the same rotational direction. *Repeat the deflection measurement at 500 Hz for two to three additional times and record the results.* The purpose of the repeated measurements is to obtain a statistical measure for the uncertainty in an individual measurement. This measure will be called the standard deviation in an individual measurement, σ_i , and can be compared to a similar measure to be obtained from the next set of data you are to take.

The next step is to measure the spot deflection for increasing values of the rotational rate. The rate can be varied continuously from 0 to about 1000 rev/s. It can then be driven at about 1500 rev/s for a short time by pressing the MAX REV/SEC button. Plan in advance which rotational rates you will use (a reading every 200 Hz is a reasonable choice) and check your choice by changing the rate and watching the deflection without measuring it. Do not attempt to take data below 100 rev/s, since the rotation speed is unstable at low values. Also, don’t struggle to set the frequency exactly on 200 Hz or 300 Hz. Pick values nearby where the rate is steady and record deflection and frequency for them. Use the maximum revolution rate with care. **The MAX REV/SEC button should be depressed for at most one minute, and the motor should be allowed to cool off for at least a minute between high speed uses.**

Finally, make a sketch of the apparatus (or use a copy of Fig. 1) and measure the positions of and distances between the optical components. You can use a tape measure to obtain the distance between M_F and M_R; some stools and wood blocks are available to keep the tape from sagging. The component positions on the optical track can be found by reading the measuring rule attached to the track.

Also record the focal lengths of lenses L_1 and L_2.

Here is a summary of the data collection tasks:

1. Set the motor for CW rotation and complete your planned set of measurements.
2. Set the motor for CCW rotation and obtain a new set of measurements (same team assignments). Take your readings as you did in 1) and avoid backlash.
3. When the first set of CW and CCW measurements is completed you may wish to obtain other sets. This depends on your stamina and the time available.
4. Measure the distances and record the focal length data needed to obtain the values of A, B, and D.

DATA ANALYSIS

For this experiment, there should be two independent sets of measurements of the spot position versus frequency (by two different microscope observers). For each set of measurements, one of the directions of mirror rotation should be plotted as positive frequencies and the other direction as negative frequencies and put on one graph.

The frequency counter measures the number of rotations per second; please plot your data versus this number and then determine the slope of each of the two sets of data from a least squares fit. You will need to estimate the uncertainty in your slope. (Several methods are described on the course website.)

You can then combine the two values for the slopes from your data and calculate the speed of light, converting the units of your slope to angular frequency for your final calculation.

A careful, well documented estimation of the uncertainty in your measurement is essential for every every analysis and report, especially this week's experiment.

Remember, your *primary* objective is to understand how well you have made your measurement, not just to obtain agreement with the defined value of c. You will need the uncertainty in each of your measurements in order to work this out. Some things to keep in mind:

- The precision of your distance measurements will depend on your ruler and on how you used it. Estimate this uncertainty, and explain in your report how and why you chose the uncertainty values that you did.
- Assume that the focal lengths of the lenses are accurate and precise to 1 mm.
- You will notice that the mirror speed fluctuates a little when you try to make the reading. But you will also notice that you cannot see the spot move during this fluctuation. The uncertainty in the spot location, which depends on the spot size, focus and resolution of the micrometer is much larger than the uncertainty in the mirror speed. Since the two quantities are used to calculate the rate $ds'/d\omega$, you can safely ignore the uncertainty in the mirror speed.
- The uncertainty in $ds'/d\omega$ will be found from the fitting uncertainty when you plot s' versus ω.

Note that Eq. (8) can be written as a linear relation between the rotational rate ω and the vertical spot position y_s. Thus, a plot of the position s' (i.e., s_2 as seen though the microscope) vs. the rotational frequency ν ($\omega = 2\pi\nu$) should give a straight line. For your report you should carry out the following tasks:

1. Obtain a computer-generated, least-squares fit of your data to a straight line.
2. Provide a plot (minimum 1/2 page in size) of the data and the fitted straight line, also computer-generated.
3. From the slope of that line and the standard deviation in the slope, along with the measurements you took to determine the distances A, B and D and their uncertainties, calculate c_{meas} and the standard deviation in c_{meas}, σ_{meas}.

The major part of your write-up will then consist of a reconciliation of your value of c_{meas} with the defined value c_{def}. Use the information you have about the uncertainties in your measurements and your knowledge of the experimental parameters.

You may want to study some of the information on uncertainty analysis on the class website. The tutorial "Notes on data analysis and experimental uncertainty" is a good place to start.

For simplification of the analysis it is OK to treat $D + B$ as independent of D^2. This will overestimate the uncertainties. If you have time to be fancy you might decide whether it is important to improve on that.

You need to realize that the relative error in D^2 is twice that of D (see examples in postings or links on course website).

Then you have four terms whose relative errors add as the square root of the sum of the squares of the individual relative errors of 4 terms:

a) the slope of displacement vs. frequency (plot CW and CCW as + and − frequencies so you fit all measurements from one set with one line);

b) the D^2 term;

c) the A term;

d) the $D + B$ term.

You might also ask yourself if you mis-estimated the position of the rotating mirror whether it affects D and B is a similar way. Note also: the lens centers are displaced from the mark on their mounting on the scale on the track.